Free Vibration of Functionally Graded Epoxy/Clay Nanocomposite Beams based on the First Order Shear Deformation Theory

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Abstract: This paper deals with free vibration of epoxy/clay nanocomposite beams for functionally graded and uniformly distributed of Nanoclay with simply supported conditions at both ends. The specimens were prepared for uniformly distributed of Nanoclay with different Nanoparticles weight percent (pure, 3 wt%, 5 wt% and 7 wt%) and functionally graded distribution. To apply the model of theoretical predictions for the Young modulus, the genetic algorithm procedure was employed for functionally graded and uniformly distributed epoxy/clay nanocomposites and then were compared with the experimental tensile results. The formulation for Young modulus includes the effect of nanoparticles weight fractions and it is modified for functionally graded distribution to take into account the Young modulus as a function of the thickness coordinate. The displacement field of the beam is assumed based on the first order shear deformation beam theory. Applying the Hamilton principle, the governing equations are derived. The influence of nanoparticles on the free vibration frequencies of a beam is presented. To investigate the accuracy of the present analysis, a compression study is carried out with the experimental free vibration results. The results have shown that there is high accuracy for the genetic algorithm procedure for theoretical predictions of the Young modulus and the free vibration frequencies for uniform distribution are generally lower than the corresponding value of the functionally graded distribution.

Keywords: First Order Shear Deformation Theory, Free Vibration, Functionally Graded Nanocomposite, Genetic Algorithm Theory

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Biographical notes: Mahdi Karami Khorramabadi received his PhD in Mechanical Engineering from Razi University in 2018. He is currently an assistant professor at the Department of Mechanical Engineering, Islamic Azad University, Khorramabad Branch, Iran. His current research interest includes preparation of functionally graded nanocomposites and modelling with theoretical predictions on mechanical properties of polymer/clay nanocomposites. His research also focuses on mechanical buckling, free vibration and dynamic stability analysis of functionally graded nanocomposite columns.

1 INTRODUCTION

Nanocomposite materials have been extensively used in structural applications, such as the aeronautical, automotive and marine industries due to their high specific strength/stiffness, high corrosion resistance, long fatigue life and good design flexibility. Their dynamic performance is a significant factor for the safety of the structures in service. There is an especially strong need for improvement of natural frequency in lightweight structural nanocomposite materials so that they may be more effectively used in the design of structures and machines.

However, in recent years the polymer matrix based nanocomposites have been drawing great attention by offering significantly better properties than those of conventional particulate or fibrous polymeric composites [1-3]. Several research efforts focus on preparation, properties and applications of polymer nanocomposite [4-7]. Jawahar et al. [8] investigated the influence of Nanoclay addition on properties of unsaturated-polyester nanocomposite gel coat system and showed that at higher clay concentration the mechanical properties decrease due to the limitation in the processing as well as the high viscosity of resin/clay/aerosol system. Chandra et al. [9] have given a review on damping characteristics of synthetic fiber reinforced composites and concluded that the major contribution of damping is due to the visco-elastic nature of the matrix. Chandradass et al. [10-11] reported that Nanoclay addition of short fiber chopped strand mat reinforced vinyl ester composite resulting in increased natural frequency and internal damping due to the large variation in fiber–matrix interface. Mohan et al. [12] investigated mechanical properties of unmodified and organically modified montmorillonite Nanoclay with epoxy matrix and found that organo-modified nanocomposites showed improved tensile modulus, natural frequency and damping factor than unmodified clay due to the addition of lower weight percentage of Nanoclay.

Based on the literature survey, it appears that the preparation and mechanical properties of uniformly distributed epoxy/ clay nanocomposite have been extensively investigated. However, no study was encountered investigating the theoretical prediction on mechanical properties based on the genetic algorithm procedure and free vibration analysis of Epoxy/clay nanocomposite beams for functionally graded (FG) and uniformly distributed of Nanoclay in the open literature. The present study aims to show the influences of nanoparticles on the free vibration frequencies of the beam. Using the genetic algorithm procedure, Young's modulus of nanocomposites for functionally graded and uniform distribution is calculated and compared with the experimental ones. The displacement field of the beam is assumed based on the first order shear deformation beam theory. Applying the Hamilton principle, the governing equations are derived. To investigate the accuracy of the present analysis, a compression study is carried out with the experimental results.

2 EXPERIMENTAL PROCEDURE

2.1. Materials

The polymer matrix used in this study was an Epoxy with trade name PR7000 from AL TANNA Co. (Germany), with density $\rho = 2.25 g/ml$. The hardener was mixed in the ratio of 10:1. The Nanofiller was US7810 from US Research Nanomaterials Inc., USA.

2.2. Mechanical Properties

The specimens were tested according to ASTM D638 with three repeats. The tension tests were carried out using a Gotech universal testing machine (Model GT-AI5000L) with a crosshead speed of 50 mm/min. The material compositions of the nanocomposites for the uniform distribution are listed in "Table 1". In this table, wt.% is considered as the weight percent.

Table 1 Sample compositions

Sample	Epoxy (wt%)	Nano Clay (wt%)
1	100	-
2	97	3
3	95	5
4	93	7

For functionally graded distribution, the preparation procedure for the uniform distribution was done for samples with 1 mm thicknesses. The thickness of each sheet is 1 mm and four sheets with different Nanoparticles weight percent (pure, 3 wt%, 5 wt% and 7 wt%) were employed to make functionally graded nanocomposite. The values of the nanocomposites Young's modulus for functionally graded and uniform distribution of Nanoclay are shown in "Fig. 1".

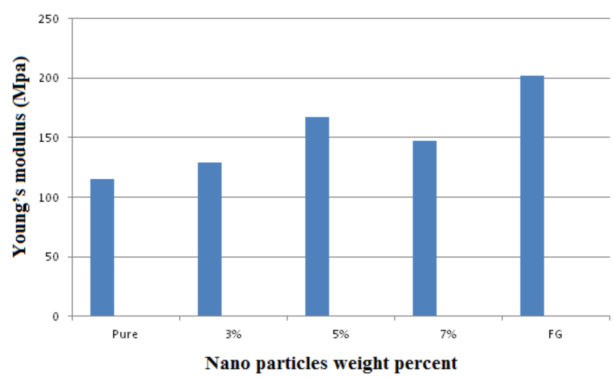

Fig. 1 The Young's modulus of nanocomposites with the different distribution of Nanoparticles.

It is shown that elastic modulus begins to increase up to 5 wt% of Nanoclay and then decreases. So, for functionally graded distribution, the elastic modulus is generally larger than the corresponding values for uniform distribution of Nanoclay. The agglomeration of Nanoclay particles on 7 wt% percentage of Nanoclay in epoxy/clay nanocomposite can be seen where this effect verifies the decrement of elastic modulus compared with other specimens.

2.3. Free Vibration Tests

The free vibration tests were carried out using the B&K testing machine from Denmark. The test specimens have a rectangular cross section 0.002m × 0.0004m and a length of 0.2m with simply supported at both ends. Once the beam is excited, the laservibrometer measures its vibration response. A Laser Doppler Vibrometer (LDV) is a scientific instrument that is used to make non-contact vibration measurements of a surface. The data acquisition card collects the signals from the laser vibrometer and sends them to the dynamic signal analyser software. Finally, the signals are processed and the Fourier was transformed to get the natural frequencies of the beam.

3 MATHEMATICAL MODELING

In computer science and operations research, a genetic algorithm (GA) is a metaheuristic inspired by the process of natural selection that belongs to the larger class of evolutionary algorithms (EA). Genetic algorithms are commonly used to generate high-quality solutions to optimization and search problems by relying on bio-inspired operators such as mutation, crossover and selection. The GA method searches for the best alternative (in the sense of a given fitness function) through chromosome evolution [13]. The basic steps in the GA analysis are shown in "Fig. 2".

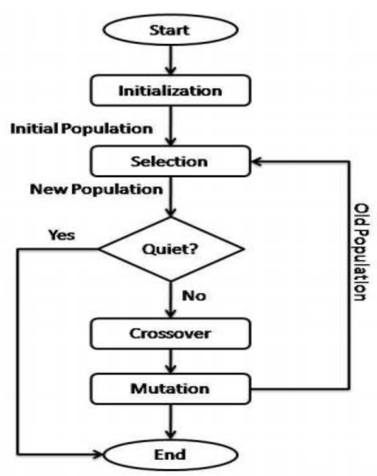

Fig. 2 High level description of the GA.

As shown above, an initial population of chromosomes is randomly selected firstly. Then each of the chromosomes in the population is evaluated in terms of its fitness (expressed by the fitness function). Next, a new population of chromosomes is selected from the given population by giving a greater chance to select chromosomes with higher fitness. This is called the reproduction operation. The new population may contain duplicates.

If the given stopping criteria (e.g., no chance in the old and new population, specified computing time, etc.) are not met, some specific, genetic-like operations are performed on chromosomes of the new population. These operations produce new chromosomes, called offsprings. The same steps of this process, evaluation and reproduction operation are then applied to chromosomes of the resulting population. The whole process is repeated until the given stopping criteria are met. The solution is expressed by the best chromosome in the final population. In this paper, the "$1-R_{adj}^2$" is introduced as the fitness function which is to be minimized. "R_{adj}^2" is accuracy criterion of an arbitrary mechanical property function (such as Young's modulus). "R_{adj}^2" is defined as a process which is demonstrated below. The mechanical property is a function of Nanoclay weight percent and "R_{adj}^2" is a function of coefficients which are introduced below. M_i is considered as the mechanical properties and "W" as the Nanoclay weight percent. The M_i is expressed as a polynomial function of "W" as follows:

$$M_i = \sum_{j=0}^{4} a_{ji} W^j \tag{1}$$

Now, the coefficients a_{ji} are found by maximizing the accuracy of the polynomial function. The equations can be written as:

$$R_{adj}^2 = 1 - \frac{VAR_E}{VAR_T} \tag{2}$$

In which:

$$VAR_E = SS_{Err}/(n-k-1) \tag{3}$$

$$VAR_T = SS_{Tot}/(n-1) \tag{4}$$

$$SS_{Tot} = \sum_{i=1}^{n} (y_i - \bar{y})^2 \tag{5}$$

$$SS_{Err} = \sum_{i=1}^{n} (y_i - M_i)^2 \tag{6}$$

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \qquad (7)$$

$$M_i(W) = a_{0i} + a_{1i}W + a_{2i}W^2 + a_{3i}W^3 \qquad (8)$$

In these equations $n = 4$ is the number of experiments and $k = 0$ is the number of duplicated experiments and y_i shows the experimentally measured mechanical properties. After minimization of "$1 - R^2_{adj}$" via MATLAB, factors a_{ji} are obtained after approximately 40 generations. Obtaining the a_{ji} coefficients, the Young's modulus can be expressed as the functions of Nanoclay weight percent as follows:

$$E = -25.145w + 14.276w^2 - 1.432w^3 + 115.108 \qquad (9)$$

Here, w is the Nanoclay weight percent. To investigate the validation of the present results, comparison studies are carried out for the Young modulus of uniform distribution nanocomposites as presented in "Table 2".

Table 2 Comparison of Young's modulus for uniform distribution nanocomposites

Nanoclay weight percent	Theoretical predictions (Mpa)	Experimental results (Mpa)
pure	115.108	115.108
3%	129.493	129.486
5%	167.283	167.258
7%	147.441	147.379

The compression between theoretical predictions and the experimental data shows the high accuracy of the present analysis. "Eq. (9)" can be used to derive the suitable relation for Young's modulus of functionally graded distribution. The specimen with functionally graded distribution consists of four perfectly bonded sheets with a total thickness of 4 mm. Each sheet has 1 mm thickness with different nanoparticles weight fractions (pure, 3 wt.%, 5 wt.% and 7 wt %). The Young's modulus can be written as:

$$E(z) = -25.145(2[z] + Sgn[z]) + 14.276(2[z] + Sgn[z])^2 - 1.432(2[z] + Sgn[z])^3 + 115.108 \qquad (10)$$

As mentioned before, the Young's modulus is assumed to vary as a function of the thickness coordinate $z(0 \le z \le 4)$. "Eq. (10)" can be verified via employing the free vibration analysis of functionally graded nanocomposite beam.

4 THEORETICAL FORMULATION

The formulation is presented based on the assumptions of the first order shear deformation beam theory. Based on this theory, the displacement field can be written as [14]:

$$\begin{aligned} u(x,z,t) &= z\phi(x,t) \\ w(x,z,t) &= w_0(x,t) \end{aligned} \qquad (11)$$

In the view of the displacement field given in "Eq. (11)", the strain displacement relations are given by [14]:

$$\begin{aligned} \varepsilon_{xx} &= \frac{\partial u}{\partial x} = z\frac{\partial \phi}{\partial x} \\ \gamma_{xz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \phi + \frac{\partial w}{\partial x} \end{aligned} \qquad (12)$$

Consider a functionally graded beam with a rectangular cross-section as shown in "Fig. 3".

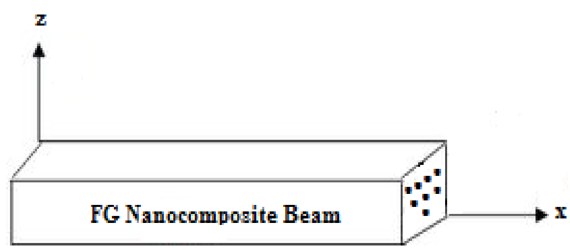

Fig. 3 Schematic of the problem studied.

The thickness, length, and width of the beam are denoted by h, L and b, respectively. The $x - y$ plane coincides with the midplane of the beam and the z-axis located along the thickness direction. The Young modulus E is assumed to vary as a function of the thickness coordinate $z(0 \le z \le 4)$. The constitutive relations for the functionally graded beam are given by [15]:

$$\begin{aligned} \sigma_{xx} &= E(z)\varepsilon_{xx} \\ \sigma_{xz} &= G(z)\gamma_{xz} \end{aligned} \qquad (13)$$

Where $\sigma_{xx}, \sigma_{xz}, E(z)$ and $G(z)$ are the normal stress, shear stress, the Young and shear modulus respectively. The shear modulus can be written as [16]:

$$G(z) = \frac{E(z)}{2(1+\nu)} \qquad (14)$$

where ν is the Poisson ratio, estimated with the aid of the relation [17]:

$$\frac{\sigma_y}{E} = \frac{1-2\nu}{6(1+\nu)} \tag{15}$$

Where σ_y is the yield strength from the tensile test result. In fact, the total yield strength for the whole of the functionally graded sample can be obtained from tensile test and calculated by using machine software like the elastic modulus for a functionally graded sample. Also, the Poisson ratio assumed to be constant through the thickness of a beam [18]. So, the "Eq. (15)" can be used to estimate the Poisson ratio for the whole of the sample. Also, u and w are the displacement components in the $x-$and z-directions, respectively. The potential energy can be expressed as [14]:

$$U = \frac{1}{2}\int_v (\sigma_{xx}\varepsilon_{xx} + \sigma_{xz}\gamma_{xz})\, dv \tag{16}$$

The width of a beam is assumed to be constant, which is obtained by integrating along y over V. Substituting "Eq. (12) and Eq. (13) into Eq. (16)" and neglecting the higher-order terms, we obtain

$$U = \frac{b}{2}\int_0^L [D(\frac{\partial\phi}{\partial x})^2 + A(\phi + \frac{\partial w}{\partial x})]dx \tag{17}$$

Where:

$$D = \int_{-\frac{h}{2}}^{+\frac{h}{2}} E(z)z^2 dz \ , \qquad A = K_S \int_{-\frac{h}{2}}^{+\frac{h}{2}} G(z)dz \tag{18}$$

Where D, A and K_s are the bending stiffness, shear stiffness and shear correction factor, respectively. The shear correction factor can be expressed as [18]:

$$K_s = \frac{5}{\left(6-\left(\nu_1 V_1 + \nu_2 V_2\right)\right)} \tag{19}$$

where ν_1 and ν_2 are Poisson's ratios of the Nanoclay and the matrix, respectively. Whereas, V_1 and V_2 are the Nanoclay and the matrix volume fractions, respectively. The quantity of $(\nu_1 V_1 + \nu_2 V_2)$ for present nanocomposite is infinitesimal and the shear correction factor can be assumed to $K_s = 5/6$ [18]. The kinetic energy can be expressed as [15]:

$$T = \frac{b}{2}\int_0^L \left[I_1(\frac{\partial w_0}{\partial t})^2 + I_3(\frac{\partial\phi}{\partial t})^2 \right] dx \tag{20}$$

Where:

$$I_1, I_3 = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \rho(z)\{1, z^2\} dz \tag{21}$$

Where I_1 and I_3 are the mass moments. We apply the Hamilton principle to derive the equilibrium equations of the beam as follows [14]:

$$\delta\int_0^t (T - U + W)\, dt = 0 \tag{22}$$

Where W is the work done by the external load. Substituting "Eq. (17) and Eq. (20) into Eq. (22)" leads to the following governing equations of the functionally graded beam:

$$A(\phi + \frac{\partial w_0}{\partial x}) = I_1 \frac{\partial^2 w_0}{\partial t^2} \tag{23}$$

$$D\frac{\partial^2\phi}{\partial x^2} - A(\phi + \frac{\partial w_0}{\partial x}) = I_3 \frac{\partial^2\phi}{\partial t^2} \tag{24}$$

5 FREE VIBRATION ANALYSIS

The boundary conditions for the simply supported beam are given by:

$$w = \frac{\partial^2 w}{\partial x^2} = \frac{\partial\phi}{\partial x}, \qquad \text{at} \quad x = 0 \quad \text{and} \quad x = L \tag{25}$$

As the term ϕ is eliminated from "Eq. (23) and Eq. (24)" and combining these two equations, the governing differential equation for free vibration of the functionally graded beam can be obtained. The transverse displacement was chosen as:

$$w(x,t) = w(x)e^{i\omega t} \tag{26}$$

Where w is the modal displacement and ω is the circular frequency. Substituting "Eq. (26)" into the governing differential equation and applying the boundary conditions, the free vibration frequency of the functionally graded beam is derived as follows:

$$\omega_n = (\frac{n\pi}{L})^2 (\sqrt{\frac{D}{I_1}}) \sqrt{\frac{A}{A + (\frac{n\pi}{L})^2 D}} \tag{27}$$

6 RESULTS AND DISCUSSION

The free vibration analysis of the simply supported functionally graded nanocomposite beams based on the first order shear deformation theory is studied. The material compositions of the nanocomposite beam are listed in “Table 1”. The effect of nanoparticles with different weight fractions on the free vibration frequency are shown in “Fig. 4”.

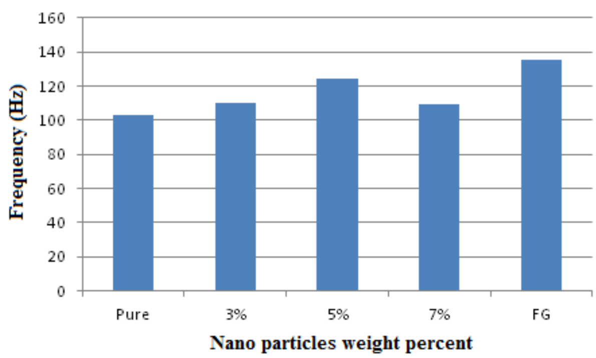

Fig. 4 The effect of nanoparticles with different weight fractions on the free vibration frequency.

It is shown that that the free vibration frequency for the beams with uniform distribution of nanocomposite is generally lower than the corresponding values for the beams with the functionally graded distribution of Nanocompite. Also, it is seen that the free vibration frequencies for the beams with uniform distribution of nanoparticles increase by increasing nanoparticles weight percent up to 5 wt%. By increasing the amount of Nanoclay more than 5 wt%, the free vibration frequency is found to decrease. To investigate the validation of the present results, comparison studies are carried out for the first mode of free vibration frequencies of uniform and functionally graded distributions nanocomposites as presented in “Table 3”.

Table 3 Comparison of the first mode of free vibration frequencies for nanocomposite beams

Nanoclay weight percent	Theoretical free vibration frequency (Hz)	Experimental free vibration frequency (Hz)
pure	103.15	98.73
3%	109.92	106.39
5%	124.33	122.19
7%	109.38	102.38
FG	135.50	131.53

The compression between theoretical and the experimental data shows the high accuracy of the present analysis.

7 CONCLUSIONS

The free vibration analysis of functionally graded and uniform distributions Epoxy/clay nanocomposite beams is studied. The Young modulus for uniformly distributed nanocomposites is calculated using a genetic algorithm procedure. The formulation is included the effect of nanoparticles weight fractions in the calculation of the Young modulus. Also, it is modified for functionally graded distribution to take into account the Young modulus as a function of the thickness coordinate. By the addition of Nanoclay particles, the elastic modulus begins to increase up to 5 wt% of Nanoclay. This may be due to the existence of strong and sufficient bonding between epoxy and Nanoclay particles. The agglomeration of Nanoclay particles may be due to the decrease of the elastic modulus for 7 wt% of Nanoclay. In other words, these particles are changed to stress concentration sites and positions for crack propagation. Also, for functionally graded distribution, the elastic modulus is generally larger than the corresponding values for uniform distribution of Nanoclay. The comparison between theoretical predictions and the experimental data for the elastic modulus of uniform and functionally graded distributions shows the high accuracy of genetic algorithm procedure. The displacement field of the beam is assumed based on the first order shear deformation beam theory. Applying the Hamilton principle, the governing equations are derived. The influence of nanoparticles on the free vibration frequencies of a beam is presented. The free vibration frequency for uniform nanoparticles distribution increases by increasing nanoparticles weight percent up to 5 wt% and then for the amount of Nanoclay more than 5 wt%, the free vibration frequency is found to decrease. Also, the free vibration frequencies for uniform distribution are generally lower than the corresponding value of the functionally graded distribution.

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