Vibration Analysis and Sensitivity Analysis of Semi-Submerged Multilayer Piezoelectric Microcantilever

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Abstract: The growing development of nanobiotechnology and its applicability resulted in a wider range of use of Microcantilevers (MCs) in liquid. Considering the applications of piezoelectric MCs in the microelectromechanical systems and Atomic Force Microscope (AFM), as well as the high performance of these beams, this article investigates the vibrating behavior of multilayer piezoelectric MCs with geometric discontinuity in liquid environment. Due to the extreme complexity of hydrodynamic forces introduced to MCs, this force may reduce their accuracy. As a result, the MC was considered to be semi-submerged in the liquid medium to reduce the effect of hydrodynamic force. In addition, to reduce the effect of hydrodynamic force on vibrating behavior of the MC, sensitivity analysis was performed on its geometric dimensions to obtain the optimal dimensions, aiming at minimizing the effect of this force. The differential equation of motion was derived using the Euler–Bernoulli theory and the Lagrange method. The hydrodynamic force was exerted on the MC through the sphere string model. The Simulation results indicated that due to reducing resonance frequency variations in the third vibrating mode, the effect of hydrodynamic force on vibrating motion is minimized in this mode and considered as the optimal vibrating mode among the first three modes. The sensitivity analysis results showed that the MC length and piezoelectric layer were geometric parameters with the greatest effect on frequency sensitivity of MC, which should be considered in semi-submerged piezoelectric MC design.

Keywords: Hydrodynamic Force, Piezoelectric Microcantilever, Semi-Submerged, Sensitivity Analysis

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Biographical notes: Mohamadreza Khosravi received his MSc in Mechanical Engineering from Islamic Azad University, Shahrekord Branch, Shahrekor, Iran in 2017. His current research interest includes piezoelectric microcantilever in liquid environment and dynamic modelling. **Reza Ghaderi** is Assistant Professor of Mechanical Engineering at the University of Islamic Azad University, Shahrekord Branch, Iran, where he has been involved in teaching and research activities in the area of vibration for the last 15 years. He received his PhD in Mechanical Engineering from Islamic Azad University, Science and Research Branch, Tehran, Iran in 2013. His current research focuses on nonlinear vibration of piezoelectric MCs, atomic force microscope and microelectromechanical systems.

1 INTRODUCTION

Today, MCs are widely used due to the growing application of nanobiotechnology and the necessity of measuring bio specimens in liquid mediums. As compared to the air and vacuum mediums, there are fewer studies into the vibrating motion of MCs in liquid mediums; however, the conduction of further studies into this medium seems essential due to the increased use of MCs in liquid mediums. In contrast to the vacuum and air mediums, the vibration of MCs in liquid mediums is affected by hydrodynamic forces. These forces not only affect the vibrating motion parameters (amplitude and resonant frequency), but also cause major complication in vibrating motion of MCs [1]. The hydrodynamic forces are generated by fluid density and viscosity. The fluid viscosity not only increases the system damping, but also changes the resonance frequency. Many mathematical models have been developed to model this force [2-8]. Eysden and Sader proposed other equations for resonance frequency variations of MC in liquid medium [9].

Typically, the vibrating motion of MCs is actuated through the base. Since MC stimulation from the base causes turbulence in liquid mediums and affects the MC performance, this method cannot be used in such mediums. The magnetic stimulation [10], acoustic stimulation [11], and piezoelectric stimulation [12-15] are among the commonly used methods in such mediums as they not cause turbulence in liquid mediums. Although the MC mass increases with increasing the number of layers in piezoelectric MCs, these layers can eliminate actuating and sensor elements in AFM. In multilayer MCs, a layer is used as the vibrating motion actuator. This layer is called the actuating layer. It is fabricated from piezoelectric materials, that the using of an alternative voltage to both sides of it causes a vibrating motion in the layer and subsequently in the MC. In addition to the actuating layer, another piezoelectric layer can also be used to measure the MC bending and resonance frequency [16-18]. The use of sensor layer is highly important in liquid mediums. This is because it can solve the laser beam refraction issues when leaving the liquid medium. In addition to the actuating and sensor layers, the multilayer MCs include inactive and passive layers. These layers are used to modify geometric and cantilever properties, such as the stiffness, and to prevent actuating layer deformation [19-20]. Layers with unequal lengths are also used in the piezoelectric MCs [21-22]. Layer length inequality results in geometric discontinuity of the cantilever and generates a unique resonance frequency relative to the single-layer MC [23-24]. Considering the superior frequency properties of multilayer MCs with geometric discontinuity, as compared to the conventional MCs, higher functional capabilities can be obtained. The multilayer piezoelectric sensors with geometric discontinuity can be used in high longitudinal-bending vibration modes, due to their accuracy at femtogram level [25-26].

Considering the use of multilayer piezoelectric MCs in fabrication of MEMS equipment, as well as Atomic Force Microscope (AFM), this article is the first to investigate the vibrating behavior of a semi-submerged multilayer piezoelectric MC in a liquid medium. In contrast to other mentioned research which investigated the vibration motion of full-submerged MC, in this study the dynamic behavior of semi-submerged MC is considered. The two piezoelectric layers with actuating and sensing functions are located at both sides of the base layer. The vibrating analysis was conducted using the Euler–Bernoulli theory, due to the geometric discontinuity of the MC. The beam was considered semi-submerged in the liquid medium, and the effect of hydrodynamic force on beam vibration was considered by the sphere string model. Since the MC resonance frequency has a significant role in the application of MEMS and AFM, this article investigated how the hydrodynamic force affects MC frequency changes.

2 DYNAMIC MODELING OF SEMI-SUBMERGED MC

As seen in "Fig. 1" , the MC was considered with two piezoelectric layers. One layer functioned as the actuator of vibrating motion, which causes the vibration of piezoelectric layer and subsequently that of the MC by applying electric potential difference P(t) to its ends. When the MC starts vibrating, the other piezoelectric layer can be used to measure deformation. Each piezoelectric layer is surrounded by two electrodes.

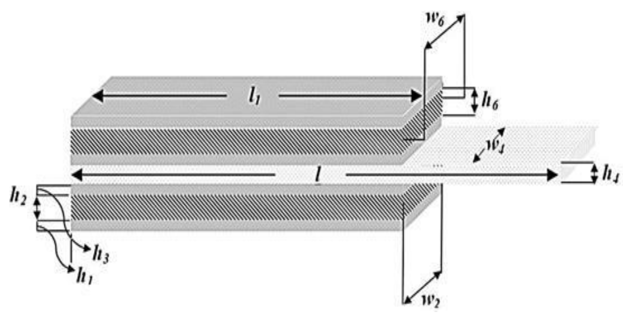


Fig. 1 Schematic of piezoelectric MC.

The Hamilton theory and Lagrange method can be used to derive the differential equation governing the vibrating motion of the MC. The differential equation governing the vibrating motion of MC in an air medium is as follows, based on the Euler–Bernoulli theory [6]:

$$m_0(x)\ddot{u} + [k(x)u'']'' + c_0\dot{u} + c_e''(x)p(t) = f_h(x,t) \qquad (1)$$

Where, t is time and $u(x,t)$ is displacement of each point of the MC. $k(x)$, m_0 and c_0 are the stiffness, mass per unit length, and is the damping per unit length, respectively. The coefficient of the differential equation of motion for a MC with two piezoelectric layers on the both sides of the main layer ("Fig. 1") can be expressed as follows:

$$m_0(x)=\sum_{i=1}^{7}\rho_i h_i w_i (H_0-H_{L_1})+\rho_4 h_4 w_4 (H_{L_1}-H_L) \qquad (2)$$

$$k(x)=\sum_{j=1}^{7}E_j h_j w_j\left\{\frac{h_j^2}{12}\left[\sum_{i=1}^{7}E_i h_i w_i\left(\sum_{j=1}^{7}h_j-\frac{h_i}{2}\right)-\left(\sum_{j=1}^{7}h_j-\frac{h_i}{2}\right)\right]^2\right\} \qquad (3)$$
$$(H_0-H_{L_1})$$

$$C_e=-w_2 d_{31}E_3\left[h_1+h_2+\frac{h_3}{2}-\sum_{i=1}^{4}E_i h_i w_i\left(\sum_{j=1}^{i}h_j-\frac{h_i}{2}\right)\Big/\sum_{i=1}^{4}E_i h_i w_i\right] \qquad (4)$$
$$(H_0-H_{L_1})$$

$$H_{L_i}=H(x-L_i) \qquad (5)$$

Where, E_i and ρ_i are the elasticity module and density of each layer, respectively; d_{31} and H are the piezoelectric constant and Heaviside function, respectively; and $f(x,t)$ is the external force applied to the MC per unit length. $f(x,t)$ for a vibrating MC in a liquid medium is the same as the fluid hydrodynamic force exerted on the MC surface. Typically, the precise calculation of fluid hydrodynamic force on vibrating MC is difficult. Several studies have been conducted on simulation of hydrodynamic force exerting on the beams [2-4]. These studies aimed at developing a simplified model for modeling the hydrodynamic force exerted on beams. Among the proposed models, the sphere string model proved high efficiency in modeling hydrodynamic forces exerted on beams with geometric discontinuity [6], [27]. In this model, the hydrodynamic model is estimated by simulating the MC as a series of spheres. The hydrodynamic force exerted on a submerged sphere in a viscous fluid is as follows:

$$f_h(x,t)=-6\pi\eta R\left(1+\frac{R}{\delta}\right)\frac{du}{dt}-3\pi R^2\sqrt{\frac{2\eta\rho_{liq}}{\omega}}\left(1+\frac{2R}{9\delta}\right)\frac{d^2u}{dt^2} \qquad (6)$$

Where, R is the sphere radius, η is viscosity, ρ_{liq} is fluid density, and u is the displacement of sphere in the fluid; in addition, δ is the penetration depth of acoustic wave, as follows:

$$\delta=\sqrt{\frac{2\eta}{\rho_{liq}\omega}} \qquad (7)$$

Based on the sphere string model, the hydrodynamic force exerted on the MC per unit length is equal to the total force applied to the sphere per unit length. In this way, the hydrodynamic force exerted on the MC can be expressed as follows:

$$f_h(x,t)=\left(\frac{1}{12}\pi\rho_{liq}w^2+\frac{3}{4}\pi b\sqrt{\frac{2\eta\rho_{liq}}{\omega}}\right)\frac{d^2u}{dt^2}-\left(3\pi\eta+\frac{3}{4}\pi b\sqrt{2\rho_{liq}\eta\omega}\right)\frac{du}{dt} \qquad (8)$$

Where, ω is the frequency of vibrating motion of MC. Based on this equation, the added mass and damping caused by the presence of a fluid can be expressed as:

$$m_a=\frac{1}{12}\pi\rho_{liq}b^2+\frac{3}{4}\pi b\sqrt{\frac{2\eta\rho_{liq}}{\omega}}\left(H_{L-L_s}-H_L\right) \qquad (9)$$

$$c_{liq}=3\pi\eta+\frac{3}{4}\pi b\sqrt{2\rho_{liq}\eta\omega} \qquad (10)$$

Where, C_{liq} is the hydrodynamic damping coefficient and and L_s is the extent of MC submergence in a liquid medium. With substituting Equations (8-10) in the differential equation of motion, we have:

$$k\frac{\partial^4 u(x,t)}{\partial x^4}+(m_0+m_a)\frac{\partial^2 u(x,t)}{\partial t^2}+(c+c_{liq})\frac{\partial u(x,t)}{\partial t}+c_e''(x)p(t)=0 \qquad (11)$$

The modal analysis was used to solve the differential equation of motion of semi-submerged piezoelectric MC in a liquid medium. The MC deformation can be presented within the place and time domains as follows, using the Galerkin method:

$$u(x,t)=\sum_{n=1}^{\infty}\Phi_n(x)q_n(t) \qquad (12)$$

Where, $\Phi_n(x)$ is comparison function of nth-order vibrating mode and $q_n(t)$ is the general coordination equation based on time. Since the MC has geometric discontinuity, because of the presence of a piezoelectric layer, and only a part of it was submerged in the liquid medium, it was divided into three homogeneous beams. Then, $\Phi_n(x)$ can be expressed as in "Eq. (13)".

Where, $\tilde{A}_n^{(i)}$, $\tilde{B}_n^{(i)}$, $\tilde{C}_n^{(i)}$, and $\tilde{D}_n^{(i)}$ are unknown coefficients determined by the boundary conditions of MC, continuous conditions, and normalization by mass. By substituting "Eq. (12) in Eq. (1)" , using the Lagrange method, the differential equation of motion turns into "Eq. (14)" :

$$\Phi_n(x)=\begin{cases}\tilde{A}_n^{(1)}\sin\beta_n^{(1)}x+\tilde{B}_n^{(1)}\cos\beta_n^{(1)}x+\tilde{C}_n^{(1)}\sinh\beta_n^{(1)}x+\tilde{D}_n^{(1)}\cosh\beta_n^{(1)}x & 0\le x\le L_1\\ \tilde{A}_n^{(2)}\sin\beta_n^{(2)}x+\tilde{B}_n^{(2)}\cos\beta_n^{(2)}x+\tilde{C}_n^{(2)}\sinh\beta_n^{(2)}x+\tilde{D}_n^{(2)}\cosh\beta_n^{(2)}x & L_1\le x\le L-L_s\\ \tilde{A}_n^{(3)}\sin\beta_n^{(3)}x+\tilde{B}_n^{(3)}\cos\beta_n^{(3)}x+\tilde{C}_n^{(3)}\sinh\beta_n^{(3)}x+\tilde{D}_n^{(3)}\cosh\beta_n^{(3)}x & L-L_s\le x\le L\end{cases} \qquad (13)$$

$$\ddot{q}_n + \omega_n^2 q_n + \sum c_{nm}\dot{q}_n + \gamma_n \mathrm{P}(t) = 0 \tag{14}$$

Where:

$$c_{nm} = \int_0^L (c + c_{liq}) \quad \Phi_n \Phi_m dx \tag{15}$$

$$\omega_n^2 = \int_0^L \Phi_n \frac{d^2}{dx^2}\left(k(x) \frac{d^2\Phi_n}{dx^2} \right) dx \tag{16}$$

$$\gamma_n = -\int_0^L \Phi_n \frac{d^2 c_e}{dx^2} dx \tag{17}$$

The ordinary differential equation of motion "Eq. (14)" for k modes can be written as:

$$M\ddot{q} + C\dot{q} + Kq = F \tag{18}$$

$$q = [q_1(t) \cdot q_2(t) \cdots\cdots q_n(t)]_{k\times k}^T \;\; ; M = I_{k\times k} \tag{19}$$

$$C = [C_{nm}]_{k\times k}; \quad K = [\omega_n^2 \quad \delta_{nm}]_{k\times k} \tag{20}$$

$$F = -P(t) \;\; [\gamma_1 \cdots \gamma_n]_{k\times 1}^T \tag{21}$$

Where, C, M, and K are the mass, damping, and stiffness matrices, and F is the force vector. "Eq. (18)" can be presented in reduced order form as follows:

$$\dot{Y} = AY + B \tag{22}$$

Where:

$$A = \begin{bmatrix} 0 & I \\ M^{-1}K & M^{-1}C \end{bmatrix}_{2K\times 2K} \;\; ; \;\; B = \begin{bmatrix} 0 \\ M^{-1}F \end{bmatrix} \;\; ; \;\; Y = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \tag{23}$$

To calculate q by means of "Eq. (22)" , the Runge–Kutta method was used in MATLAB.

3 SIMULATION AND DISCUSSION

To investigate the vibrating behavior of semi-submerged piezoelectric MC in a liquid medium, a MC was considered with two piezoelectric layers on sides of the main layer ("Fig. 1"). The MC and piezoelectric layers were made of silicon and ZnO. The piezoelectric layers were completely surrounded by two electrodes Ti/Au at both sides. The geometric and mechanical properties of the simulated piezoelectric MC are presented in "Table 1" .

Table 1. Geometric and mechanical specifications of simulated piezoelectric MC [24]

	L (μm)	w (μm)	h (μm)	E (N/m^2)	ρ (kg/m^3)
Piezoelectric Layer	300	130	4	180	6390
Electrode Layers	330	130	0.25	78	19300
Base Layer	375	250	4	104	2330

To simulate the vibrating motion of the semi-submerged MC in a liquid medium, it was vertically located, part of which inside the liquid medium. A piezoelectric layer, as an actuator, was under an alternative potential difference. The other piezoelectric layer was used as sensor. A water-glycerin solution was used with different concentrations to investigate the effect of liquid medium on vibrating behavior of MC. Figure 2 presents the effect of fluid density on resonance frequency drop of MC in the first three modes. The extent of MC submergence in the liquid medium was considered to be 50μm. $\Delta\omega$ indicates changes in resonance frequency of MC in the liquid medium relative to the air medium, and ω is the resonance frequency in an air medium. The hydrodynamic force exertion on the MC increases with increasing the fluid density of the MC. Based on the sphere string model, the hydrodynamic force is comprised of velocity and acceleration terms. As a result, the beam is subjected to greater damping and added mass with increasing fluid density and hydrodynamic force, resulting in a greater resonant frequency drop. The results of "Fig. 2" indicate an increase in frequency variations with increasing fluid density in the first three vibrating modes.

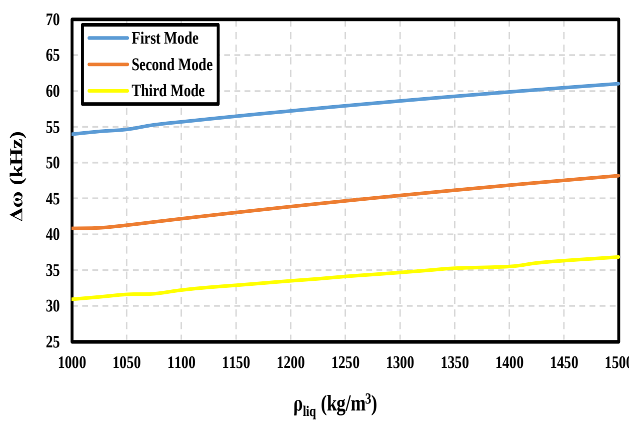


Fig. 2 Effect of liquid density on resonance frequency drop in MC.

In addition to the fluid density, the extent of MC submergence in a liquid medium increases fluid hydrodynamic force and subsequently results in resonance frequency drop of MC. Figure 3 shows the effect of MC submergence in a fluid on frequency drop. According to this figure, the frequency drops increase with increasing the MC submergence in the liquid medium (water); however, the rate of frequency changes reduces with greater increase in submergence.

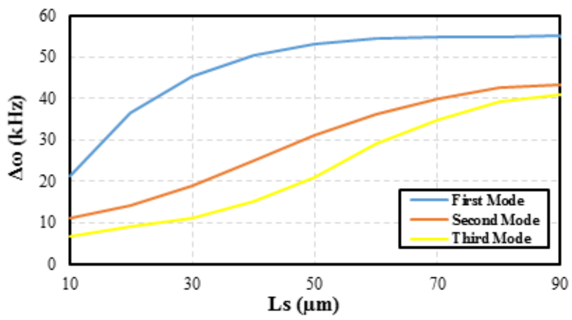


Fig. 3 Effect of the extend of MC submergence in liquid medium.

According to "Figs. 1 and 2" , frequency changes of the third vibrating mode are lower than that of the first and second modes. Low frequency changes indicate lower effect of fluid hydrodynamic force on vibrating motion of MC. Since the precise calculation of this force is difficult, the MC measurement results would be more accurate when the effect of this force on vibrating behavior of MC is lower. Therefore, it can be concluded that the third vibrating mode is more appropriate one considering the semi-submerged MC performance in the fluid, due to the low effect of hydrodynamic force on MC vibration.

3.1. Effect of Geometric Dimensions of MC On Frequency Drop

As mentioned earlier, reduction in hydrodynamic force acting on the MC not only reduces its effect on vibrating motion, but also increases MC readings, which is due to the difficulty of hydrodynamic force calculation. Reduction in hydrodynamic force acting on MC reduces resonance frequency changes while the MC is submerging in the liquid medium. As a result, lower frequency drop improves MC performance. Undoubtedly, the geometric dimension of the MC is a factor affecting hydrodynamic force. The selection of appropriate geometric dimensions for MC is effective in improving its performance in a liquid medium. To investigate the effect of geometric dimensions of MC on its frequency drop, the sensitivity analysis was used. The eFAST-based sensitivity analysis can be used to investigate the effect of some parameters on the main variable at the same time [28]. Figure 4 shows the effect MC layer thickness on frequency changes.

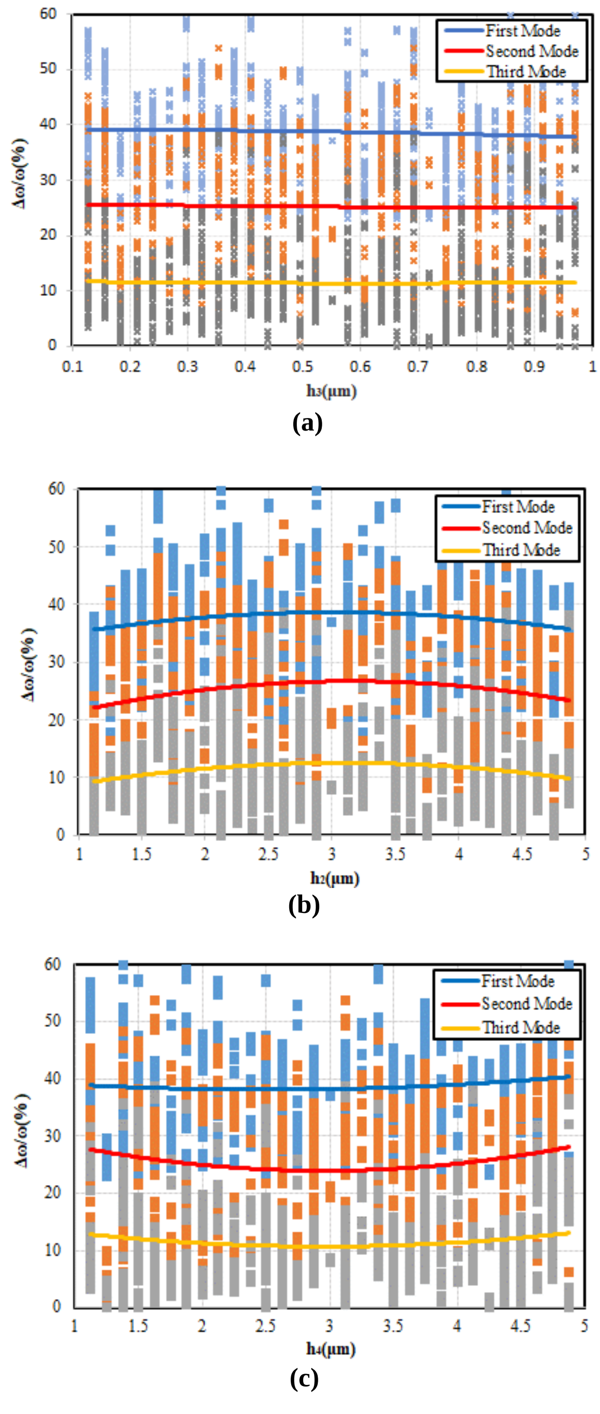
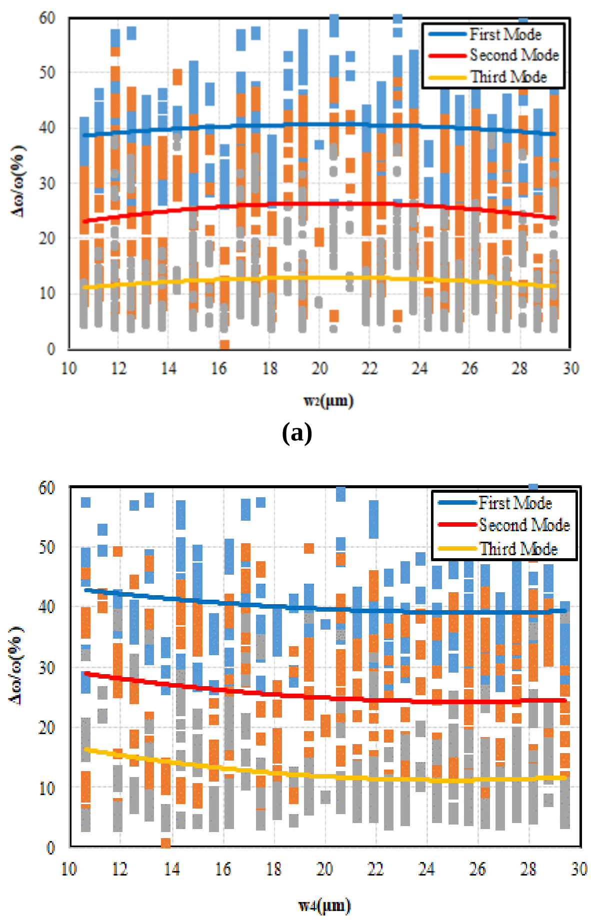


Fig. 4 Effect of layer thickness on frequency sensitivity of: (a): Electrode layer, (b): Piezoelectric layer, and (c): main layer.

The extent of MC submergence in liquid medium (water) was considered to be 50μm. Figure 4a shows the effect of electrode thickness on frequency sensitivity of MC submerged in the fluid. According to this figure, due to the low thickness of these electrodes, they have low effects on frequency sensitivity of the MC.

According to “Fig. 4b” , the frequency sensitivity varies with the thickness of actuating piezoelectric layer. Simulation results show that in the first three vibrating modes, the frequency sensitivity is maximized at the thickness of 3.3µm. According to “Fig. 4c” , the frequency sensitivity in the first three vibrating modes is minimum at the thickness of 3µm. As a result, the effect of liquid medium in MC is minimal at this thickness. Since the aim in designing these MCs is to reduce the effect of hydrodynamic forces on MCs, 3µm is the optimal thickness for the main layer.

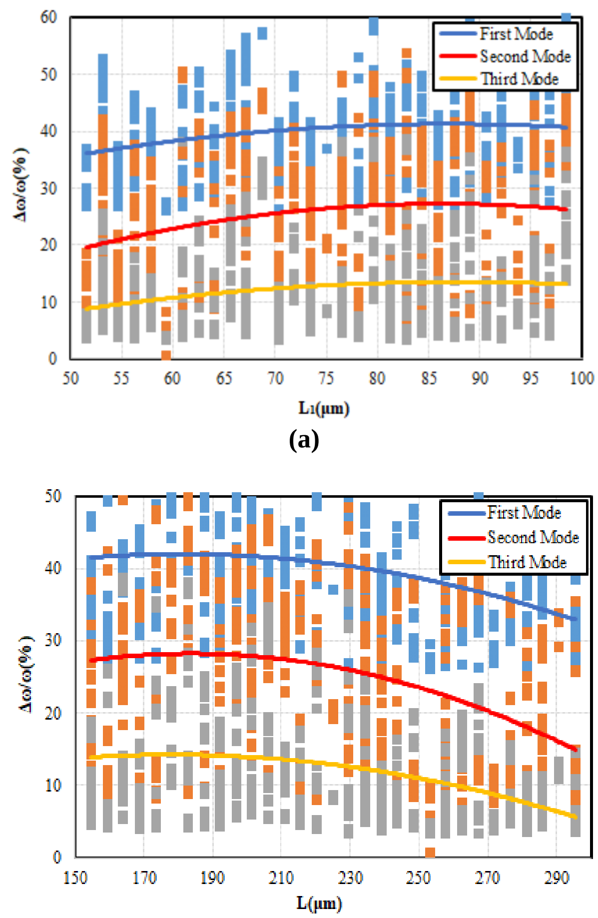
Figure 5 shows the effect of the piezoelectric MC layer width on MC frequency sensitivity in the first three vibrating modes. Figure 5a presents the effect of piezoelectric layer width on frequency sensitivity. According to this figure, the maximum frequency sensitivity occurred at the width of 21.8µm in the first and second modes and at the width of 20.3µm in the third mode. Figure 5b shows that the minimum frequency sensitivity occurred at the main layer width of 26.1µm.



(b)

Fig. 5 Effect of layer width on frequency sensitivity of: (a): piezoelectric layer width, and (b): main layer width.

Figure 6 shows how the lengths of main and piezoelectric layers affect frequency sensitivity in the first three vibrating modes. Since the lengths of electrode and piezoelectric layers are assumed equal, “Fig. 6a” shows the effect of this layer's length on frequency sensitivity of MC. According to this figure, the frequency sensitivity is maximized in the first to the third modes at the lengths of 86.1, 87.2, and 89 µm, respectively. Figure 6b presents the effect of the main layer length of MC on frequency sensitivity. According to this figure, the frequency sensitivity reduces with increasing the length of the main layer. As a result, acting fluid hydrodynamic force reduces and MC performance improves with MC length.



(b)

Fig. 6 Effect of layer length on frequency sensitivity of: (a): piezoelectric layer length, and (b): main layer length.

The degree of each geometric parameter of MC affects the frequency sensitivity has an effective role in the optimal design and selection of MCs. Undoubtedly, the geometric parameters with a greater effect on frequency sensitivity of the MC can be regarded as the main parameters in designing the beams.

Figure 7 shows the effect of each geometric parameter of piezoelectric MC on frequency sensitivity. Results were obtained based on the Sobol's sensitivity analysis and statistical data. According to this figure, the MC length and piezoelectric layer are geometric parameters with the greatest effect on frequency sensitivity of MC, which should be considered in design.

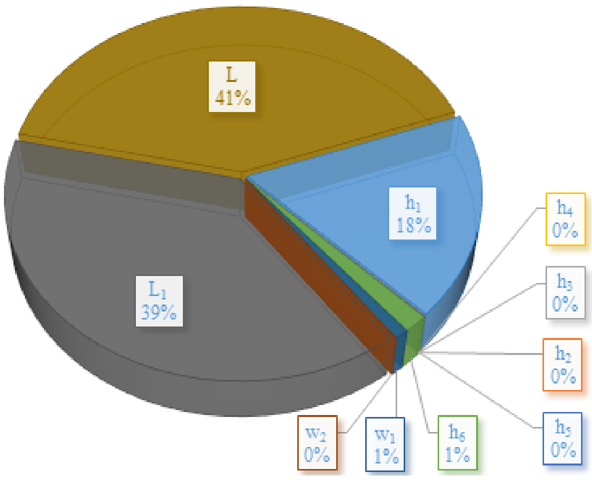


Fig. 7 Effect of geometric dimensions of piezoelectric MC on frequency sensitivity.

4 CONCLUSION

The semi-submerged MC in a liquid medium was investigated with two piezoelectric layers on both sides of it. In this MC, a layer functions as an actuator and the other as a sensor. The hydrodynamic force exerted from the liquid medium on the MC was obtained using the sphere string model. In addition, the differential equation governing the vibrating motion of the MC was derived using the Euler–Bernoulli theory and Lagrange method. To investigate the effect of geometric dimensions of MC on the resonance frequency variation in a semi-submerged state, the Sobol's sensitivity analysis was used. This equation can be used to investigate the effect of all geometric parameters on the frequency drop of MC at the same time. The simulation of vibrating motion of semi-submerged MC with two piezoelectric layers in a liquid medium produced following results:

1- Since resonance frequency variations reduce in the third vibrating mode, the effect of hydrodynamic force on vibrating motion is minimized in this mode and considered as the optimal vibrating mode out of the first three modes.

2- According to this figure, due to the low thickness of these electrodes, they have low effect on frequency sensitivity of the MC.

3- Simulation results show that in the first three vibrating modes, the frequency sensitivity is maximized at piezoelectric layer thickness of 3.3µm. In addition, it is minimized at piezoelectric layer thickness of 3.3µm for the main layer.

4- The frequency sensitivity is maximized at piezoelectric layer lengths of 81.1, 87.2, and 89µm for the first, second, and third modes, respectively.

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