Vibration Analysis of Laminated Composite Plates Carrying Rotating Circular Mass

R. Akbari Alashti
Department of Mechanical Engineering,
Babol University of Technology, Iran
E-mail: raalashti@nit.ac.ir

R. Alibakhshi*
Department of Mechanical Engineering,
Babol University of Technology, Iran
E-mail: sh_a_bakhshi@yahoo.com
*Corresponding author

Received: 13 December 2013, Revised: 6 July 2013, Accepted: 27 July 2013

Abstract: In this paper, the free vibration response of a laminated composite rectangular plate supporting a rotating circular patch mass is studied. The two variables refined plate theory which has the number of unknown functions involved is only four, as against five in case of other simple shear deformation theories, is applied to define the third order displacement field of a composite rectangular plate. The plate is considered to have simply supported boundaries. The first variation of the Lagrangian (Hamilton's principle) is used to obtain the equations of motion for the rectangular plate. Due to the significance of fundamental frequency of the plate, its variation with respect to the non-dimensional geometrical parameters such as aspect ratio of the plate, size, location and angular velocity of the rotating patch mass, is investigated. It will be shown herein that the proposed theory is simple in solving the free vibration problems of plates with patch masses.

Keywords: Composite Plates, Free Vibration, Rotating Mass, Refined Plate Theory

Reference: Akbari Alashti, R., Alibakhshi, R., "Vibration Analysis of Laminated Composite Plates Carrying Rotating Circular Mass", Int J of Advanced Design and Manufacturing Technology, Vol. 7/ No. 3, 2014, pp. 1-9.

Biographical notes: R. Akbari Alashti has received his PhD in Mechanical Engineering, Applied Design from Tarbiat Modares University, Tehran, Iran, in 2006. His PhD program was focused on the limit load analysis of cylindrical shells with opening under combined loading using constrained minimization techniques. His major fields of study are stress analysis in shells and plates, nonlinear analysis of structures, limit load and damage analysis. Since 2006, he has been working as a faculty member of the Babol University of Technology, Mechanical Engineering Department. **R. Alibakhshi** has the Master degree in mechanical engineering from Babol University of Technology. His field of research is on vibration analysis of composite and FG materials.

1 INTRODUCTION

Plate structures made of composite materials, are mainly a combination of two or more different materials that may provide superior and unique mechanical and physical properties. These materials have high specific strength to weight ratio in comparison to other materials. In the past three decades, researchers have paid significant attention to the behavior of plates and proposed a variety of plate theories, in which plates are subjected to different types of loadings. In addition to transverse and in-plane loadings, these materials may be exposed to other loading conditions such as distributed patch mass. Therefore, it is necessary to develop an appropriate model being capable of accurately predicting behaviors of these materials in designing of plate type structures.

Srinivas and Rao studied the bending, vibration and buckling behaviors of simply supported thick orthotropic rectangular laminated plates and obtained normal and shear stress fields under the effect of a uniformly distributed transverse loading [1]. It is important to know that they have presented a three-dimensional linear, small deformation theory of elasticity solution for static as well as dynamic analysis of isotropic, orthotropic and laminated simply-supported rectangular plates. The Reissner-Mindlin's first-order shear deformation plate theory (FSDT) assumes first-order displacement functions with a shear correction factor for alleviating the discrepancy of having non-zero transverse shear strain on the top and bottom surfaces. This theory was employed by Whitney and Pagano to study vibration and bending of anisotropic plates [2].

They investigated free vibration response of a composite plate using this theory and employed the Yang-Norris-Stavski (YNS) theory to study the cylindrical bending of anti-symmetric cross-ply and angle-ply plate strips with sinusoidal loading. Bert and Chen presented a closed form solution for the free vibration of simply supported anti-symmetric rectangular plates based on the YNS theory [3]. Shankara and Iyengar obtained finite element solutions of free vibration of laminated composite plates by higher-order shear deformation theory [4]. Because the structures designed based on classical laminate plate theory (CLPT) may be unsafe and the CLPT overestimates the buckling load of the laminated composite plates, Reddy used finite element method (FEM) to carry out free vibration of anti-symmetric angle-ply laminated plates considering the effect of transverse shear deformation [5].

In other work, Reddy introduced a set of equilibrium equations for the kinematic models proposed by Levinson and Murthy and also developed a third-order shear deformation theory (TSDT) for composite laminates based on assumed displacement fields (third-order in-plane and constant out-of-plane displacement) [6]. Khdeir and Reddy obtained a complete set of linear equations of the second order theory to analyze the free vibration behavior of cross-ply and anti-symmetric angle-ply laminated plates [7]. Wong studied the effect of distributed patch mass on the plate vibration response [8]. In his work, effects of shear deformation and rotary inertia were not considered and the Rayleigh-Ritz method was applied to find the response of a rectangular plate.

Recently, a two variable refined plate theory (RPT) was first developed for isotropic plate by Shimpi, and was extended to orthotropic plates by Shimpi and Patel and Kim et al., [9], [10], [17], [14]. The most interesting feature of this theory is that it does not require shear correction factor, and has strong similarities with the CLPT in some aspects such as governing equation, boundary conditions and moment expressions. Alibeigloo et al., applied the third order shear deformation theory (TSDT) to solve the free vibration of a simply supported laminated composite plate with distributed patch mass using the Hamilton's principle by means of a double Fourier series [11].

In this article, free vibration characteristics of plates carrying distributed attached mass with arbitrary size and location on the rectangular plate are investigated. Alibeigloo and Kari also studied the forced vibration response of anti-symmetric laminated rectangular plates with distributed patch mass [12]. Seung-Eock et al., employed the two variable refined plate theory (RPT2) for plates under the action of transverse and in-plane forces and obtained the stiffness and mass matrices using the Hamilton principle [13]. They compared the non-dimensional deflection obtained by various theories namely the classical laminate plate theory, the first order shear deformation theory, the higher order shear deformation theory and the refined plate theory. They showed that the RPT2 model gives more accurate results of deflection and buckling load than the HSDT in comparison with the three-dimensional elasticity solution.

Seung-Eock et al., also carried out buckling analysis of isotropic and orthotropic plates using the two variable refined plate theory [14]. A closed form solution of a simply supported rectangular plate subjected to in-plane loading has been obtained using Navier's method. Huu-Tai and Seung-Eock developed analytical solutions of deflection and stress fields for orthotropic plates using the two variable refined plate theory and showed their strong similarities with those obtained by the classical plate theory [15]. In other work, Huu-Tai and Seung-Eock developed the two variable refined plate theory for free vibration of composite plates using Navier's technique [16]. Based on the computed results for free vibration of composite plates, RPT2 was found

to give more accurate results than the TSDT compared with the exact solutions of three-dimensional elasticity theory [16]. Alibakhshi extended the two variable refined plate theory for a rectangular composite plate carrying a rectangular patch mass and showed that this theory has more consistency with the third order shear deformation theory compared with the finite element method, Yang-Norris-Stavski and higher order shear deformation theories [18].
This paper aims to extent the RPT2 theory for vibration analysis of laminated composite plates carrying a rotating circular patch mass. The non-dimensional first natural frequency of a plate with simply supported boundary conditions is obtained under the effect of a patch mass with arbitrary dimensions, positions and angular velocities. The simultaneous effect of various parameters such as dimensionless position of the patch mass, angular velocity and aspect ratios of circular patch mass on free vibration response of the plate is also investigated.

2 REFINED PLATE THEORY

Let us consider a rectangular plate with length, width and total thickness equal to a, b and h, respectively. The plate supports a circular patch mass, M_{mass}, with radius of R, that is located in an arbitrary position (x', y') as shown in Figure 1. The mass is considered to be placed on the upper surface of the plate. The origin of the global Cartesian coordinate system is chosen to be at the corner and on the middle plane of the plate, z=0.

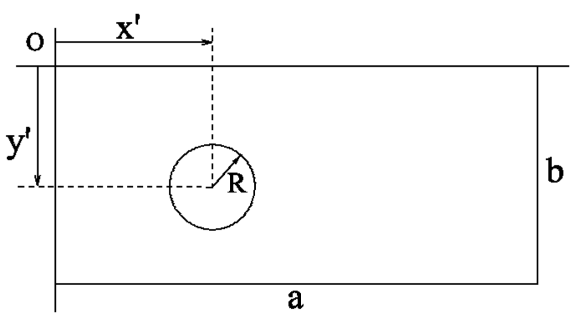

Fig. 1 Geometry of a rectangular plate with a circular patch mass

Therefore, the domain of plate is defined as $0 \le x \le a$, $0 \le y \le b$ and $-h/2 \le z \le h/2$. In order to proceed with the formulation of the problem using two variable refined plate theory (RPT2), it is assumed that the displacement components (u, v, w) of the plate are small in comparison with the thickness of the plate, hence the strains involved are considered to be infinitesimal. On the other hand, the transverse normal stress in the z-direction, σ_z, is assumed to be very small in comparison to the in-plane stress components, σ_x and σ_y. As a consequence of the above definition, the stress–strain relations can be reduced from a 6×6 matrix to a 5×5 matrix that may significantly reduce the complexity of the problem. The total displacement of the plate in the z-direction (*w*) is assumed to be consisting of three components, w_a (extension), w_b (bending) and w_s (shearing) which are functions of x, y coordinates and the time [13].

$$w(x, y, z, t) = w_a(x, y, t) + w_b(x, y, t) + w_s(x, y, t) \tag{1}$$

The displacement components in the x and y-directions are also defined as [13]:

$$\begin{aligned} u(x, y, z, t) &= u_0(x, y, t) + u_b(x, y, t) + u_s(x, y, t) \\ v(x, y, z, t) &= v_0(x, y, t) + v_b(x, y, t) + v_s(x, y, t) \end{aligned} \tag{2}$$

The bending components of the displacement function i.e. u_b and v_b are assumed to be similar to the displacements based on the classical plate theory (CPT). It means that [10]:

$$u_b = -z(\partial w_b / \partial x), \; v_b = -z(\partial w_b / \partial y) \tag{3}$$

Considering the fact that the shear stresses τ_{zx} and τ_{zy} are zero at upper and lower faces of the plate, i.e. $z = +h/2$ and $z = -h/2$ respectively, the shear displacement u_s and v_s can be written as [10]:

$$\begin{aligned} u_s &= z\left((1/4) - (5/3)(z/h)^2\right)(\partial w_s / \partial x), \\ v_s &= z\left((1/4) - (5/3)(z/h)^2\right)(\partial w_s / \partial y) \end{aligned} \tag{4}$$

Each layer is assumed to have orthotropic material property, hence the stress-strain relations in the direction of the principle axes of orthotropy are found to be [9]:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \tag{5}$$

Where Q_{ij} are the components of the reduced stiffness matrix and are expressed in terms of material properties of each layer [13].

$$Q_{11} = E_1/(1-\upsilon_{12}\upsilon_{21}),\ Q_{22} = E_2/(1-\upsilon_{12}\upsilon_{21}),\ Q_{12} = \upsilon_{12}Q_{22},\ Q_{13} = Q_{23} = Q_{33} = 0,\ Q_{44} = G_{23}, Q_{55} = G_{13} \text{ and } Q_{66} = G_{12}. \quad (6)$$

Equation (5) represents the stress-strain relations in an especially orthotropic material, where the principle axes of orthotropy are parallel to the geometric axes of the plate (x, y), i.e. the direction of the load application. In order to define the stress-strain relations in the geometrical coordinate system of the plate, that is the global Cartesian coordinate system, components of the reduced stiffness tensor are transformed according to the transformation law of fourth order tensors. Hence, the stress-strain relations in the global coordinate system are [13]:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^{(k)} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^{(k)} \quad (7)$$

Where k indicates the layer number and $\bar{Q}_{ij}$ is the material constants of the k^{th} lamina in the laminate coordinate system. In order to obtain the equations of motion by the Hamilton principle, the strain energy and the kinetic energy of the plate are first defined. The definition of the strain energy is as follows:

$$U_{plate} = \frac{1}{2}\int_V \sigma_{ij}\varepsilon_{ij}dV \quad (8)$$

The strain energy of the plate can be written as [13]:

$$U_{plate} = \frac{1}{2}\int_{A_{pltae}} (N_x\varepsilon_x^0 + N_{xy}\gamma_{xy}^0 + N_y\varepsilon_y^0 + M_x^b\kappa_x^b + M_{xy}^b\kappa_{xy}^b + M_y^b\kappa_y^b + M_x^s\kappa_x^s + M_{xy}^s\kappa_{xy}^s + M_y^s\kappa_y^s + Q_{yz}^s\gamma_{yz}^s + Q_{xz}^s\gamma_{xz}^s + Q_{yz}^a\gamma_{yz}^a + Q_{xz}^a\gamma_{xz}^a)dxdy \quad (9)$$

Where $\{N\}$, $\{M\}$ and $\{Q\}$ are the stress resultants of the total N layers of the plate which are defined in the Appendix. The total kinetic energy is the summation of the kinetic energies of the plate, the uniformly distributed patch mass and that of the rotating mass acting on the top surface of the plate [11].

$$T = T_{mass} + T_{plate} \quad (10)$$

The kinetic energy of the plate is defined as [11]:

$$T_{plate} = \frac{1}{2}\int_V \rho\left((\partial U/\partial t)^2 + (\partial V/\partial t)^2 + (\partial W/\partial t)^2\right)dxdydz \quad (11)$$

Substituting equations (1) to (4) into Equation (11), and considering the limits of integration in the plate, the kinetic energy of plate can be written as [13]:

$$T_{plate} = (1/2)\int_0^b\int_0^a I_0\left[\dot{u}^2 + \dot{v}^2 + (\dot{w}_a + \dot{w}_b + \dot{w}_s)^2\right]dxdy + (1/2)\int_0^b\int_0^a I_2\left[(\partial\dot{w}_b/\partial x)^2 + (\partial\dot{w}_b/\partial y)^2\right]dxdy + (1/2)\int_0^b\int_0^a (I_2/84)\left[(\partial\dot{w}_s/\partial x)^2 + (\partial\dot{w}_s/\partial y)^2\right]dxdy. \quad (12)$$

The kinetic energy of the circular patch mass (M_{mass}) that is located on the top surface of the plate $(z = h/2)$ can be written as:

$$T_{mass} = \frac{1}{2}\int_{x'-\sqrt{R^2-y^2}}^{x'+\sqrt{R^2-y^2}}\int_{y'-R}^{y'+R} \rho_m h_m \left(\left[\dot{u} - \left(\frac{h}{2}\right)\left(\frac{\partial\dot{w}_b}{\partial x}\right) - \frac{h}{12}\left(\frac{\partial\dot{w}_s}{\partial x}\right)\right]^2 + \left[\dot{v} - \left(\frac{h}{2}\right)\left(\frac{\partial\dot{w}_b}{\partial y}\right) - \frac{h}{12}\left(\frac{\partial\dot{w}_s}{\partial y}\right)\right]^2 + [\dot{w}_a + \dot{w}_b + \dot{w}_s]^2 \right) dydx + \frac{1}{2}IJ^2 \quad (13)$$

Where I and J denote the inertia momentum (i. e., $M_{mass}R^2/2$) and angular velocity of the mass, and I_0, I_2 are inertia terms as below [16]:

$$(I_0, I_2) = \int_{-h/2}^{h/2} \rho(1, z^2)dz \quad (14)$$

Substituting the displacements field in the relevant strain energy and kinetic energy terms, integrating the results and obtaining their first variation, the equations of motion are found. Finally, by collecting the coefficients of parts, the governing equation of plate vibration is obtained as below [11]:

$$([S] - [M]\omega^2)\{\lambda\} = \{0\} \quad (15)$$

Where [S], [M], ω and λ are the stiffness (see ref. [13]), mass matrices, natural frequency and the vector of unknown coefficients respectively. For convenience, the non-dimensional natural frequency of the plate is defined as [11]:

$$\overline{\omega} = \omega\left(a^2/h\right)\sqrt{(\rho/E_2)} \quad (16)$$

3 PROBLEM DEFINITION

Now, a set of boundary conditions namely the SS-2 boundary condition of the following form is applied to an anti-symmetric angle-ply laminate [13]:
For $i = a, b, s$, $m = 0, a$ and $n = 0, b$

$$\begin{aligned} &u(m,y) = v(x,n) = 0, w_i(m,y) = w_i(x,n) = 0, \\ &\partial w_i/\partial y\,(m,y) = \partial w_i/\partial x\,(x,n) = 0, \\ &N_{xy}(0,y) = N_{xy}(x,0) = M_x^b(0,y) = \\ &M_y^b(x,0) = M_x^s(0,y) = M_y^s(x,0) = 0, \\ &N_{xy}(a,y) = N_{xy}(x,b) = M_x^b(a,y) = \\ &M_y^b(x,b) = M_x^s(a,y) = M_y^s(x,b) = 0. \end{aligned} \quad (17)$$

In order to satisfy the boundary conditions, the following displacement fields are assumed [13].

$$\begin{Bmatrix} u \\ v \\ w_b \\ w_s \\ w_a \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}\cos\alpha_m x \sin\beta_n y \\ V_{mn}\sin\alpha_m x \cos\beta_n y \\ W_{bmn}\sin\alpha_m x \sin\beta_n y \\ W_{smn}\sin\alpha_m x \sin\beta_n y \\ W_{amn}\sin\alpha_m x \sin\beta_n y \end{Bmatrix} \quad (18)$$

Where $\alpha = m\pi/a$, $\beta = n\pi/b$ and U_{mn} , V_{mn} , W_{bmn} , W_{smn} , W_{amn} are coefficients.

4 NUMERICAL SCHEME

The following properties are used for computing the non-dimensional natural frequencies:
Material data:

$$E_1/E_2 = 40, G_{12}/E_2 = 0.6, G_{23}/E_2 = G_{13}/E_2 = 0.5, \nu_{12} = 0.25.$$

At first, the effect of a non-rotating circular patch mass, located at $x' = a/2, y' = b/2$, is considered (i. e. J=0). In Fig. 2, the influence of the length to thickness ratio, a/h , on the non-dimensional natural frequency is shown. As this figure shows, the non-dimensional natural frequency increases by increasing the length to thickness ratio. It is evident that the non-dimensional natural frequency reaches to a constant value at $a \prec b$ and $a/h \geq 20$ (i. e. thin plate's behaviour).

In the case of free vibration of a plate with circular mass, the ratio of the radius of the patch mass to the length of the plate, R/a , is an effective factor should be considered. As shown in Fig. 3, by increasing the radius to the length ratio (with constant mass ratio), the non-dimensional natural frequency increases.

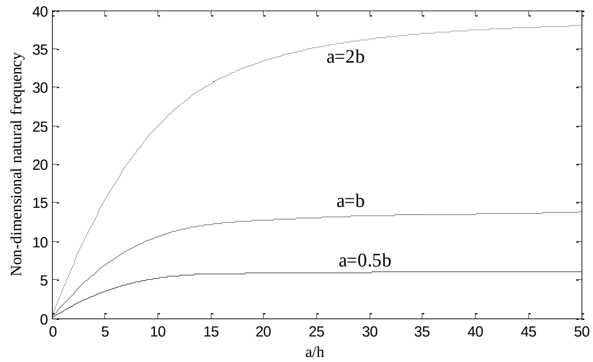

Fig. 2 Influence of length to thickness ratio on non-dimensional natural frequency, $[45/-45]_2$, $R/a = 0.1$, $M_{mass}/M_{plate} = 0.5$.

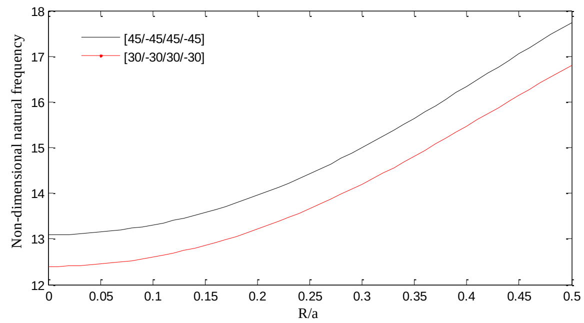

Fig. 3 Influence of radius to length ratio on non-dimensional natural frequency of a square plate, $a/h = 30$, $M_{mass}/M_{plate} = 0.5$.

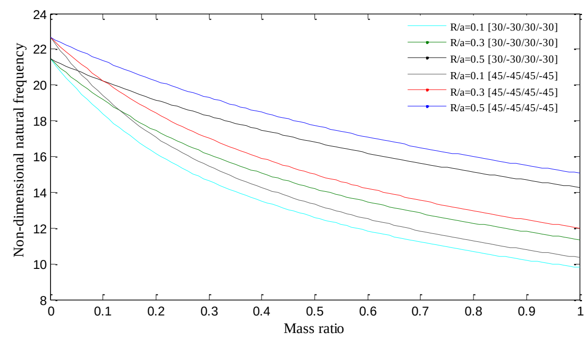

Fig. 4 Influence of mass ratio on non-dimensional natural frequency of a square plate, $a/h = 30$.

The effect of the mass ratio on the non-dimensional natural frequency for different radius to length ratio is presented in Fig. 4. According to this figure, by increasing the amount of local mass, the stiffness of the plate decreases and consequently the non-dimensional natural frequency decreases. The effect of rotating circular mass (with constant mass ratio) on the non-dimensional natural frequency with a/h as a parameter is presented in Table 1. According to the table, by increasing the angular velocity of the circular patch mass, it can be seen that the non-dimensional natural frequency decreases. It is also found that increasing the angular velocity of circular mass has more decreasing effect on the non-dimensional natural frequency of rectangular plate than square plate.

Table 1 The Non-dimensional natural frequency for $[45/-45]_2$ with various a/h and $R/a = 0.1.$ ratios, $M_{mass}/M_{plate} = 0.5$, $R/a = 0.1$.

	a/h				
a = b	10	20	30	40	50
$J=0$	10.5839	12.7370	13.3076	13.5268	13.6321
$J=5$	9.9171	12.4773	13.0465	13.2623	13.3656
$J=15$	4.4532	10.8166	11.3943	11.5878	11.6788
$J=30$	2.2441	7.7627	8.4897	8.6407	8.7094
a = 2b					
$J=0$	24.9523	33.4194	36.2502	37.4328	38.0224
$J=5$	18.8416	32.7755	35.7306	36.9141	37.4986
$J=15$	6.7277	24.7533	32.1952	33.4150	33.9646
$J=30$	3.3756	12.9396	24.7117	26.3261	26.7986

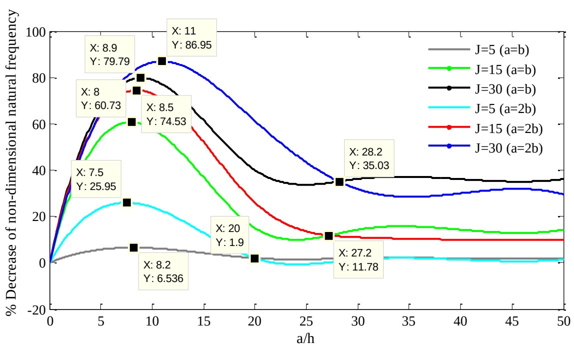

Fig. 5 The effect of angular velocity on the percentage decrease of natural frequency, $[45/-45]_2$, $M_{mass}/M_{plate} = 0.5$, $R/a = 0.1$.

The percentage decrease of non-dimensional natural frequency for three loading cases ($J = 5, 15, 30$ rad/sec) is shown in Fig. 5. It is shown that as the angular velocity of the patch mass increases, percentage decrease of natural frequency increases for small values of a/h (i.e. $a/h \prec 10$). In other words, increasing the angular velocity leads to an increase in the kinetic energy of the patch mass (see Eq. (13)), and consequently to increase the effective mass acting on the plate. Percentage decrease of the non-dimensional natural frequency of a rectangular plate is higher than that of a square plate (with constant angular velocities). It is noteworthy to notice that there is a singular point in which the percentage decrease of the non-dimensional natural frequency of square and rectangular plates are the same. Considering different values of angular velocity of the patch mass (i.e. $J = 5, 15, 30$ rad/s), these singular points are found to be at $a/h = 20, 27.2, 28.2$ respectively. As depicted in Fig. 5, by increasing the angular velocity of the patch mass and a/h ratio, the percentage decrease of the non-dimensional natural frequency of square and rectangular plates approaches to the same value. In order to reveal the effect of aspect ratio of the patch mass on the behavior of laminated composite plates, the contour plots of non-dimensional natural frequencies are presented.

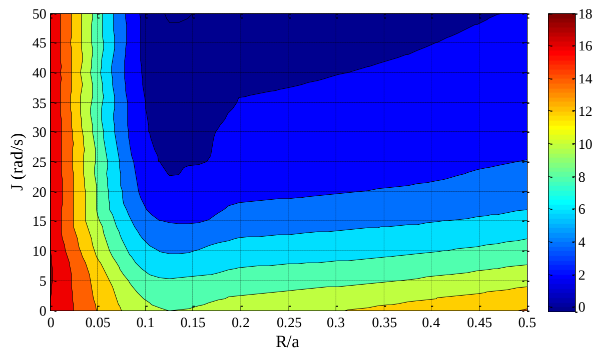

Fig. 6 Variation of non-dimensional natural frequency with radius to length ratio and angular velocity, $[45/-45]_2$, $M_{mass}/M_{plate} = 0.5$, square plate, $a/h = 10$.

Fig. 6 shows simultaneous effect of the radius of the patch mass to the length and the angular velocity of the patch mass (mass ratio is taken as 0.5) on the non-dimensional natural frequencies of a square plate. It is evident from Fig. 6 that the non-dimensional natural frequency decreases as both the R/a ratio and the angular velocity increase.

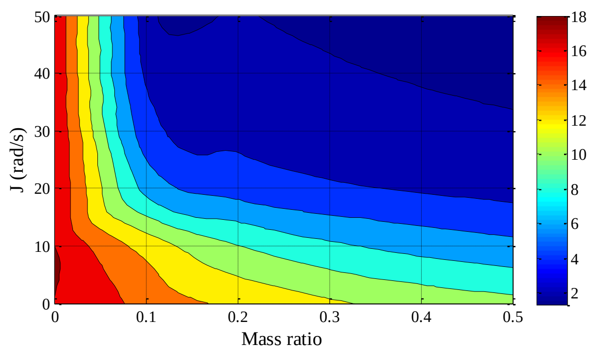

Fig. 7 Variation of non-dimensional natural frequency with the mass ratio and angular velocity, $[45/-45]_2$, $R/a = 0.1$, square plate, $a/h = 10$.

Simultaneous effect of the mass ratio and the angular velocity of patch mass (ratio of the radius to length is taken as 0.1) on the non-dimensional natural frequency of a square plate is shown in Fig. 7.

As shown in this figure, the non-dimensional natural frequency decreases as both the mass ratio and the angular velocity increase. Fig. 8 presents simultaneous effect of the x-position and the angular velocity of a circular patch pass (with constant mass ratio) on non-dimensional natural frequency of a rectangular plate. It is found that by increasing the distance between the mass and x=0 edge of the plate (i. e., x/a ratio) and also the angular velocity of mass, the stiffness of the plate decreases, and finally the non-dimensional natural frequency decreases.

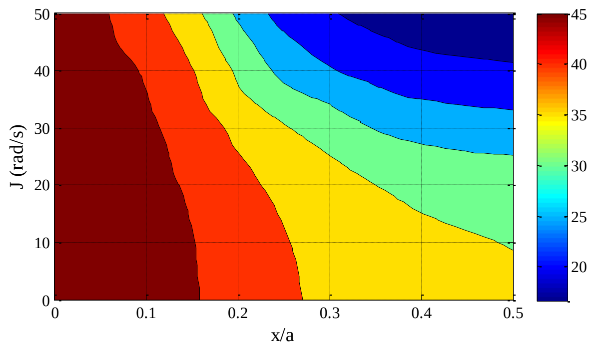

Fig. 8 Variation of non-dimensional natural frequency with x-position and angular velocity, $[45/-45]_2$, $R/a=0.05$, $M_{mass}/M_{plate}=0.5$, rectangular plate, $a/h=30$.

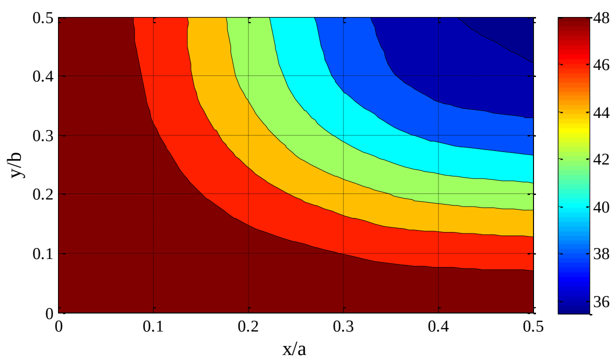

Fig. 9 Variation of non-dimensional natural frequency with x and y-positions, $[45/-45]_2$, $R/a=0.05$, $M_{mass}/M_{plate}=0.5$, rectangular plate, $a/h=30$, $J=0$ rad/s.

Figures 9 and 10 present simultaneous effect of the *x*-position and the y-position of the circular patch pass (with constant mass ratio) on the non-dimensional natural frequency of the rectangular plate for J=0 and 50 respectively. According to these figures, as the distance between the mass and x=y=0 edges of the plate increases, the stiffness of the plate decreases, and consequently the non-dimensional natural frequency decreases. Due to the symmetry imposed by the boundary conditions of the plate, it is observed that the longitudinal and transversal movement of the patch mass would have the same effect on the non-dimensional natural frequencies of the plate.

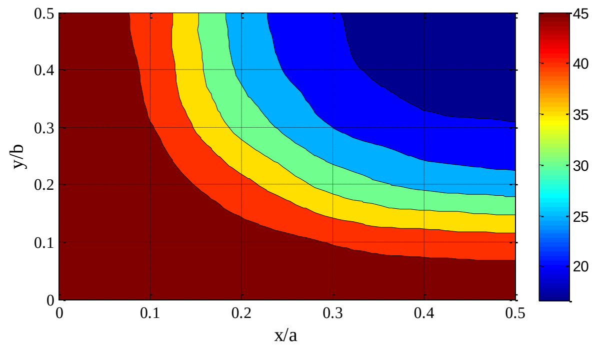

Fig. 10 Variation of non-dimensional natural frequency with x and y-positions, $[45/-45]_2$, $R/a=0.05$, $M_{mass}/M_{plate}=0.5$, rectangular plate, $a/h=30$, $J=50$ rad/s.

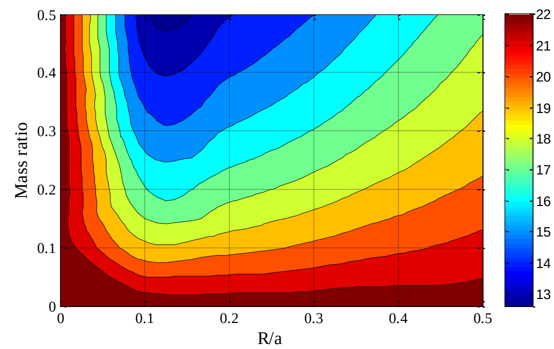

Fig. 11 Variation of non-dimensional natural frequency with radius to length and mass ratio, $[45/-45]_2$, square plate, $a/h=30$, $J=0$ rad/s.

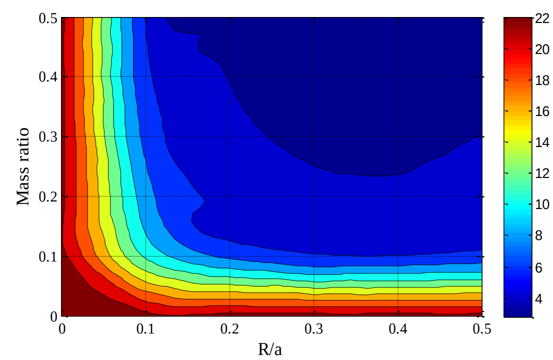

Fig. 12 Variation of non-dimensional natural frequency with radius to length and mass ratio, $[45/-45]_2$, square plate, $a/h=30$, $J=50$ rad/s.

Figures 11 and 12 show simultaneous effect of the radius of the patch mass to the length of the plate and the mass ratio of the circular patch mass on non-dimensional natural frequency of the square plate for J=0 and 50 respectively. By comparing the contour plots, it is observed that the non-dimensional frequencies have a significant decrease when the angular velocity of the mass approaches to a large value. Simultaneous effect of the *x*-position and the mass ratio of a circular patch mass on the non-dimensional natural frequency of a square plate for J=0 and 50 are shown in Figures 13 and 14, respectively. It is observed that the non-dimensional frequencies significantly decrease when the angular velocity approaches to a very large value.

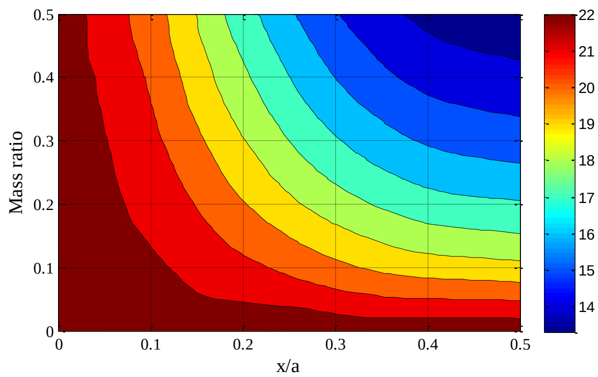

Fig. 13 Variation of non-dimensional natural frequency with x-position and mass ratio, $[45/-45]_2$, square plate, $R/a = 0.1$, $a/h = 30$, $J = 0$ rad/s

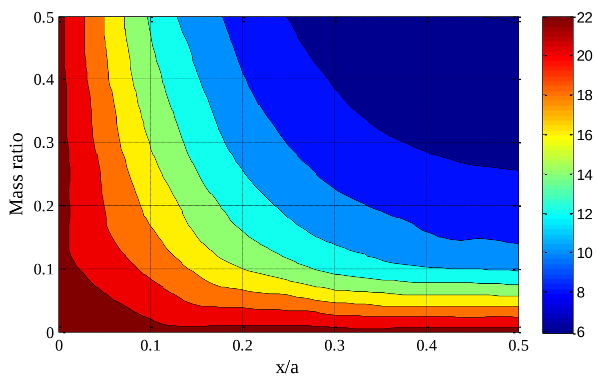

Fig. 14 Variation of non-dimensional natural frequency with x-position and mass ratio, $[45/-45]_2$, square plate, $R/a = 0.1$, $a/h = 30$, $J = 50$ rad/s

The influence of both the radius of the patch mass to the length of the plate and the x-position of the circular mass on the non-dimensional natural frequency of a square plate for J=0 and 50 are shown in Figs. 15 and 16, respectively. It can be seen that, by increasing the angular velocity of the mass, there is a sudden change in the natural frequency. Finally, comparative results show that the simultaneous influence of radius of the patch mass to the length of the plate and mass ratio (Figs. 11-12), and also the x-position of the circular mass and radius of the patch mass to the length of the plate (Figs. 15-16) have more decreasing effect on non-dimensional natural frequency than other parameters.

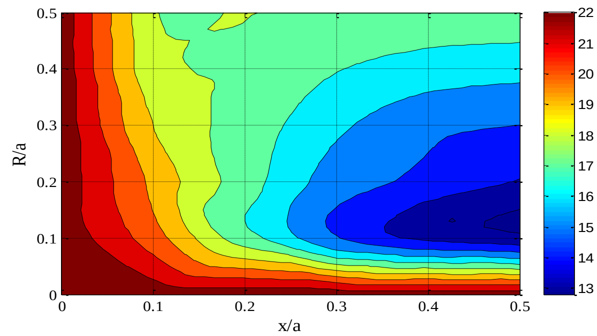

Fig. 15 Variation of non dimensional natural frequency with x-position and radius to length ratio, $[45/-45]_2$, $M_{mass}/M_{plate} = 0.5$, square plate, $a/h = 30$, $J = 0$ rad/s.

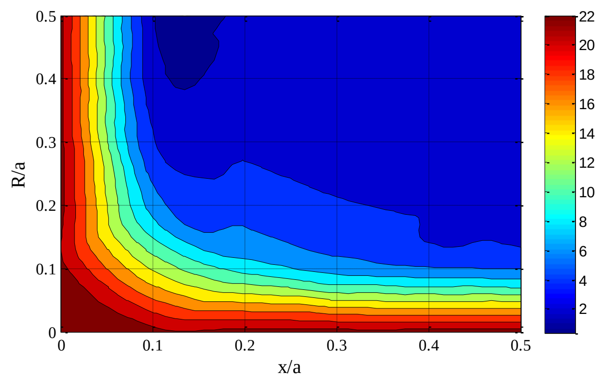

Fig. 16 Variation of non-dimensional natural frequency with x-position and radius to length ratio, $[45/-45]_2$, $M_{mass}/M_{plate} = 0.5$, square plate, $a/h = 30$, $J = 50$ rad/s.

5 CONCLUDING REMARKS

In the present study, the two variable refined plate theory was developed for vibration analysis of laminated composite plates with rotating circular patch mass. First, the governing equation of vibration of the rectangular plate with a patch mass was obtained by this theory. Simultaneous effects of various parameters such as the size and the location of the rotating circular patch mass, and the aspect ratio of the plate on the non-dimensional natural frequency of the plate was studied. The main conclusions are listed as follows:

▪ The ratio of the radius of the patch mass to the length of the plate has higher effect on the non-dimensional natural frequency for $[45/-45]_2$ than that of $[30/-30]_2$ composite plates.

■ By increasing the value of the angular velocity of the mass, the kinetic energy of patch mass and finally the effective patch mass on the plate increases.

■ Considering large values of a/h and small values of angular velocities, the percentage decrease of non-dimensional natural frequencies for square and rectangular plates carrying a circular mass is the same.

■ Considering the large values of a/h and angular velocities, the percentage decrease of non-dimensional natural frequency of a square plate is greater than that of a rectangular plate carrying a rotating circular mass.

■ Considering constant angular velocity, Percentage decrease of the non-dimensional natural frequency for a rectangular plate is greater than that of a square plate.

■ By increasing the radius of the patch mass to the length of the plate and the angular velocity, the non-dimensional natural frequency decreases.

■ By increasing the mass ratio and the angular velocity of the circular patch mass, the non-dimensional natural frequency decreases.

■ As the distance between the added mass and x,y=0 edges of the plate and angular velocity of the mass increases, with constant mass ratio, the stiffness of the plate decreases, and finally the non-dimensional natural frequency decreases.

6 APPENDIX

$$\begin{Bmatrix} (N_x, N_y, N_{xy}) \\ (M_x^b, M_y^b, M_{xy}^b) \\ (M_x^s, M_y^s, M_{xy}^s) \end{Bmatrix} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} (\sigma_x, \sigma_y, \sigma_{xy}) \begin{Bmatrix} 1 \\ z \\ J \end{Bmatrix} dz, \quad J = -z\left((1/4) - (5/3)(z/h)^2\right)$$

$$\begin{Bmatrix} (Q_{xz}^a, Q_{yz}^a) \\ (Q_{xz}^s, Q_{yz}^s) \end{Bmatrix} = \sum_{k=1}^{N} \int_{Z_k}^{Z_{k+1}} (\sigma_{xz}, \sigma_{yz}) \begin{Bmatrix} 1 \\ I \end{Bmatrix} dz, \quad I = (5/4) - 5(z/h)^2$$

$$(\gamma_{xz}^a, \gamma_{yz}^a, \gamma_{xz}^s, \gamma_{yz}^s) = (\partial w_a/\partial x, \partial w_a/\partial y, \partial w_s/\partial x, \partial w_s/\partial y)$$

$$(\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0) = (\partial u/\partial x, \partial v/\partial y, \partial u/\partial y + \partial v/\partial x)$$

$$\kappa_x^b = -(\partial^2 w_b/\partial x^2), \ \kappa_y^b = -(\partial^2 w_b/\partial y^2), \ \kappa_{xy}^b = -2(\partial^2 w_b/\partial x \partial y)$$

$$\kappa_x^s = -(\partial^2 w_s/\partial x^2), \ \kappa_y^s = -(\partial^2 w_s/\partial y^2), \ \kappa_{xy}^s = -2(\partial^2 w_s/\partial x \partial y)$$

REFERENCES

[1] Srinivas, S., Rao, AK., “Bending, vibration and buckling of simply supported thick orthotropic rectangular plates and laminates”, International Journal of Solids and Structures, Vol. 6, No. 11, 1970, pp. 1463-1481.

[2] Withney, J. M., Pagano, N. J., “Shear deformation in heterogeneous anisotropic plates”, ASME Journal of Applied Mechanics, Vol. 37, No. 4, 1970, pp. 1031-1036.

[3] Bert, C. W., Chen, T. L. C., “Effect of shear deformation on vibration of antisymmetric angle-ply laminated rectangular plates”, International Journal of Solids and Structures, Vol. 14, No. 6, 1978, pp. 465-473.

[4] Shankara, C. A., Iyengar, N. G., “A C^0 element for the free vibration analysis of laminated composite plates”, Journal of Sound and Vibration, Vol. 191, No. 5, 1996, pp. 721-738.

[5] Reddy, J. N., “Free vibration of anti-symmetric angle-ply laminated plates including transverse shear deformation by the finite element method”, Journal of Sound and Vibration, Vol. 66, No. 4, 1979, pp. 565-576.

[6] Reddy, J. N., “A simple higher order theory for laminated composite plates”, ASME Journal of Applied Mechanics, Vol. 51, No. 4, 1984, pp. 745-752.

[7] Khdeir, A. A., Reddy, J. N., “Free vibrations of laminated composite plates using second-order shear deformation theory”, Journal of Computers and Structures, Vol. 71, No. 6, 1999, pp. 617-626.

[8] Wong, W. O., “The effect of distributed mass loading on plate vibration behaviour”, Journal of Sound and Vibration, Vol. 252, No. 3, 2002, pp. 577-583.

[9] Shimpi, R. P., patel, H. G., “A two variable refined plate theory for orthotropic plate analysis”, International Journal of Solids and Structures, Vol. 43, No. 22-23, 2006, pp. 6783-6799.

[10] Shimpi, R. P., patel, H. G., “Free vibrations of plate using two variable refined plate theory”, Journal of Sound and Vibration, Vol. 296, No. 4-5, 2006, pp. 979-999.

[11] Alibeigloo, A., Shakeri, M., and Kari, M. R., “Free vibration of anti-symmetric laminated rectangular plates with distributed patch mass using third-order shear deformation theory”, Journal of Ocean Engineering, Vol. 35, No. 2, 2008, pp. 183-190.

[12] Alibeigloo, A., Kari, M. R., “Forced vibration analysis of anti-symmetric laminated rectangular plates with distributed patch mass using third order shear deformation theory”, Journal of Thin-Walled Structures, Vol. 47, No. 6-7, 2009, pp. 653-660.

[13] Seung-Eock, K., Huu-Tai, T., and Lee, J., “A two variable refined plate theory for laminated composite plates”, Journal of Composite Structures, Vol. 89, No. 2, 2009, pp. 197-205.

[14] Seung-Eock, K., Huu-Tai, T., and Lee, J., “Buckling analysis of plates using the two variable refined plate theory”, Journal of Thin Walled Structures, Vol. 47, No. 4, 2009, pp. 455-462.

[15] Huu-Tai, T., Seung-Eock, K., “Analytical solution of a two variable refined plate theory for bending analysis of orthotropic Levy-type plates”, International Journal of Mechanical Sciences, Vol. 54, No. 1, 2012, pp. 269-276.

[16] Huu-Tai, T., Seung-Eock, K., “Levy-type solution for free vibration analysis of orthotropic plates based on two variable refined plate theory”, Journal of Applied Mathematical Modeling, Vol. 36, No. 8, 2012, pp. 3870-3882.

[17] Shimpi, R. P., “Refined plate theory and its variants”, AIAA Journal, Vol. 40, No. 1, 2002, pp. 137-146.

[18] Alibakhshi, R., “The effect of anisotropy on free vibration of rectangular composite plates with patch mass”, International Journal of Engineering, Vol. 25, No. 3, 2012, pp. 223-232.