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# Implementation of Performance-Based Budgeting Using the Combined Technique of Best-Worst Method (BWM) and Robust Optimization

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ABSTRACT

One of the priorities of reforming the economic structure of the country is to reform the budgeting of state-owned companies. Performance-based budgeting (PBB) emphasizes the importance of identifying measurable results from costs and can enhance the transparency and accountability of public-sector budgeting. The main purpose of the research is to present a model of budgeting in Hormozgan Province Gas Company that maximizes the desirability of budget allocation in terms of budgeting constraints. The present study, by investigating the theoretical foundations and factors affecting the PBB, uses the linear programming problem by the BWM with robust optimization approach. To achieve this goal, the linear programming mathematical model was applied to implement PBB and the executive units officials (administrators) survey and interview tools and questionnaires based on the best worst method (BWM) was used to solve the problem of budgeting and to determine short-term goals, executive programs and products. The research results showed that the model can reduce the budget deviation index and improve the performance of the company by forecasting the budget of operational activities. Regarding the uncertainty of the company's resources, the model is capable of predicting the optimal budget for the coming years.

## 1 Introduction

New Public Management is the most actual public administration management system in the world and is being realized in almost all developed countries. It is a global phenomenon, aimed at improving efficiency and effectiveness, enhancing responsiveness, and improving the managerial accountability of public organizations [43]. The quest for a leaner and more efficient bureaucracy through performance-based reform has been ongoing, gaining steady momentum since the 1970’s [12]. These reforms focused on ensuring that agency operations and resource allocation leads to the achievement of a high level of mission-based goals, and resultantly the production of the desired outcomes [9]. Many

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of these reforms, including the current, include a linkage between agency performance, expenditure, and funding. Therefore, they are not just performance reforms, but they are also budget reforms as well [12]. Budgeting is a strategic tool for economic and financial discipline of governments and in its present form provides a competent and accountable government and encourages citizen participation [3] thus it has significant effects on the economic growth of a country [24]. Performance-based budgeting (PBB) is an approach in the budgeting system that take into consideration the interrelationship between the funding and the expected performance, and also the efficiency in achieving that performance. Performance is an achievement of work in the form of output from an activity or the outcome of a program with measured quantity and quality [22]. Prior to the introduction of Performance-based budgeting system, the budgeting method used was the traditional method or line item budget. The way this budget is drafted is not based on an analysis of the series of activities that must be linked to the stated objectives, but rather focuses on the need for expenditure and the system of accountability is not examined and examined whether the funds have been used effectively and efficiently or not. The benchmark for success is only demonstrated by the balance of budgets between income and expenditure but if the budget deficit or surplus means that the budget execution fails [16]. The performance-based budgeting approach is designed to try to overcome the weaknesses of traditional budgeting, especially the weaknesses caused by the absence of benchmarks that can be used to measure performance in the achievement of public service objectives and targets [17]. Budget with a performance approach emphasizes the concept of value for money and supervision of output performance, and is based on goals and performance goals. Therefore, the budget is used as a means to achieve the goal. Performance appraisal is based on the implementation of value for money and budget effectiveness [27]. Ineffective budget and not performance-oriented will be able to thwart the plans that have been prepared. Continuous performance measurements will provide feedback, so that continuous improvement efforts will be successful in the future [6]. Also saving costs and collecting funds from low-productivity units and directing them to high productivity is a strategic and important Policy [26] in performance-based budgeting.

An idea beyond performance-based budgeting is that if policymakers make financial decisions objectively based on efficiency and effectiveness, then both they and people can have a clearer judgment about the performance of the government. In fact, performance-based budgeting enhances government accountability to legislators and people by linking budget decisions and government performance [1]. Furthermore, PBB is regarded as an organizational practice whose success depends on the possibility of organizational individuals to become actors, i.e., to build a set of construct causalities on their work world by integrating four dimensions, namely facts, values, possibilities and communication [14]. The Performance-based budgeting as a subsystem of results-based management, with the aim of improving the efficiency and effectiveness of organization resource consumption, has drawn serious attention of State-owned companies. The PBB system includes three main elements of planning, costing and performance management. Performance-based budgeting plays a key role in paving the way for the implementation of advanced management accounting techniques such as activity based costing (ABC) by identifying cost factors and measuring them accurately through the link between costs and activities. Activity-based costing is able to provide the PBB system's information requirements, including accurate sharing of overhead costs, and to calculate the unit cost of each activity in the past year, as well as to forecast the unit cost of each activity for the next year and to utilize unused capacities of companies. As a result, activity-based costing assists in managing cost and reducing cost per unit of the activity. On the other hand, in performance-based budgeting system, using performance indicators performance management plays the role of linking the two elements of “planning” and “costing”. Applying the performance indicators of input, output and results, performance management analyzes relationship between “inputs”, “costs”, “plans and activities” and “goals and strategies” in three dimensions of “economic”, “efficiency” and “effectiveness” and also analyzes the firm's performance from the stand point of finance, stakeholders, quality, process and growth and learning. Finally, the results of the performance analysis are the basis for the improvement of future plans and the performance agreement for next year's budget [11]. According to what was said and also the necessity of implementing different laws related to the establishment of a performance-based budgeting system, with the aim of making the connection between planning, costing and performance management by all executive agencies, and in order to improve the efficiency and effectiveness of enterprise resource consumption, implementation of PBB is put on the agenda of state-owned companies including Hormozgan Province Gas Company. Provincial Gas Company aims to provide continuous service, safe gas supply, quality service improvement and efficient use of company resources. This will be achieved by changing the budgeting system and reducing production costs, at the same time with maintaining product quality. Thus, the main issue of the research is to present a model that can maximize the utility of the total budget of the company and consider the constraints in its budgeting. Therefore, the purpose of this research is to present the optimal PBB model in Hormozgan Province Gas Company by examining the theoretical foundations of PBB and budget modeling techniques and selecting the appropriate performance evaluation criteria. That's why we seek to answer the following questions:

1. What is the appropriate PBB model for implementation in Hormozgan Province Gas Company?
2. Can the selected PBB model be implemented at Hormozgan Gas Company?
3. Does the PBB model lead to reduce budget diversion and improve performance at Hormozgan Province Gas Company?

**Research Innovation:** The present study is an applied research that has applied a deterministic linear programming problem and robust counterpart optimization to implement performance-based budgeting in Hormozgan Province Gas Company by reviewing the budgeting literature as well as mathematical programming models. In this research, a linear programming model with the objective function of maximizing efficiency and effectiveness has been applied. It's the first time that the best-worst method is implemented to calculate the coefficients of the objective function and to determine the weights of short-term goals, executive programs and products and also to determine product performance evaluation index from the productivity index (total factor productivity) using the Cobb Douglas production function. This have not been observed in previous researches.

## 2 Theoretical Foundations and Literature Review

**Performance-based budgeting (PBB):** The past two decades have witnessed a growing interest in performance management and budgeting reforms, in response to louder public demands for government accountability in industrial countries [45]. While the triggers for introducing reforms vary across countries, the major reform motivators can be summarized as a financial crisis, pressure to reduce public expenditures, and a change in political administration [29]. Performance-based budgeting aims to improve the efficiency and effectiveness of public expenditure by linking the funding of public sector organizations to the results they deliver, making systematic use of performance information [39]. The effect of performance information on decision making and resource allocation in the budget process is mixed [20]. In performance budgeting, the key issue is the extent to which resources should be linked to results [44]. Performance budgeting has potentially two things to offer the government under the circumstances: the first is a clear link between government priorities and resource allocation in the budget. The second is increased fiscal space through carefully targeted cuts to baseline expenditure [21].

Although there is no standard definition of PBB but in a general definition, it can be said: performance-based budgeting is a kind of budgeting that links the funds allocated to measurable results [30]. Managers may find performance measurement helpful in achieving eight specific managerial purposes. As part of overall management strategy, public managers can use performance measures to evaluate, control, budget, motivate, promote, celebrate, learn, and improve. Ultimate purpose of using performance information is to improve the performance of public organizations [7]. Performance information, in the budgeting process, can be used for three purposes: it can be used for development or preparation of the budget, justification of the budget proposal, and performance information can be used to fulfill the requirements of the upper level officials [19]. Performance information can contribute in three primary ways. Firstly, it helps government to improve expenditure. Secondly, it can place more pressure on line ministries/agencies to improve the effectiveness of their programs. Thirdly, when it is available and used, can help in increasing efficiency of goods produced and services delivered by government [40]. A performance budget clearly links goals with costs for achieving targeted levels of performance. This is not an old-style budget that simply seeks funding for one or a category of programs. Instead, the new- style budget is based on a target level or quantity of important outcomes or results-an agency’s strategic performance goals. Producing such financial plans requires interesting strategy, accounting, budgeting and performance measurement [25].

Robinson and Last [39] provide a basic model of PBB for the following two categories of countries: those that wish to introduce a performance-based budgeting system while minimizing the complexities and costs of doing so; and those with limited resources and capacity, including appropriate low-income countries (LICs). The key common elements of the PBB model of this research include 1) a “strategic” priority setting phase early in the budget cycle, 2) an expenditure process review, 3) a systematic scrutiny of new spending proposals, 4) information on efficiency and effectiveness to support budget submissions, 5) introduction of program budget structure, 6) increased managerial flexibility. The research emphasizes necessary preconditions for any move to performance-based budgeting recognizing that performance-based budgeting, even in its basic form, should not be considered in countries with seriously dysfunctional public financial management (PFM) and governance systems. In the study carried out by Azar et al [4], the feasibility of using intelligent DSS approach for performance based budgeting based was examined by identifying the factors which have a role in failure of budgeting systems and are relevant to lack of intelligent system approach and extracting information classes in PBB process as well as semi-structured factors of the process. The results show that this system can improve the indicator of budget deviation in the organization. In another research, Azar et al [5] presented a mathematical model for PBB that can maintain optimality and justification of Budget allocation under uncertain conditions. Studies show that the cost estimation and cost drivers are the most important factors that cause uncertainty in the budgeting space. Therefore, a robust planning approach is investigated to present the performance-based mathematical model in an Iranian bank. The research indicates that using robust models of PBB could improve the indicator of budget deviation. Zamfirescu and Zamfirescu [49] develop a mathematical model that allow a deeper understanding of the complex mechanisms and criteria for assessing the ongoing programs within the budgeting system. The paper considers only a very small part of this process, namely the final allocation of public funds. The model has been implemented in a spreadsheet tool and builds on one of the most widely used multi objective approach in management science, namely goal programming optimization method. The paper stressed the prescriptive use of this tool to analyze data and recommend action as regards the change in the amount of allocated funds for a specific program. The simulation results show that this model driven DSS may be a useful tool to support fact-based decisions for the public funds allocation. Widodo [48] investigate the implementation of PBB in Indonesia. The research examines the implementation of PBB by government officials/practitioners of performance information in the planning and budget making process. In addition, it assesses the impact that performance results have on budget allocations and vis-a-vis other factors affecting budget allocation decisions. The research has involved a combination of qualitative and quantitative analysis, with three different data collection methods deployed. Interviews and questionnaire were conducted with government officials. A key conclusion from the study is that Indonesia has made significant efforts in the direction of using performance information in its budgetary planning processes but as elsewhere around the world, the impact of performance information on resource allocation decisions has, so far, been quite limited. Aramesh et al [2] have introduced a performance-based budgeting implementation model by adopting a fuzzy Delphi technique for introducing cost effective means of resource sharing into police activities and using the FAHP technique to determine their importance through calculating the weights. The results indicate that in successful implementation of budget-based performance in this section several factors are involved in the sharing of awareness police sources to the scientific discovery of crimes, among which judicial records, personnel, specialized equipment, accused persons and vehicles are of special importance.

Nasrabadi et al [28] have used robust optimization to deal with uncertainty in budget allocation. Their research results demonstrate that the performance of robust optimization is an effective way to address uncertainty in budget allocation. Tat Kei Ho [47], in his research, examines the decades-long practices of performance budgeting in different countries and their associated challenges from a multilayered institutional framework. This study recommends an array of strategies to address institutional and organizational barriers. It also proposes to reconceptualize performance budgeting as a performance budget management system and suggests how multiyear budget planning, financial risk assessment, policy planning, the departmental budget cycle, the program budget cycle, stakeholder engagement, regular spending reviews, and performance audits should be integrated more closely to address the long-term fiscal challenges faced by many governments and to respond to the public pressure on agencies to do more with less. Guzman [15] analyzes performance budgeting systems with top-down approach in Chile. Despite its top-down approach, is often regarded as successful. Research findings suggest that while top-down performance budgeting systems often fail to meet the needs of individual ministries and are prone to principal-agent issues, under certain conditions such as involving third party experts and having a professional civil service the performance information from those systems might still be useful and/or might generate positive spillovers for line ministries. Amini et al [1] by reviewing maturity reference models as well as two performance-based budgeting maturity models presented in budgeting literature, have attempted to highlight weaknesses of the recent models in order to present the developed performance-based budgeting maturity model. This study develops a multi-layer data envelopment analysis model to measure PBB maturity index using optimization approach in constructing the composite indicator. By calculating the maturity score of each decision unit, the weight of sub-criteria in the macro indicator will be calculated.

**Performance-based budgeting models:** Since the beginning of the PBB system, various models have been proposed and implemented for this system. All PBB models have two main objectives: 1) compilation of budget by prioritizing consumptions and expenditures so that resources are allocated efficiently and effectively, 2) improving organizational performance.

Some of the most important PBB implementation models are mentioned in the following:

**Shah Model:** A governmental institutional model that provides a set of ideas about the factors that influence the successful implementation of a performance-based budgeting system. This model emphasizes three factors: (a) ability (b) authority and (c) acceptance. Studies suggest that the interaction of these factors determines the reform atmosphere in PBB implementation [13].

**Diamond Model:** A model based on research literature and laws and regulations in Iran and consists of three main elements and three empowering elements. The main elements of the diamond model are: 1) planning, 2) cost analysis and 3) performance management. The empowering elements involve: 1) change management, 2) accountability system and 3) motivational system [18].

**Mathematical Models:** In many situations, such as a manufacturing system, a production process or a service system, inputs and outputs are volatile and complex. Thus, it is difficult to measure them accurately [23]. In fact, many real-life applications face with uncertain data, which may influence the results of efficiency [31]. Two important general issues in PBB are "optimization" and "forecasting". On one hand, due to the complexity of the decision environment and the large number of decision variables and parameters in solving general optimization and allocation problems, and on the other hand, because of the quantitative and objective nature of mathematical models versus subjective aspects, the tendency for mathematical methods in PBB has increased. In this regard, different mathematical methods such as linear programming, meta-heuristic algorithms, multi-criteria decision-making techniques, multi-objective mathematical models, fuzzy mathematical models, and hybrid methods and techniques can be mentioned as a basis for the formation of mathematical models of PBB that in recent years have received serious attention.

**Linear programming models:** The problem of deterministic linear programming or deterministic linear optimization is one of the most important mathematical modeling techniques which can be used to find the minimum or maximum (optimal) value of a linear function with limited sources. This technique has been useful for guiding quantitative decisions in planning, achieving the best outcome and optimal allocation of limited resources.

One of the primary concerns in most decision making problems is the uncertainty associated with the input parameters. The existence of uncertainty may lead to some unrealistic results, which may make the final decision even more difficult [42]. Therefore in the real word, we are often conformed to vague and uncertain data and performance evaluation by usual methods in the presence such data may lead errors in decision-making process [41]. Overall, in the presence of imprecise and vague data, using the models that can measure performance of decision making units are essential [35]. Accordingly, applications of linear programming, one cannot ignore the possibility that a small uncertainty in the data can make the usual optimal solution completely meaningless from a practical viewpoint. The need naturally arises to develop models that are immune, as far as possible, to data uncertainty. The solution approaches of problem of linear programming that are immune to data uncertainly [10], are robust and this kind of optimization is called robust optimization. Robust Optimization (RO) is one of the applicable and popular methods that can be used to deal with uncertainty in optimization problems [34].

The following table presents the most important researches in the field of mathematical model for optimal budget allocation and performance-based budgeting:

**Table 1:** Most important researches in the field of mathematical model of PBB

| Researchers | Research topic | Research method | Mathematical model and type of objective function | Results |
|---|---|---|---|---|
| Azar et al [5] | A Linear Programming Model with Robust Approach For PBB | Reviewing the robust optimization approach to present the PBB mathematical model | Bertsimas and Sim's model and the objective function is Maximize | To improve the indicator of budget deviation by robust optimization models |
| Zamfirescu and Zamfirescu [49] | Goal Programming as a Decision Model for Performance-based budgeting | Use of decision support systems (DSS) based on spreadsheet solutions to partially implement the PBB principles for a public body (goal programming optimization method) | Goal Programming Model and the objective function is Minimize | Being useful model driven DSS to support fact-based decisions for the public funds allocation |
| Aramesh et al [(2] | Presentation of the Pattern of PBB by Hybrid Technique of Fuzzy Delphi and Fuzzy Analysis Hierarchy Process | Use of a fuzzy Delphi technique for introducing cost effective drivers of resource sharing into police activities and FAHP technique to determine their importance by calculating the weights | - | To determine multiple cost drivers in allocating resources to activities and their importance by fuzzy Delphi and FAHP techniques |
| Nasrabadi et al [28] | Robust Optimization for PBB Allocation at Payam Noor University | Use of robust optimization to deal with uncertainty in budget allocation | Bertsimas and Sim'model and the objective function is Minimize | Performance of robust optimization as an effective way to address uncertainty in budget allocation |
| Amini et al [1] | Developing a PBB Maturity Model and Constructing a DEA-Based Composite Indicator to Measure It's Score | Reviewing maturity reference models as well as PBB maturity models For presenting budgeting maturity model | DEA-based CI model (this model is the same as CCR model with constant input) and the objective function is Maximize | To develop a multi-layer data envelopment analysis model to measure PBB maturity index using optimization approach |

**The importance of PBB in Hormozgan Province Gas Company:** According to the National Gas Company's announcement schedule and guidelines, Hormozgan Province Gas Company's budgeting method is Zero-Based Budgeting (ZBB). Despite the advantages of zero-based budgeting, Hormozgan Province Gas Company has decided to change the budgeting approach to the PBB method due to several disadvantages of ZBB such as the difficulty of activities evaluation and determination of their different levels as well as the long-term implementation process of this method besides legal requirements to implement PBB. Therefore, linear programming mathematical model with robust approach is chosen to be applied in the present study, as a consequence of studying the literature and theoretical foundations and investigating different methods of PBB implementation, considering the complexities of the process of budgeting and allocation of resources to needs and goals, as well as uncertainties of future financial resources and large number of variables in the Gas company of Hormozgan province, in addition to widespread use of mathematical models for optimal allocation of resources limited to activities.

## 3 Conceptual Model of Research

According to the theoretical foundations, three main elements of a PBB system include planning, costing, and performance management. The planning element is a set of items involving macro objective, long-term goals, short-term goals, executive programs, activities and products. At the planning stage, in order to achieve corporate perspective that is macro goals, long-term goals, which are usually a five-year target, are designed. In addition, aiming to reach long term goals, short-term goals are also identified and subsequently, the implementation plans that help companies meet short-term goals, and also activities that make it possible to implement executive programs for the production of products or services, are determined. In the cost element, the purpose is to calculate the cost of the activity accurately. Activity based costing (ABC) is one of the most important techniques that can help companies to calculate cost of each activity and improve performance-based cost. In the ABC method, after identifying activities related to the products or services, the centers and costs of each activity are determined. Then various bases are used to determine the relationship between activities and costs (the cost drivers are utilized to assign costs to activity centers), and also the relationship between activities and cost objects (the activity drivers are used to allocate the activity centers to cost objects). After allocating the costs by the activity drivers, the overhead rate and the amount of allocation to the products are settled. The performance management element analyzes the relationships between the planning and costing elements in three dimensions of “economy”, “efficiency” and “effectiveness” by using input, output and outcome indicators.

Outcome indicators are used to evaluate the effectiveness of executive programs and are calculated based on the quantity of goals and programs. Input indicators are applied to evaluate the economy dimension and are measured based on the amount of cost drivers. Output indicators are utilized to estimate the efficiency of activities and are calculated based on the value of activity drivers. The optimal and simultaneous operation of the elements of the PBB system results in the augmentation of the economy, efficiency and effectiveness dimensions, and subsequently the improvement of these dimensions increases the responsibility of accountability. Therefore, in order to answer the research question, in accordance with the literature and theoretical foundations of PBB and mentioned items, a conceptual model is presented in Fig. 1 to better understand the process of PBB implementation.

## 4 Research Method

The present study is an applied research in terms of orientation and its methodology is the combination of qualitative and quantitative approaches. In order to present the PBB model, the Deterministic Linear Programming Model of Azar et al [5] is selected in consequence of studying the theoretical foundations and various budgeting models, according to the application of mathematical models in optimal allocation of limited resources as well as the nature of financial and non-financial information and restrictions on Hormozgan Province Gas Company. Regard to the uncertainty of the model parameters in the aforementioned company, after solving the deterministic linear optimization model, robust optimization models are investigated and the most appropriate model is determined with regard to the worst-case scenario approach and with the purpose of optimization based on the best justified answer for all data.

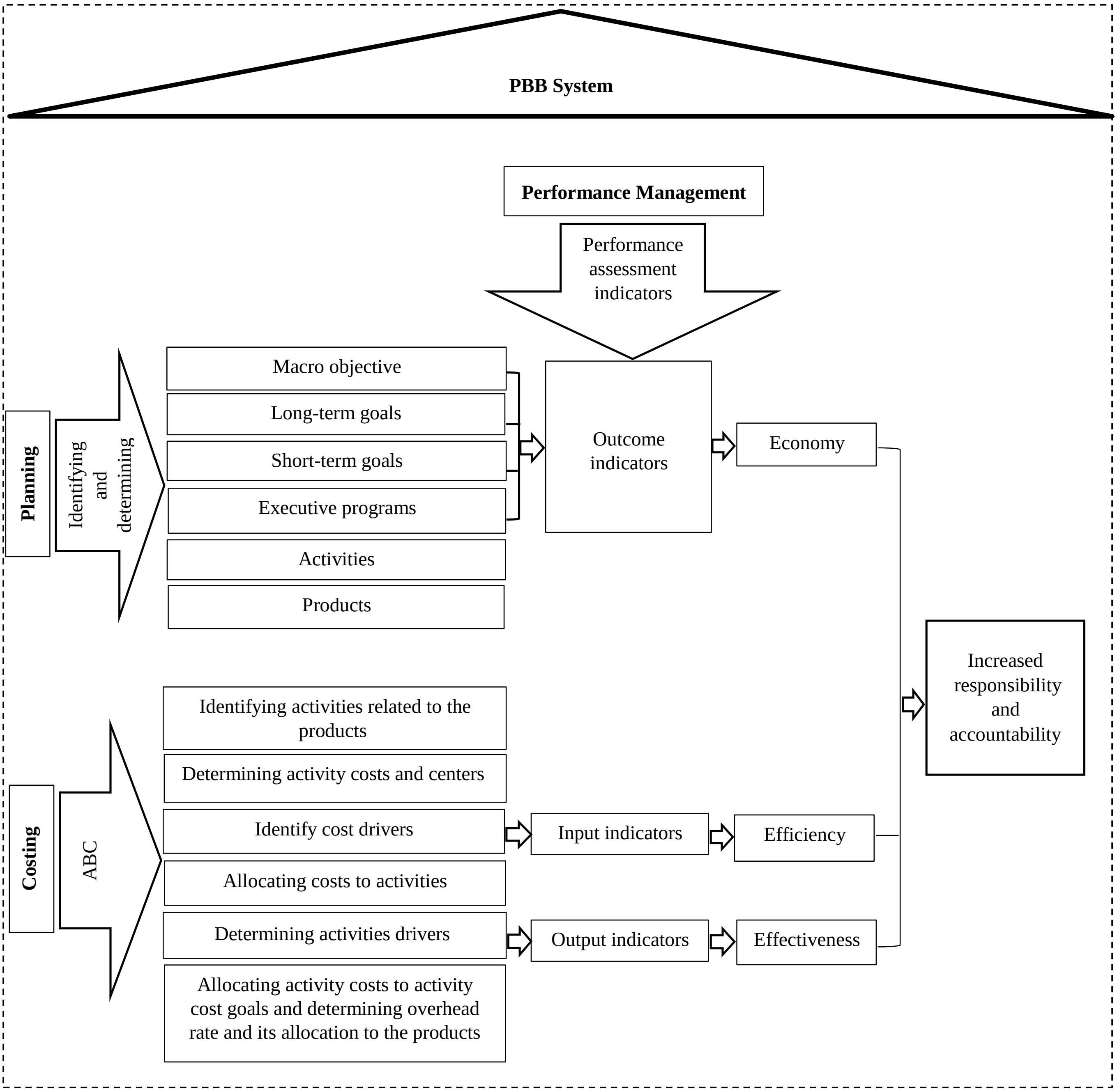


**Fig 1:** The Conceptual Model of PBB and the Relationships Between Its Constituent Elements

The best worst method (BWM) is applied in order to implement PBB, from the mathematical model of LP and to calculate the objective function of the LP problem and to determine the weights of short-term goals, executive programs and products. After determining the weight of each of the indices required in the model by BWM and collecting the model remainder information in Excel file format, Lingo software is used with the aim of solving the model and achieving optimal budget allocation with maximum efficiency and effectiveness according to the constraints on cost allocation. The following steps are taken in order to implement the PBB model in Hormozgan Province Gas Company:

**First step:** Presenting the deterministic LP model of PBB and expanding it as well as examining a variety of robust optimization models and selecting the appropriate robust model according to the variable parameters of the Company.

**Second step:** Introducing the BWM and its implementation steps to determine short-term goals, executive programs, and process outputs (products) to complete objective function of the PBB model.

**Third step:** Introducing the productivity index of total production factors using Cobb-Douglas production function as a criterion for determining the product performance evaluation index (Ik) to complete the final part of the objective function of PBB model.

**Fourth step:** Introducing resources, activities and products, and identifying share of cost and activity drivers, real and predicted sources using the company's activity based costing system (ABC).

**Fifth step:** Putting all parameters in the model and executing the model using LINGO software, analyzing and concluding.

## 5 Implementation steps of the PBB model

**First step:** The deterministic linear programming model of PBB for optimal budget allocation is as follows:

Maximize

$$\sum_{j=1}^{n_k}\sum_{q=1}^{Q}\sum_{p=1}^{P}\sum_{k=1}^{K} f_{rg} e_{qr} d_{pq} c_{kp} I_k z_k \tag{1}$$

Subject to:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}x_j \le \left(\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}\right) {S_i}'\big/S_i \tag{2}$$

$$\sum_{i=1}^{n} b_{jk}x_j \ge z_k \; ; K = 1 , \dots , n$$

Boundary Constraints:

$X_j, Z_k \ge 0; X_j, Z_k \in Z \ \forall i, k, j$

**Indexes used in the model include the following:**

g: index of macro objective, g = 1, 2 …G

r: long-term goals index, r = 1, 2 … R

q: short-term goals index, q = 1, 2 … Q

p: index of executive programs, p = 1, 2 … P

k: products index, k = 1, 2 … K

j: activity unit index, j = 1, 2… m

i: source unit index, i = 1, 2 … n

**The variables used in the model are:**

**1. Objective Function Variables:**

$f_{rg}$: share of long-term goal driver (r) in the macro objective (g)

$e_{qr}$: share of short-term goal driver (q) in the long-term goal (r)

$d_{pq}$: share of executive program driver (p) in the short-term goal (q)

$c_{kp}$: share of product driver (k) in the executive program (p)

$I_k$: result of evaluating product's performance (k)

$Z_k$: output (product) (k)

**2. Constraint variables:**

$a_{ij}$: share of cost driver (i) in activity (j)

$X_j$: volume of activity (j)

$b_{jk}$: share of activity driver (j) in product (k)

$S_i$: real amount of source (i) in the performance period

$S'_i$: predicted amount of source (i)

**The objective function:**

The objective function is to maximize the effectiveness of the company's operating activities or in other words the achievement values of the macro objectives. Achievement values of macro objectives is calculated by measuring performance indicators related to long-term and short-term goals, and executive programs. The macro objectives score ($f_{rg}$) is obtained by multiplying performance indicators of long-term goals in share of long-term goal driver (r) in the macro objective (g). Long-term goals score ($e_{qr}$) is attained by multiplying performance indicators of short-term goals in share of short-term goal driver (q) in the long-term goal (r). Short-term goals score ($d_{pq}$) is equal to the score of executive programs multiplied by share of executive program driver (p) in the short-term goal (q) and executive programs score ($c_{kp}$) is the result of multiplying product evaluation indices in share of product driver (k) in the executive program (p). These steps are repeated for all the macro objectives and then the final scores of all the macro objectives are added together. Finally, by calculating the index of product performance evaluation k ($I_k$) and multiplying it by the ultimate score of macro objectives, the factor (or score) of each product (k) is obtained. The factor (or score) of each product (k) is the coefficient of $Z_k$. Therefore, the objective function, which is the sum of the coefficients of $Z_k$ multiplied by $Z_k$, is completed in this way.

**Constraints:**

The first-row constraint is the sources constraint. Activities consume various sources (such as manpower, energy, materials, etc.). The number of sources is available in accounting systems in the headings of cost and by using a prediction method, their value can be estimated for the next period. The use amount of each activity from each source is defined as the share of cost driver ($a_{ij}$), which represents the coefficient of variables or volume of activities ($X_j$) in the first constraint. In budgeting, the consumption of each source (for different activities) ($S_i$) should be equal to or less than the predicted amount of each source ($S'_i$). The second-row constraint is to balance as well as to establish a logical relationship between the volume of activity ($X_j$) and the number of products ($Z_k$). To produce any output or product ($Z_k$), a certain volume of different activities ($X_j$) must be performed, this is considered in the form of a constraint that the coefficients of this constraint are the same as the activity driver ($b_{jk}$). The third-row constraint is assigned to boundary constraints and indicates that the presented model is an integer programming problem ($X_j$ is the number of repetitions of an activity to produce each unit of product $Z_k$) and also these are designed to prevent the model from the becoming zero.

After solving the model and determining the optimal amount of activity volume ($X_j$) and number of products ($Z_k$), the following can be obtained from the model [3]:

$$\text{Activity budget} = \left(\sum_{i=1}^{m}\sum_{j=1}^{n} S'_i a_{ij} X_j\right) \Big/ \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij} \tag{3}$$

The basic premise of classical mathematical programming is to develop a model, which input data are certain values [10]. However, this assumption is often violated in real-world problems. This problem can be either attributed to the fact that the parameters used in the model are only the estimates of the real parameters or in a more general state due to the effect of uncertainty on some parameters. Uncertainty is a prominent feature of real world problems. Data uncertainty can dramatically change final results and unit performance classification [36].

The optimization issues affected by non-deterministic or uncertain parameters have been in the focus of attention from a long time ago. Two important factors in problems involving uncertainty are quality and feasibility of the solution. The optimization models occurring in uncertainty conditions may produce solutions so far from the optimal solution or even infeasible. Therefore, it seems natural to look for designing of solution methods capable of securing the planning models against uncertainty, or in other words, the solutions that are “robust” [10]. In this way, this optimal solution was called robust optimization. According to Ben Tal and Nemirovski [8], in many cases the feasibility of the usual optimal solution to a Linear Programming can be heavily affected by quite small, from the practical viewpoint, perturbations of the data. Robust approach to solving linear optimization problems with uncertain data was proposed in the early 1970s and has recently been extensively studied and extended [10]. This approach provides a systematic and computationally reasonable way to construct reliable solutions, those capable to withstand data uncertainty of a given level [8]. Under this approach, we are willing to accept a suboptimal solution for the nominal values of the data in order to ensure that the solution remains feasible and near optimal when the data changes [10]. Therefore, robust optimization is a certain measure against uncertainty of LP model parameters. The robust optimization approaches proposed by different researchers have taken into consideration different levels of conservatism. The robust model selection depends on the type of parameter or parameters that are variable in the desired system. The uncertain data of a robust optimization problem vary in an uncertainty set. Soyster [46] provided a robust optimization approach that the objective function coefficients as well as the right-hand sides of the constraints are deterministic, and the coefficients matrix in the constraints are uncertainty. This assumption does not make any sense to the problem.

Because if the model has uncertainty in the objective function, the objective function can also easily be converted to a constraint. If the right-hand sides of the constraints are uncertainty, the right-hand sides are treated like the parameters on the left-hand side of the constraint. So that it can be assumed that the right-hand sides are multiplied by the constant value of one. The Ben Tal and Nemirovski’s model [8] Shows less conservatism than the Soyster’s model and feasible solution obtained in it, is also feasible solution in the Soyster’s model. The Ben Tal and Nemirovski’s model focus on ellipsoidal uncertainty that is a type of conic quadratic program with (n+2k) variables and (m+2k) constraints. Since this model is a nonlinear one, it is not particularly attractive for solving robust discrete optimization models [10]. The Bertsimas and Sim [10] proposed linear optimization problems, and their approach readily generalizes to discrete optimization problems. In their model, it is assumed that the objective function coefficients as well as the coefficients matrix in the constraints are uncertainty, and the right-hand sides of the constraints are deterministic (without loss of generality). The important thing is that the coefficients matrix is independent, symmetrically distributed and bounded random variable. In addition, they developed an approach that the right-hand sides are uncertain in a symmetric interval. In this case, the right-hand sides behave like the coefficients matrix, then the robust solution will be feasible deterministically.

It should be noted that assume that the input output data are certain and precise. However, crisp inputs and outputs data sometimes are unavailable in many real-world applications [32]. Therefore, according to what was said, the deterministic Linear Programming can only calculate optimal budget allocation, and is not able to provide optimal allocation solutions under uncertainty. Due to the necessity of considering the uncertainty [33], to allocate the budget under uncertainty, after solving the deterministic Linear Programming and finding the optimal budget allocation, the appropriate robust optimization approach should be chosen. The most appropriate robust optimization approach depends on the uncertain system parameters. Since the objective function coefficients in the Gas Company are deterministic and the uncertainties are related to the right-hand sides (sources), after solving the deterministic LP model, the Soyster's robust solution is used to ensure the optimization and feasibility of the model under uncertainty of the sources. The Soyster's robust solution is as follows:

Maximize ĆX
Subject to:

$$\sum_j \mathrm{a}_{ij} x_j + \sum_j \hat{\mathrm{a}}_{ij} y_{ij} \leq b_i \quad \forall_i \qquad (4)$$
$$-y_j \leq \mathrm{X_i} \leq y_j \quad \forall_i , \mathrm{L} \leq \mathrm{X} \leq \mathrm{u}, \mathrm{y} \geq 0$$

In the above model, Ji are the set of uncertain coefficients in the row i of matrix A. According to the deterministic Linear Programming, the robust counterpart optimization, on the assumption of the presence of the uncertain right-hand sides (sources) based on the Soyster model, is as follows:

$$Max \sum_{r=1}^{r}\sum_{q=1}^{q}\sum_{p=1}^{p}\sum_{k=1}^{k} e_{qr} d_{pq} c_{kp} I_k Z_k \qquad (5)$$

S.t:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}\, x_j - \hat{\mathrm{S}}_i\, y \leq \left(\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}\right)\frac{S'_i m_i}{S_i} \; \forall_i, S_i \in U \; Uncertainty\ Set: U \qquad (6)$$

$$\text{-y} \leq m_i \leq \text{y}$$
$$\sum_{j=1}^{n} b_{jk}\; x_j \geq z_k \text{ ; k =1, 2 … n}$$

Boundary Constraints:

$$\text{y}, m_i, x_j, Z_k \geq 0; X_j, Z_k \in \mathrm{Z} \;\; \forall_i, \text{k, j}$$

That: $\forall_i: S_i \in [S_i - \hat{\mathrm{S}}_i, S_i + \hat{\mathrm{S}}_i]$

(In this model, $\hat{S}_i$ is the amount of budget deviation).

### 5.3 Best Worst Method

In order to implement the PBB model, different levels of information should be identified, including: share of long-term goal driver r in the macro objective g ($f_{rg}$), share of short-term goal driver q in the long-term goal r ($e_{qr}$), share of executive program driver p in the short-term goal q ($d_{pq}$), share of product driver k in the executive program p ($c_{kp}$), result of evaluating product's performance k ($I_k$), share of cost driver i in activity j ($a_{ij}$), share of activity driver j in product k ($b_{jk}$), real amount of source i in the performance period ($S_i$) and predicted amount of source I ($S'_i$). Initially, information related to the macro objectives and long-term goals and the share of long-term goal driver r in the macro objective g ($f_{rg}$), short-term goals, executive programs, process outputs (products) were all received from the company. In addition, using the organization's process information, short-term goals, executive programs, process outputs (products) were extracted, that at this stage should be calculated the share of these factors drivers. For this purpose, the best worst method (BWM) was used. The best worst method (BWM) was proposed by Rezaei [37]. The best-worst method (BWM) is proposed to solve multi-criteria decision-making (MCDM) problems. In an MCDM problem, a number of alternatives are evaluated with respect to a number of criteria in order to select the best alternative(s). The method needs only 2n-3 comparisons, which makes it easier to use. In fact, best-worst method (BWM) is an MCDM method, where the decision maker chooses the best and the worst criteria and two pairwise comparison vectors for the best and the worst criteria are provided by the decision maker so this method requires fewer comparisons [42]. According to BWM, the best (e.g. most desirable, most important) and the worst (e.g. least desirable, least important) criteria are identified first by the decision-maker.

Pairwise comparisons are then conducted between each of these two criteria (best and worst) and the other criteria. A maximin problem is then formulated and solved to determine the weights of different criteria. The weights of the alternatives with respect to different criteria are obtained using the same process. The final scores of the alternatives are derived by aggregating the weights from different sets of criteria and alternatives, based on which the best alternative is selected. A consistency ratio is proposed for the BWM to check the reliability of the comparisons. The salient features of the proposed method, compared to the existing MCDM methods, are: (1) it requires less comparison data; (2) it leads to more consistent comparisons, which means that it produces more reliable results [37]. The steps of BWM that can be used to derive the weights of the criteria, are as follows:

**Step 1.** Determine a set of decision criteria. In this step, the criteria $\{C_1, C_2, ... C_n\}$ should be used to arrive at a decision.

**Step 2**. Determine the best (e.g. most desirable, most important) and the worst (e.g. least desirable, least important) criteria*. In this step, the decision-maker identifies the best and the worst criteria in general. No comparison is made at this stage.

**Step 3.** Determine the preference of the best criterion over all the other criteria using a number between1 to 9. The resulting Best-to-Others vector would be: $a_B = (a_{B1}, a_{B2}, \dots a_{Bn})$, where $a_{Bj}$ indicates the preference of the best criterion B over criterion j. It is clear that $a_{BB} = 1$.

**Step 4.** Determine the preference of all the criteria over the worst criterion using a number between 1 to 9. The resulting Others-to-Worst vector would be: $a_w= (a_{1w}, a_{2w}, \dots, a_{nw})^T$, where $a_{jW}$ indicates the preference of the criterion j over the worst criterion w. It is clear that $a_{WW} = 1$.

**Step 5.** Find the optimal weights ($w_1$*, $w_2$*, … $w_n$*). The optimal weight for the criteria is the one where, for each pair of $w_B/w_j$ and $w_j/w_W$ the following relation is set: $w_B/w_j = a_{Bj}$ and $w_j/w_W = a_{jW}$.

To satisfy these conditions for all j, should be find a solution where the maximum absolute differences $|w_B/w_j - a_{Bj}|$ and $|w_j/w_w - a_{jB}|$ for all j is minimized. Considering the non-negativity and sum condition for the weights, the following problem (1) is resulted:

$$\text{Min Max}_j \{|w_B - a_{Bj}w_j|, |w_j - a_{jW}w_W|\}$$
s.t.
$$\textstyle\sum_j w_j = 1$$
$$W_j \geq 0, \text{ for all } j$$

Problem (1) can be transferred to the following linear programming problem (2):

$$\text{Min } \xi^L$$

* If more than one criterion is considered to be the best or the worst, one can be chosen arbitrary.

s.t.
$|w_B - a_{Bj}w_j| \le \xi^L$ , for all j
$|w_j - a_{jW}w_W| \le \xi^L$ , for all j
$\sum_j w_j = 1$
$W_j \ge 0$, for all j

Problem (2) is a linear problem, which has a unique solution. By solving problem (2), the optimal weights ($w_1$*, $w_2$*…$w_n$*) and $\xi^{L*}$ are obtained. For this model $\xi^{L*}$ can be directly considered as an indicator of the consistency of the comparisons. Values of $\xi^{L*}$ close to zero show a high level of consistency [38].

**Table 2:** Objectives, goals and executive programs and of the company

| Objective | Long-term goals | Short-term goals | Executive programs | Products |
|---|---|---|---|---|
| Expansion of gas supply infrastructure | Exploitation development | Increase in natural gas branch relative to commitments | Supply of appropriate infrastructure conditions | Distribution of gas to the industrial sector |
| | | | Zoning gas supply area | |
| | | Reduction of average time of installation scattered branches | Field visit of rescuers | |
| | | | Reviewing conditions of requests | |
| | | Increasing the amount of subscription relative to commitments | Supply of appropriate infrastructure conditions | |
| | | | Zoning gas supply area | |
| | | | Advertising and Informing | |
| | | | Completion of the infrastructure holes | |
| | Improving the natural gas consumption Measurement system | Install of upgraded meters | Replace of faulty gas meters | |
| | | | Supply of upgraded meters | |
| | | Per capital of modification resulting from wrong readings | Use of upgraded meters | Distribution of Gas to the power plant |
| | | | Issuance of instant bill | |
| | | | Calibration of measuring equipment | |
| | | | Reduction of the time Consumption approval | |
| | | Review of subscribers covered by reading | Meter reading | |
| | | | Print of the bills | |
| | | Checking on lack of reading | Increase in meter reading forces | |
| | | | Increase in the number of times monitoring and reading | |
| | Optimal utilization of assets | Completion of the carbon roadmap | Identification of gas wasted locations | |
| | | | Feasibility study and analysis of emissions | |
| | Optimal maintenance and development of equipment and facilities | Implementation of leak detection program | Schedule of networking and power lines | Distribution of gas to household, public, commercial, agricultural and ... |
| | | | Schedule of leak detection of stations | |
| | | | Execution of leak detection operations | |
| | | | Providing of leak detection reports | |
| | | | Re-inspection of leaks | |
| | | Checking on routine and non-routine repair plans | Weekly visit of stations | |
| | | | Iinspection of regulators, valves, gauge… | |

## 5.4 Implementation of the Best-Worst Method

According to surveys, in Hormozgan Province Gas Company, seven (7) macro objectives and thirty (30) long-term goals, one hundred and twenty-two (122) short-term goals and three hundred and sixty-seven (367) executive programs have been designed. In order to examine the model in Hormozgan Province Gas Company, only the data of the operations department have been used. Thus, macro objectives, long-term goals, short-term goals, executive programs, products and the most important activities of the operations department and the relationships between them have been collected. According to the studies, to achieve macro objectives in 1397, six (6) macro objectives, fifteen (15) long-term goals, forty-four (44) short-term goals and one hundred and thirty-three (133) executive programs are intended for the operations department of the company. In the table (2), the first macro objective of the company in the operational department (expansion of gas supply infrastructure) along with its subset (long-term and short-term goals and executive programs) and also process outputs are given as an example. Due to the high volume of other objectives, they are waived here.

First, a hierarchical tree, including levels of macro objectives, long-term goals, short-term goals, executive programs, and process outputs related to the company's operational activities in accordance with the table (2) was prepared to determine the share of short-term goal driver ($e_{qr}$), the share of executive programs driver ($d_{pq}$) and share of product driver in the executive program ($c_{kp}$) of the operations department. Then to provide share of drivers, in two steps, the questionnaires were given to the executive units officials (administrators) of the company:

**Step 1:**

1) Determination of the most important and least important criterion of each level in general (using the first questionnaire).

2) Determination of the preference of the best criterion over all the other criteria, using a number between 1 to 9 and also determination of the preference of all the criteria over the worst criterion, using a number between 1 to 9 (using the second questionnaire).

**Step 2:**

Determination of the optimal weights of the criteria (share of drivers) using problem (2), in such a way that the optimal share of drivers were calculated according to the BWM linear programming problem (problem 2) in LINGO17 software. For example according to the administrators, “increase in natural gas branch relative to commitments” was determined as the most important short-term goal related to the long-term goal of “exploitation development”, and on the other hand “increasing the amount of subscription relative to commitments” was determined as the least important short-term goal related to this long-term goal. After solving the problem (2), the results are as follows:

**Table 3:** Relative weight of short-term goals

| Criteria name (short-term goal) | Criteria weight |
|---|---|
| Increase in natural gas branch relative to commitments | 0.45 |
| Reduction of average time of installation scattered branches | 0.3 |
| Increasing the amount of subscription relative to commitments | 0.25 |
| Consistency Index: 0.056 | |

According to the Table 3, the consistency index is 0.056, which is less than or equal to 0.1, and it indicates that the consistency of results is acceptable. In addition, “increase in natural gas branch relative to commitments” with a weight of 0.45, as the first priority, “reduction of average time of installation scattered branches” with a weight of 0.3, as the second priority and “increasing the amount of subscription relative to commitments” with a weight of 0.25, as the third priority among the short-term goals are related to the long term goal of “exploitation development”. Thus, for each of the short-term goals, executive programs and process outputs, the above steps were taken. After solving the BWM linear programming problem (problem 2), the driver share of each of the short-term goals ($e_{qr}$), executive programs ($d_{pq}$) and process outputs ($c_{kp}$) were determined (regarding to the fact that the operations department of the company includes 44 short-term goals and 133 executive programs, thus mentioning other goals and programs are waived here).

**Third step:**

In this study, using Cobb-Douglas production function, the productivity index (total factor productivity (TFP)) is applied to determine the performance evaluation index ($I_k$), as the equation (7):

$$VA = A\,L^{\alpha}\,K^{\beta} \quad (7)$$

Where, VA is adjusted value added, A is the productivity of total production factors, L is the number of active labor force, K is the value of adjusted capital (value of fixed assets), α is the labor force share of value added and β is the capital factor share of value added. In equation (7) α and β are calculated as follows:

$$\beta = 1 - \alpha,\ \alpha = (\text{compensation of employees})/VA$$

By taking the natural logarithm of the e constant (Euler's number) from equation (7), the equation (7) transfers to equation (2):

$$Ln\,(A) = Ln\,(VA) - \alpha\,Ln\,(L) - \beta\,Ln\,(K) \quad (8)$$

To calculate the share of each factor of production, the average of the last 3 years of production factors is used.

**Fourth step:**

According to the studies, Hormozgan Province Gas Company has three main products and, in the operations department, has eight main activities and eight general sources as follows (Table 4):

**Table 4:** Outputs, activities and sources of the company

| Source (i) | i | Activity (j) | i | Product (k) | $Z_k$ |
|---|---|---|---|---|---|
| Rent, water, electricity and telephone, education, insurance, … | 1 | Health, Safety and the Environment (HSE) | 1 | Distribution of gas to the industrial sector | $Z_1$ |
| Administrative and Organizational | 2 | Repairs and maintenance | 2 | | |
| Depreciation | 3 | Technical inspection | 3 | | |
| Salaries, wages and benefits | 4 | Relief | 4 | Distribution of gas to the power plant | $Z_2$ |
| Transportation | 5 | Telecommunications and telemetry | 5 | | |
| Received services | 6 | Transportation | 6 | | |
| termination benefits | 7 | Measurement and monitoring | 7 | Distribution of gas to household, public, commercial, agricultural… | $Z_3$ |
| Materials and goods | 8 | Gas Subscription Service | 8 | | |

Next, in order to determine the required information related to the first and second constraints of the linear programming model, the amount of sources in the performance period ($S_i$), the predicted amount of source ($S'_i$), the share of source (cost) driver ($a_{ij}$) and the share of activity driver ($b_{jk}$) were extracted from the company's activity-based costing system (ABC). After calculating all the elements of the model and preparing the information in Excel file according to the table 4, the model was ready to run in LINGO software (it is worth mentioning that due to the large volume of titles of macro objectives, long term and short-term goals and executive programs, also, with regard to the insertion of the information of the first macro objective in Table (2), as an example, mentioning other objectives and their subset have been avoided). Table 5 describes mathematical symbols, outputs, activities, and sources.

Each parameter of the model was calculated according to the steps mentioned earlier. According to the extracted values of each of the parameters of the linear planning model, the deterministic linear programming model of PBB in the operations department of Hormozgan Province Gas Company is as follows:

**Table 5:** Information related to the model of PBB and their mathematical symbols

| Macro objective (g) | Long- term goal (r) | $f_{rg}$ = share of long-term goal driver (r) in the macro objective (g) | Short-term goal (q) | $e_{qr}$ = share of short-term goal driver (q) in the long-term goal (r) | executive program (p) | $d_{pq}$ = share of executive program driver (p) in the short-term goal (q) | Output (product) (k) | $c_{kp}$ = share of Product driver (k) in the executive program (p) | Activity (j) | Source (i) | $a_{ij}$= share of cost driver (i) in activity (j) | $b_{jk}$ = share of activity driver (j) in product (k) | $X_j$ = volume of activity (j) | $Z_k$ = Number of output (product) (k) | $S_i$ = real amount of source) i (in the performance period | $S'_i$ = Predicted amount of source (i) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | $F_{1,1}$ | 1 | $e_{1,1}$ | 1 | $d_{1,1}$ | 1 | $c_{1,1}$ | 1 | 1 | $a_{1,1}$ | $b_{1,1}$ | $X_1$ | $Z_1$ | $S_1$ | $S'_1$ |
| 2 | 2 | $F_{2,1}$ | 2 | $e_{2,1}$ | 2 | $d_{2,1}$ | 2 | $c_{1,2}$ | 2 | 2 | $a_{1,2}$ | $b_{1,2}$ | $X_2$ | $Z_2$ | $S_2$ | $S'_2$ |
| 3 | 3 | $F_{3,1}$ | 3 | $e_{3,1}$ | 3 | $d_{3,1}$ | | $c_{1,3}$ | 3 | 3 | $a_{1,3}$ | $b_{1,3}$ | $X_3$ | | $S_3$ | $S'_3$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | 3 | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | $Z_3$ | ⋮ | ⋮ |
| 6 | 15 | $F_{15,6}$ | 44 | $e_{44,15}$ | 133 | $d_{133,44}$ | | $C_{3,133}$ | 8 | 8 | $a_{8,8}$ | $b_{8,3}$ | $X_8$ | | $S_8$ | $S'_8$ |

Max= $0.32 z_1 + 0.31 z_2 + 0.05 z_3$;
!source Constraints;
$0.1x_1 + 0.06x_2 + 0.3x_3 + 0.45x_4 + 0.025x_5 + 0.053x_6 + 0.01x_7 + 0.002x_8 <= (66.5)$ ;
$0.015x_1 + 0.15x_2 + 0.035x_3 + 0.0682x_4 + 0.285x_5 + 0.006x_6 + 0.44x_7 + 0.0008x_8 <= (10.25)$ ;
$0.2x_1 + 0.2x_2 + 0.01x_3 + 0.02x_4 + 0.4x_5 + 0.01x_6 + 0.15x_7 + 0.01x_8 <= (7.5)$;
$0.1x_1 + 0.2x_2 + 0.06x_3 + 0.03x_4 + 0.1x_5 + 0.01x_6 + 0.5x_7 + 0x_8 <= (9)$;
$0.015x_1 + 0.25x_2 + 0.04 x_3 + 0.072x_4 + 0.25x_5 + 0.12x_6 + 0.003x_7 + 0.25x_8 <= (11)$;
$0.07x_1 + 0.17x_2 + 0.010x_3 + 0.02x_4 + 0.45x_5 + 0.007x_6 + 0.27x_7 + 0.003x_8 <= (7)$;
$0.195x_1 + 0.11x_2 + 0.004x_3 + 0.035x_4 + 0.3x_5 + 0.002x_6 + 0.35x_7 + 0.004x_8 <= (8)$;
$0.020x_1 + 0.063x_2 + 0.011x_3 + 0.002x_4 + 0.043x_5 + 0.017x_6 + 0.844x_7 + 0x_8 <= (2)$;
! Activity Constraints;
$0.527x_1 + 0.578x_2 + 0.848x_3 + 0.148x_4 + 0.169x_5 + 0.143x_6 + 0.233x_7 + 0.254x_8 >= z_1$;
$0.54x_1 + 0.25x_2 + 0.33x_3 + 0.26x_4 + 0.16x_5 + 0.10x_6 + 0.95x_7 + 0.31x_8 >= z_2$;
$0.054x_1 + 0.354x_2 + 0.476x_3 + 0.345x_4 + 0.91x_5 + 0.008x_6 + 0.431x_7 + 0.322x_8 >= z_3$;
!boundary constraints;
$x_1>=1;x_2>= 1;x_3>=1;x_4>= 1;x_5>=1;x_6>=1;x_7>=1;x_8>=1;z_1>=0;z_2>=0;z_3>=0$; $x_1$-$x_8$, $z_1$-$z_3 \in Z$;

Regard to the fact that the company's sources ($S_i$) are faced to changes and uncertainty in later periods budgeting, therefore, the optimization of the deterministic LP model (the above solved model) is overshadowed by this uncertainty and one of the ways to consider uncertainty conditions in LP models is robust optimization models. As previously mentioned, by examining the different robust optimization approaches, the Soyster approach was selected as a robust counterpart optimization. According to the first step, in Soyster's robust counterpart optimization, the right-hand sides are as follows:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}\frac{S'_i m_i}{S_i} \quad (9)$$

Furthermore, the amount of source deviation in uncertainty conditions ($\hat{S}_{iy}$) is added to the right-hand sides and it can be presented as a negative number to the unequal left as follows:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}\, x_j - \hat{S}_i\, y \le \left(\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}\right)\frac{S'_i m_i}{S_i} \quad (10)$$

Therefore, by applying the above conditions, the robust counterpart optimization of PBB is solved and presented as follows:

Max= $0.32\, z_1 + 0.31z_2 + 0.05\, z_3$;
!source Constraints;
$0.1x_1 + 0.06x_2 + 0.3x_3 + 0.45x_4 + 0.025x_5 + 0.053x_6 + 0.01x_7 + 0.002x_8 - 1325y_1 <= (66.5m)$;
$0.015x_1 + 0.15x_2 + 0.035x_3 + 0.0682x_4 + 0.285x_5 + 0.006x_6 + 0.44x_7 + 0.0008x_8 - 52y_2 <= (10.25m)$;
$0.2x_1 + 0.2x_2 + 0.01x_3 + 0.02x_4 + 0.4x_5 + 0.01x_6 + 0.15x_7 + 0.01x_8 - 35000y_3 <= (7.5m)$;
$0.1x_1 + 0.2x_2 + 0.06x_3 + 0.03x_4 + 0.1x_5 + 0.01x_6 + 0.5x_7 + 0x_8 - 7250y_4 <= (9\ m)$;
$0.015x_1 + 0.25x_2 + 0.04\, x_3 + 0.072x_4 + 0.25x_5 + 0.12x_6 + 0.003x_7 + 0.25x_8 - 3000y_5 <= (11m)$;
$0.07x_1 + 0.17x_2 + 0.010x_3 + 0.02x_4 + 0.45x_5 + 0.007x_6 + 0.27x_7 + 0.003x_8 - 2000y_6 <= (7m)$;
$0.195x_1 + 0.11x_2 + 0.004x_3 + 0.035x_4 + 0.3x_5 + 0.002x_6 + 0.35x_7 + 0.004x_8 - 1500y_7 <= (8m)$;
$0.020x_1 + 0.063x_2 + 0.011x_3 + 0.002x_4 + 0.043x_5 + 0.017x_6 + 0.844x_7 + 0x_8 - 3500y_8 <= (2m)$;
! Activity Constraints;
$0.527x_1 + 0.578x_2 + 0.848x_3 + 0.148x_4 + 0.169x_5 + 0.143x_6 + 0.233x_7 + 0.254x_8 >= z_1$;
$0.54x_1 + 0.25x_2 + 0.33x_3 + 0.26x_4 + 0.16x_5 + 0.10x_6 + 0.95x_7 + 0.31x_8 >= z_2$;
$0.054x_1 + 0.354x_2 + 0.476x_3 + 0.345x_4 + 0.91x_5 + 0.008x_6 + 0.431x_7 + 0.322x_8 >= z_3$;
!boundary constraints;
$x_1>=1;x_2>=1;x_3>=1;x_4>=1;x_5>=1;x_6>=1;x_7>=1;x_8>=1;z_1>=0;z_2>=0;z_3>=0;m-y_1<=0;m+y_1>=0;$
$my_2<=0;\ m+y_2>=0;\ m-y_3<=0;\ m+y_3>=0;\ m-y_4<=0;\ m+y_4>=0;\ m-y_5<=0;\ m+y_5>=0;\ m-y_6<=0;\ m+y_6>=0;$
$my_7<=0;\ m+y_7>=0;\ m-y_8<=0;m+y_8>=0;\ y_1>=0;y_2>=0;\ y_3>=0;y_4>=0;\ y_5>=0;\ y_6>=0;y_7>=0;y_8>=0;\ x_1>=1;\ x_2>=1;\ x_3>=1;\ x_4>=1;\ x_5>=1;x_6>=1;\ x_7>=1;\ x_8>=1;\ z_1>=0;z_2>=0;\ z_3>=0;\ x_1\text{-}x_8,\ z_1\text{-}z_3 \in Z;$

After solving the model of PBB and its robust counterpart (approach proposed by Soyster) with the help of Lingo17 software, the software output includes Xj or the number of repetitions activity (j) to produce product $Z_k$. To determine the optimal budget of activities, Xj (for all j) in the first step formula is given as follows:

$$\text{Activity Budget} = \left(\sum_{i=1}^{m}\sum_{j=1}^{n} S'_i a_{ij}\, X_j\right) \Big/ \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij} \quad (11)$$

As mentioned earlier, the share of the cost drivers (aij) and the predicted amount of source (S'i), have been extracted from the company's ABC system. Therefore, the optimal budget for each activity is obtained by placing the values of Xj, aij and S'i in the formula. Similarly, the optimal budget for products, executive programs, and short-term goals, long-term goals, and macro objectives can be calculated according to its own formula (see first step). After making these calculations, the optimal budget of activities and the optimal number of products were determined and the results of solving deterministic model and robust counterpart were compared with the real and predicted budget of the company. The summary of the results, in Table 6 is extracted from the model. As illustrated in table 6, the deterministic LP model and its robust counterpart have been compared with the actual and predicted budget. It is noteworthy that the information of the year 2018 has been extracted based on the actual budget. However, due to the non-realization of the actual amounts so far, the predicted amounts of the budgets of years 2019 and 2020 have been used as a basis for comparison with the optimal budget. In accordance with Table 6, the results of the implementation of deterministic LP model of PBB and its robust counterpart (approach proposed by Soyster) indicate that maintaining the feasibility and optimality budget in deterministic LP model and its robust counterpart, the optimal budget has been reduced compared to the actual and predicted budget.

**Table 6:** Compare real budget with optimal budget

| Amounts is in millions of Rial | | | |
|---|---|---|---|
| Budget / Year | 2018 | 2019 | 2020 |
| Real and predicted budget | 429,080 | 695,002 | 948,954 |
| Deterministic optimal budget | 407,733 | 530,053 | 689,069 |
| Robust optimization budget | 423,518 | 550,573 | 715,745 |

The results of the implementation of PBB model using deterministic LP model and its robust counterpart are presented in Table 7. In order to compare the results of the model, the actual budget and the amount of budget deviation, the percentage of budget deviation has been used (the percentage of budget deviation is equal to difference between the actual budget and the optimal budget divided by the actual budget). As can be seen in Table 7, the results show an improvement in budget deviation and optimal allocation using robust counterpart optimization by Soyster's approach.

**Table 7:** Results of the PBB model and comparison with the actual and predicted budget

| Deviation of Actual Budget year 2018 | | Deviation of Predicted Budget year 2019 | | Deviation of Predicted Budget year 2020 | |
|---|---|---|---|---|---|
| Optimal Budget | Robust Budget | Optimal Budget | Robust Budget | Optimal Budget | Robust Budget |
| -%5 | - %1.3 | -%24 | -%20 | -%27 | -%24 |

Tables 6 and 7 and Fig 2 and reviewing the results of the implementation of PBB model indicate the following:

1) Maintenance of feasibility and optimality budget in deterministic LP model and its robust counterpart
2) Reduction of the budget in deterministic LP model and its robust counterpart, in comparison with the actual budget
3) Improvement of the budget deviation and optimal allocation using the Soyster's robust counterpart

Thus, using the Soyster's robust counterpart, despite the possible changes and uncertainty in the sources of company, the optimal of Budget allocation has been unchanged during the years 2018-2020.

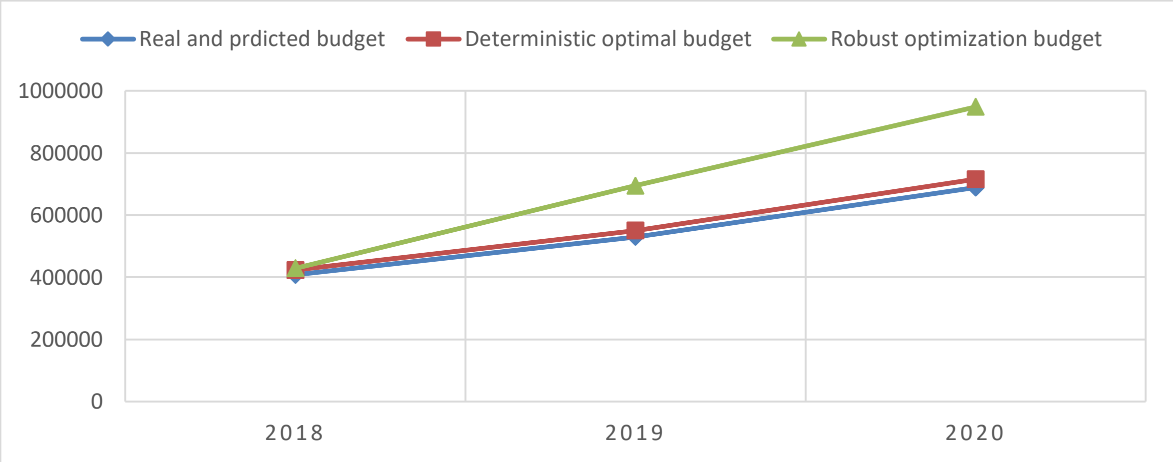


**Fig 2:** Comparison of the Budget with The Results of Solving Model

Furthermore, Table 8 and Fig. 3 show the results of comparing the actual budget related to the company's operational activities in year 2018 with the deterministic LP model and its robust counterpart. As can be seen, in robust counterpart, while maintaining the optimality of the model in the case of uncertainty on the right-hand sides, the budget allocation is less than the actual budget for operational activities no. 2,3,4,7,8 (Repairs and maintenance, Technical inspection ,Relief, Measurement and monitoring, Gas Subscription Service) and also the budget allocation is more than the actual budget for operational activities no. 1,5,6 (Health, Safety and the Environment (HSE), Telecommunications and telemetry, Transportation). Therefore, the results can draw the attention of the company's decision makers to change the attitude and review on the level of importance of various operational activities and the budget allocated to them.

**Table 8:** The results of comparing the actual budget of operational activities and PBB model- year 2018

| Activity / Budgeting Method | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Optimal Budget | 10,671 | 131,171 | 13,870 | 52,558 | 4,697 | 1,946 | 22,872 | 169,948 |
| Robust Budget | 48,512 | 140,370 | 11,769 | 22,604 | 4,697 | 1,546 | 28,872 | 165,148 |
| Actual Budget | 3,256 | 145,121 | 6,212 | 23,061 | 41 | 1,614 | 66,083 | 183,691 |

According to what has been said, the robust optimization model of PBB with the Soyster approach in the present study has the following features:

Reducing the complexity of the PBB system due to the use of mathematical model, using the maximum objective function with the aim of maximizing the efficiency and effectiveness of budget allocation and calculating the objective function coefficients by the best-worst method to determine the weight of short-term goals, executive programs and products as well as using the productivity index for determining the performance evaluation index of products, in terms of uncertainty of available resources to predict the optimal budget for the coming years, reducing the budget deviation index, forecasting the budget of activities and the optimal budget allocation to activities.

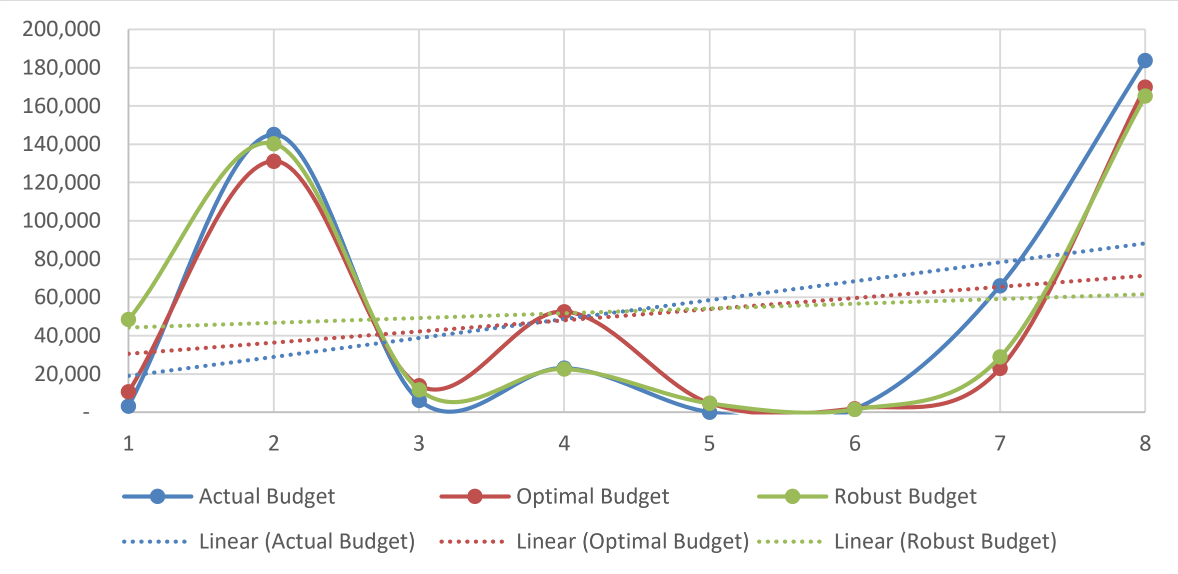


**Fig 3:** Comparing the Budget of Operational Activities with The Results of Solving Models

Therefore, regard to the uncertain condition of decision making and along with the probabilities in companies, using the robust optimization approach of Soyster can provide an appropriate basis for programming, allocating and optimal using of limited resources of the company and controlling it to achieve the company's macro objectives. In a situation where the complexity of budgeting and allocating resources to the needs, goals and consumption of future financial facilities is so great that the usual mental methods cannot achieve the appropriate level of satisfaction, a combination of models and mathematical techniques can be useful.

## 6 Conclusion

Reviewing the theoretical foundations and literature of PBB, the present study examined the optimal allocation of budget in Hormozgan Province Gas Company in the form of linear programming model and Soyster's robust optimization. The study sought to answer two research questions: Whether the selected PBB model can be implemented in Hormozgan Province Gas Company? And will the implementation of the PBB model reduce budget deviations and improve performance in the company? In this regard, after describing the model and extracting the required data, the model was solved and analysed in Lingo.17 software. In order to answer the research questions, it can be stated that while the model can be implemented in Hormozgan Province Gas Company, the results of solving the model show that Soyster's robust counterpart, considering the resources uncertainty, is capable of allocating the optimal budget, predicting the optimal budget for the coming years and reducing the budget deviation index. In addition, optimal budget allocation results in improvement of the allocation of available resources and expenditures in accordance with the framework of objectives and executive programs. Eventually, the improvement of the allocation of resources and expenditures can lead to improved performance. Furthermore, the results indicate that the use of this model can reduce the complexity of the PBB system, increase the effectiveness of operational activities by predicting the budget of operational activities, improve the performance of the budgeting system and thus increase accountability. Therefore, this model can be an appropriate basis for implementing PBB in Hormozgan Province Gas Company. According to the performance-based budgeting system, the budget structure of any organization should be in line with its perspective and goals, in such a way that the budget of the operational activities should be allocated to the performance of each activity according to the set goals, programs and policies and not to the tasks defined for it. Consequently, the research results can draw the attention of organizational decision makers to change their attitude and viewpoint toward the level of importance and budget allocated to different operational activities according to the performance of each activity.

Regard to the importance of proper costing system in performance-based budgeting, the ability of costs reporting according to activities and the essential role of activity-based costing system as one of the main elements of the PBB system as well as the significance of calculating the real cost of products and services, it is suggested that the share of cost drivers and activity drivers be accurately identified using mathematical methods and meta heuristic algorithms, to facilitate the implementation of performance-based budgeting system in its true sense.

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