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Jump Phenomenon Analysis in Vehicle and Chaos Control of Active Suspension System via Extended Pyragas Algorithm

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Abstract: In this paper, the nonlinear phenomenon including hump and chaos analysis along with chaos control of an active suspension in vehicles has been studied. The unstable periodic orbits of the system are stabilized using the novel developed delay feedback control algorithm based on the fuzzy sliding mode system. The chaotic Equations of motions are derived via Newton-Euler relations then, the nonlinear phenomenon such as jump and chaos in the vehicle dynamics has been confirmed using forcing frequency method. The results of the forcing frequency demonstrate the changes in system behaviour from the periodic to the irregular chaotic responses. In order to eliminate the chaotic responses in the vertical dynamics of the vehicle, a new fuzzy sliding delay feedback control algorithm is designed on the active suspension. The controller gain of the sliding feedback control is online estimated via fuzzy logic causing to rejection of the chattering phenomenon in the sliding mode algorithm besides the improvement in the responses of the feedback system. Simulation results of the control system depict a reduction of settling time and energy consumption along with eliminating the overshoots and chaotic vibrations.

Keywords: Chaotic Dynamics, Chaos Control, Extended Pyragas, Jump Phenomenon

Biographical notes: Yavar Nourollahi received his BSc and MSc in Mechanical Engineering at the Islamic Azad University of Tabriz, where he is now studying PhD in Mechanical Engineering, Applied Solid Design at the Islamic Azad University of Qazvin, Iran since 2016. His research interest includes nonlinear dynamics and control of specialized chaotic systems. **Mahdi Abtahi** received his PhD in Mechanical Engineering from K. N. Toosi University of Technology in 2013. He is currently an Assistant Professor at the Department of Mechanical Engineering, Islamic Azad University branch of Qazvin in Iran. His current research interest includes dynamic and control system, nonlinear mechanics, and Chaos.

Research paper

1 INTRODUCTION

Nonlinear phenomena including jump, quasi-periodic, and chaos can occur in the heave motion of vehicles. Therefore, some oscillations in the suspension system are returned to this nonlinear phenomenon. Most algorithms in chaos control are designed on the basis of stabilization of unstable orbits that require a complex solution of unstable orbits. Pyragas controller based on the delay feedback control can control the chaotic system without solving the complex orbits [1-4].

Many papers have been recently published about the chaotic dynamics and chaos control of suspension systems. The chaotic dynamics and control of vehicles under the unevenness of the road surface are investigated using numerical and analytical procedures [5-7]. The effect of damping coefficients and the passengers on the chaotic dynamic behavior are considered via the bifurcation diagrams in the vertical model of vehicles under the excitation force of the road surface [8-11]. Pyragas method based on delay feedback control has been used in chaotic systems because of the simple structure and good performance [12]. Zhang et al controlled the chaotic lateral dynamic in an active steering system using the adaptive time delay feedback method which led to the reduction of bounce vibrations [13-15].

In this research, the jump and chaotic responses in heave motion are studied and controlled using the delay feedback control algorithm developed by the fuzzy sliding mode system. The nonlinear jump phenomenon and chaos are analyzed via the forcing frequency diagrams. In order to stabilize the chaotic dynamical system, the Pyragas method is integrated with a new sliding mode algorithm which fuzzy inference system extends the sliding delay feedback. In this novel controller, by online calculation of the controller's coefficient in the developed Pyragas control system based on the sliding mode, the appropriate value of the control gain was estimated via the fuzzy system that the chattering phenomenon caused by the sliding mode behaviour around the sliding surface can be eliminated. The simulation results of the feedback system demonstrate the control of the suspension system without chaos.

2 MATHEMATICAL MODELING

The model of vertical motion is shown in "Fig. 1" that the body has two state variables consisting of the vertical displacement x_b and rotation θ. The rotation around the longitudinal axis of the body and the rotation perpendicular to the passageway can be eliminated due to their small effects. The tire model with nonlinear damping and springs is modeled as unsprung masses. The front and rear actuators' forces are u_f and u_r in the active suspension system.

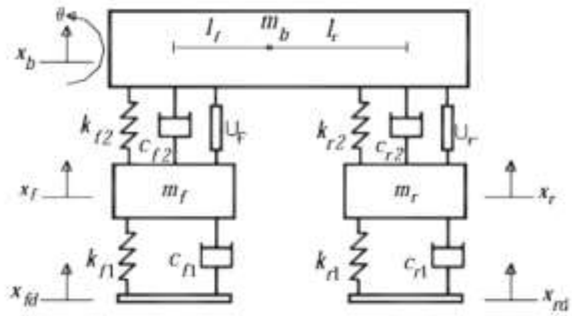


Fig. 1 Vertical model with active suspension.

Nonlinear spring and damper suspension relations are as follows [16]:

$$
\begin{aligned}
F_S &= K_1 X + K_2 X^3 = K_1(x_b - x_0) + K_2(x_b - x_0)^3 \\
F_{SC} &= C_1 V + C_2 V^3 = C_1 \frac{d}{dt}(x_b - x_0) + C_2\left(\frac{d}{dt}(x_b - x_0)\right)^3
\end{aligned} \tag{1}
$$

Where, k and c are the stiffness and dampers coefficient, X_0 is the displacement of the input excitation from the road surface that is expressed by $X_{fd}=A\sin(2\pi ft)$ and $X_{rd}=A\sin(2\pi ft+\alpha)$ for the front and rear tires, where A is the amplitude, f is the frequency of the excitation force and α represents the time delay between the displacement applied by the road surface affecting the front and rear tires. Also, vehicle tires are modeled with nonlinear spring and viscous damper, and the mathematical relationship for tire spring force equals $f_s=k_s\zeta_s$ and $f_{tc}=c_t\zeta\dot{x}_t$ as tire damper force. By applying the Newton-Euler laws, the vehicle motion Equations are as follows:

$$
\begin{aligned}
m_b\ddot{x}_b = &-k_{f2}(\zeta_{bf2}) - c_{f2}(\dot{\zeta}_{bf2}) \\
&- k_{r2}(\zeta_{br2}) - c_{r2}(\dot{\zeta}_{br2}) - m_b g + u_f + u_r
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
j\ddot{\theta} = &(k_{f2}(\zeta_{bf2}) + c_{f2}(\dot{\zeta}_{bf2}))(l_f\cos\theta) \\
&- (k_{r2}(\zeta_{br2}) + c_{r2}(\dot{\zeta}_{br2}))(l_r\cos\theta)
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
m_f\ddot{x}_f = &k_{f2}(\zeta_{bf2}) + c_{f2}(\dot{\zeta}_{bf2}) \\
&- k_{f1}(\zeta_{bf1}) - c_{f1}(\dot{\zeta}_{bf1}) - m_f g + u_f
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
m_r\ddot{x}_r = &k_{r2}(\zeta_{br2}) + c_{r2}(\dot{\zeta}_{br2}) \\
&- k_{r1}(\zeta_{br1}) - c_{r1}(\dot{\zeta}_{br1}) - m_r g + u_r
\end{aligned} \tag{5}
$$

Where $\zeta_{bf1} = x_f - \zeta_{sf1} - x_{fd}$, $\zeta_{bf2} = x_b - \zeta_{sf2} - x_f - l_f\sin\theta$, and that ζ_{sf} are the static length variations of the suspension springs in tires. The simulation parameters' numerical values are shown in “Table 1”.

Table 1 Values of parameters in the numerical solutions

System parameters	Value
M_b	1180 kg
J	633.6 kg m^2
M_f	50 kg
M_r	45 kg
K_{f2}	36952 N/m
K_{r2}	30130 N/m
K_{f1}, K_{r1}	140000 N/m
C_{f2}	500 kg/s
C_{r2}	360kg/s
C_{f1}, C_{r1}	10kg/s
l_f	1.123 m
l_r	1.377 m

3 JUMP ANALYSIS

After simulation of the system based on the values of parameters in “Table 1” and the input excitation of the road surface as A=0.08m and phase α=π/9 rad, the responses show the occurrence of chaos in the system, which the forcing frequency diagrams is used to prove the nonlinear vibrations. The natural frequencies of the system are calculated as $f_{n1}=1.0750\,\text{Hz}$, $f_{n2}=1.8234\,\text{Hz}$, $f_{n3}=9.4976\,\text{Hz}$, and $f_{n4}=9.8139\,\text{Hz}$ that *f*=10823Hz is the dominant frequency of the system. In order to verify the values of the natural frequencies, the results obtained in this research were compared with the results of reference [10] in “Table 2”, the results of the comparisons showed that the natural frequencies obtained in this research are in the same range as the reference that clearly indicated the correctness of the calculations.

Table 2 Comparison of the results of natural frequencies obtained with reference [10]

Comparison	f_{n1}	f_{n2}	f_{n3}	f_{n4}
Natural Frequency	1.0750 Hz	1.823 Hz	9.4976 Hz	9.8139 Hz
Natural Frequency in Ref [10]	1.28 Hz	1.80 Hz	9.08 Hz	9.26 Hz

The values of the vehicle speed, taking into account the reference [6] and the values of the first and second natural frequency and the jump frequency [*f*=1.075, 1.8234, 3.6] Hz are equal to [v=13.93, 23.63, 46.66] km/h.

The dynamic analysis of the frequency control parameter of the road surface excitation force is shown in the graphs of “Fig. 2”, which includes the maximum absolute value of the displacement of the state variables according to the frequency control parameter of the road surface excitation force, which indicates different behaviors in the responses of the system with increasing and decreasing frequency of the driving force. The range of increasing and decreasing frequency, for example, in the frequency range 1Hz<*f*< 6Hz and in parts of the frequency range 3Hz<*f*<5Hz, we see the behavior of frequency jumping.

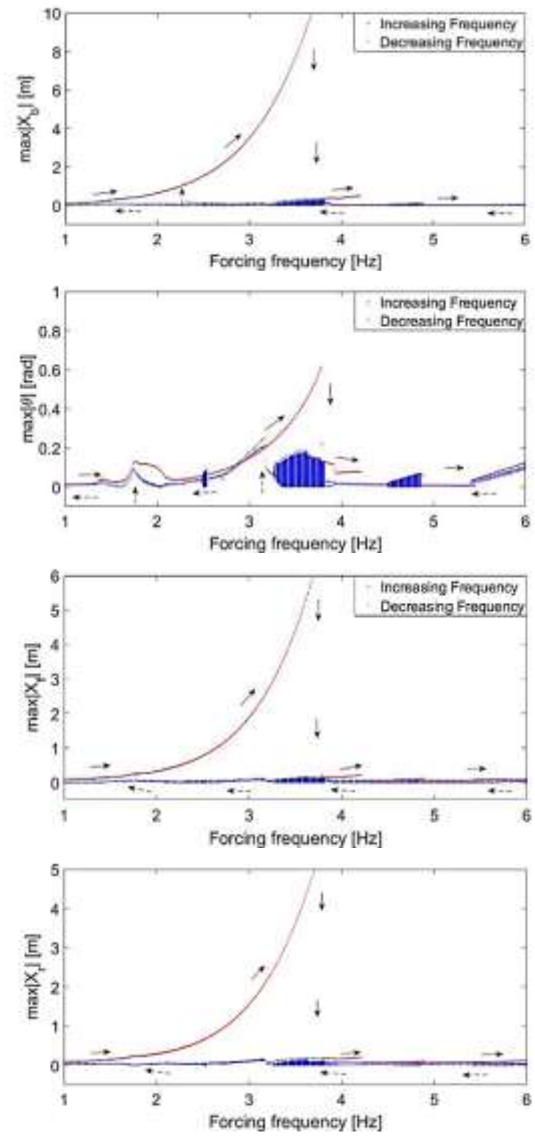


Fig. 2 Frequency response of system.

4 CHAOS CONTROL

Sliding-Pyragas controllers based on the delay feedback control are used to stabilize the unstable orbits which is estimated by a time-delay state variable. The feedback is the difference between the state and its time delay τ, and the delay time constant is estimated as the periodic of orbits. The main advantage of this method is no need to calculate the orbits. Therefore, the control input signal is derived as u(t)=k[y(t-τ)-y(t)] that k is control gain [3]. The Pyragas algorithm is a linear feedback to stabilize unstable orbits. If the dynamic system is defined as:

$$x^{(n)} = f(t, \underline{x}) + g(t, \underline{x})u \tag{6}$$

Where, x is the state vector, u is the control input signal, f and g are uncertain functions, and in u=0, the system has chaotic behavior [17]. For this reason, at first, the delay state is defined. It is $\tilde{\underline{x}}(t) = \underline{x}(t-T)$. It is obvious that the delay state should meet the following:

$$\tilde{x}^{(n)} = f(t-T, \tilde{\underline{x}}) + g(t-T, \tilde{\underline{x}})\tilde{u} \tag{7}$$

Where, $\tilde{u} = u(t-T)$. The dynamics of the error system is obtained by distinguishing between two Equations (6) and (7) as follows:

$$x^{(n)} - \tilde{x}^{(n)} = f(t, \underline{x}) - f(t-T, \tilde{\underline{x}}) + g(t, \underline{x})u - g(t-T, \tilde{\underline{x}})\tilde{u} \tag{8}$$

Where, $\underline{e} = \underline{x} - \tilde{\underline{x}}$ the error and the differential Equation of the error system are expressed as follows:

$$e^{(n)} = f(t, \underline{e} + \tilde{\underline{x}}) - f(t-T, \tilde{\underline{x}}) + g(t, \underline{e} + \tilde{\underline{x}})u - g(t-T, \tilde{\underline{x}})\tilde{u} \tag{9}$$

Therefore, the stability of orbits in a chaotic system according to Equation (8) leads to the stabilization of the error dynamics (9) that in order to increase the speed of convergence of the system to its stable fixed points, due to the uncertainties of the system, the robust control strategy based on the sliding mode has been used with definition of the sliding surface as follows [17]:

$$S = \sum_{i=1}^{n+1} \alpha_i \left[\int_T^t e_i(s)ds = \int_T^t e^{(i-1)}(s)ds \right] \tag{10}$$

Where $\alpha_i \rangle 0$ and for stability of sliding mode, the system must be placed in $S = 0$ that is defined $V = (1/2)S^2$ as the Lyapunov function. Assuming $g(t, \underline{x}) \rangle 0$ and simplifying the calculations, the control input u is extracted as follows [17]:

$$u = -\frac{1}{\alpha_n g_m(t, \underline{x})}[\alpha_n \hat{f}(t, \underline{x}) - \alpha_n \hat{f}(t-T, \tilde{\underline{x}}) + \sum_{i=1}^{n} \alpha_i e_i + K\text{sign}(S)] \tag{11}$$

Where, k in order to satisfy the Lyapunov stability condition of the system must be applied to the following inequality.

$$K \ge \left(\frac{g_M(t, \underline{x})}{g_m(t, \underline{x})} - 1\right)\left|\sum_{i=1}^{n} \alpha_i e_i + \alpha_n f(t, \underline{x}) - \alpha_n f(t-T, \tilde{\underline{x}})\right| + \alpha_n F(t, \underline{x}) + \alpha_n F(t-T, \tilde{\underline{x}}) + g_M(t-T, \tilde{\underline{x}})|\tilde{u}| + \theta \tag{12}$$

5 SIMULATION OF CONTROL SYSTEM

The behaviour of the feedback system under the fuzzy-sliding Pyragas controller is simulated according to "Fig. 3".

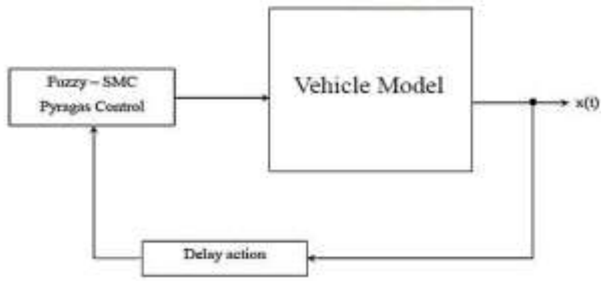
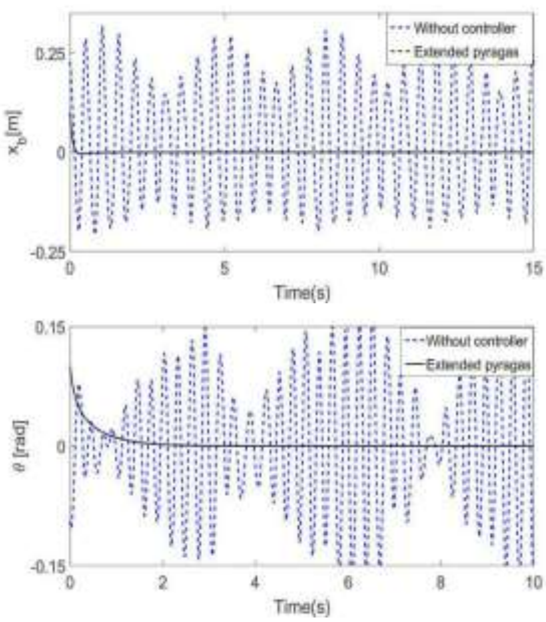


Fig. 3 Block diagram of the Control system.

Responses of the feedback system along with the chaotic open loop of state variables in active suspension are demonstrated in "Fig. 4" that stat the rejection of chaos in the feedback responses.



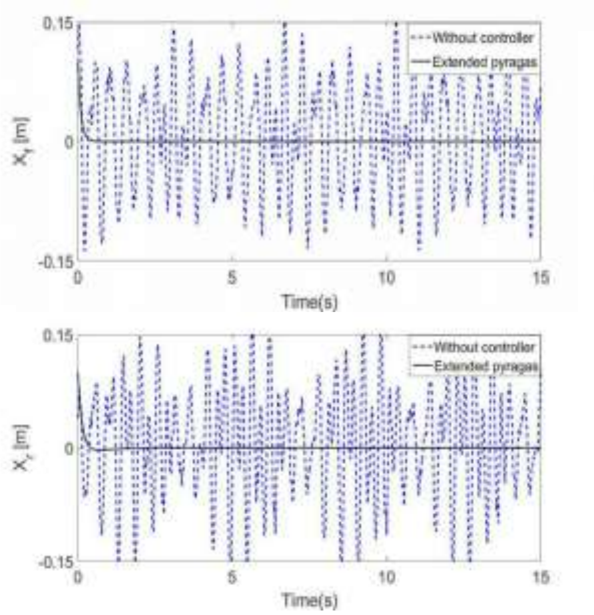


Fig. 4 Responses of the feedback control system.

The responses of the actuators in the active suspension under the Extended Pyragas controller are shown in "Fig. 5" below.

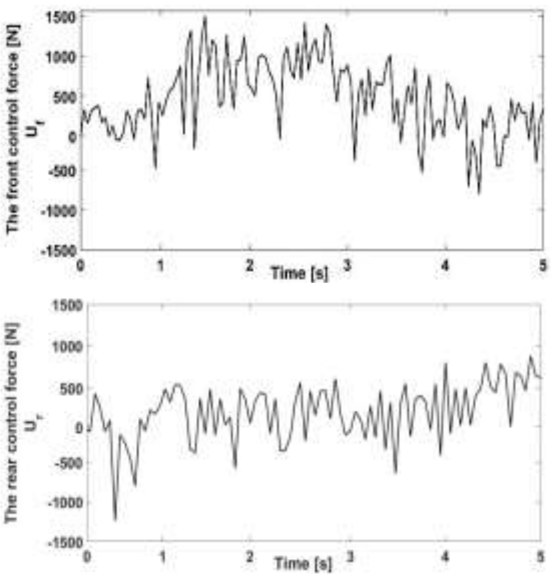
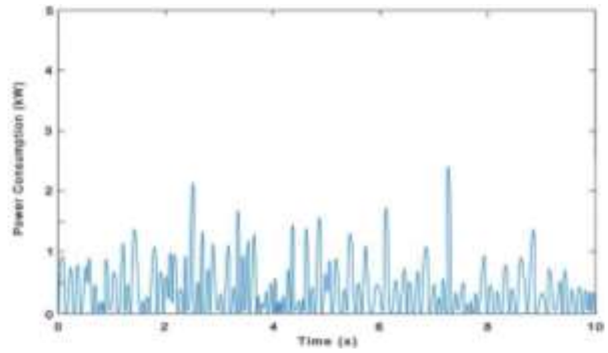


Fig. 5 The front and rear actuators of active suspension.

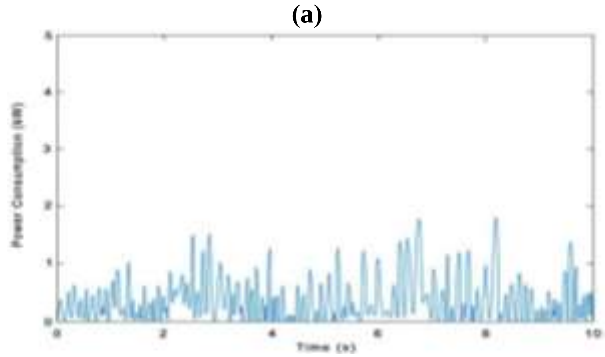
Power consumption of active suspension is determined as the following [18]:

$$P_{ac} = \frac{\int_0^T \left[U(t) \cdot (S\dot{W}S)(t) \right] dt}{T} \tag{13}$$

Where, U is the control force and SWS is the suspension deflection of the actuator. Figure 6 shows the average power consumption of the Extended Pyragas controller for the front and rear suspension systems which is equal to 1.2117kW and 0.4114kW.



(a)



(b)

Fig. 5 Power consumption of the controller system: (a): front suspension, and (b): rear suspension.

For stability analysis of the feedback system, after linearization of the system around the fixed point, all of the eigenvalues are placed in the left hand of the complex plane, which depicts the stability of system.

For robustness analysis of the controller against the parametric uncertainties, the inertia of the system model is increased as 8% of simulated values and the new simulation results show the appropriate performance of the feedback controller. Also, in order to analyze the structural robustness of the controller, the control system is applied to the Carsim model as a SUV vehicle with 27 degrees of freedom based on "Fig. 3" that results of this simulation showes the entirely adaptation of responses.

6 CONCLUSIONS

The nonlinear vibrations of the bounce model of vehicle in the face of uneven road surface are investigated in this work and then the irregular oscillations are rejected via a new sliding delay feedback fuzzy controller. Therefore, after simulation of the open loop system, forcing frequency diagrams are used to analyze the jump phenomenon and chaos in the nonlinear dynamics, quasi-periodic and chaotic behavior are demonstrated in the uncontrolled system relative to changing frequency. In order to control chaos, the delay feedback strategy is developed in the active suspension system. To increase the system's rapid stabilization, a sliding mode control is used in the structure of the Pyragas controller. Also, to eliminate the chattering phenomenon in the sliding mode and to online estimate the controller gain accurately, the fuzzy inference system is combined with the sliding delay feedback system. The simulation results of the Fuzzy SMC-Pyragas controller indicate the repid stabilization along with the elimination of chaos by reducing the settling time without any overshoot in the responses. Comparison of results in this research with respect to [19] depicts a 15% reduction at the settling time besides rejection of the overshoot. Also in control signals responses, in addition to a 26% reduction in actuators effort and 34% decrease in the energy consumption, the saturation problems in suspension actuators are solved. Consequently, these results are compared with [20] which depicts a 20% reduction in the settling time by the overshoot rejection, a 35% reduction in the amplitude of the controller input in suspension actuators, and 20% decrease in energy consumption is illustrated in the control signal.

7 NOMENCLATURES

α	:the time delay between the road roughness to the front and rear tire
m_b	vehicle body mass:
J	:vehicle body inertia
m_f	:front unsprung mass
m_r	:rear unsprung mass
$x_b(t)$	:displacement of m_b
$\theta(t)$	:angular displacement of m_b
)x_f(t	:displacement of m_f
$X_r(t)$	:displacement of m_r
$x_{fd}(t)$	:excitation to the front tire
$x_{rd}(t)$	:excitation to the rear tire
l_f	:front length
l_r	:rear length
k_{f2}	:front suspension spring stiffness
c_{f2}	:front suspension damping coefficient
k_{r2}	:rear suspension spring stiffness
c_{r2}	:rear suspension damping coefficient
k_{f1}	:front tire stiffness
c_{f1}	:front tire damping coefficient
k_{r1}	:rear tire stiffness
c_{r1}	:rear tire damping coefficient
k_s	:stiffness of the suspension springs
A	:amplitude of the excitation force
f	:frequency of the excitation force

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