Optimal Routing of Rocket Motion using Genetic Algorithm and Particle Swarm Optimization

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Abstract: In this paper, a new approach to the use of genetic algorithms and the predictive control method, for goal tracking is presented. A hypothetical rocket is modelled for the analyses. Rocket guidance algorithm is developed to achieve a desired mission goal according to some performance criteria and the imposed constraints. Given that goals can be fixed or moving, we have focused and expanded on this issue in this study and also the dynamic modelling of flying objects with six-degrees-of-freedom (DOF) is used to make the design more similar to the actual model. The predictive control method is used to predict the next step of rocket and aim movement. At each step of the problem, the rocket distance to the aim is obtained, and a trajectory is predicted to move the rocket towards the purpose. The objective function of this problem, in addition to the distance from the rocket position to the target, are also parameters of the dynamic model of the rocket. Therefore, these parameters are optimized at each step of the problem solving. Ultimately, the rocket strikes the intended aim by following this optimal path. Finally, for the validation of the model, numerical results are obtained for both Genetic Algorithms (GA) and Particle Swarm Optimization (PSO). Simulation results demonstrate the effectiveness and feasibility of the proposed optimization technique.

Keywords: Genetic Algorithm, Optimal Path Design, Optimization Algorithms, Particle Swarm Algorithm, Predictive Control

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1 INTRODUCTION

Aerospace instruments are one of the most sophisticated systems in terms of design, and construction. It is not possible to design and test aerospace guidance and control systems without a trajectory simulation based on a system model. On the other hand, with the advancement of computer science and increasing processing power, efforts to model different subsystems and topics involved in design have improved the design process. In recent years, many methods for missile path planning have been proposed on the basis of some intelligence algorithms, such as Genetic, robust control, predictive control, Nonlinear Guidance, Optimization. Predictive control [1-6] Optimization [7-13], intelligence algorithms [14-22], Nonlinear Guidance [23-24].

An article entitled "Designing a Guided Interceptor Missile Using Genetic Algorithm" is presented in [25]. The design goals of this paper include reducing crash error, reducing tracking time, and reducing lift-off weight. Evaluation of GA for solving the optimization problem of the aerodynamic form of the missile is presented by investigating single-purpose and multipurpose different objective function. In this paper, the researchers have used objective functions determination to evaluate the objective functions in different flight conditions and the sum of these functions for all conditions. That is, the ultimate goal of the process is to maximize the sum of an aerodynamic coefficient in all flight conditions.

It should be noted, that this article is well-positioned as one of the first steps in optimizing the aerodynamic form of missiles. In [21], research entitled "Route Finding Based on Genetic for Tactical Missiles" studied the application of the genetic algorithms to path design. Increased speed, increased range, and improved flight time were investigated as objectives of the study. In [26], a study was conducted, entitled "Trajectory Optimization Using Genetic Algorithm Simulation". In this paper, the trajectory data used for the optimization process are generated by simulating the motion Equations. The track optimization technique has been tested on a moving supersonic missile in the vertical plane. The results show the success of the GA program in trajectory issues. In [27], a study titled "Route Design, Optimization and Guidance of Launchers in the Final Launch Phase" is presented. This study shows a method in route design, optimization, and guidance. The methods used in the three-degrees-of-freedom simulation are evaluated. Routes designed to analyze the final phase of flight are evaluated and used to develop the guidance program. In [28], using simulated Annealing algorithm and two-DOF simulation, they optimized wing aerodynamics and engine parameters of a missile using analytical relationships in the engine case and the MD code of conduct in the aerodynamics case to increase the missile range. Then, the parameters of the simulated Annealing algorithm are evaluated to achieve the desired performance.

In [29], researchers optimized the aerodynamic levels of the Canard; and tail for a rocket in two unguided and guided modes to achieve maximum range. In [30], the optimization of tail and canards control fins with a specific geometry is performed for a supersonic missile using beam responses for four different two-dimensional performance functions for the target functions. In this study, the authors defined two different configurations (one with a single tail and the other with a tail and canard) for a specific missile assuming a large exponential coefficient of tail and canard surfaces size and deflection angle of these surfaces as design parameters, optimizing maneuverability, missile target range, and missile static stability as performance functions.

In [31], the six-DOF dynamic model controls a balanced missile. In this paper, the uncertainties in the dynamic model and the aerodynamic conditions are approximated using the Monte Carlo method. In [32], a paper entitled "Nonlinear Predictive Control of a Missile with Sustainable Terminal Restriction", the predictive control method was used with the aid of a nonlinear prediction model to bring the controlled missile closer to its operational range. The use of the elliptic terminal constraint has been sustained. In [33], an article is presented on predicting missile control parameters in six degrees of freedom simulation. In this paper, by predicting the positional parameters (x, y, z) as well as the angular rotations (φ, θ, ø) of the missile, the position, and the velocity is estimated every moment. The obtained parameters help the missile to hit the target more accurately. In [34], an SDRE model based on the suboptimal guidance law is presented for a fixed purpose. In this method, all the angular altitude limitations of the missile are guided, and the changes in the missile's flight mode are controlled. The article is presented to optimize the flight range of the missile. In [7], optimization the performance of missiles at supersonic speeds was examined. The genetic algorithm is used in this reference. Baran in [14], in her master thesis used GA algorithm for trajectory optimization of the tactical missile. In this reference, to modify the precision, Conjugate Gradient Method is used for good tuning.

The purpose of this paper is to simulate the flight of a rocket and to determine the optimal direction. First step, Dynamic modelling based on six degrees of freedom is investigated. Next step, at each step of the problem, the distance from the rocket to the target is obtained and a trajectory is predicted to move the rocket towards the target, with predictive control method. Then, the predicted path is optimized using the optimization algorithms used in this study by two genetic meta-heuristic and particle swarm algorithms.

Ultimately, the rocket strikes the intended target by following this optimal path, given the background of research that has mostly involved the final phase of the route as well as the design of the rocket configuration and the comparison of the results of the optimization methods. For this research, we designed and implemented a route for the mid-flight phase of the Ground-to-Air Rocket (GAR) flight that would provide the shortest and fastest route for the rocket to achieve the desired goal. In order to solve the problem of optimal control, the comparative method is used. The configuration of this research is as follows. After the introduction, theoretical topic related to algorithms are presented. In the next part, the Equations of six degrees of freedom in the form of point mass has been expressed. In the next part, the numerical results of the simulation are presented and at the end, the conclusion is presented.

2 THEORETICAL TOPICS

This section presents a brief overview of the missile guidance procedures and the predictive control methods and the algorithms used.

2.1. The Guidance Steps

The guidance of a missile consists of three phases including, launch, intermediate, and final phase. The launch phase aims to increase the speed of the flight vehicle so that it can be aerodynamically stable and controllable in the shortest possible time and away from the launcher. In this phase, the missile is usually not guided, or its guidance algorithm is open loop. For example, ground-to-air missiles of the type of oblique launch are generally not guided during the launch phase (the first few seconds after launch). The middle phase is usually the most extended phase of the flight. The goal of the middle phase is to get the vehicle closer to the target, moving on an optimal path to reduce the energy loss of the device and sometimes keeping it hidden. Figure 1, represents the velocity vector in the earth and body coordinate system.

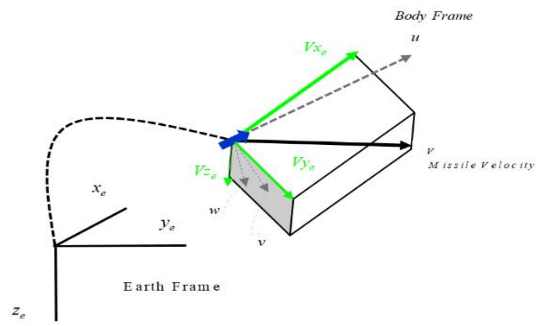

Fig. 1 The velocity vector in the earth and body coordinate system [35].

2.2. Predictive Control

The Predictive control term does not mean the application of a single control method, but rather a wide range of control methods that explicitly uses the process model to achieve the control signal and to minimize the objective function [36].

1- Future outputs for a given horizon of N, called the forecast horizon, are predicted using the process model at any time of t. These predicted outputs $(y(t+k|t))$ for k = 1… N will depend on the specified values up to t (past outputs and inputs) and the future signals $(u(t+k|t)); k = 1, 0, ..., N-1$. These signals are actually the inputs that are to be sent to the system and calculated by the control method. In the above notation, if we are in the moment $t, (y(t+k|t))$ is the predicted value of the y variable at the moment $(t+k)$.

2- The set of future control signals is calculated by modifying a specific criterion to bring the process output closer to $w(t+k)$. $w(t+k)$ can be a reference path or an approximation close to the reference path. This criterion is generally predicted as a quadratic function of the errors between the estimated output signal and the reference path. In most cases, control efforts are defined as a term in the objective function. If the criterion is quadratic, the linearity of the model, and the absence of constraints, an explicit solution will be obtained, but otherwise, an iterative optimization method should be used. In some cases, the structure of the future control law is speculated, for example, this law is likely to become stable after a certain time.

The control signal of $u(t|t)$ s is sent to the process while the subsequent control signals are discarded. At the time of the next sampling, $(y(t+1))$ is known and step 1 is repeated with these new values and the results are updated accordingly.

Therefore, $(t+1|u(t+1))$ is calculated using a creep horizon concept, which originally, due to the existence of new information, it is different from $u((t+1)|t)$. Figure 2 shows the MPC strategy.

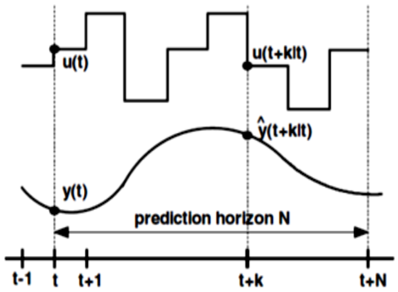

Fig. 2 MPC strategy [36].

2.3. Optimization Algorithms

The algorithms used in this article are heuristic. The approach of these algorithms is that by modeling the genetic evolution, they provide patterns for problem solving. These algorithms provide a robust search method in very large spaces which ultimately leads to an optimized orientation to find the answer.

1- Genetic Algorithm: Genetic algorithm is one of a variety of evolutionary algorithms inspired by biological science such as inheritance, mutation, sudden selection, natural selection and composition. Evolution begins from an early population and is repeated in subsequent generations. The important thing in the function of the genetic algorithm is to choose the most appropriate in each generation, not the best.

2- Particle Swarm Algorithm: The algorithm for mass movement of birds is a hyper-heuristic algorithm. In many cases, this technique operates similar to the genetic algorithms such as evolutionary computational techniques. In this way, the system also starts with a population of a number of initial responses and tries to find the optimal response by moving these responses over consecutive iterations. Unlike genetic algorithms, the algorithm does not have a group motion of evolutionary operator particles such as mutation and recombination [37].

3 EXTRACTIONS OF THE MOTION EQUATIONS WITH SIX-DOF AS THE POINT MASS

In general, the Equations of motion are divided into two categories:

1- Kinematic Equations: These Equations express the geometric relationship between the variables of the motion.

2- Dynamic Equations: These Equations are derived using physical laws, the fundamental physical laws related to flight dynamics, laws of motion, Newton's gravity, and the aerodynamic principles that based on it, forces, aerodynamic and propulsion moments are calculated. First, using Newton's second law, we write the transitional Equations of an aerospace device that is exposed to aerodynamic forces, thrust and gravity [35]. The basic formula for transitional Equations of motion is as follows:

$$F = mV^{\cdot} \tag{1}$$

In which, F includes the sum of external forces (aerodynamics, pressure thrust and gravity). In missile flight simulation, the usual method used to solve the transitional Equations of motion in “Eq. (1)” is to calculate the sum of F forces, based on aerodynamics, propulsion, and gravity data and F replacement is fixed in the Equation of motion to solve the acceleration of the reference V Expressing “Eq. (1)” that results in the system of physical coordinates and solving the acceleration components:

$$\left(\dot{v}_{x_{Body}}\right)_{inertial} = \frac{F_{x_{Body}}}{M}$$
$$\left(\dot{v}_{y_{Body}}\right)_{inertial} = \frac{F_{y_{Body}}}{M}$$
$$\left(\dot{v}_{z_{Body}}\right)_{inertial} = \frac{F_{z_{Body}}}{M} \tag{2}$$

Although “Eq. (2)” expresses the acceleration components in the fixed reference, the acceleration vectors are expressed in the rotational coordinates of the physical reference. At this point, the goal is to calculate the mass center of the missile. The velocity is the integral of the cases in the left-hand side of “Eq. (2)”. However, the fact that the reference coordinate is rotatable must be taken into account in integrating practices. “Eq. (3)” must be applied to find the absolute velocity where the integral is applied to the rotating machine.

$$\left(\frac{dB}{dt}\right)_{inertial} = \left(\frac{dB}{dt}\right)_{rotitional} + (\omega \times B) \tag{3}$$

Substituting V by B in “Eq. (4)” results:

$$\dot{V}_{rotitional} = \dot{V}_{inertial} - (\omega \times B) \tag{4}$$

The ratio of the angles of the rotating machine relative to the inertial reference device is expressed by ω. $\dot{V}_{rotitional}$ acceleration is equal to the ratio of force to mass. If the integral is applied to the rotating machine, $\dot{V}_{rotitional}$ is a vector that must be taken to reach the integral velocity. For integrating “Eq. (4)”, it must be expressed in the selected device which is the body coordinate system in this case. The first case is on the right side of "Eq. (4)", which is expressed in physical coordinates by "Eq. (2)". Vectors V and ω are expressed in the physical coordinates as follows:

$$\omega = pi_{Body} + qj_{Body} + rk_{Body} \tag{5}$$

The following Equation can be used to calculate the angle of attack:

$$\alpha = \arctan\left(\frac{w}{u}\right) \tag{6}$$

And with the external multiplication, we will have:

$$\omega \times V = (qw - rv)i_{Body} + (ru - pw)j_{Body} + (pv - qu)k_{Body} \tag{7}$$

$v_{z_{Body}}$ with w, $v_{x_{Body}}$ with u, and $v_{y_{Body}}$ with v, we will have:

$$\begin{cases} \dot{v}_{x_{Body}} = \frac{F_{x_{Body}}}{M} - (q \times v_{z_{Body}} - r \times v_{y_{Body}}) \\ \dot{v}_{y_{Body}} = \frac{F_{y_{Body}}}{M} - (r \times v_{x_{Body}} - p \times v_{z_{Body}}) \\ \dot{v}_{z_{Body}} = \frac{F_{z_{Body}}}{M} - (p \times v_{y_{Body}} - q \times v_{x_{Body}}) \end{cases} \tag{8}$$

"Eq. (8)" are the transition motion Equations expressed in the body coordinates that rotate. By replacing the discrete forces in "Eq. (8)", the Equations of the final transient motion will be obtained as follows:

$$\begin{cases} \dot{v}_{x_{Body}} = \left(\frac{1}{M}\right)(T + F_{x_{Aero}} + F_{x_{Gravity_{Body}}}) \\ -(q \times v_{z_{Body}} - r \times v_{y_{Body}}) \\ \dot{v}_{y_{Body}} = \left(\frac{1}{M}\right)(F_{y_{Aero}} + F_{y_{Gravity_{Body}}}) \\ -(r \times v_{x_{Body}} - p \times v_{z_{Body}}) \\ \dot{v}_{z_{Body}} = \left(\frac{1}{M}\right)(F_{z_{Aero}} + F_{z_{Gravity_{Body}}}) \\ -(p \times v_{y_{Body}} - q \times v_{x_{Body}}) \end{cases} \tag{9}$$

Where, M is the mass of the device, T is the thrust and v_{Body} is the mass center of the body device. The left-hand side of the above Equations are easily calculated in the body coordinates, through which the components of the acceleration of the device in the body device will be calculated. Figure 3 shows the aerodynamic forces. The image rocket from [38] is used. By integrating the above system of Equations, for the initial conditions of zero, the components of velocity will be obtained in the body device.

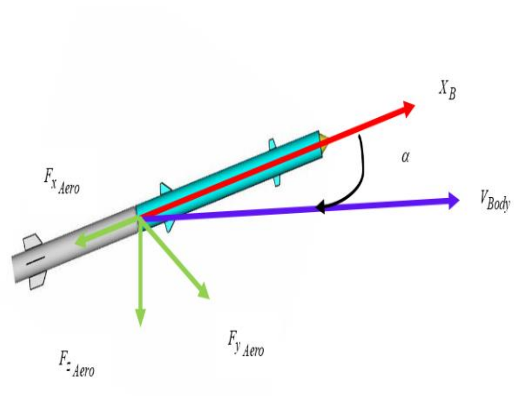

Fig. 3 Aerodynamic forces.

The forces are exerted by a variety of factors such as aerodynamics, propulsion, control system, gravity, fuel particle movement and disturbances caused by atmospheric and internal factors of the aircraft itself. The aerodynamic forces are as follows [39].

$$F_{X_{Aero}} = \frac{1}{2}\rho V^2 S C_X$$
$$F_{Y_{Aero}} = \frac{1}{2}\rho V^2 S C_Y$$
$$F_{Z_{Aero}} = \frac{1}{2}\rho V^2 S C_Z \tag{10}$$

In this respect, S is the characteristic level, ρ is the density, C_X, C_Y and C_Z are the axial, lateral and normal force coefficients, respectively, and V^2 ,is the velocity of the mass center of the body device. The propulsion force of the engine is solid fuel which is calculated using "Eq. (2)".

$$Thrust = \dot{m}V_e + (p_{exit} - p_{ambient})A_e \tag{11}$$

Which in this relation, p_{exit} is the outlet pressure of the combustion gases, $p_{ambient}$ is the atmospheric pressure, A_e is the nozzle span area, and V_e is the velocity of the nozzle exhaust gases. The model used in this simulation is as follows, which has only one component in z direction in the terrestrial system.

$$G = g_0\left(\frac{R_e}{R_e + H}\right)^2 \tag{12}$$

Where, G is the acceleration of gravity at the surface of the earth. Since the Equations of motion are written in the body device, the force of gravity must be transferred from the terrestrial system to the body device using a transformation matrix.

$$F_{Gravity_{Earth}} = \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix} \tag{13}$$

$$\begin{pmatrix} F_{X_{Gravity}} \\ F_{Y_{Gravity}} \\ F_{Z_{Gravity}} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix} \tag{14}$$

$$\begin{cases} F_{X_{Gravity}} = -mg\sin\theta \\ F_{Y_{Gravity}} = 0 \\ F_{Z_{Gravity}} = mg\cos\theta \end{cases} \tag{15}$$

The degrees of the rotational freedom are subject to the Euler law, which states that the time rate of angular momentum variation is equal to the applied external torques. In order to fit the transitional Equations, we consider the body device as an inertial framework [39]. The basis of the Equations of state is as follows:

$$M = \frac{dh}{dt} \quad \text{(N.m)} \tag{16}$$

In this relation, M is the vector of the sum of the external torques and h is the angular momentum vector. On the one hand, the angular momentum vector has the following relation to the angular velocity vector of ω:

$$h = [I]\ \omega \quad \text{(N.m.s)} \tag{17}$$

In this relation, [I] is the inertia moment tensor which is as follows:

$$[I] = \begin{pmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{pmatrix}, \quad (\text{kg.m}^2) \tag{18}$$

The principal diameter components of this relation are called inertia moment, and the other components are called the multiplicative moments of inertia moment, which are considered to be zero when considering the choice of physical coordinates reference. By inserting I_x, I_y, I_z into the original diameter, the inertia moment tensor relation is rewritten as follows:

$$[I] = \begin{pmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{pmatrix}, \quad (\text{kg.m}^2) \tag{19}$$

By deriving from the relation (17), we have:

$$\dot{h} = [I]\ \dot{\omega} + [\dot{I}]\ \omega \tag{20}$$

Due to $[\dot{I}]$ smallness, we can neglect it and also, with respect to the relation (3), we have:

$$\dot{h}_{Inertl} = \dot{h}_{rot} + \omega \times h \tag{21}$$

By placing relation (21) in Equation (16), we obtain "Eq. (22)". Given "Eq. (20)", we have:

$$M = \dot{h}_{rot} + \omega \times h \tag{22}$$

From the external multiplication of the second right side of the "Eq. (21)", considering the following relationship:

$$\dot{h}_{rot} = [I]\ \dot{\omega} \tag{23}$$

Relationships (17) and (5) and Relationship (24) are obtained:

$$\omega \times h = qr(I_z - I_y)i_b + pr(I_x - I_z)j_b + pq(I_y - I_x)k_b \tag{24}$$

By placing the "Eq. (24) and (23)" in relation (21), the angular acceleration components are obtained as the relation (25):

$$\begin{cases} \dot{p} = [L - qr(I_z - I_y)]/I_x & , \text{rad/s}^2 \\ \dot{q} = [M - pr(I_x - I_z)]/I_y & , \text{rad/s}^2 \\ \dot{r} = [N - pq(I_y - I_x)]/I_z & , \text{rad/s}^2 \end{cases} \tag{25}$$

In this relation, $\dot{p}, \dot{q}, \dot{r}$ are the angular acceleration vector elements of $\dot{\omega}$ along the torsion and pitch and Yaw axes, respectively, and L, M, N are the elements of the total torque of M, along the torsion and pitch and Yaw axes, respectively. The elements of the total torque vector of M can be divided into two parts; aerodynamic torque and propulsion torque, so the relation (25) can be rewritten as follows:

$$\begin{cases} \dot{p} = [L_A + L_P - qr(I_z - I_y)]/I_x & , \text{rad/s}^2 \\ \dot{q} = [M_A + M_P - pr(I_x - I_z)]/I_y & , \text{rad/s}^2 \\ \dot{r} = [N_A + N_P - pq(I_y - I_x)]/I_z & , \text{rad/s}^2 \end{cases} \tag{26}$$

The aerodynamic torque elements of N_A, M_A, L_A are calculated using the following Equation. In these relationships C_l is the aerodynamic roller coefficient of torque, Cm is the aerodynamic pitch torque coefficient, Cn is the aerodynamic Yaw torque coefficient, S is the aerodynamic reference area, Vm is the missile mass center velocity vector size and ρ is the air density.

$$L_A = \frac{1}{2}\rho V_M^2 C_l S d \quad ,\text{N.m} \tag{27}$$

$$M_A = \frac{1}{2}\rho V_M^2 C_m S d \quad ,\text{N.m} \tag{28}$$

$$N_A = \frac{1}{2}\rho V_M^2 C_n S d \quad ,\text{N.m} \tag{29}$$

In most cases, the multiplication moment for the propulsion system is zero. To simulate missiles, a torque is generated by a thrust force whose elements are obtained using the following Equation.

$$\begin{cases} L_P = 0 \\ M_P = F_{P_{z_b}} l_P \\ N_P = -F_{P_{y_b}} l_P \end{cases} \quad , \text{N.m} \tag{30}$$

In this relation, $F_{P_{y_b}}$ is the y-axis element in F_P thrust force vector in the body coordinate device and $F_{P_{z_b}}$ is the z-axis element in the F_P thrust force vector in the body coordinate device. l_p is also the distance from the center of mass to the tip of the missile. Missile theory requires a number of simulation functions including calculation of attack angle, search crown angles, sight angles and cap spray pattern, which is obtained by integrating the set of Euler angle rate relationships.

$$\begin{cases} \dot{\phi} = p + (q\sin\phi + r\cos\phi)\tan\theta \\ \dot{\theta} = q\cos\phi - r\sin\phi \\ \dot{\psi} = (q\sin\phi + r\cos\phi)/\cos\theta \end{cases} \quad , \text{rad/s} \tag{31}$$

In this relation, $\dot{\phi},\dot{\theta},\dot{\psi}$ are the change rate of the Euler angle on the roller, the pitch and the Yaw and ϕ,θ,ψ are the Euler angles on the roller, the pitch and the Yaw, respectively. Quaternions are vectors in four-dimensional space. According to the Euler rotation theorem, four parameters can be specified for an arbitrary rigid rotation. The three components of the n rotation axis together with the specific rotation angle ε, constitute these four parameters.

$$Q = q_0 + q_1 a_x + q_2 a_y + q_3 a_z \tag{32}$$

$$[n] = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \tag{33}$$

The rotation quaternion has four coordinates including q_0, q_1, q_2, q_3 that are directly related to the Euler, ε and n parameters. These quaternions are obtained using the following relationships.

$$q_0 = \cos\left(\frac{\varepsilon}{2}\right) \tag{34}$$

$$q_1 = \sin\left(\frac{\varepsilon}{2}\right).a_x \;;\; q_2 = \sin\left(\frac{\varepsilon}{2}\right).a_y \;;\; q_3 = \sin\left(\frac{\varepsilon}{2}\right).a_z \tag{35}$$

Using quaternions, the rotation matrix can be computed as follows:

$$R = \begin{pmatrix} 1-2q_2^2-2q_3^2 & 2q_1q_2-2q_0q_3 & 2q_1q_3+2q_0q_2 \\ 2q_1q_2+2q_0q_3 & 1-2q_1^2-2q_3^2 & 2q_2q_3-2q_0q_1 \\ 2q_1q_3-2q_0q_2 & 2q_2q_3+2q_0q_1 & 1-2q_1^2-2q_2^2 \end{pmatrix} \tag{36}$$

Using the rotation matrix, we can determine the vectors determining the roller, the pitch and the side axes of the missile in its current direction as follows:

$$\begin{cases} Y_A = RY_{A,0}^T \\ P_A = RP_{A,0}^T \\ R_A = RP_{A,0}^T \end{cases} \tag{37}$$

In this relation, $Y_{A,0}, P_{A,0}, R_{A,0}$ are the reference axes of the roller, the pitch and the Yaw of the missile, respectively. We also obtain the angular velocity vector ω using the rotation matrix R as follows:

$$\omega = R[I]^{-1} R^T M^T \tag{38}$$

Using quaternions, Euler angles can be calculated as follows:

$$\begin{cases} \psi = \tan^{-1}\left(\dfrac{2q_1q_2+2q_0q_3}{1-2q_2^2-2q_3^2}\right) \\ \phi = \tan^{-1}\left(\dfrac{2q_2q_3+2q_0q_1}{1-2q_1^2-2q_2^2}\right) \\ \theta = \sin^{-1}(-2q_1q_3+2q_0q_2) \end{cases} \tag{39}$$

4 NUMERICAL RESULTE

The MATLAB software was used for coding and simulation to reach the target of the proposed model in previous section. The purpose function is to determine the optimal trajectory of the missile's motion in order to destroy the target. Some fixed parameters in the problem such as initial reference of the Yaw, the roller and the pitch axes, the ground mass, global gravity constant and gravitational acceleration are given to the program by the user. Design variables such as linear and rotational speed and acceleration are optimized by the program. The objective function is determined by the location of the missile and the target, as well as by the prediction made by the calculation optimization algorithms and the optimal trajectory of the missile's motion. The problem is thus solved as a predictive control. This problem is solved as a fixed point and moving object, using genetic algorithms and particle swarm.

4.1. Input Parameters

"Table 1" contains the fixed parameters for input to the program.

Table 1 Fixed parameters used in the dynamic model

Explanation	Value	symbol
Reference temperature	$291.15\ K$	Θ_0
Reference dynamic density	$1.827e-7\ Pa.s$	μ_0
Earth Mass	$5.974e24\ kg$	M_E
Earth radius	$6378100\ m$	r_E
Universal gravity constant	$6.673e-11\ m^3 / kg.s$	G
Reference axes of the roll	$[1,\ 0,\ 0]$	$Y_{A,0}$
Reference axes of the pitch	$[0,\ 1,\ 0]$	$P_{A,0}$
Reference axes of the Yaw	$[0,\ 0,\ 1]$	$R_{A,0}$

4.1.1. Genetic Algorithm Parameters

Table 2 shows the parameters of the genetic optimization algorithm used in the program.

Table 2 Genetic algorithm parameters

Value	Explanation
40	Generations
20	Population Size
0.1	Mutation
0.8	Fraction Crossover

4.1.2. Genetic Algorithm Parameters

Table 2 contains the parameters of the particle swarm optimization algorithm presented in the program.

Table 3 Particle swarm algorithm parameters

Value	Explanation
50	max iteration
20	number of particles
0.9	inertia weight
2	individual co speed
2	social co

4.2. Simulation Results

The problem model is designed using MATLAB software. The results has been presented in two form. In the first case, the target is considered as a fixed point. To solve the problem in this case, a genetic optimization algorithm has been used. In the second part, the goal is assumed to be a moving point and the problem for this part is solved in two different scenarios using a genetic algorithm and the results are presented in the article. In these scenarios, by changing the location of the target, the program is able to detect this change and by tracking of target, draws the optimal path of the rocket to the desired point. Finally, the problem-solving steps are repeated using the particle swarm optimization algorithm and they are compared with the results obtained from the genetic algorithm.

Part One: Fixed Target (GA) Method

In this section, the results based on fixed aim is presented. To optimize the objective function, the genetic algorithm with the parameters given at the beginning of the section is used from "Table 1". The following results are obtained by running the software. Figure 4 shows the trajectory of rocket movement along the Y axis. As shown in the diagram, after 40 s, the rocket hit the target.

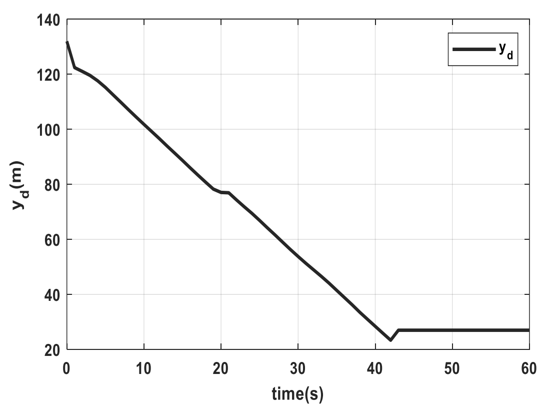

Fig. 4 Predicted trajectory diagram of the rocket moving along the Y-coordinate axis the fixed target.

Figure 5 shows the path of the rocket in the direction of the x-coordinate axis and Figure 6 shows trajectory diagram of the missile in the X-Y coordinate system. In Figure 7, rocket trajectory locomotion has been presented.

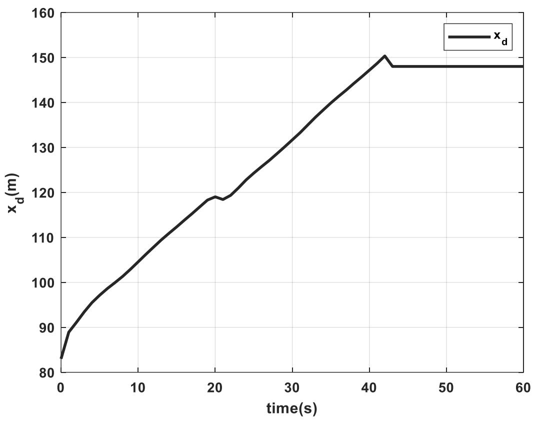

Fig. 5 Predicted trajectory diagram of the rocket moving along the X- Coordinate axis of a fixed target.

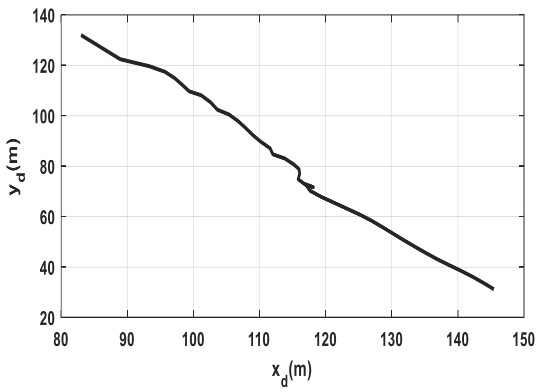

Fig. 6 Predicted trajectory diagram of the rocket moving along the X-Y Coordinate axis of a fixed target.

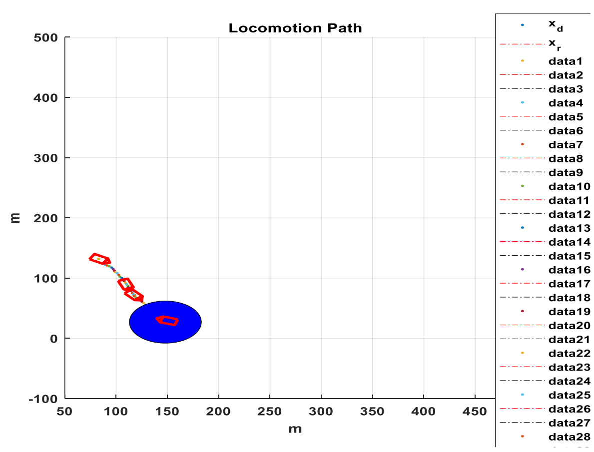

Fig. 7 Designed track diagram of the rocket.

Figure 8 and 9 show Diagram of linear velocity and angular velocity variations of the rocket mass center over time for a fixed purpose, respectively.

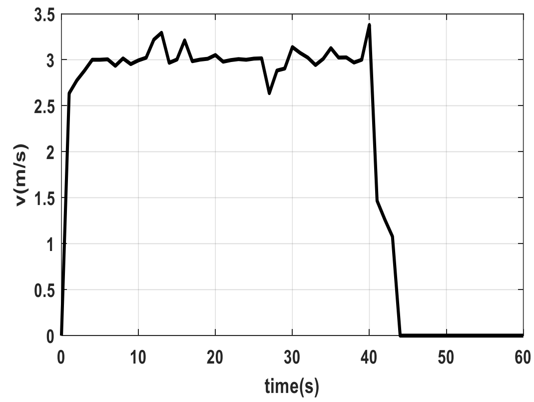

Fig. 8 Diagram of linear velocity of a fixed target.

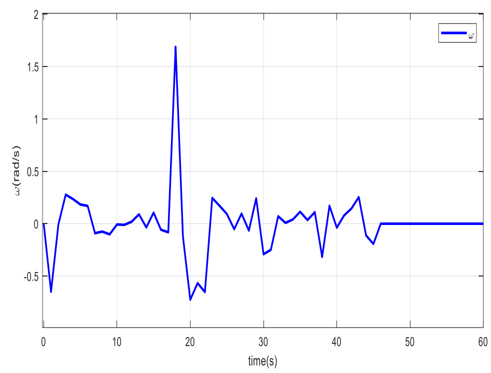

Fig. 9 Diagram of angular velocity of a fixed target.

The results for this section showed that the rocket hit the target by following a smooth and straight path, which is the shortest path to the target. The diagrams presented show that with the least changes in the control parameters of the rocket, including the angles θ and φ, see "Figs. 10 and 11" as well as the linear velocity and Angular velocity of the center of mass of the rocket, we have achieved the desired target.

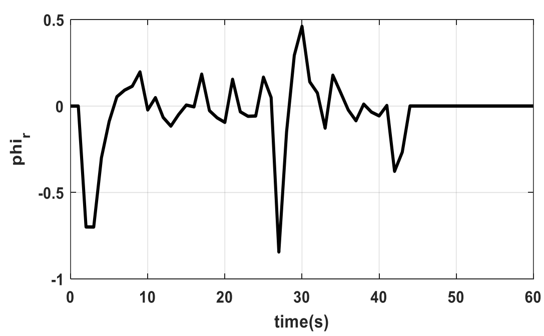

Fig. 10 Diagram of angle changes φ of a fixed target.

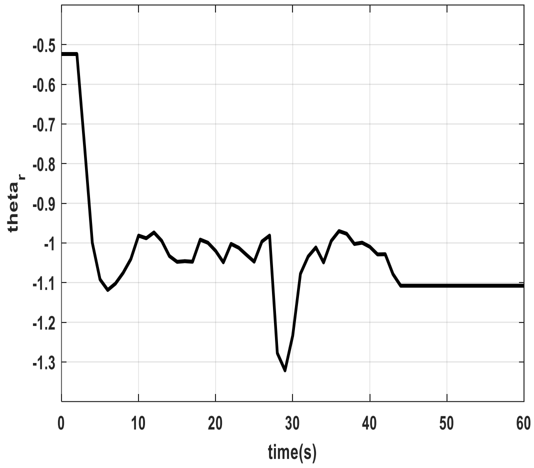

Fig. 11 Diagram of angle changes θ of a fixed target.

Part Two: Variable Target

In the continuation of the article process, the target is considered as moving. The target point of the program was executed for 2 movement scenarios. The results obtained from software implementation for scenarios 1 and 2 using GA are as follows.

A) First Motion scenario: In this scenario, the target moves in a straight line

Output form, the moving target in a straight line and the GA method are presented. In "Fig. 12", Target location variations has been presented. Also, in "Figs. 13 and 14" Predicted trajectory diagram of the rocket in the y coordinate system and x coordinate system has been shown, respectively.

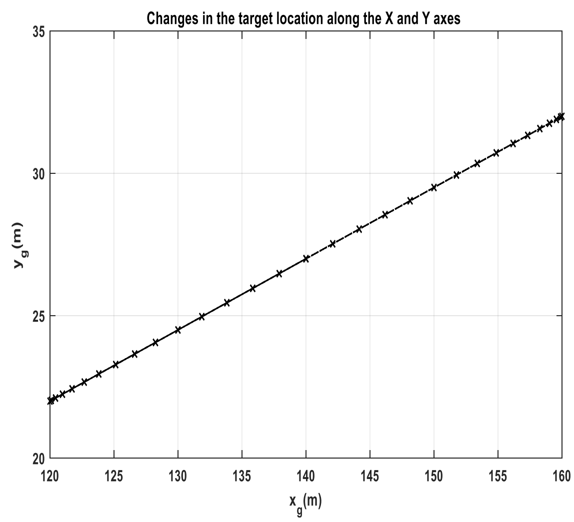

Fig. 12 Target location variations along the x and y axis scenario 1.

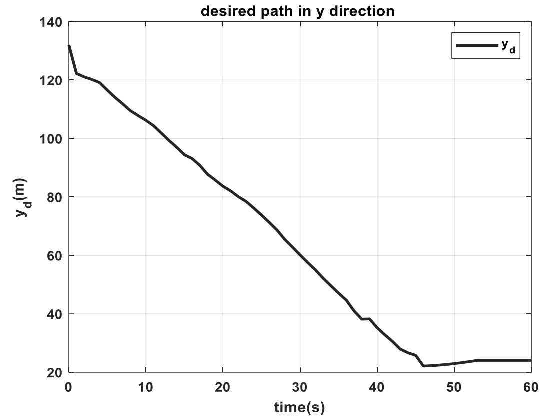

Fig. 13 Predicted trajectory diagram of the rocket in the y coordinate system.

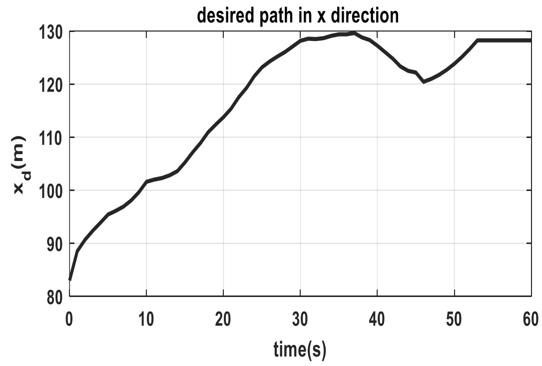

Fig. 14 Predicted trajectory diagram of the rocket in the x coordinate system.

The results obtained for Variable target test segments are shown. As you can see, Changes in angular velocity and linear velocity are shown in "Figs. 15 and 16", respectively. Figure 17 shows a plot of the simulated angle changes θ and φ for scenario 1.

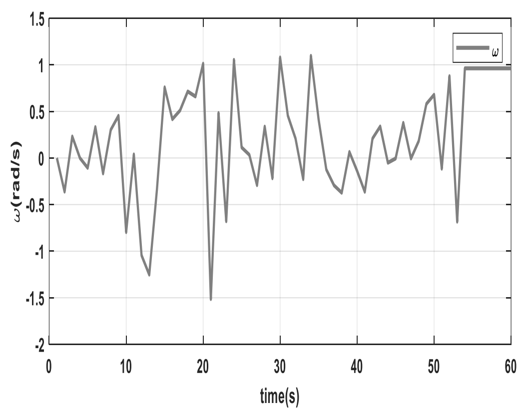

Fig. 15 Diagram of the angular velocity scenario 1.

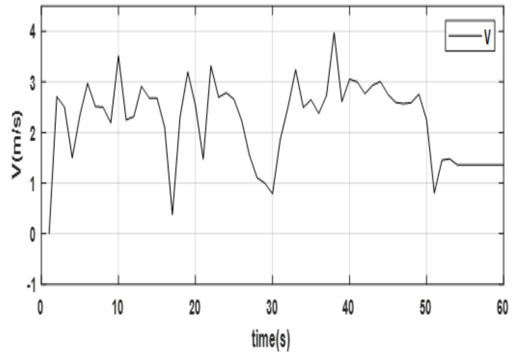

Fig. 16 Diagram of the linear velocity variations of the mass center of the rocket.

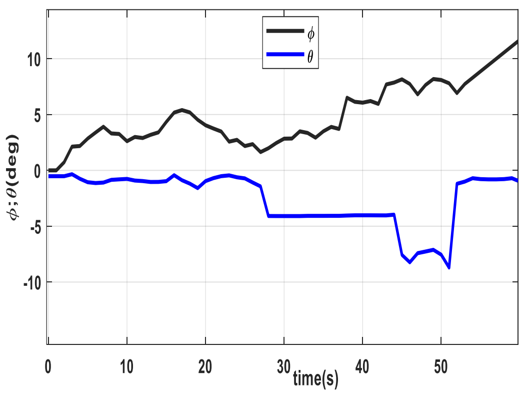

Fig. 17 Diagram of angle changes θ and φ for scenario 1.

B) Second Motion Scenario: In this scenario, the target moves in the zigzag

Target location variations along the x and y axis are shown in “Fig. 18”. It has predicted trajectory diagram of the rocket in the x and y coordinate system shown in “Figs. 19-20”. The zigzag path is complex due to variable maneuvering, but it is well illustrated in “Fig. 20” that it hit the target after 50 seconds.

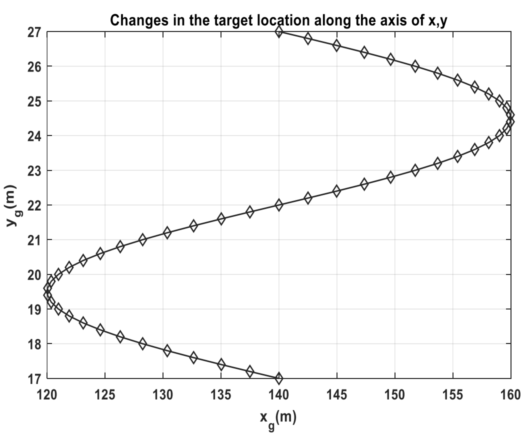

Fig. 18 Target location variations along the x and y axis: zigzag scenario.

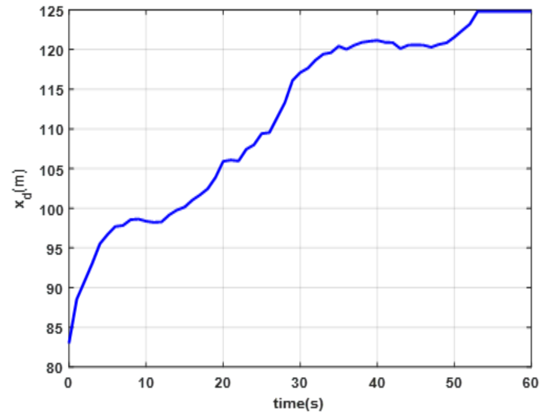

Fig. 19 Predicted trajectory diagram of the rocket in the x coordinate system.

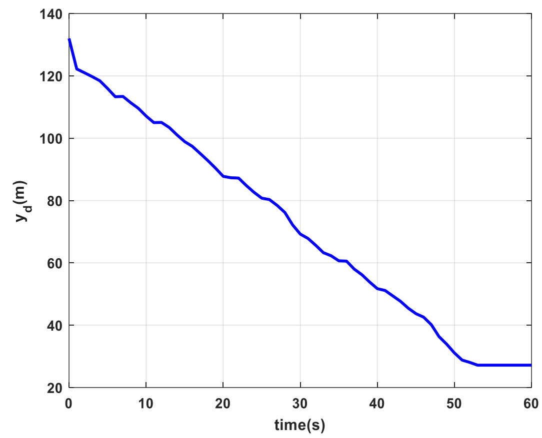

Fig. 20 Predicted trajectory diagram of the rocket in the Y coordinate system.

Figures 21 and 22, show the variations of the problem control variables (angular and linear velocity of the rocket mass center) for this scenario, also “Fig. 23” shows control parameters including the angles θ and φ.

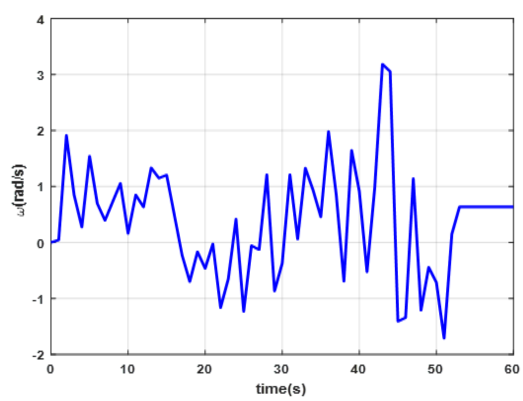

Fig. 21 Diagram of the angular velocity.

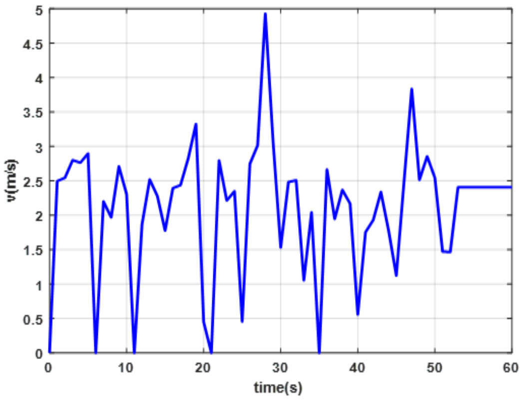

Fig. 22 Diagram of the linear velocity.

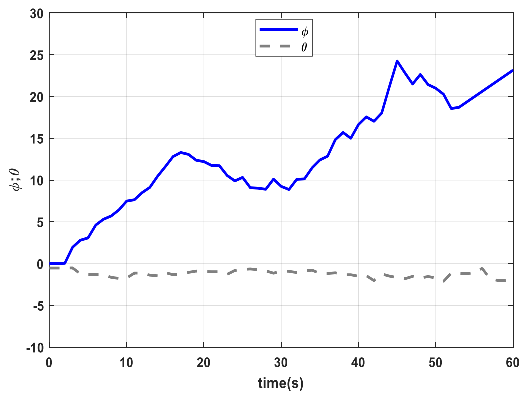

Fig. 23 Diagram of angle changes θ and φ.

C) Third part: comparison method

In this section, a functional comparison is performed in the target that moves in a straight-line scenario with the particle algorithm.

As shown in the graphs obtained from the implementation of the particle swarm optimization algorithm for the described moving target, this algorithm has an acceptable performance. Figure 24 shows a comparison between the results of the two GA and PSO algorithms in the moving target state. In this figure, the optimal paths predicted for both algorithms are similar with slight variation. The comparison between the results of the two GA and PSO algorithms of the moving target, along the x-coordinate axis is shown in "Fig. 25". Figures 26 and 27 also show the variations of the problem control variables (linear and angular velocity of the rocket mass center) for the two algorithms. It can be seen that these variations are more evident for the PSO algorithm. The results obtained for Different tests, PSO and GA algorithm, for control parameters including the angles φ and θ are shown in "Figs. 28 and 29", respectively. Therefore, it can be concluded that with a little approximation, the GA optimization algorithm yields better results than the PSO algorithm. But on the other hand, since the computational cost of the GA algorithm is higher than PSO, evolutionary particle swarm algorithm can be a good alternative to the genetic algorithm in optimizing the objective function of the problem.

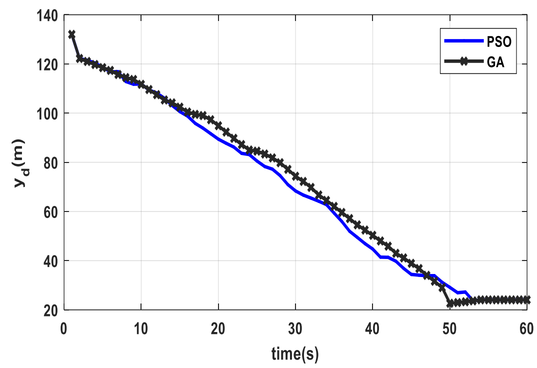

Fig. 24 Diagram of the predicted trajectory of the rocket movement for PSO and GA algorithms along the y-coordinate axis.

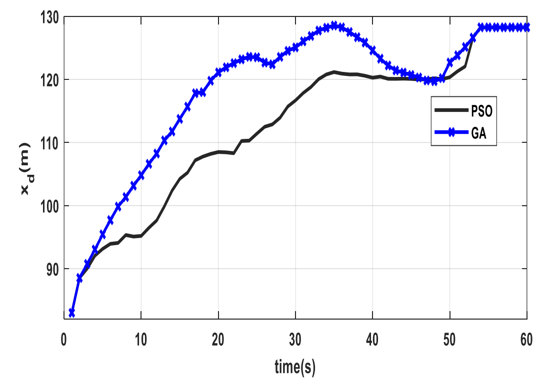

Fig.25 Diagram of the predicted trajectory of the rocket movement for PSO and GA algorithms along the x-coordinate axis.

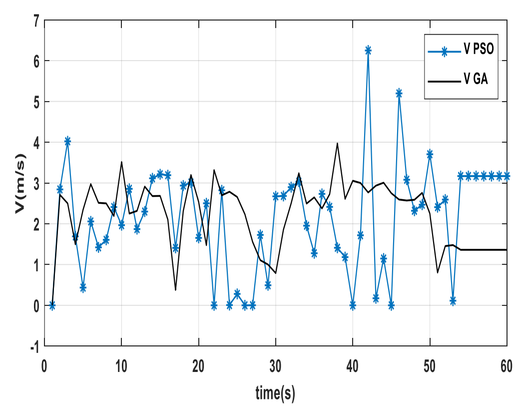

Fig. 26 Diagram of linear velocity variations of the rocket mass center in PSO and GA algorithms.

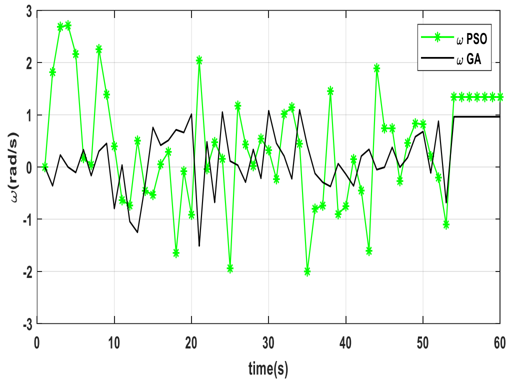

Fig. 27 Diagram of the angular velocity variations of the rocket mass center in PSO and GA algorithms.

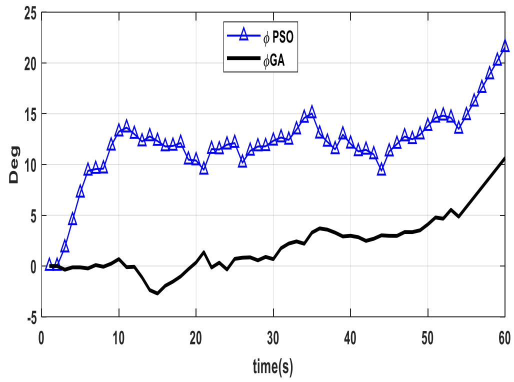

Fig. 28 Diagram of angle changes φ for PSO and GA algorithms.

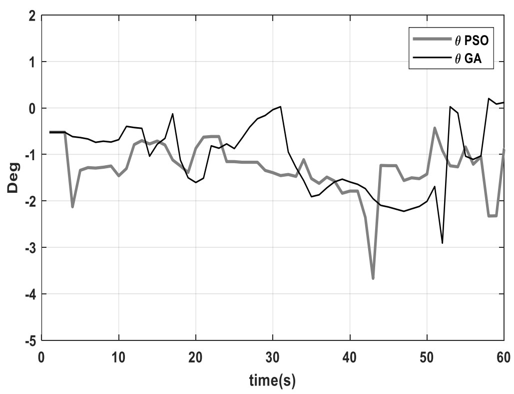

Fig. 29 Diagram of angle changes θ for PSO and GA algorithms.

5 CONCLUSIONS

The objective function of the optimal trajectory problem of the rocket was determined to destroy the target. In this provided software (MATLAB), some fixed parameters in the problem such as initial reference of the axes of the side, the roller and the pitch, groundmass, global gravity constant, and gravitational acceleration are given to the software by the user. Design variables such as linear and rotational speed and acceleration are optimized by the program. The objective function is determined according to the location of the rocket and the target, as well as the prediction made by the optimization algorithms and the optimal trajectory of the rocket motion.

The problem is thus solved as a predictive control. The problem was solved for several different states, and the desired results were obtained. The first case study was performed for the target as a fixed point and with a genetic optimization algorithm. The obtained results for this section showed that the rocket hit the target by running a straight and direct path that is the shortest path to the target.

The presented diagrams show that we achieved the desired target with the least changes in the rocket control parameters including the angles θ and φ as well as the linear and angular velocities of the mass center of the rocket. Then, considering the target point, as a moving object, the program was performed for the moving target. In this section, the desired results were obtained using both genetic optimization and particle swarm algorithms.

Due to the target's motion, the rocket also traversed the non-uniform path to reach the target. It should be noted that the fluctuations in the linear and angular velocity variations of the rocket as well as the angles θ and φ were higher. By comparing the results of both algorithms, we can conclude that both algorithms are efficient. However, due to the computational cost of the genetic algorithm, the particle swarm algorithm can be substituted.

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