Implementation of the Quasi-Brittle Damage Model for 2024 Aluminum Alloy under Periodic Loading

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Abstract: Damage mechanics is one of the most important parts of mechanical engineering that determines the time life for different mechanical elements. The most various models that have been provided so far in damage mechanics, are related to ductile or brittle damage. Nowadays, the investigation of materials by ductile-brittle damage behavior has been considered by researchers. Kintzel quasi-brittle damage model is one of the best damage models in this field. Therefore, in this paper, due to the application of 2024 Al alloy in different industries especially aerospace and the ductile-brittle damage behavior of this alloy, the implementation of the Kintzel quasi-brittle damage model is presented. For this purpose, by writing an explicit user subroutine VUMAT in finite element software (ABAQUS), a test sample under periodic loading has been modeled. The results of this research showed that the complete failure occurs after the 12th cycle under a periodic loading. Also, 2024 Al alloy showed a good ultimate tensile strength (about 400 MPa) under periodic loading. The magnitude of ductile and brittle damage variables are 0.23 and 0.38, respectively.

Keywords: 2024 Al Alloy, Damage Mechanism, Kintzel Damage Model, Periodic Loading

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1 INTRODUCTION

Damage mechanics is one of the branches of mechanical engineering that predict and model the occurrence of cracks in the materials or predict failure during industrial processes. Damage mechanics researches can be classified into four general categories of ductile, brittle, creep, and fatigue damages. Ductile damage occurs when the failure is along with the significant plastic deformation. The damage phenomenon can be called brittle, whenever it occurs on a mesoscale, no significant plastic strain occurs. At high temperatures (above one-third of the melting temperature), which is important for the creep phenomenon, the material may be deformed under constant stress.

In addition, in this case, if the strain is large enough, separation occurs within the grain that could lead to creep damage. When the material is affected by repetitive loading, the damage develops with the repeating plastic strain which leads to fatigue damage. Some materials show ductile and brittle damage behavior at the same time in different directions. These materials can be called ductile-brittle. These materials will first show plastic strain under different loading conditions. The micro-porosity is created in the piece and they are joined together to make micro-cracks, and afterwards, due to the growth of the micro-cracks and voids, the brittle damage will be activated.

Ultimately, the overall damage will be caused as quasi-brittle damage or ductile-brittle damage. One of these materials is the 2024 aluminum alloy, which is largely used in the aerospace industry. Figure 1 shows the fracture cross sections of the 2024 Al alloy in various directions. As in the “Fig.1” is shown, the samples along the longitudinal and transversal direction have a 45 degree break cross section (ductile damage), while the prepared sample from small transversal direction had 90 degrees break cross section (brittle damage). Therefore, it can be said that the 2024 Al alloy will show ductile-brittle damage behaviour.

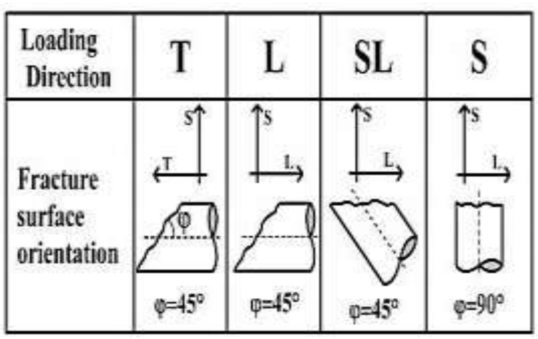

Fig. 1 2024 Al alloy fracture cross-section along different direction [16].

At first, the damage patterns such as Chabanet activities were defined by the growth of voids in the volumetric elements of the material [1]. Then, the basic principles of damage mechanics were established by Kachanov [2]. In the following, the finite element models such as Verhoosel et al were considered [3]. Lemaitre is among the people who have done a lot of work in the field of damage mechanics. Lemaitre damage model is the most famous ductile damage model. In this model, the damage is associated with a high plastic strain. In the Lemaitre damage model, a thermomechanical variable, which indicates load tolerance reduction in an irreversible process, is introduced as the damage variable [4-6].

In addition to the Lemaitre who used the equivalent strain principle to present a model for ductile damage, another model for the ductile damage was presented based on the equivalent energy. Gurson could be mentioned among other people who worked in damage mechanics [7]. The plastic flow analysis is carried out in the Gurson model in a porous medium assuming that the behaviour of the material is continuous. In this model, voids are presented indirectly and only affect the overall behaviour of the material. This effect is averaged in the material and its effect on the yield condition of the material is considered. The initial formulation of this model was presented by Gurson and was then modified by Tvergaard and Needleman [8].

Rice and Tracey [9] defined material failure in cases where the damage variable grows and reaches a certain threshold. However, over the past three decades, the study of materials with a dual ductile and brittle behaviour has been highly considered. In this regard, Quan et al. [10] first studied the microstructure of the 2024 Al alloy. Steglich et al. [11] investigated the behaviour of the 2024 aluminium alloy under uniform vertical loading. Vyshnevskyy et al. [12-13] used the continuum damage mechanics Lemaitre model for predicting of cyclic lifetime 2024 Al alloy. Finally, Kintzel examined it to provide a model for quasi-brittle materials and succeeded in providing a homogeneous elastoplastic damage model based on minimizing stress intensity integral [14-16].

Berto and Lazzarin [17] reviewed some local applicable approaches near the stress raisers in the notched components. Afterwards, they developed a new approach based on the volume strain energy density (SED), which has been applied to assess the brittle failure of a large number of materials. Ren et al. [18] proposed a rate-dependent model for the simulation of quasi-brittle materials. They developed an explicit integration algorithm to implement the proposed model in the structural simulation. Finally, the results were validated by a series of numerical tests that cover a wide variety of stress conditions and loading rates.

Wang and Waisman [19] proposed a coupled continuous/discontinuous approach to model two failure phases of quasi-brittle materials in a coherent way. Their approach involved an integral-type nonlocal continuum damage model coupled with an extrinsic discrete interface model. Riccardi et al. [20] obtained a numerical simulation of two-dimensional fracture processes of quasi-brittle materials by means of the Embedded Finite Element Method. They presented a modified crack-tracking algorithm, considering the evolution of the root for the identification of the crack path. Pereira et al. [21] presented a numerical study towards the propagation and branching of the cracks in the quasi-brittle materials, using a new effective rate-dependent damage model.
As it can be seen in the literature review, most of past researches in damage mechanics are based on single damage mechanisms like ductile or the brittle one. In new researches for ductile-brittle material like 2024 Al alloy, fewer numerical studies based on FEM by writing a user subroutine to show the ductile-brittle behaviour of this material under periodic loading have been conducted. Therefore, in this paper, the Kintzel quasi-brittle damage model was implemented by writing an explicit user subroutine VUMAT. After that, the behaviour of 2024 Al alloy round bar was obtained under periodic loading in ABAQUS software. Finally, by computing ductile and brittle damage variables, the effect of ductile part and brittle part on total damage of material was investigated.

2 CONTINUUM DAMAGE MECHANICS

Damage mechanics is suitable in the fields of modeling and material damage expression in order to predict the onset or growth of material degradation, whereas it is very complicated for practical engineering analysis. Damage mechanics is broadly related to continuum mechanics. In most researches in damage mechanics, a series of state variables have been used to study the influence of damage resulting from the thermomechanical loading on the engineering component lifetime. From the physical point of view, the damage can be defined in terms of the reduction of the effective cross-sectional area due to the cracks and voids in a volumetric element of the material ("Eq. (1)").

$$D = \frac{A_D}{A} \;\; ,0 < D < 1 \tag{1}$$

Where, A_D represents the cross-section of the cracks and voids and A is the total cross-section. According to this definition, the quantity variable of damage is between zero and one, where $D=0$ indicates no-damage state and $D=1$ indicates a breakdown of the cross section. In experimental works, critical damage variable D_{cr} will be less than one.

2.1. Kintzel Quasi-brittle Model

In this section, the Kintzel quasi-brittle model will be briefly described along with the governing relationships [15]. At first, the ductile and brittle damage models are presented separately and then the composite damage model is expressed. For ductile damage, a model that is essentially similar to the model proposed by Lemaitre is used. In this model, the damage growth is based on the amount of energy released by Y in accordance with "Eq. (2)" [15].

$$Y = \left(\frac{\varepsilon^e : C : \varepsilon^e}{2} + H_k \frac{\alpha_k : \alpha_k}{2} + H_i \frac{\alpha_i^2}{2} \right) \tag{2}$$

In this equation, Y is a scalar as the amount of released energy. Also, α_k and α_i are strain internal variables, which are strain due to the kinematic hardening and the isotropic hardening, respectively. H_k is the kinematic hardening modules and H_i is the isotropic hardening modules. C is an elastic stiffness tensor, which is based on the lame's constants and (:) is expressed double product multiply. D is the damage variable which has been already described it (a quantity that is always between zero and one). Eq. (3) expresses the relationship between the stresses caused by hardening and the Cauchy's stress tensor.

$$\hat{\sigma} = \frac{\sigma}{(1-D)} \qquad \mathrm{Q}_k = \frac{\mathrm{Q}_k}{(1-D)} \qquad \mathrm{Q}_i = \frac{\mathrm{Q}_i}{(1-D)} \tag{3}$$

Where σ is the Cauchy's stress tensor of the material without damage. Q_k and Q_i are stress variables due to the kinematic and isotropic hardenings that are conjugate to the internal strains α_k and α_i, respectively. Accordingly, the effective stress levels of the damaged materials are given in "Eq. (3)". $\varnothing^p$, yield function surface, which indicates the space of allowed stresses, will be introduced in accordance with "Eq. (4)". In this model, the type of von-Mises function is considered as follows:

$$\varnothing^p = \sqrt{\frac{3}{2} dev\left(\tilde{\sigma} - Q_k\right) : dev\left(\tilde{\sigma} - Q_k\right)} - \left(Q_i + Q_o^{eq}\right) \le 0 \tag{4}$$

In "Eq. (4)", dev is the abbreviation of the deviatoric part of each tensor. Q_o^{eq} is the initial yield threshold. By making some corrections to consider the damage and hardening on the model, the new form of the yield function will be in the form of "Eq. (5)".

$$\overline{\varnothing^p} = \varnothing^p + \frac{Y^M}{MS_1(1-D)} + \frac{B_k}{H_k}\frac{Q_k:Q_k}{2} + \frac{B_i}{H_i}\frac{Q_i^2}{2} \tag{5}$$

In this equation, the parameters S_1 and M have constant values and will be obtained from the matching of the experimental and simulation results. B_k and B_i are also defined as the saturation values of hardening stresses Q_k and Q_i , respectively. By applying this yield function, we can express the growth rate of damage variable based on a differential equation, which λ^p is a plastic multiplier ("Eq. (6)").

$$\dot{D} = \frac{\lambda^p}{(1-D)}\frac{Y^{M-1}}{S_1} \tag{6}$$

In addition to ductile damage that is caused by the growth of vacancy in the sample, the brittle damage will also affect the material failure. To describe this phenomenon, a new model is presented, which in this model also a function called energy released is initially defined in the form of "Eq. (7)" [15].

$$\psi = (1-D)Y + H_\Gamma\frac{\alpha_\Gamma}{2} + H_b\frac{\alpha_b}{2} \tag{7}$$

Where Y is the amount of released energy. Also, α_Γ and α_b are internal strain variables that are related to damage. It should be noted that H_Γ and H_b are the relative modulus of hardening.

$$\begin{aligned} Y &= -\partial_D\psi \\ \Gamma &= -\partial_{\alpha_\Gamma}\psi = -H_\Gamma\alpha_\Gamma \\ Q_b &= -\partial_{\alpha_b}\psi = -H_b\alpha_b \end{aligned} \tag{8}$$

Γ is so-called shift tensor which has been introduced for describing cyclic loading effects (similar to Armstrong-Frederick-type hardening) [15]. Based on these variables, the $\varnothing^b$ function is similar to the stress permissible space defined in the ductile damage part and is introduced in "Eq. (9)".

$$\varnothing^b = \frac{\left|Y^N - \Gamma\right|}{S_2} - (Q_b + Q_{b0}) \le 0 \tag{9}$$

In "Eq. (9)", N and S_2 are constant parameters of the material. Like the yield function presented in the ductile damage part, the brittle damage function can also be corrected by adding new terms in the form of "Eq. (10)".

$$\bar{\varnothing}^b = \varnothing^b + \frac{B_\Gamma}{H_\Gamma}\frac{\Gamma^2}{2} + \frac{B_b}{H_b}\frac{Q_b^2}{2} \tag{10}$$

Where B_b and B_Γ are the parameters of the material. Based on the above equations, the differential equation for the development of brittle damage variable can be written similar to the ductile damage part ("Eq. (11)").

$$\dot{D} = \lambda^b\partial_y\bar{\varnothing}^b = \lambda^b\frac{sign\left(Y^N - \Gamma\right)}{S_2}NY^{N-1} \tag{11}$$

Where, λ^b is similar to the plastic multiplier of the ductile part and it is used to solve the differential equation. Finally, the overall damage is achieved by a combination of variables of ductile D^p and brittle damage D^b . In the Kintzel quasi-brittle damage model, the overall damage variable is obtained from the linear combination of ductile and brittle damage variables according to "Eq. (12)" [15]. γ^p and γ^b are the coefficient of the ductile and brittle damage variable, respectively.

$$D = \gamma^p D^p + \gamma^b D^b , \qquad \gamma^b = 1 - \gamma^p \tag{12}$$

3 IMPLEMENTATION OF THE KINTZEL QUASI-BRITTLE DAMAGE MODEL

In this section, a flowchart for an implementation of the Kintzel quasi-brittle damage model will be shown based on the equations presented in Section 2. According to this flowchart, this model can be used in ABAQUS by writing a user subroutine VUMAT. The Kintzel quasi-brittle damage algorithm is shown in "Fig. 2".

As discussed before, the problem can be broken to ductile and brittle parts. For solving this numerical problem, the computational plasticity approach should be used. In this approach, the first-order forward Euler explicit integration scheme has been used. At first, the elastic predictor should be defined as the trial stress in ("Eq. (13)"). After that, by using the effective stress ("Eq. (14)"), the Von-Mises yield surface should be calculated ("Eq. (4)"). In order to find the plastic strain, the plastic flow direction ("Eq. (15)") and the plastic multiplier ("Eq. (16)") are needed. Then, by having a plastic strain, the ductile state variables can be obtained ("Eq. (18)"). Finally, the ductile damage variable ("Eq. (19)") could be fined by solving the differential equation ("Eq. (6)"). The same method was used for solving the brittle part to find brittle damage variable. In the end, the linear combination of these two damage variable made the total damage variables. By having total damage variable, the new effective stresses and the new yield surface could be calculated to run the flowchart again.

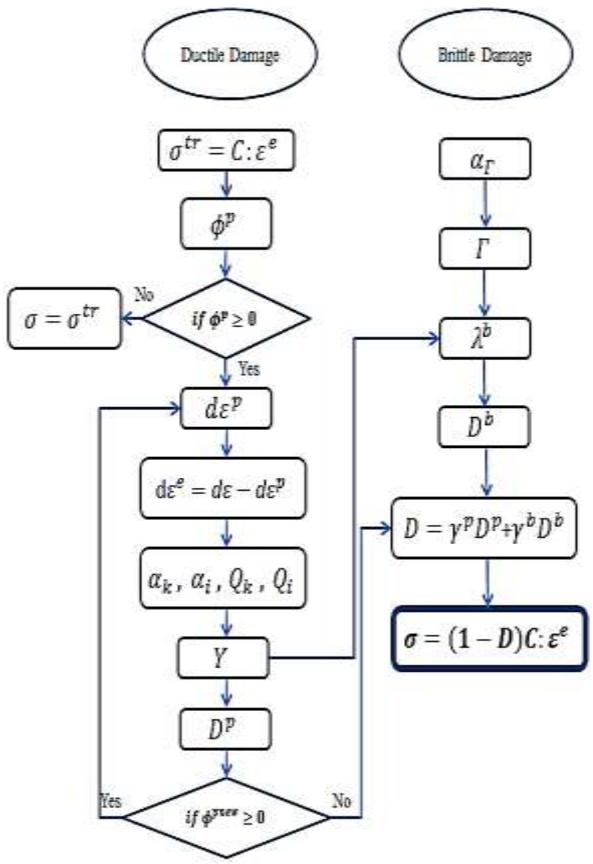

Fig. 2 The Kintzel quasi brittle damage model algorithm.

$$\tilde{\sigma}^{trial} = \mathbf{C} : \boldsymbol{\varepsilon}^{\mathbf{e}} \tag{13}$$

$$\tilde{\sigma}^{eff} = \sqrt{\frac{3}{2} dev\left(\tilde{\sigma} - \tilde{Q}_k\right) : dev\left(\tilde{\sigma} - \tilde{Q}_k\right)} \tag{14}$$

$$\mathbf{n} = \frac{dev(\tilde{\boldsymbol{\sigma}}) - \tilde{\mathbf{Q}}_{\mathbf{k}}}{\tilde{\boldsymbol{\sigma}}^{eff}} \tag{15}$$

$$\lambda^p = \frac{-\phi^P}{\partial_{\lambda^p}\phi^P} = \frac{\phi^P}{\mathbf{n}:\mathbf{C}:\mathbf{n} + H_k(\frac{3}{2} + B_k\mathbf{n}:\boldsymbol{\alpha}_{\mathbf{k}}) + H_i(1 + B_i\alpha_i)} \tag{16}$$

$$\mathbf{d}\dot{\boldsymbol{\varepsilon}}^{\mathbf{p}} = \lambda^p.\mathbf{n} \tag{17}$$

$$\boldsymbol{\alpha}_{\mathbf{kn+1}} = \frac{\boldsymbol{\alpha}_{\mathbf{kn}} - \boldsymbol{\Delta\varepsilon}^{\mathbf{p}}}{1 + B_k\lambda^p}, \quad \alpha_{in+1} = \frac{\alpha_{in} - \lambda^p}{1 + B_i\lambda^p}. \tag{18}$$

$$C^D = \lambda^p \frac{Y_{n+1}^{M-1}}{S_1}, D^P{}_{n+1} = D^P{}_n + C^D \tag{19}$$

$$\boldsymbol{\sigma} = (\mathbf{1} - \mathbf{D}^{\mathbf{p}})\mathbf{C} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathbf{p}}) \tag{20}$$

$$\lambda^b = \frac{\frac{sign\left(Y^N - \Gamma\right)}{S_2} N Y^{N-1} \dot{Y}}{B_r H_r \frac{sign\left(Y^N - \Gamma\right)}{S_2} + B_b H_b} \tag{21}$$

$$\alpha_{\Gamma n+1} = \frac{\alpha_{\Gamma n} - \Delta\lambda^b \frac{sign\left(Y^N - \Gamma\right)}{S_2}}{1 + B_\Gamma \Delta\lambda^b}, \quad \alpha_{bn+1} = \frac{\alpha_{bn} - \Delta\lambda^b}{1 + B_b\Delta\lambda^b} \tag{22}$$

$$D_{n+1} = D_n + \Delta\lambda^b \frac{sign\left(Y^N - \Gamma\right)}{S_2} N Y_{n+1}^{N-1} \tag{23}$$

4 RESULT

In this section, the results of the implementation of the Kintzel quasi-brittle damage model are presented for the 2024 Al alloy under periodic loading in ABAQUS. In order to implement the quasi-brittle damage model in this software, the finite element coding as a user subroutine VUMAT has been used. As shown in "Fig. 3", this sample is used for numerical simulation.

All sample dimensions shown in this figure are in mm. The mechanical properties of the alloy related to elastic and plastic part, ductile and brittle damage are presented in "Table 1", which will be used as inputs. The sample is subjected to a periodic loading in term of displacement. The value of this displacement is 0.0625 mm and will be applied along the axis to the upper edge of the sample. So that at a time, the sample will be under tension and after that it will be under compression.

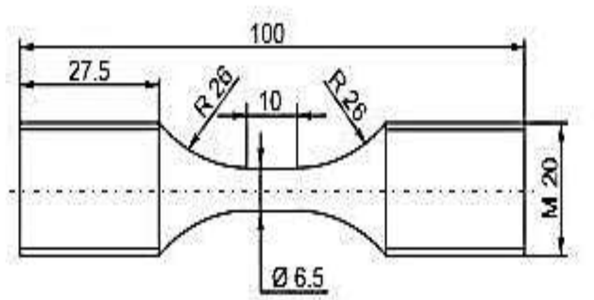

Fig. 3 Geometrical dimension of simulation sample [16].

One of the most important factors affecting the analysis of mechanical problems by finite element software is the structural elements.

In this modelling and for the analysis, an $CAX4R$ element that represents a 4-node bilinear axisymmetric element has been used. With the repetition of modelling in finite element software and having about 200 to 300 elements in the sample, a good convergence is achieved in the results. In this section, the results of using the Kintzel quasi-brittle damage model under periodic loading are presented and the results are compared with the results of Kintzel's experimental work.

Table 1 Mechanical & Damage properties of Al 2024 [16]

Parameter	Value
E	6700 (MPa)
v	0.30
Q_o^{eq}	225 (MPa)
H_i	501.252 (MPa)
H_k	180000 (MPa)
B_i	6.84 (MPa)
B_k	3050 (MPa)
M	1.6
S_1	1.5
N	1.25
S_2	1.25
H_Γ	0.992 (MPa)
B_Γ	121.50 (MPa)
γ^d	0.86
γ^b	0.14

At first, to verify the results, the stress-strain curve obtained through the experimental works is compared to the curve obtained from the simulation in "Fig. 4".

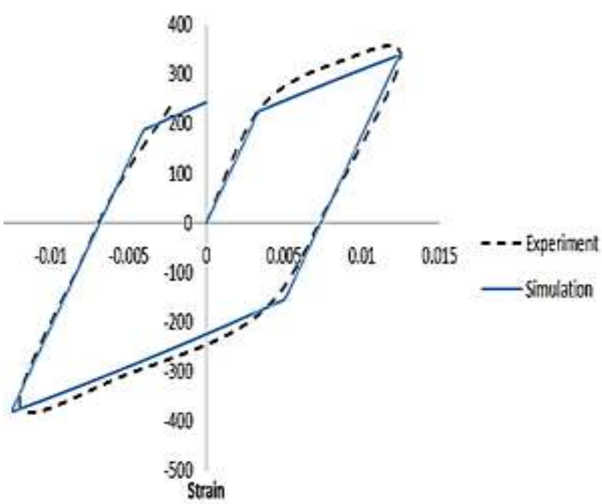

Fig. 4 Comparison of the stress-strain curve for experimental data [16] & simulation data for 2024 Al alloy under cyclic loading.

It can be said that with starting from the zero load and stretching the sample, the stress values will increase up to about 250 MPa. Then, by changing the type of loading into compression state, the stress values will gain negative amounts. Finally, in a tensile state, the ultimate tensile stress will be about 400 MPa, and in compression mode it will be a little more than that, which eventually will fracture through the repetition. In "Fig. 4", in order to make it easier to display, only a comparison of a cycle is shown. An appropriate validation between the experimental and the numerical results was observed. The reason for the slight difference between the numerical and experimental data can be seen in the difference between the values of ductile and brittle damage coefficients.

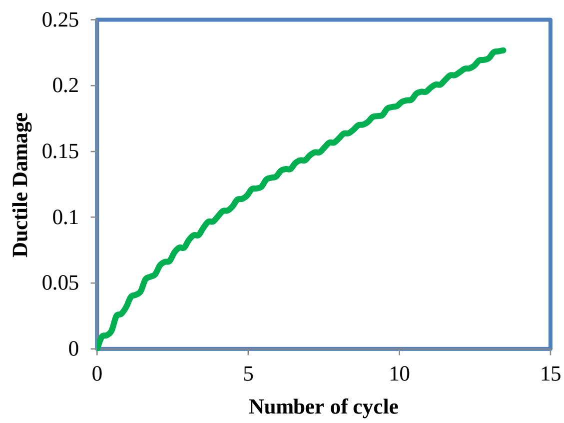

Fig. 5 Ductile damage variable versus the number of the cycle for 2024 Al alloy under strain 0.04.

The variations in the ductile damage, the brittle damage and the total damage in terms of the number of cycles for the strain range of 0.04 are shown in "Fig. 5 up to Fig. 7", respectively.

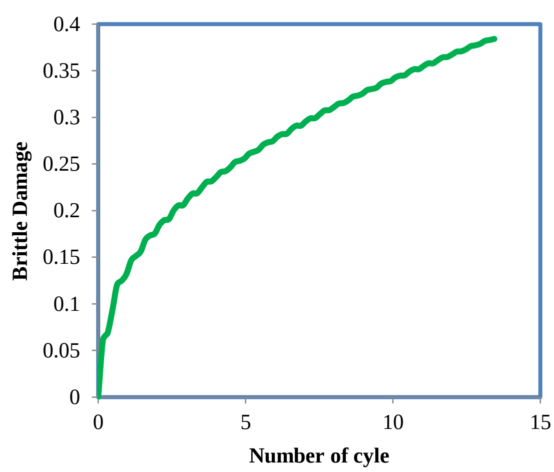

Fig. 6 Brittle damage variable versus the number of the cycle for 2024 Al alloy under strain 0.04.

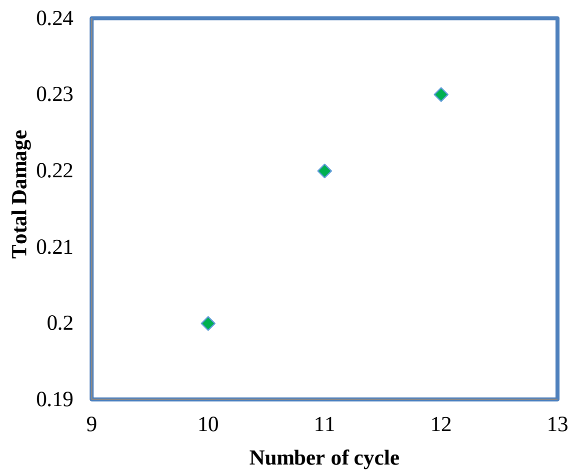

Fig. 7 Total damage variable versus the number of the cycle for 2024 Al alloy under strain 0.04.

As shown in these figures, the values of the damage variable increase with a slight gradient from zero and the damage growth continued until the twelfth cycle. By increasing the load on the sample, the micro-voids and the micro-cracks are created in the sample; thus the plastic strain in the material is increasing. In this way, the ductile damage begins to occur and at the same time, with the creation of micro-cracks in the material and their interconnection based on the amount of energy, the brittle damage begins to be activated. Finally, due to the strain range of the sample, a complete failure occurs in the 12th cycle. The value of the ductile damage variables will increase to 0.23, but the brittle damage variable increased up to 0.38. The reason for the stress values will gain the negative amounts value of variations can be found in the dependence of these two variables on the amount of released energy.

As shown in the equations related to definition of the brittle and ductile damage variables, the slope of energy released changes in the brittle damage are more than the ductile one, which causes the differences in the values of these two variables. Also, since the 2024 Al alloy has a high strength to weight ratio, it can be said that in the uniaxial tensile test, this aluminium alloy has less elongation compared to other aluminium alloys. Therefore, the most aluminium alloys failure occurs slowly with rupture, but the 2024 Al alloy has more sudden breakdown rather than other Al alloys. According to "Fig. 5", the value of 0.23 can be identified as the critical value of the ductile damage variable. After that, the increase in the ductile damage variable, and ultimately the total damage variable and inducing the high stresses to the sample, cause a sudden failure of the sample.

The critical value of the total damage variable is also determined by "Eq. (12)", which according to "Fig. 7", this value will be about 0.22. In "Fig. 8", the effect of strain on the total damage variable is observed in one cycle. As expected, with the increase of strain due to the plastic deformation in the material, the amount of energy released should be increased.

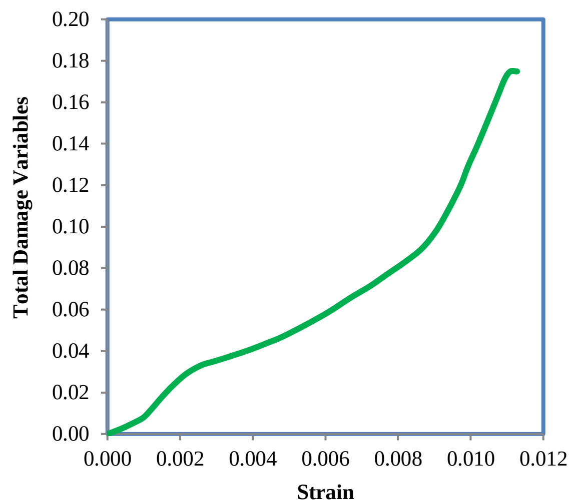

Fig. 8 Total damage variable versus induce strain for 2024 Al alloy.

It is clear that the contribution of the ductile damage variable was greater than the brittle damage variable because of the difference between their happening mechanisms. As discussed before (section 1), the plastic strain causes the ductile damage by creating micro-voids, micro-cracks and joining together until failure.

5 CONCLUSION

As shown in this study, a quasi-brittle damage model can be used to describe the behavior of 2024 aluminum alloy as a ductile-brittle material under periodic loading. Additionally, one can note that the 2024 Al alloy exhibits an acceptable resistance to periodic loading. The following can be generalized for damage variables of this model:

- Higher brittle damage variable rather than ductile damage variable under periodic loading until failure.
- Significant impact of applied strain on the ductile damage in comparison to brittle damage.
- Efficiency of the Kintzel quasi-brittle damage model for estimating the life of mechanical components under periodic loading.

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