Determination of Material Properties Components used in FEM Modeling of Ultrasonic Piezoelectric Transducer

Abbas Pak*
Department of Mechanical Engineering,
University of Bu-Ali Sina, Iran
E-mail: a.pak@basu.ac.ir
*Corresponding author

Received: 4 December 2018, Revised: 2 January 2019, Accepted: 10 May 2019

Abstract: Ultrasonic transducers have found new applications such as ultrasonic assisted micromachining, micro forming, surface treatment, welding, etc. Apart from the transducer's shape and size, the resonant frequencies and amplitude are seriously affected by materials properties used for transducer components. A further problem with the material is that their properties may vary from batch to batch and may also depend on the size of the raw stock. In this work using modal analysis, the material properties are calculated based on the frequency response method, which is more accurate than the nominal one. The finite element modelling was employed for both 2D and 3D FEM analysis to observe the behaviour of the cylindrical test rods and two sandwich-type piezoelectric transducers with the nominal frequency of 20 kHz and 30 kHz to find the validity of these properties. The obtained results showed that the modal analysis method could accurately determine the bar speed, Poisson's ratio and elastic modulus of the ultrasonic transducer components. The accuracy of this method increases by considering more vibration mode. Based on the results, obtained errors for FEM modelling of two ultrasonic transducers with the frequency of 20 kHz and 30 kHz are 0.15% and 0.33%, respectively.

Keywords: FEM Modeling, Modal Analysis, Ultrasonic Transducer, Young's Modulus

Reference: Pak, A., "Determination of Material Properties Components used in FEM Modeling of Ultrasonic Piezoelectric Transducer", Int J of Advanced Design and Manufacturing Technology, Vol. 12/No. 2, 2019, pp. 75-81.

Biographical notes: Abbas Pak received his Ph.D. in Manufacturing Engineering from Tarbiat Modares University. He is currently Assistant Professor at the Department of Mechanical Engineering, Bu-Ali Sina University, Hamedan, Iran. His current research interest includes Advanced Manufacturing Processes and Precision Machining.

1 INTRODUCTION

Determining the components of transducer's material properties such as elastic modulus, Poisson's ratio, and density is important in the design of ultrasonic transducers. Several methods are proposed for transducer design, among them, FEM is an efficient tool which was used in this study [8-14]. FEM cannot correctly predict transducer performance unless the components of transducer's material properties are known. The resonant frequencies and amplitude are affected by the properties of materials used for transducer components [14-15]. For transducers with small lateral dimensions, the density and the modulus of elasticity determine the resonant frequencies. For larger ultrasonic transducers, the resonant frequencies are affected somewhat by Poisson's ratio. Depending on the transducer component's shape, the amplitude can also be significantly changed by Poisson's ratio [16]. Furthermore, the material properties may vary from batch to batch and may also depend on the size of the raw stock.

The theoretical basis is established by Lundberg [17] for a general way of determining mechanical material properties from a one-dimensional wave equation and measured response to impact at two different cross-sections of a rod specimen. Hillstrom et al. [18] worked on the identification of complex modulus by strains which are known at three sections of an axially impacted bar specimen. However, the sound speed and the Poisson's ratio were not determined in these methods. The complex Poisson's ratio was identified by measured strain in the circumferential and axial directions at a single section of a cylindrical test bar subjected to axial impact by Mousavi et al. [19-20]. Advantages of these methods are that they use short bar specimens. A disadvantage is that they require at least two independent measurements. Another is that they may be mathematically complex. Thus, the equation which relates the wave propagation coefficient to the measured quantities has several or a large number of solutions and, it is not always evident which one to choose. Therefore, it is necessary to use an accurate and simple method to determine the properties of the components of transducer's material. In this work based on the modal analysis, the frequency response method is used to calculate the material properties such as sound speed, elastic modulus and Poisson's ratio with the best accuracy from a one-dimensional wave equation. Determined material properties would be the input parameters of FEM modelling to compute the correct resonance frequencies of ultrasonic transducers.

2 THEORY

By considering the lateral oscillation of an isotropic bar in longitudinal vibration due to Poisson's ratio effect, Love's theory for the longitudinal vibration is [21]:

$$\frac{\partial^2 u}{\partial x^2}+\frac{\nu^2 k^2}{C_0^2}\frac{\partial^4 u}{\partial x^2 \partial t^2}=\frac{1}{C_0^2}\frac{\partial^2 u}{\partial t^2} \tag{1}$$

In which u, t, x and υ are the particle displacement, time, coordinate and the Poisson's ratio, respectively. K is the radius of the gyration of the bar cross-sectional, $C_0=\sqrt{\frac{E}{\rho}}$

$C_0=\sqrt{\frac{E}{\rho}}$, is the bar speed, in which E and ρ are the elastic modulus and the density, respectively.

For a free-free bar with length L, the boundary conditions are $\frac{\partial u_{(0,t)}}{\partial x}=0$ and $\frac{\partial u_{(L,t)}}{\partial x}=0$. By assuming a particle displacement u = A $e^{i\gamma(ct-x)}$ + B $e^{i\gamma(ct+x)}$ and substituting the above boundary conditions into "Eq. (1)", one obtains A = B. Cn is the sound speed at the nth mode, and it can be calculated as:

$$C_n = 2 f_n L / n,\ n = 1,2,3,... \tag{2}$$

Where A is a constant, i $=\sqrt{-1}$, f_n is the resonant frequency, and C_n is the bar speed at the nth mode. Then, substituting the particle displacement u into "Eq. (1)", we arrive at:

$$-\gamma_n^2+\frac{\nu^2 k^2}{C_0^2}\gamma_n^4 C_n^2+\frac{\gamma_n^2 C_n^2}{C_0^2}=0 \tag{3}$$

Where $\gamma_n = \omega_n/C_n$ and ω_n is the nth angular resonant frequency. From "Eqs. (2) and (3)", the bar speed (C_n) and resonant frequency (f_n) of a free-free bar can be solved as:

$$f_n=\frac{C_0 n}{2}\frac{1}{\sqrt{\frac{L^2}{n^2}+\pi^2\nu^2 k^2}} \tag{4}$$

In "Eq. (4)", with two resonant frequencies (f_n, f_m) obtained from the experiments, the bar speed can be as follows [22]:

$$C_0=\frac{2 f_n f_m L}{nm}\sqrt{\frac{m^2-n^2}{f_m^2-f_n^2}} \tag{5}$$

3 EXPERIMENTAL AND RESULTS

Two cylinders from aluminum 7075-T6 and stainless steel 304 which were used mostly to fabric the ultrasonic transducers were selected in this study. Aluminum sample, L_1, had a diameter of 50.8 mm and a length of 1005 mm, and stainless steel sample, L_2, had a diameter of 38 mm and a length of 973 mm. The experimental set-up and frequency response curve obtained from the test are illustrated in “Figs. 1 and 2”. The frequency response functions for vibration modes of the bar were obtained between the vibration exciter (shaker) and the accelerometer sensor, and the resonant frequencies were extracted from the frequency response functions. Also, the sample suspension way was selected based on the modal testing requirements, the same. Experimental instruments used for extraction of frequency modes have been given in “Table 1”.

Table 1 Experimental instruments used in the modal tests

Instrument	Vibration exciter (Shaker)	Accelerometer	Signal analyser
Type	B&K Type 4809	DJB A952	B&K 3536

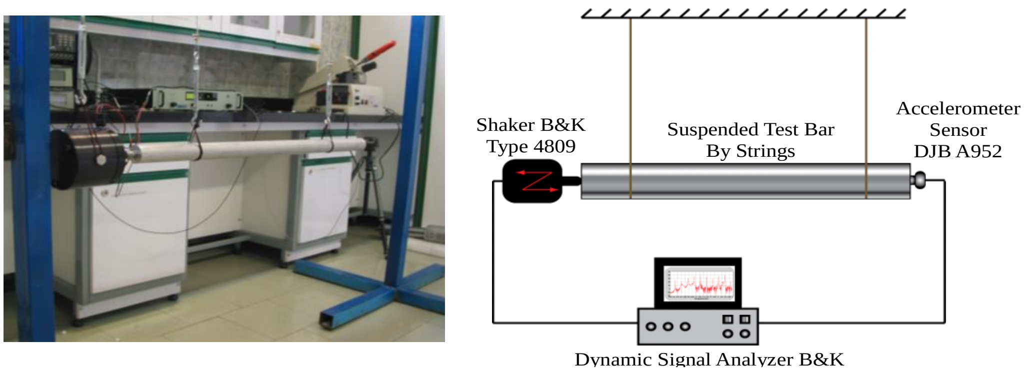

Fig. 1 Experimental set-up to measure vibration mode frequencies.

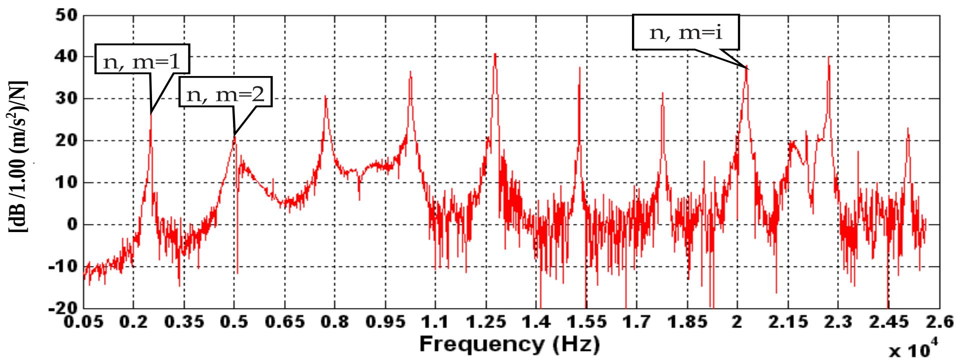

Fig. 2 Frequency response curve obtained from the modal test.

3.1. Determination and Measurement of Bar Speed and Density

Based on “Eq. (5)”, the calculated sound speeds of the samples are shown in “Tables 2 and 3”. As shown in “Tables 2 and 3”, the averages bar speed, $(Co)_{Ave}$, are 5134.6 m/s and 5054.4 m/s for aluminum and stainless steel bars, respectively.

Table 2 Bar speeds, C_0, of the aluminum sample (L_1=1005 mm, d=50.8 mm), as calculated by the mode number pair (m,n)

Mode		Bar speed (m/s)							
Frequency (Hz)	(m,n)	1	2	3	4	5	6	7	8
2552	1	-	5132.4	5130.2	5130.6	5130.6	5130.7	5130.8	5131.0
5095.3	2	5132.4	-	5118.1	5123.2	5124.2	5124.8	5125.5	5126.3
7648	3	5130.2	5118.1	-	5138.0	5136.2	5136.2	5136.9	5138.3
10176	4	5130.6	5123.2	5138.0	-	5132.2	5133.5	5135.6	5138.6
12694	5	5130.6	5124.2	5136.2	5132.2	-	5136.0	5139.5	5144.6
15190	6	5130.7	5124.8	5136.2	5133.5	5136.0	-	5145.3	5153.2
17652	7	5130.8	5125.5	5136.9	5135.6	5139.5	5145.3	-	5165.5
20060	8	5131.0	5126.3	5138.3	5138.6	5144.6	5153.2	5165.5	-
$(C_{oAl})_{Ave}$=5134.6 m/s									

Table 3 Bar speeds, C_0, of stainless steel sample (L_2=973 mm, d=38 mm), as calculated by the mode number pair (m,n)

Mode Frequency (Hz)	Bar speed (m/s)							
	(m,n)	1	2	3	4	5	6	7
2580.9	1	-	5024.2	5022.6	5022.9	5022.9	5023.0	5023.1
5156.5	2	5024.2	-	5014.1	5017.8	5018.6	5019.0	5019.6
7740.8	3	5022.6	5014.1	-	5028.1	5027.2	5027.3	5028.0
10310	4	5022.9	5017.8	5028.1	-	5025.2	5026.0	5027.9
12874	5	5022.9	5018.6	5027.2	5025.2	-	5027.5	5031.0
15426	6	5023.0	5019.0	5027.3	5026.0	5027.5	-	5036.7
17954	7	5023.1	5019.6	5028.0	5027.9	5031.0	5036.7	-
$(C_{oSt})_{Ave}$=5054.4 m/s								

The exact value of the density and the dilatational bar speed were accurately measured for two bars. The density of specimens was obtained as:

ρ_{Al} =2823 Kg/m3, ρ_{st} =7944 Kg/m3

Ultrasonic equipment ASCANWIN, E2.58 was employed to measure the dilatational sound speed in the NDT Laboratory. The time of flight (TOF) of the transmitted pulse by a 2MHz probe was measured. By knowing the thickness of the specimens, the dilatational sound speeds could be obtained by a simple calculation. The experimental dilatational sound speeds were 6067.803 m/s and 5720 m/s for aluminum and stainless steel samples, respectively.

3.2. Determination of Poisson's ratio and Young Modulus

The dilatational bar speed, C_b, can be calculated as follows [22]:

$$C_b = C_0\sqrt{\frac{1-\upsilon}{(1+\upsilon)(1-2\upsilon)}} \tag{6}$$

Thus, by "Eq. (6)" the Passion's ratio for aluminum 7075-T6 and stainless steel 304 were determined to be:

$$6067.803 = 5134.6\times\sqrt{\frac{1-\upsilon_{Al}}{(1+\upsilon_{Al})(1-2\upsilon_{Al})}} \Rightarrow \upsilon_{Al} = 0.31244$$

$$5720 = 5024.4\times\sqrt{\frac{1-\upsilon_{St}}{(1+\upsilon_{St})(1-2\upsilon_{St})}} \Rightarrow \upsilon_{St} = 0.29$$

and from the equation, $C_0=\sqrt{\frac{E}{\rho}}$, Young modulus for these materials was calculated to be:

$$E_{St} = (C_{0St})^2{}_{Ave}\times\rho_{St} = (5024.4)^2\times 7944 = 200.5\times10^9 \quad N/mm$$

$$E_{Al} = (C_{0Al})^2{}_{Ave}\times\rho_{Al} = (5134.6)^2\times 2823 = 74.426\times10^9 \quad N/mm^2$$

4 VALIDATION AND APPLICATION

To verify the resonant frequency to determine material properties, the Poisson's ratio and elastic modulus were inputted in the general-purpose finite element package ANSYS to compute the resonance frequencies at the test bars, and the results are shown in "Tables 4 and 5". The maximum difference between results from the experimental and FEM modeling are 0.807% and 0.663%, but they are not significantly different in the longitudinal mode (Mode No. 1) since the error of these results are extremely small, i.e., 0.08% and 0.039% for aluminum and stainless steel bar, respectively.

Table 4 Results obtained from FEM modeling and experimental modal analysis for aluminum 7075-T6 bar

Mode No.	Frequency from FEM modeling (Hz)	Frequency from Modal Test (Hz)	Error (%)
1	2554.1	2552	0.08
2	5105.9	5095.3	0.208
3	7652.9	7648	0.064
4	10193.	10176	0.167
5	12722.	12694	0.220
6	15239.	15190	0.322
7	17740.	17652	0.498
8	20222.	20060	0.807

Table 5 Results obtained from FEM modeling and experimental modal analysis for stainless steel 304 bar

Mode No.	Frequency from FEM modeling (Hz)	Frequency from Modal Test (Hz)	Error (%)
1	2581.9	2580.9	0.039
2	5163.8	5156.5	0.141
3	7745.7	7740.8	0.063
4	10328.	10310	0.174
5	12909.	12874	0.272
6	15491.	15426	0.421
7	18073.	17954	0.663

To demonstrate the resonant frequency for a real application in design and modelling of the ultrasonic transducer, two high power ultrasonic transducers with the nominal frequency of 20 kHz and 30 kHz were designed, modeled and fabricated from tested materials. Determined material properties were the input parameters in the ANSYS program to compute the longitudinal resonance frequency of the designed transducers. In this regard, 2D and 3D FEM simulations were performed for the transducer to determine their resonance frequencies (“Figs. 3 and 4”). For the design of the transducers discussed in this paper, PZT-SA was chosen as a piezoelectric material. TAMURA Inc. lists the material properties of PZT-SA as [23]:

Dielectric Relative Permittivity Matrix at Constant Strain, $\left[\varepsilon_r^S\right]$ (polarization axis along Y-axis):

$$\left[\varepsilon_r^S\right]=\begin{bmatrix} 874 & 0 & 0 \\ 0 & 718.06 & 0 \\ 0 & 0 & 874 \end{bmatrix}$$

Dielectric charge constant matrix (Strain developed/electric field applied at constant stress), [d] (polarization axis along Y-axis):

$$[d]=\begin{bmatrix} 0 & -131 & 0 \\ 0 & 286 & 0 \\ 0 & -131 & 0 \\ 387 & 0 & 0 \\ 0 & 0 & 387 \\ 0 & 0 & 0 \end{bmatrix}\times 10^{-12}\ m/V$$

Compliance Matrix [s] for PZT-SA under constant electric field, $[s^E]$ (polarization axis along Y-axis):

$$\left[s^E\right]=\begin{bmatrix} 11.7 & -5.17 & -3.63 & 0 & 0 & 0 \\ -5.17 & 14.3 & -5.17 & 0 & 0 & 0 \\ -3.63 & -5.17 & 11.7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 33.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 33.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 30.7 \end{bmatrix}\times 10^{-12}\ m^2/N\ a$$

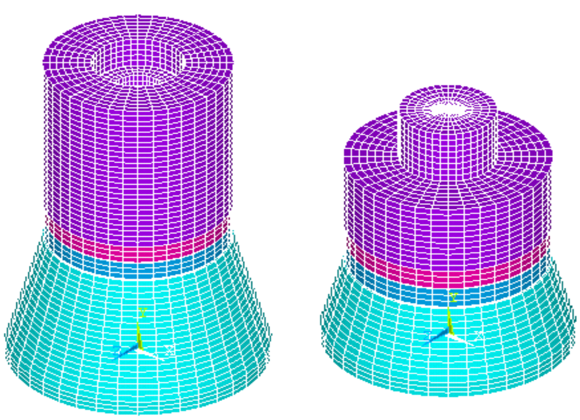

Fig. 3 Full 3D modelling with SOLID5 elements.

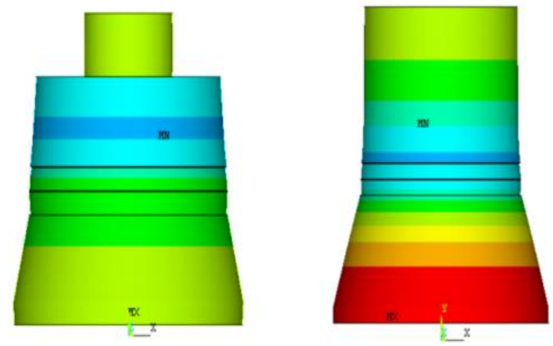

Fig. 4 Mode shape from modal analysis of full 3D modelling.

5 TEST OF THE FABRICATED TRANSDUCER

To measure the actual resonance frequency of the designed and fabricated ultrasonic transducers, a network analyzer of ROHDE & SCHWARZ was employed, the sweeping frequency of this device was within 9kHz-4GHz with a resolution of 10Hz. The sweeping frequency was adjusted between 15 kHz to 35 kHz while series and parallel frequencies were illustrated in the phase-versus-frequency diagram shown in “Fig. 5”. The measurement was performed on the unloaded transducer. The summarized results of the FEM simulations for two ultrasonic transducers are shown in “Table 6”.

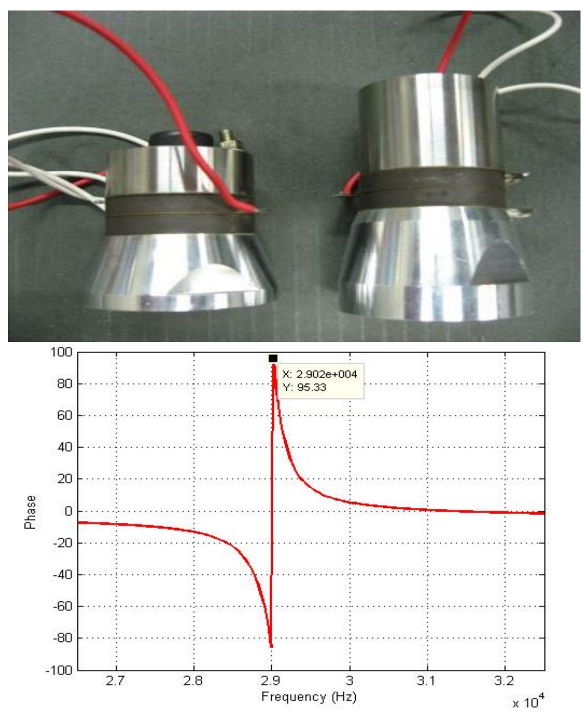

Fig. 5 Fabricated transducer and the diagram of phase versus frequency generated by Network Analyzer.

Table 6 The results obtained from FEM modelling of two ultrasonic transducers with the nominal frequency of 20 kHz and 30 kHz

Nominal Resonance Freq. (kHz)	Modeling Type	Polarized Axis	Element Type of other Components	Element Size (mm)	Resonance Freq. (kHz) From FEM-	Measured Resonance Freq. (kHz)	Error (%)
20	2D	Y	PLANE13	1	20.019	19.970	0.15
20	3D	Y	SOLID5	2	20.043	19.970	0.15
30	2D	Y	PLANE13	1	30.090	29.900	0.33
30	3D	Y	SOLID5	2	30.156	29.900	0.33

7 CONCLUSION

It has been shown that sound speed (a rod that used for fabrication high power ultrasonic transducers and mechanical transmitter in mass production) can be measured accurately by the resonant frequency method. The employed method was robust in the field test and having high accuracy for the determination of Poisson ratio and elastic modulus which is an important factor on the resonant frequencies of ultrasonic transducers and mechanical transmitter as well as accurate FEM simulation of the transducers to understand their mechanical behavior. In this research, obtained errors for FEM modelling of bars and two sandwich-type piezoelectric transducers are 0.08 %, 0.15%, and 0.33%, respectively. The accuracy of this method increases by considering more vibration mode. The obtained results showed that the capabilities of the ANSYS software could be used successfully as a powerful and reliable tool for prediction of behavior of sandwich-type piezoelectric transducers and comparison of the actual network analyzer results and modal analysis results for two different 2D and 3D modeling techniques proves that there is a good agreement between the frequencies obtained.

8 NOMENCLATURE

A	Constant	-
B	Constant	-
C_0	Bar speed	m/s
C_b	Dilatational bar speed	m/s
C^n	Bar speed at the nth mode	m/s
E	Young's modulus	N/mm^2
f_n	The nth resonant frequency	Hz
f_s	The resonant frequency of the Transducer	Hz
k	The radius of the gyration	m
L	Length of bar	m
t	Time	sec
u	Displacement	m
ρ	Density	Kg/m^3
ν	Poisson's ratio	-
ω_n	The nth angular resonant frequency	Hz

ACKNOWLEDGMENTS

The Author would like to thank Eng. Shahidi for assistance with methodology, and Dr. Abdullah for comments that greatly improved the manuscript.

REFERENCES

[1] Frederick, R., Ultrasonic Engineering, John Wiley and Sons, New York, USA, 1965.

[2] Kumar, S., Wu, C. S., Padhy, G. K., and Ding, W., Application of Ultrasonic Vibrations in Welding and Metal Processing: A Status Review, Journal of Manufacturing Processes, Vol. 26, 2017, pp. 295-322.

[3] Langevin, P., French Patent, Application No. FR575435D filed 27, December 1923.

[4] Mason, W. P., Electromechanical Transducers and Wave Filters, Van Nostrand, New York, USA, 1942.

[5] Krimholtz, R., Leedom, D. A., and Mattaei, G. L., New Equivalent Circuits for Elementary Piezoelectric Transducer, Electron, Vol. 6, 1970, pp. 398–399.

[6] Redwood, M., Experiments With the Electrical Analog of a Piezoelectric Transducer, Journal of the Acoustical Society of America, Vol. 36, No. 1, 1964, pp. 1872–1880.

[7] Al-Budairi, H., Lucas, M., and Harkness, P., A Design Approach for Longitudinal–Torsional Ultrasonic Transducers, Sensors and Actuators A: Physical, Vol. 198, 2013, pp. 99-106.

[8] Kagawa, Y., Yamabuchi, T., Finite Element Simulation of a Composite Piezoelectric Ultrasonic Transducer, IEEE Transactions on Sonics and Ultrasonics, Vol. 26, No. 2, 1979, pp. 81 – 87.

[9] Jian, S. W., Dale, F. O., A Finite Element-Electric Circuit Coupled Simulation Method for Piezoelectric Transducer, IEEE Ultrasonics Symposium, Caesars Tahoe, NV, USA, 1999, pp. 1105 – 1108.

[10] Cunningham, P. M., Use of the Finite Element Method in Ultrasonic Applications, Ultrasonic Industry Association Symposium, Ohio, USA, 2000.

[11] Kocbach, J., Finite Element Modeling of Ultrasonic Piezoelectric Transducers- Influence of Geometry and Material Parameters on Vibration, Response Functions and Radiated Field, Ph.D. dissertation, Department of Physics, University of Bergen, Bergen, 2000.

[12] Moreno, E., Acevedo, P.,. Fuentes, M., Sotomayor, A. Borroto, L., Villafuerte, M. E., and Leija, L., Design and Construction of a Bolt-Clamped Langevin Transducer, 2nd International Conference on Electrical and Electronics Engineering, Mexico City, Mexico, 2005, pp. 393 – 395.

[13] A. I. Fernando, M. Pappalardo, and J. Gallego, Finite Element Three-Dimensional Analysis of the Vibrational Behavior of the Langevin-Type Transducer, Ultrasonics, Vol. 40, 2002, pp. 513-517.

[14] Abdullah, A., Pak, A., Correct Prediction of the Vibration Behavior of a High Power Ultrasonic Transducer by FEM Simulation, The International Journal of Advanced Manufacturing Technology, Vol. 39, 2008, pp. 21–28.

[15] Abdullah, A., Pak, A., Abdullah, M. M, Shahidi, A., and Malaki, M., Study of the Behavior of Ultrasonic Piezo-Ceramic Actuators by Simulations, Electronic Materials Letters, Vol. 10, No. 1, 2014, pp. 37-42.

[16] Culp., D. R., Ultrasonic Resonator Design Using Finite Element Analysis, Available, 2002: http://www.krellengineering.com/fea/fea_info/fea_resonator_design.htm#FEA%20Procedure, [2002].

[17] Lundberg, B., Blanc, R. H., Determination of Mechanical Material Properties From the Two-Point Response of an Impacted Linearly Viscoelastic Rod Specimen, Journal of Sound and Vibration, Vol. 126, No. 1, 1988, pp. 97-108.

[18] Hillstrom, L., Mossberg, M., and Lundberg, B., Identification of Complex Modulus From Measured Strains on an Axially Impacted Bar Using Least Squares., Journal of Sound and Vibration, Vol. 230, No. 3, 2000, pp. 689-707.

[19] Mousavi, S., Nicolas, D. F., and Lundberg, B., Identification of Complex Moduli and Poisson's ratio From Measured Strains on an Impacted Bar, Journal of Sound and Vibration, Vol. 277, No. 4–5, 2004, pp. 971-986.

[20] Mousavi, S., Hillström, L., and Lundberg, B., Identification of Complex Shear Modulus From Measured Shear Strains on a Circular Disc Subjected to Transient Torsion at Its Centre, Journal of Sound and Vibration, Vol. 313, No. 3-5, 2008, pp. 567-580.

[21] Graff, K. F., Wave Motion in Elastic Solids, London, UK, Oxford University Press, 1975.

[22] Hen, S., Resonant Frequency Method for the Measurement and Uncertainty Analysis of Acoustic and Elastic Properties, Ultrasonics, Vol. 38, 2000, pp. 206–221.

[23] TAMURA Co., Piezoelectric Ceramics for High Power Applications Data Sheet, Available: https://www.tamurass.co.jp, 2017.