Modal Analysis of Complex Structures via a Sub-Structuring Approach

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Abstract: In this paper, the problems arising from determining the modal properties of large and complex structures are investigated. For this purpose, the free interface component mode synthesis method has been used. In the following, Singular-Value Decomposition (SVD) is applied as a powerful mathematical tool to determine the appropriate coordinates to participate in the coupling process. Also, the effective error resources including modal shear error and the continuous systems overlapping error and their solution are introduced. Initially, a discrete system has been employed to investigate the free interface component mode synthesis method. Eventually, the studied main samples in this research are beam, plate and cylindrical shell. It is worth noting that the application of this method on the cylindrical shell has not been observed in previous researches.

Keywords: Component Mode Synthesis, Free Interface Method, Modal Analysis, Substructures, Singular-Value Decomposition Method

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1 INTRODUCTION

The necessity of dynamic analysis of structures in industry in order to predict the response of structures under different loads is very high. Access to the dynamic characteristics of the structure is one of the most basic parts of dynamic analysis. Therefore, modal analysis has become a comprehensive knowledge with the aim of determining, improving and optimizing the dynamic characteristics of engineering structures [1]. The finite element method is a great step for analyzing the static and dynamic problems of highly complex structures. In order to increase the speed of solving many problems, instead of modeling the equations of a problem for the whole structure, another method can be used. The basic principle of this approach relies on the assumption that the entire structure consists of unique components such as beam, plate, and shell so that the model of the entire structure will be assembled of these components in the final stage. Therefore, some information is needed about the properties of its components in order to accurately analyze the structure, which is not available at the early stages of the design. As a result, the use of sub-structuring methods has recently become commonplace for the modal analysis of large structures in various industries. The Component Mode Synthesis method (CMS) is one of the best and most accurate sub-structuring techniques for the modal analysis of a structure. The main goal of using this method is to reduce the computational costs and conduct the modal test of large and multi-part structures. The entire structure response is obtained by assembling the analysis results of each of the substructures. CMS allows to modify a single subsystem without changing the full problem formulation [2]. The component mode synthesis can be used for vibrational analysis of the entire structure. In fact, it uses the eigenvalues and eigenvectors' combinations to model the dynamic behavior of the components. This method is a two-part analytical method. In the first stage, the components are analyzed separately and their answers are obtained. In the next step, these responses are combined together with observing the condition of continuity between the interfaces and create an eigenvalue problem, using which, the eigenvalues and eigenvectors of the entire structure can be extracted. The analyzers can reduce the order of the substructures used in the entire structure. To do so, only the modes of components are considered that have an important effect on the overall structure response [3-4].

The impedance coupling method is used for many dynamic problems. Extensive research has been done in this field [3-7]. Lutes and Heer [8] solved this problem by filtering the elements of the frequency response matrix. Other methods have been presented in the works of Ewins [9], Gleeson [10], and Robb [11], so that the inconsistency is resolved using the raw data modal analysis and then by employing the frequencies response rebuilt to retrieve and improve the data. The modal coupling method is another coupling approach. Its main basis is similar to previous methods; i.e., it uses the reduced-order components model to reduce the order of the matrices associated with the assembled structure. There are basically two modal coupling methods. These methods include fixed-interface and free-interface approaches. The elastic modes dependent on the fixed-interfaces of components are examined in the fixed-interface methods, while the modes are examined with the assumption of the freely-supported components vibration in the free-interface methods. The main idea behind the fixed-interface methods is based on the Przemieniecki static method [12]. Hurty [13] suggested the method of normal mode or the component mode synthesis. At this time, he was working on the dynamic systems and examining the elastic properties of the components along with their mass properties. Craig and Bampton [14] modified the Hurty method formulas by simplifying the selection of the modes for the transformation matrix. In the free interface method, the natural modes necessary for a transformation matrix are those obtained from the freely-supported vibrations of subsystems. However, the simplest conditions were used to simulate the test conditions, which involved an interesting combined method from theoretical and empirical analysis of the dynamic system. Gladwell and Goldman have used the free interface modes in their studies and called this method the branch-mode analysis method [15-16]. Some studies have explicitly pointed out that the use of free interface methods has less accuracy compared to the fixed interface methods [17-21]. A number of researchers have also used the theoretical methods for coupling [22-26], while others have tried to use the experimental modal properties to formulate the motion equations of subsystems. After that, several studies have been presented with different ideas including identification of modal parameters for large and complex structures by numerical and experimental modal analysis approach [27-33]. Karpel and Ricci [34] have studied an experimental coupling technique in a large structure. The validation of this method was evaluated by applying two different sub-structure. An and Lee [35] have improved a component mode synthesis method according to frequency response functions to solve a security problem consisting of two sub-structures. Yangui et. al [36] have used the experimental and numerical modal analysis to identify the eigenfrequencies first five of the blade by the Craig-Bampton sub-structure theory.

In this research, the free interface component mode synthesis method has been used for modal analysis of beams, plates and cylindrical shells. In order to investigate this method, first the modal analysis of the

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discrete system has been performed. In this analysis, the discrete system was divided into several parts, and assuming the full use of all the modes, the modal analysis was done. Then, this method was used to examine the continuous systems. It is not possible to access all modes in the continuous systems. Therefore, the reduction in the number of modes shape was studied in this section. After examining the beam model, the plate model was studied. Finally, this method was implemented on a cylindrical shell sample and the error rate of this method was assessed. For validation, the results obtained from component mode synthesis method were compared with the modal results of the integrated structure in each of stages (beam, plate and cylinder) and the resulting error was investigated.

2 GOVERNING EQUATIONS

2.1. The Free Interface Component Mode Synthesis Method

The methods, in which, the normal free interface modes are used for moving from physical to modal space, are known as free interface methods. In this section, the free interface method proposed by Hou [37] is described assuming that the systems are non-damping and rigidly connected [38]. In the free interface method, the shape of the modes of each substructure was initially extracted. Using the degrees of freedom of the interfaces of this form of modes, the required transformation functions were extracted followed by obtaining the matrices of mass and stiffness equivalent to the original system. By solving the eigenvalue of these matrices, the mode shapes and natural frequencies of the main system can be obtained. Finally, using the inverse transformation, the mode shapes were obtained in the main modal coordinates for the main structure. Consider two non-damping substructures of A and B that are rigidly connected to each other. The degrees of freedom of each substructure were classified into internal degrees of freedom of i and boundary degrees of freedom of c. The equilibrium equation of any substructure in the physical domain, assuming that no force is applied to the internal degrees of freedom, is as follows:

$$\begin{bmatrix} M_{ii} & M_{ic} \\ M_{ci} & M_{cc} \end{bmatrix} \begin{Bmatrix} \ddot{x}_i \\ \ddot{x}_c \end{Bmatrix} + \begin{bmatrix} K_{ii} & K_{ic} \\ K_{ci} & K_{cc} \end{bmatrix} \begin{Bmatrix} x_i \\ x_c \end{Bmatrix} = \begin{Bmatrix} 0 \\ f_c \end{Bmatrix} \quad (1)$$

For each substructure, the normal modes were calculated by placing the boundary forces of f_c equal to zero and solving the following eigenvalue equation:

$$(-\omega^2 [M] + [K])\{x\} = \{0\} \quad (2)$$

Assuming that the modes are normalized according to the mass and due to their perpendicularity feature toward mass and stiffness, it is concluded:

$$\begin{aligned} [\phi_m]^T [M][\phi_m] &= [I] \\ [\phi_m]^T [K][\phi_m] &= diag(\omega_r^2) \end{aligned} \quad (3)$$

Any vector of displacement in the physical space can be transferred to the modal space by this form of orthogonal modes. The new coordinates are referred to as modal coordinates or generalized coordinates and their components are the coefficients of the form constituting the linear composition of the displacement vector:

$$\begin{Bmatrix} x_i \\ x_c \end{Bmatrix} = \begin{bmatrix} \phi_{im} \\ \phi_{cm} \end{bmatrix} \{p_m\} = [\phi_m]\{p_m\} \quad (4)$$

If "m" is the number of all modes, “Eq. (4)” is perfectly accurate. However, if the first k modes are considered, the answer for each substructure will be approximately as follows:

$$\begin{Bmatrix} x_i \\ x_c \end{Bmatrix} = \begin{bmatrix} \phi_{ik} \\ \phi_{ck} \end{bmatrix} \{p_k\} = [\phi_k]\{p_k\} \quad (5)$$

Therefore, the equation of motion of the entire structure can be written as follows for two separate substructures A and B:

$$[I] \begin{Bmatrix} {}_A P_k \\ {}_B P_k \end{Bmatrix} + \begin{bmatrix} diag({}_A \omega_{rk}^2) & 0 \\ 0 & diag({}_B \omega_{rk}^2) \end{bmatrix} \begin{Bmatrix} {}_A P_k \\ {}_B P_k \end{Bmatrix} = \begin{bmatrix} {}_A \phi_{ck}^T & 0 \\ 0 & {}_B \phi_{ck}^T \end{bmatrix} \begin{Bmatrix} {}_A f_c \\ {}_B f_c \end{Bmatrix} \quad (6)$$

By writing the force equilibrium and displacement conditions, the two substructures can be connected together. These conditions include:

$$\begin{aligned} \{{}_A f_c\} &= -\{{}_B f_c\} \\ \{{}_A X_c\} &= \{{}_B X_c\} \end{aligned} \quad (7)$$

In the modal space, it can be stated as follows:

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$$\begin{bmatrix} {}_A\phi_{ck} & \cdots & -{}_B\phi_{ck} \end{bmatrix} \begin{Bmatrix} {}_A P_k \\ {}_B P_k \end{Bmatrix} = [S]\{p\} = \{0\} \quad (8)$$

Then, for further reduction, the matrix S is classified into two parts:

$$\begin{bmatrix} S_d & \cdots & S_i \end{bmatrix} \begin{Bmatrix} P_d \\ P_i \end{Bmatrix} = \{0\} \quad (9)$$

Where, S_d is a non-singular square matrix and S_i is the remainder of the matrix S. Then, one can write:

$$\begin{Bmatrix} {}_A P \\ {}_B P \end{Bmatrix} = \begin{Bmatrix} P_d \\ P_i \end{Bmatrix} = \begin{bmatrix} -S_d^{-1} S_i \\ I \end{bmatrix} \{p_i\} = [T]\{q\} \quad (10)$$

2.2. Selection of The Acceptable Coordinates for Connecting Points

During the coupling process, especially when the matrices are inverted, the numerical problems are seen better and clearer in the inversion process. Otherwise, the numerical error due to these problems still remains in methods of coupling that do not need the inversion stage. In the inversion process during the coupling process, when the matrix is singular or close to singular, the predicted dynamic properties for the final structure are meaningless and have a large error. In general, the more redundancy in the coordinates participated in the coupling process, more unfavorable the answers become, which will ultimately become totally meaningless. The extra degrees of freedom must be eliminated before the coupling process to solve this numerical problem. However, this may cause error as well since the degrees of freedom may be removed from the connection points, which has a greater and more important effect and the coupling results with those degrees of freedom become more accurate. To this end, a powerful algorithm should be used to categorize and select the degrees of freedom in order of priority. One of the important tools used for this purpose is the decomposition of the singular value that not only determines the order of a matrix but also can be used to reverse the matrix whether it is singular or not [38].

2.2.1. The Order and Inverse of a Matrix

2.2.1.1. The Singular-Value Decomposition (SVD) and The Invertible Matrix

Here, this tool is used to obtain the right coordinates (or to detect the additional coordinates) as well as to calculate the inverse of an incomplete matrix [38]. The singular-value decomposition of a real matrix such as [A] results in three matrices, which are related to each other as follows:

$$[A]_{m\times n} = [U]_{m\times m} \begin{bmatrix} \begin{bmatrix} \ddots & & \\ & \Sigma & \\ & & \ddots \end{bmatrix} \\ 0 \end{bmatrix}_{m\times n} [V]^T_{n\times n} \quad (11)$$

By performing SVD on any of the FRF or modal matrices, [U] and [V] will gain the following properties:
$[U]m\times m$, the left singular vectors, represent the most appropriate coordinates for displaying the answer. Thus, the first singular vector, $\{u_1\}$, is the easiest direction that the system can change within. The second singular vector $\{u_2\}$, is the easiest next direction, and so on.
$[V]n\times n$, the right singular vectors, represent the most suitable coordinates for representing forces or modes. The first singular vector, $\{v_1\}$, represents a combination of forces (or modes) that have the greatest effect on the system. The $\{v_2\}$ is the most effective next combination, and it goes on for the rest of the vectors. If the sides from "Eq. (11)" are inversed, then, the pseudo-inverse of matrix [A] is obtained, which is shown by $[A]^+ m\times n$.

$$[A]^+_{n\times m} = [V]_{n\times n} \begin{bmatrix} \ddots & & \\ & \Sigma & \\ & & \ddots \end{bmatrix}^{-1}_{n\times m} [U]^T{}_{m\times m} \quad (12)$$

2.2.1.2. Rank Degeneracy

The singular-vale decomposition is a reliable tool for numerical calculation of a matrix rank. For a matrix such as $[A]m\times n$ with a rank of $(r<n)r$, it is assumed that some of the elements may experience error. It is unlikely that after the error occurrence, the matrix r rank remains intact. In other words, with a slight change in some of the elements (due to the error), the matrix can turn into a full rank matrix. However, when the matrix A is undergone numerical algorithms, it shows a behavior like incomplete order matrices. Therefore, to fix this problem, a range should be assigned for determining the rank. In fact, a contractual parameter such as δ must be defined, that according to which, if the σ_i applies to the following conditions, then, the rank of the matrix will be r.

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$$\sigma_1 \geq \sigma_2 \geq ... \geq \sigma_r \geq \delta \geq \sigma_{r+1} \geq ... \geq \sigma_n \qquad (13)$$

Globe and Van Lewin proposed the δ as follows:

$$\delta = 10^{-2} \|A\|_\infty \qquad (14)$$

The ‖A‖_∞ is the infinite norm of a matrix. This criterion is appropriate when there is no precise separation between small and large values in the eigenvalues.

2.2.1.3. Orthogonality in The Continuous Systems
For discrete systems, when speaking of orthogonality, the displacement of any degree of freedom is included in the mode shape. If X, is considered as the displacement of each degree of freedom, the orthogonality relationship between the two modes "a" and "b" is as follows for an n-degree of freedom system:

$$\sum_{i=1}^{n} m_i X_i^a X_i^b = 0 \qquad a \neq b \qquad (15)$$

For a continuous system, the orthogonality relationship must be written for each component. For example, the orthogonality relationship for two modes "a" and "b" in the beam is as follows:

$$\int_0^1 \rho\phi(x)\phi_b(x)dx = 0 \quad , \quad a \neq b \qquad (16)$$

The ρ is the mass of the unit of length. As can be seen, the mode shape is defined for each point of the beam and is a function of the longitudinal variable of the beam (x). For a continuous system like the plate, which components are expanded in two directions, the orthogonality relation for the two modes "a" and 'b" is written as follows:

$$\int\int \rho\phi_a(x,y)\phi_b(x,y)dxdy = 0 \quad , \quad a \neq b \qquad (17)$$

3 EXAMINING A DISCRETE SAMPLE WITH SEVERAL COMMON DEGREES OF FREEDOM

Figure 1 shows an asymmetric structure with 4 degrees of freedom, which consists of two substructures of 4 and 3 degrees of freedom. The unit of masses is kg and the unit of hardness is KN/m, and the connection between the common degrees of freedom is rigid.

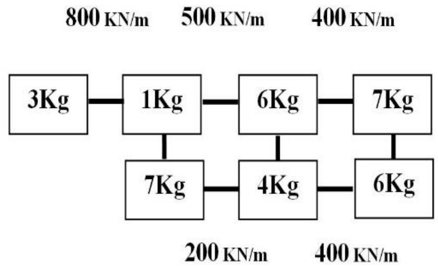

Fig. 1 An asymmetric sample model with 4 degrees of freedom.

For the model presented in "Fig. 1", the matrices of the mode shape of the two substructures are as follows:

$$[\phi_{left}] = \begin{bmatrix} 0.242 & 0.375 & 0.306 & -0.200 \\ 0.242 & 0.273 & 0.078 & 0.927 \\ 0.242 & 0.070 & -0.315 & -0.057 \\ 0.242 & -0.260 & 0.128 & 0.002 \end{bmatrix}$$

$$[\phi_{rightt}] = \begin{bmatrix} 0.242 & -0.281 & 0.068 \\ 0.242 & 0.105 & -0.424 \\ 0.242 & 0.258 & 0.203 \end{bmatrix}$$

In the mode shape matrix, each row represents a degree of freedom and each column represents a mode shape. The last three rows of the mode shape matrix of the left substructure and all the rows of the mode shape matrix of the right substructure are related to the degrees of freedom of the interfaces. Using this, the matrices S and T are formed as follows.

$$[S] = \left[\begin{array}{ccc|cccc} 0.242 & 0.273 & 0.078 & 0.927 & -0.242 & -0.281 & -0.068 \\ 0.242 & 0.0702 & -0.315 & -0.057 & -0.242 & -0.105 & 0.424 \\ 0.242 & -0.260 & 0.128 & 0.002 & -0.242 & -0.258 & -0.203 \end{array}\right]$$

$$[T] = \begin{bmatrix} -1.212 & 1 & 0.167 & 0.001 \\ -1.876 & 0 & -1.053 & -0.131 \\ -1.530 & 0 & -0.440 & 1.315 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The first element of matrix S forms the square section, i.e., the S_d, and the remaining elements generate the matrix S_i. The T transformation matrix affects a state of the equation of motion, in which, only the modal properties of the system exist.

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The graph of the point frequency response function for two main system modes and the coupling of the two substructures by the free interface method for the degree of freedom 2 (masses of 1 and 7 kg) are shown in “Fig. 2”.

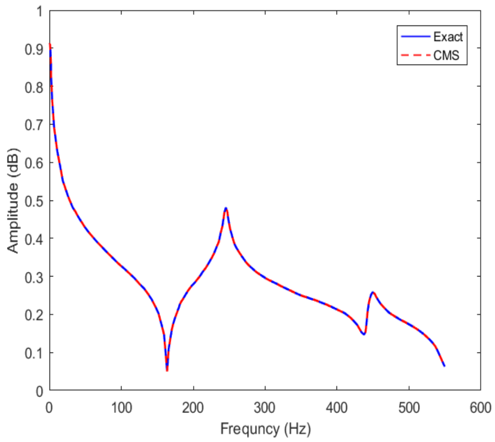

Fig. 2 Comparing the point frequency response function of the degree of freedom 2 by two Exact and CMS methods.

4 MODELING AND EXAMINING THE CONTINUOUS SYSTEMS

In this section, the performance of component mode synthesis method in continuous systems is investigated. The major difference between discrete and continuous systems is in the degrees of freedom used in the computing. Thus, the component mode synthesis method is expected to involve error after discrete-making of a continuous system since it cannot use the finite mode shapes in its calculations.

4.1. Studying The Beam System

In this section, the component mode synthesis method on the beam structure is implemented. Initially, the natural frequencies of this structure are extracted analytically and finite element method and compared with each other. After validation of the finite element method, the component mode synthesis method has been implemented on the beam and its performance in this system has been evaluated. The natural frequency of a Euler-Bernoulli beam in the general state is in accordance with the following equation:

$$\omega = \beta^2 \sqrt{\frac{EI}{\rho}} \tag{18}$$

The first 10 first answers from “Eq. (18)” for βL of a two free-end beam are given in “Table 1” (L is the length of the beam). The answers vary to examine a system in the finite element method depending on the structure dividing into several elements. Usually the answer converges to its final value by increasing the number of elements. However, there may be a difference between the convergence value and the exact answer as well. On the other hand, there may also be errors due to the numerical calculations.

The finite elements results are displayed for a 2-meter and two-dimensional beam in “Table 2”, which natural frequencies are calculated once with 10 elements and again with 100 elements. Finally, their values were compared with the analytical results. The cross-section area of the examined beam and its moment of inertia are $4 \times 10^{-4}\ m^2$ and $1.33 \times 10^{-8}\ m^4$, respectively. The beam is made of steel with a 200Gpa Young's Modulus and a density of $7800\ kg/m^3$.

As can be seen, the discrepancy (difference rate) has declined by increasing the number of elements, which indicates the proper function of the finite elements code. The beam information was used as a continuous system for the substructures. It was assumed in the first sample that two 1-meter beams with a square cross-section of 2-Cm side are connected at a point in a rigid form in accordance with “Fig. 3”. It should be noted that the shape of the modes must be normalized relative to the mass before using in the coupling process.

Different scenarios can be imagined depending on the number of participating modes from each substructure in the coupling process. “Table 3” shows the comparison of the first five natural frequencies generated by the coupling of two subsystems of “Fig. 3” with the substructure modal analysis results. In the three cases studied, 5, 10, and 15 first elastic modes (other than rigid body motions) of each substructure are included in the coupling process. The information related to the mode shape in each substructure is read at 10 points with equal distances from each other. As can be seen, the disparity rate is negligible with the component mode synthesis method.

Table 1 Ten first responses of “Eq. (18)”

Row	1	2	3	4	5	6	7	8	9	10
βL	4.73	7.853	10.996	14.137	17.278	20.420	23.562	26.703	29.845	32.987

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Table 2 Comparison of the natural frequency resulting from the finite element of two different elements with analytical results in Hz

Number of modes	Analytical results	10 elements	Difference percentage	100 elements	Difference percentage
1	26.022	26.023	0.003	26.022	0.0003
2	71.731	71.748	0.024	71.731	0
3	140.621	140.750	0.092	140.621	0.0002
4	232.452	233.012	0.241	232.454	0.001
5	347.244	349.008	0.5082	347.246	0.0008
6	484.997	489.476	0.923	484.997	0.0001
7	645.706	655.295	1.485	645.707	0.0002

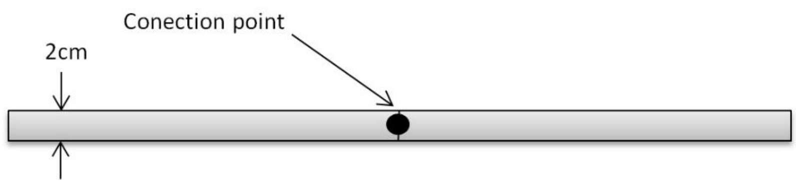

Fig. 3 Two one-meter beams connected together.

Table 3 Comparison of natural frequency obtained from the component mode synthesis method with the results of numerical analysis in Hz

Number of modes	Results of numerical analysis	Component mode synthesis with 3 elements in each part	Difference percentage
1	6.51	6.507	0.044
2	17.93	17.963	0.184
3	35.16	35.356	0.559
4	58.11	58.846	1.267
5	86.81	87.810	1.152

4.2. Examining the Plate System

The number of points of interfaces is the main difference between the plate and the beam. The number of points increases in the plate, which increases the error of the component mode synthesis methods. Thus, some methods are used to reduce the error of this method. The results of this numerical method for a plate with a distant joint were compared with the natural frequencies extracted from the analytical method and the accuracy of the numerical method was investigated. Finally, the free interface method was implemented on the plate. The plate modeling results were compared with the analytical solution of the plate to examine the suitable number of elements. The natural frequencies of a rectangular plate with the length of “a” and width of “b” with simply support boundary conditions around are represented by “Eq. (19)”.

$$\omega_{mn} = \sqrt{\frac{D}{\rho}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] \tag{19}$$

Where, ρ is the density of the plate and D is the flexural strength of the plate ($D = \frac{Et^3}{12(1-\upsilon^2)}$). Therefore, the natural frequencies of a square plate of steel with a thickness of 10 mm and a length of one meter were obtained. Considering a square plate with a length “a”, the natural frequency of the dimensionless plate was then defined in “Eq. (20)”. Ultimately, the dimensionless natural frequencies of a simply support square plate were obtained. As shown in “Table 4”, the numerical method used is acceptable and its results are very consistent with the analytical results.

$$\bar{\omega} = \omega_{mn} a\sqrt{\frac{\rho}{G}} \tag{20}$$

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Table 4 The dimensionless frequencies of a simply support square plate by a numerical method

Mode No.	M	N	Numerical method with 20×20 mesh	Analytical method
1	1	1	0.096	0.096
2	2	1	0.243	0.240
3	1	2	0.243	0.240
4	2	2	0.389	0.384
5	3	1	0.492	0.480
6	1	3	0.492	0.480
7	3	2	0.638	0.624
8	2	3	0.638	0.624
9	4	1	0.855	0.815
10	1	4	0.855	0.815
11	3	3	0.885	0.864
12	4	2	0.999	0.959

4.2.1. Component Mode Synthesis for the Plate

An important point to note is that the matrix S_d must be inverted in the process of the free interface method. This matrix is often singular in the plates. This phenomenon causes the calculations error to increase significantly. The Singular-Value Decomposition (SVD) method was used to avoid this phenomenon. In this section, the examined sample was first considered as two square steel plates with a one-meter side and a thickness of 1 mm in accordance with "Fig. 4".

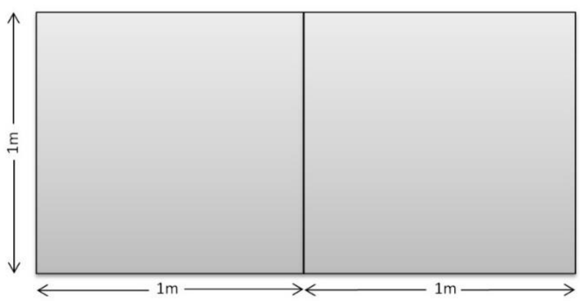

Fig. 4 Two square plates connected to each other.

The component mode synthesis method was applied for modal analysis of each of the squares separately. Also, by using this method the modal analysis of two interconnected squares was performed. To verify the accuracy of the results, the modal analysis with a 2×1 rectangular was performed and the results were compared with the component mode synthesis method. This was done with 21×21 meshing.

4.2.1.1. 21×21 Meshing

In this section, each square was meshed with 21× 21 elements (Fig. 5). The results of this analysis are presented in Table (5). The results of the component mode synthesis method with two square plates were compared with the results of a rectangular integrated plate with 21×42 elements in Fig. 6. As can be seen, with the increase in the number of elements in the plate, the results of the component mode synthesis method become more accurate and all the modes are acceptable except for the mode No. 3. Therefore, the component mode synthesis method is a quite proper and practical method for the modal analysis in the plate.

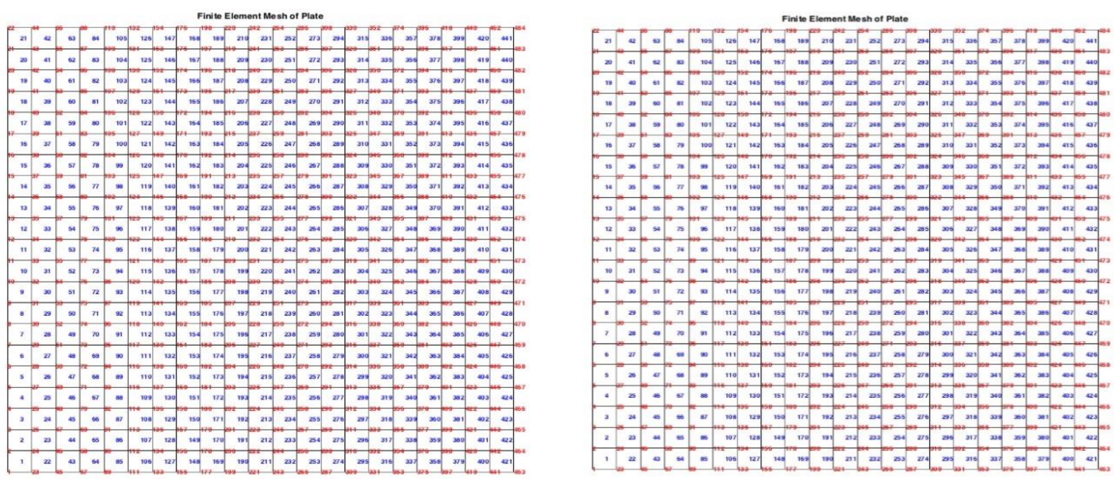

Fig. 5 Two square plates connected to each other with 21×21 elements.

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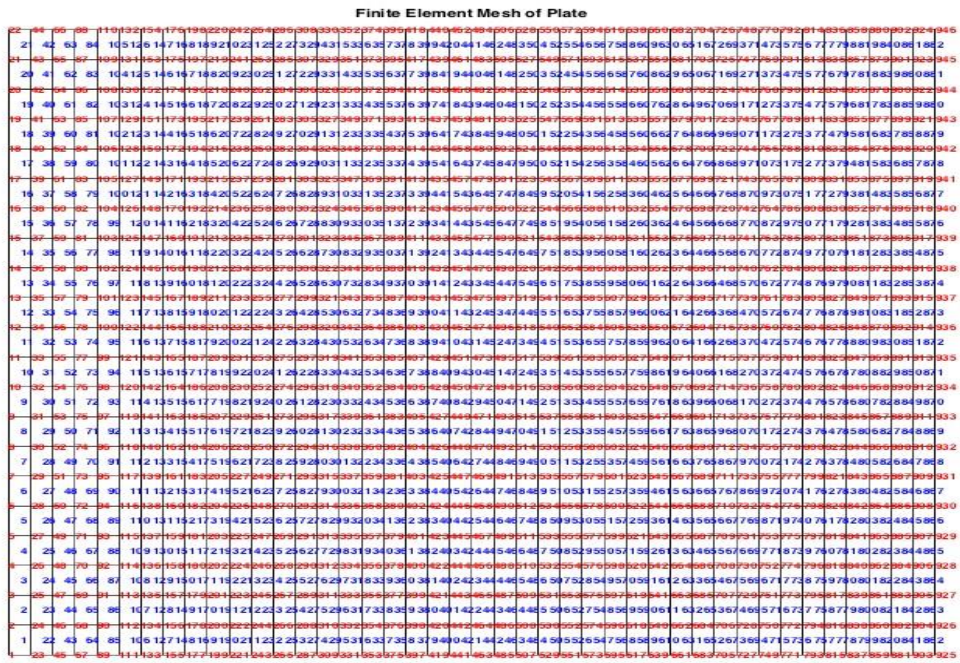

Fig. 6 A rectangular plate with 21×42 elements.

Table 5 Frequencies obtained from the CMS method and their comparison with the integrated rectangular plate mode

Rows	The natural frequency of the CMS method (Hz)	The natural frequency of solving the finite element of an integral sheet (Hz)	Difference in percentage
1	229.154	227.356	0.791
2	234.608	276.763	15.231
3	346.547	320.907	7.989
4	398.387	391.735	1.698
5	408.881	403.025	1.453
6	468.798	441.321	6.226

4.3. Examining the Cylindrical Shell System

In this section, the reduction of the modal rank as well as the reduction of the degrees of freedom were examined followed by studying the performance of the component mode synthesis method in the extraction of the frequency properties of cylindrical shells. In order to properly evaluate the CMS method, a single grid has been used in each step. Thus, the modal properties of the substructures and the entire structure were extracted from a certain grid. Hence, the difference between the component mode synthesis method and solving the integrated shell is merely due to the implementation of the component mode synthesis method. The main reason for choosing the shell is its extensive use in the industries as well as the difficulty of doing its modal test at large dimensions.

4.3.1. The Component Mode Synthesis Method for The Cylindrical Shell

What should be noted is that the matrix S_d needs to be inverted in the process of the free interface method. In the plates, this matrix is often singularized. This phenomenon causes the calculation error to increase significantly. The Singular-Value Decomposition (SVD) was used to avoid this phenomenon. In this study, two steel shells with a length of one meter, a thickness of 10 mm, and a radius of 10 cm were examined. To apply the CMS method, modal analysis each of the cylindrical shells was conducted separately. Also, by using the CMS method, the modal analysis of two interconnected cylindrical shells was performed. To verify the accuracy of the results, the modal analysis of a two-meter-long cylindrical shell was performed and compared with the results of the component mode synthesis method. This was done with two single and 11×21 meshing.

4.3.1.1. Single Meshing

In this section, the cylindrical shell was meshed with an element along the cylinder ("Fig. 7"). The nodes 13, 14, 15, 16, 17, 18, 19, and 20 of the lower cylindrical shell are connected together with the nodes 1, 2, 3, 4, 5, 6, 7, and 8 from the upper cylindrical shell. The results of the component mode synthesis method with two cylindrical shells ("Fig. 7") were compared with the results of the two-meter long cylindrical shell ("Fig. 8") in "Table 6". As can be seen, the results indicate the accuracy of the function of the component mode synthesis method and the difference between these two is due to singularity of the matrix S_d. The Singular-Value Decomposition (SVD) was used to compensate for the difference.

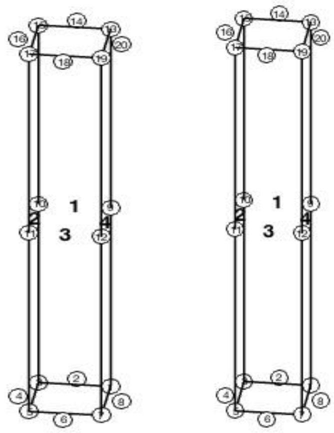

Fig. 7 Two cylindrical shells connected together with one element.

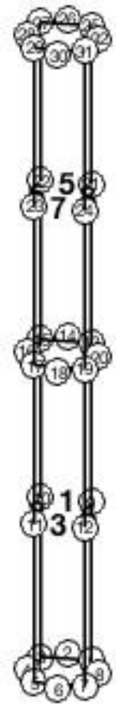

Fig. 8 A two-meter long cylindrical shell with two elements.

Table 6 Investigation of the component mode synthesis method in a cylindrical shell with an element in length

Row	Natural frequency of the component mode synthesis method (kHz)	Natural frequency from solving the finite element of the integrated cylindrical shell (kHz)	Difference in percentage
1	1.371	1.370	0.021
2	2.154	2.127	1.269
3	2.576	2.505	2.837
4	2.92	3.014	3.138
5	3.273	3.230	1.315

4.3.1.2. The 11×12 Meshing

The above process was meshed with an 11×12 matrix (Figs. 9 and 10). The results of this analysis are presented in "Table 7". It is observed that as the number of elements in the plate increases, the results of the component mode synthesis method become more accurate and all the modes are acceptable except for the mode No. 5. Therefore, the component mode synthesis method is a quite perfect and applied approach for modal analysis in the cylindrical shell. As can be seen, the error rate decreases in the component mode synthesis method with increasing number of the elements. This occurs since the number of modes used in this method increases with the increasing number of elements, and thus, the accuracy of this method enhances. To this end, the mean error rate of this method was provided for different numbers of elements in "Table 8". As can be seen, the average error rate decreases with the increased use of further longitudinal elements in the modes synthesis process. This phenomenon is applicable to each of the various shapes of modes.

In general, these errors arise from the fact that a continuous system is simulated with a discrete system, and thus, the number of degrees of freedom, and consequently, the number of modes used in the modes synthesis calculations has been reduced from an infinite value to a very limited value. Therefore, the resulting error is related to the reduction in the number of modes used in the calculations. Then, the number of modes shape is expected to increase by the increased degree of freedom. This error decreases to the boundary of zero. However, in practical activities and where the results of the experimental mode shape are used, it is inevitable to use a limited number of mode shapes and the occurrence of such an error is inevitable. Some solutions to remove this error such as the use of residual have been suggested in the activities undertaken in recent years by researchers [39].

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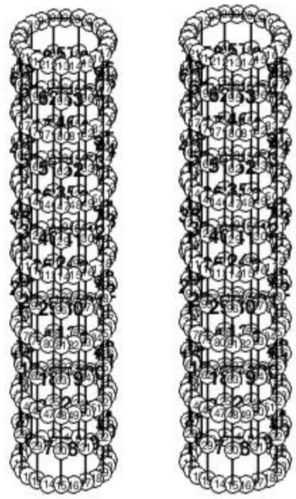

Fig. 9 Two cylindrical shells connected to each other with 6×11 elements.

Fig. 10 A cylindrical shell with 11×12 elements.

Table 7 Examining the component mode synthesis method in the cylindrical shell with six elements in length

Row	The natural frequency of the component mode synthesis method (kHz)	The natural frequency from solving finite element of the integrated cylindrical shell (kHz)	Difference in percentage
1	4.189	4.174	0.358
2	4.203	4.198	0.116
3	4.343	4.32	0.538
4	4.722	4.723	0.027
5	4.859	4.814	0.909

Table 8 The error of the component mode synthesis method in terms of the number of elements used in the calculations

Row	Number of longitudinal elements	Mean error (percent)
1	Two elements	1.7
2	Seven elements	1.05
3	Twelve elements	0.4
4	Fifteen elements	0.2

5 CONCLUSIONS

In this research, to achieve the technical knowledge of sub-structuring in the modal domain, it has been attempted to introduce the effective error resources including modal shear error (considering a limited number of modes) and the continuous systems overlapping error and their solution. As mentioned, the modal coupling methods often are recognized as Components Modes Synthesis or Components Modal Synthesis (CMS). To this end, the free interface component mode synthesis method was used for modal analysis of beams, plates and the cylindrical shells. The first employed the discrete system to implement this method and the free interface method was implemented in the discrete systems with multiple points of the fixed interface. Unlike the fixed interface method, this method is applied when the physical properties of finite elements model are not available and instead the modal properties of the system such as natural frequencies and its modes form are used as substructure data. If all data of the modal associated with all degrees of freedom of substructures is available, the modal coupling results are completely consistent with the accurate results. However, if the modal properties of a limited number of modes are available, the error arising from the deficiency will be observed in the coupling results. The fewer the available modes, the more error there will be in the coupling. The number of frequencies and the shape of modes resulting from the coupling method are equal to the sum of the modes extracted from each of the substructures minus the common degrees of freedom between the two substructures. Therefore, among the necessary conditions of this method is that the extracted modes of substructures are totally more than the number of degrees of freedom of interface.

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Concerning the deficiency of the degrees of freedom, the most important point is that only the degrees of freedom of interfaces affect the results of coupling and lack of the data internal degrees of freedom has no effect on the coupling results. In the following, the continuous samples of beam, plate and cylindrical shell were studied. Further, the natural frequencies of the beam system were extracted analytically. Then, the modal analysis was done on two one-meter substructures by the components modes Synthesis, which results were compared with the modal analysis of an integrated two-meter structure. The analysis results showed an acceptable accuracy rate.

This method was also implemented on the plate. In this system, the number of connecting points increases and this phenomenon increases the method error rate. The Singular Value Decomposition (SVD) method was used to reduce the error of this method in the matrix inverting process. Finally, the components modes Synthesis was implemented on the cylindrical shell. According to the results of this research, the method of components modes Synthesis appears to be a reliable approach for implementation on various structures, including the cylindrical shell structure. However, by developing this method, the test costs will be reduced and the computational volume will be much lower. Working out with the samples of plate and cylindrical shell, it was found that despite assumptions, increasing the number of degrees of freedom of the substructures interfaces in the coupling process leads to the increase of results error. For the reason, the process of choosing acceptable degrees of freedom for connection points was introduced. The main tool was Singular Value Decomposition (SVD) method. Results obtained from coupling have less error in the view of the provided criteria in this process.

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