Using the Bees Algorithm with the Boundary Elements Method to Solve the Inverse Problem of Transient Heat Conduction

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Abstract: In this paper, a new population-based search called the Bees Algorithm (BA) is presented to estimate the time-dependent heat transfer coefficient and the corresponding heat flux at the boundaries of a two-dimensional body subjected to transient heat conduction, using the temperature measurements at discrete nodal locations on the boundaries, where heat flux is specified as the boundary condition. In the forward problem, a two dimensional transient heat conduction problem subjected to heat flux boundary conditions is solved for temperature distribution at the boundaries using the boundary elements method. In the inverse problem the heat transfer coefficient (h) at the boundaries where thermal conditions are over specified is estimated by minimizing an objective function which is defined as the sum of the squared differences between the measured and computed temperatures at the nodal locations on the boundary. The Bees algorithm which is a new global evolutionary optimization method is used to investigate the inverse problem. The average value of the heat transfer coefficient at the boundaries is assumed over each time interval from initial time until the final steady-state time. The optimum parameters of Bees algorithm are found and used to estimate the heat transfer coefficient as a function of time. The effect of temperature measurement errors on the identification process is also investigated.

Keywords: Bees Algorithm, Boundary Elements Method, Heat Transfer Coefficient, Inverse Problem

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1 INTRODUCTION

In this study, temperature measurements at the surface of a two-dimensional grid where heat flux is specified as the boundary condition are used to determine convective boundary heat transfer coefficient (h). Heat flux (q) at those boundaries could then be determined using Newton's cooling law. Techniques currently used in industry rely on the equation for the surface temperature history of a semi-infinite conducting medium. This approach is valid only in regions where heat conduction could be approximated as one dimensional, contrary to many actual cases such as the stagnation region of an impinging jet, regions downstream of film cooling holes and corner and step regions encountered at blade tips and end walls. Transient heat transfer experiments have traditionally been used to evaluate h for steady-state processes where h is invariant. This approach is difficult to use in a real time-dependent process where h is a function of time. To determine accurate values of h as a function of time, which is critical in the design of thermal systems, there is a need to accurately model the transient heat transfer problem.

In the case of irregular domain geometry such as a turbine blade or a combustor wall, the analytical solution for the field problem which is required at each step of the inverse problem, is impossible to achieve, therefore numerical methods are implemented. The boundary elements method (BEM) which only requires surface mesh is adopted in this identification problem.

There are several other advantages in using the BEM to solve the inverse problem. In the BEM, the Boundary Integral Equations are solved for the unknown boundary temperatures and/or heat fluxes. The heat fluxes are found directly as part of the solution and not by post-processing numerical differentiation which is done in the Finite Elements Method (FEM). In the BEM-based inverse algorithms, there is no need for internal temperature measurement which is intrusive and destructive when thermocouples are placed inside the body which could significantly affect the temperature field.

An inverse BEM-based approach is presented by [1, 2] to retrieve angular distribution of h from steady-state temperature measurements from a cylinder surface. A second order regularization scheme is adopted to stabilize results of the numerical simulations. Similar work is done by [3] in which angular variation of h over a rough heated horizontal cylinder is estimated by conducting an experiment which steady-state surface temperatures are measured by infrared scanning.

In [4] steady-state BEM-based approach is followed to retrieve h values from simulated steady-state temperature measurements. Singular value decomposition method is used to stabilize the retrieval process when noisy input data are simulated. [5] has employed an inverse BEM/genetic algorithm based approach to retrieve multi-dimensional h within film cooling holes/slots. Steady temperature measurements (actual or simulated) are used in these studies to estimate h, however, transient surface temperature measurements are commonly used in experiments, [6] developed a BEM-based inverse method to retrieve surface heat flux values from one-dimension transient temperature measurements. [7, 8] developed a BEM-based inverse algorithm to retrieve multidimensional varying h from transient surface temperature measurements. [9-11] developed an inverse algorithm to reconstruct multi-dimensional surface heat flux using Levenberg-Marquardt method.

In the present study, a BEM/BA based approach is developed to identify heat transfer coefficient as a function of time. The heat transfer coefficient (h) is assumed to have an averaged value over space coordinates. Multi dimensional boundary heat fluxes are then computed as a function of time. Unlike the previous studies in which boundary heat flux measurements are used as over specified conditions to solve the inverse problem, temperature measurements which are easier to get and less erroneous, are used to solve the inverse problem. It is shown that BA is a reliable and appropriate method to be used in the BEM-based transient problems. It does not have the problem of getting stock in local minima as is the case when local optimization methods such as L-M or Conjugate Gradient Method (CGM), etc encounter. Appropriate stopping criteria and optimum Bees algorithm parameters such as population, number of selected sites, number of elite sites, initial patch size, number of bees around elite points and number of bees around other selected points are found and used to identify h accurately. The effect of noisy data i.e. measurement errors on the identification process which is over looked in previous studies is also investigated.

2 FORWARD PROBLEM

There are three common BEM formulations for transient heat conduction problems: 1) The Laplace transfer method, 2) the dual reciprocity BEM, and 3) the time-dependent fundamental singular solution method. The latter is used to solve the forward problem. The BEM formulation is briefly stated as follows:

The governing differential equation for a two-dimensional transient heat conduction problem is:

$$k\nabla^2 T(x_1,x_2,t)=\rho c\frac{\partial T(x_1,x_2,t)}{\partial t} \quad (1)$$

which governs the temperature field in domain Ω, k, ρ and c are thermal conductivity, density and specific heat respectively. The boundary conditions as shown in Fig. 1 and initial conditions are:

$$T=T_0 \quad at\ t=0\ in\ \Omega \quad (2)$$

$$T=\bar{T} \quad at\ t>0\ on\ \Gamma_1 \quad (3)$$

$$q=h(T_S-T) \quad at\ t>0\ on\ \Gamma_2 \quad (4)$$

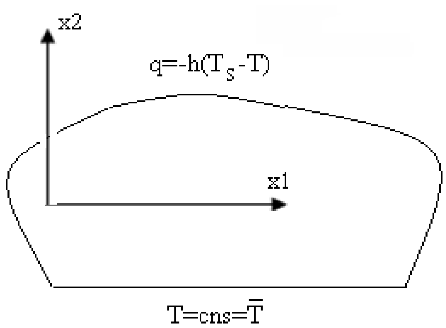

Fig. 1 The schematic of the boundary conditions

The numerical BEM solution of T is obtained by minimizing the error introduced by weighting the governing and boundary equations by a new function T* which is the fundamental singular solution or Green's function which satisfies the following equations:

$$\nabla^2 T^*-\frac{1}{\alpha}\frac{\partial T^*}{\partial t}=\delta(x_1^i,x_2^i,t) \quad (5)$$

where $\delta(x_1,x_2,t)$ is Dirac delta function. As the problem is time-dependent, we shall also have to weight the equations with respect to time.

The temperature distribution (T_i) over the domain Ω is given by [12].

$$T_i+\frac{1}{\rho c}\int_0^\tau\int_\Gamma T^*q\,d\Gamma dt=\frac{1}{\rho c}\int_0^\tau\int_\Gamma q^*T\,d\Gamma dt+\left[\int_\Omega T^*T\,d\Omega\right]_{t=0} \quad (6)$$

where q and q^* are given as:

$$q=-k(\partial T/\partial n)$$

$$q^*=-k(\partial T^*/\partial n)$$

The boundary integral equation which gives the temperature distribution, T_i at any point x_1^i, x_2^i on the boundary Γ is given by the boundary integral equation as:

$$c_iT_i+\frac{1}{\rho c}\int_0^\tau\int_\Gamma T^*q\,d\Gamma dt=\frac{1}{\rho c}\int_0^\tau\int_\Gamma q^*T\,d\Gamma dt+\left[\int_\Omega T^*T\,d\Omega\right]_{t=0} \quad (7)$$

where the coefficient c_i is a function of the solid angle of the boundary at point x_1^i,x_2^i on the boundary [13] and the integral involving q^* is evaluated in the Cauchy principal value sense. The fundamental singular solution or Green's function which is the solution to the equation (5) is:

$$T^*=\frac{1}{[4\pi\alpha.\tau]}\exp\left[-\frac{R^2}{4\alpha\tau}\right] \quad (8)$$

$$R=\left[(x_1-x_1^i)^2+(x_2-x_2^i)^2\right]^{1/2}$$

where R is the radial distance from the source point x_1^i,x_2^i to the field point x_1,x_2 under consideration [14].

3 CREDIBILITY OF THE BEM CODE

A long fin of rectangular cross section shown in Fig. 2 is subjected to transient heat convection at the boundaries.

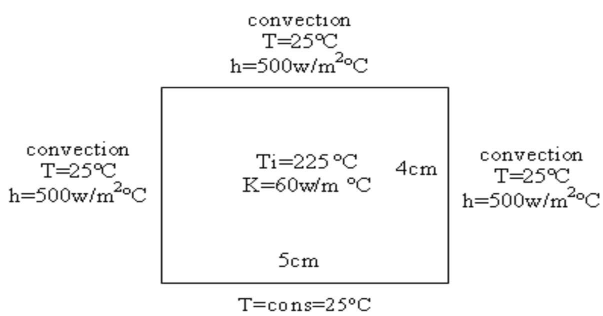

Fig. 2 Rectangular cross section of the long fin

The thermo-physical parameters of the fin are:

$k=60\,w/m^\circ C$, $\rho=7850\,kg/m^3$ and the initial temperature is 225ºC. The surrounding temperature is 25ºC and the averaged value of h over the boundary is taken to be $h=500\,w/m^{2\circ}C$. The temperatures after two minutes at two nodal locations on the boundary are computed by the BEM program. The boundary is discretized into a) 5×4, b) 10×8, c) 15×12 linear isoparametric elements and results which are the temperature distribution in the x and y directions on the boundary are compared with the results obtained by employing ANSYS software for the same problem with the identical boundary nodal locations. The results are shown in Fig. 3.

It is shown that as the boundary is divided into fine elements, the percent error in obtained numerical results decreases from 6.12 % for the 5×4 to 1.58 % for the 15×12 mesh.

As the right side is considered isolated, the same problem with different boundary conditions is solved as the second test case problem and the temperature distribution in the x and y directions on the boundary is plotted and compared; as shown in Fig. 4. It is observed that %error decreases from 6.27 % for the 5×4 mesh to 1.69 % for the 15×12 mesh.

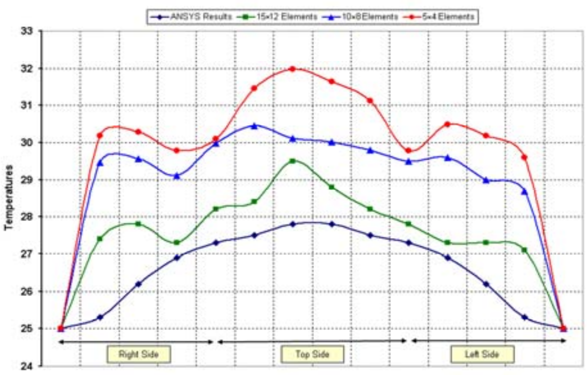

Fig. 3 Comparison of ANSYS temperature results with BEM results in the first example

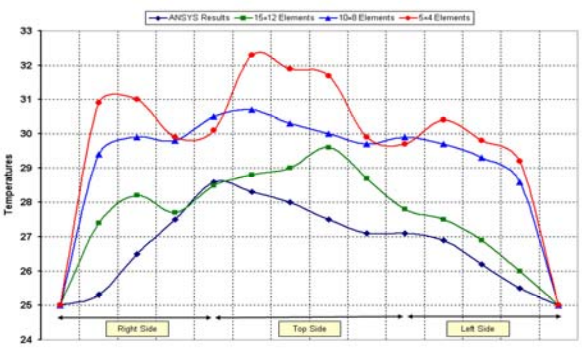

Fig. 4 Comparison of ANSYS temperature results with BEM results in the second example

4 INVERSE PROBLEM

In the inverse problem, temperatures at r locations, on the boundary where heat flux is specified as the boundary condition are measured and used as extra information to estimate h for that boundary. Let T_{ci}^{k} be the computed temperatures at time k corresponding to a guessed value of unknown h, solved by the BEM program, and T_{mi}^{k}, be the measured values of temperature at time k at the same nodal locations on the boundary. An averaged value of h on the boundary is estimated at time k, such that an objective or fitness function called Z is minimized.

$$Z = \sum_{i=1}^{r} (T_{ci}^{k} - T_{mi}^{k})^{2} \qquad (9)$$

The inverse problem is solved using the Bees algorithm which is a global optimization technique. The Bees algorithm and its important parameters involved in this technique are briefly presented in the following section.

5 THE BEES ALGORITHM

5.1. Intelligent swarm-based optimization

Swarm-based optimization algorithms (SOAs) mimic nature's methods to drive a search towards the optimal solution. A key difference between SOAs and direct search algorithms such as hill climbing and random walk is that SOAs use a population of solutions for every iteration instead of a single solution. As a population of solutions is processed in each iteration, the outcome is also a population of solutions. If an optimization problem has a single optimum, SOA population members can be expected to converge to that optimum solution.

However, if an optimization problem has multiple optimal solutions, an SOA can be used to capture them in its final population. SOAs include the Ant Colony Optimization (ACO) algorithm [15], the Genetic Algorithm (GA) [16] and the Particle Swarm Optimization (PSO) algorithm [17].

5.2. Bees in nature

A colony of honey bees can extend itself over long distances (more than 10 km) and in multiple directions simultaneously to exploit a large number of food sources [18, 19]. A colony prospers by deploying its foragers to good fields. In principle, flower patches with plentiful amounts of nectar or pollen that can be collected with less effort should be visited by more bees, whereas patches with less nectar or pollen should receive fewer bees [20-21].

The foraging process begins in a colony by scout bees being sent to search for promising flower patches. Scout bees move randomly from one patch to another. During the harvesting season, a colony continues its exploration, keeping a percentage of the population as scout bees [19]. When they return to the hive, those scout bees that found a patch which is rated above a certain quality threshold (measured as a combination of some constituents, such as sugar content) deposit their nectar or pollen and go to the "dance floor" to perform a dance known as the "waggle dance" [18]. This mysterious dance is essential for colony communication, and contains three pieces of information regarding a flower patch: the direction in which it will be found, its distance from the hive and its quality rating (or fitness) [18, 21]. This information helps the colony to send its bees to flower patches precisely, without using guides or maps. Each individual's knowledge of the outside environment is gleaned solely from the waggle dance. This dance enables the colony to evaluate the relative merit of different patches according to both the quality of the food they provide and the amount of energy needed to harvest it [21]. After waggle dancing on the dance floor, the dancer (i.e. the scout bee) goes back to the flower patch with follower bees that were waiting inside the hive. More follower bees are sent to more promising patches. This allows the colony to gather food quickly and efficiently. While harvesting from a patch, the bees monitor its food level. This is necessary to decide upon the next waggle dance when they return to the hive [21]. If the patch is still good enough as a food source, then it will be advertised in the waggle dance and more bees will be recruited to that source.

5.3. Proposed Bees algorithm

As mentioned, the Bees Algorithm is an optimization algorithm inspired by the natural foraging behavior of honey bees to find the optimal solution [17].

1. Initialize population with random solutions.

2. Evaluate fitness of the population.

3. While (stopping criterion not met), Forming new population.

4. Select sites for neighborhood search.

5. Recruit bees for selected sites (more bees for best e sites) and evaluate fitness.

6. Select the fittest bee from each patch.

7. Assign remaining bees to search randomly and evaluate their fitness.

8. End While.

The steps above shows the pseudo code for the algorithm in its simplest form. The algorithm requires a number of parameters to be set, namely: number of scout bees (b), number of sites selected out of b visited sites (m), number of best sites out of m selected sites (e), number of bees recruited for the best e sites (nep), number of bees recruited for the other (m-e) selected sites (nsp), initial size of patches (ngh) which includes the site and its neighborhood and stopping criterion. The algorithm starts with the b scout bees being placed randomly in the search space. The fitness of the sites visited by the scout bees are evaluated in step 2. In step 4, bees that have the highest fitness are chosen as "selected bees" and sites visited by them are chosen for neighborhood search. Then, in steps 5 and 6, the algorithm conducts searches in the neighborhood of the selected sites, assigning more bees to search near to the best e sites. The bees can be chosen directly according to the fitness associated with the sites they are visiting.

Alternatively, the fitness values are used to determine the probability of the bees being selected. Searches in the neighborhood of the best e sites which represent more promising solutions are made more detailed by recruiting more bees to follow them than the other selected bees. Together with scouting, this differential recruitment is a key operation of the Bees Algorithm. However, in step 6, for each patch only the bee with the highest fitness will be selected to form the next bee population. In nature, there is no such restriction. This restriction is introduced here to reduce the number of points to be explored. In step 7, the remaining bees in the population are assigned randomly around the search space scouting for new potential solutions. These steps are repeated until a stopping criterion is met. At the end of each iteration, the colony will have two parts to its new population representatives from each selected patch and other scout bees assigned to conduct random searches.

5.4. Optimum parameters of the Bees algorithm

The appropriate values of the BA parameters are obtained by examining different values of these parameters and making many trial and error runs. The following parameter values were set for this problem: population n=500, number of selected sites m=200, number of elite sites e=100, initial patch size ngh=5, number bees around elite points nep=50, number of bees around other selected points nsp=30. Note that ngh defines the initial size of the neighborhood in which follower bees are placed. For example, if x is the position of an elite bee in the i^{th} dimension, follower bees will be placed randomly in the interval $x_{ie} \pm ngh$ in that dimension at the beginning of the optimization process. As the optimization advances, the size of the search neighborhood gradually decreases to facilitate fine tuning of the solution.

These values are appropriate since they result in the best rate of convergence to a correct solution and are kept constant throughout this investigation. The details of this investigation could be found in [22].

6 RESULTS OF THE INVERSE PROBLEM

The same long fin of rectangular cross section presented in the section 3, subjected to the same boundary conditions is considered. A typical grid and its corresponding boundary elements and nodal locations are shown in Fig. 5.

The initial temperature is 225ºC and the surrounding temperature is 25 ºC.

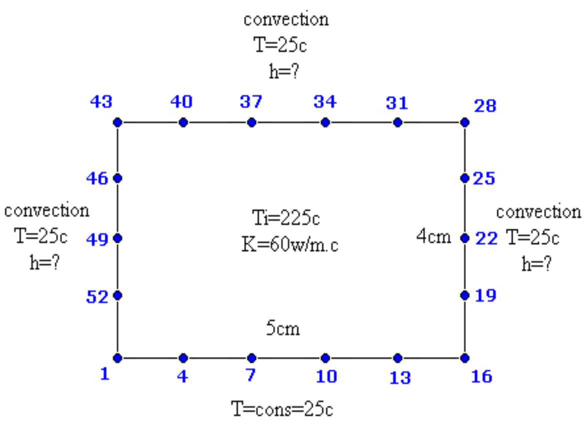

Fig. 5 Typical grid of problem modelling

An averaged value of h over each convective boundary is estimated as a function of time, from initial time until the time of 200 seconds in which the value of h reaches steady-state condition. The estimation procedure is such that an averaged value of h over each boundary for the first time interval of zero to twenty seconds is considered and based on this value the temperature distribution corresponding to that boundary is computed by the forward solution of the BEM program. These temperatures (with or without noise) are taken to be the measured temperatures which are to be used as specified conditions used by the inverse algorithm (BA) to estimate the corresponding h value for the same time interval. This step is repeated for the remaining time intervals.

The stopping criterion for the BA is either $|Z \le 10^{-10}|$ or the Max number of iterations being 50. Usually for error free temperature measurements, first criterion is satisfied, and for erroneous experimental temperature measurements the second criterion is satisfied. The reason for the selection of this stopping criterion is based on our knowledge of the characteristics of the real parameters and by playing with the value of the parameters and by trial and error. In the BA procedure after a number of iterations, similar solutions are obtained which is taken to be the best converged results. When error free measurements are used, often the first criterion i.e. $|Z \le 10^{-10}|$ gives accurate converged value of estimated parameters, but when erroneous measurements are used, since random errors are added to the temperatures, the possibility of reaching this tolerance does not exist and with more iterations, improvements of results is not obtained. With the size of population (n=500), the algorithm proceed even with more number of iterations, but improvements in results is not noticeable. Therefore to avoid long computational time, $iTer_{\max} = 50$ has been chosen as an optimum choice. The estimated values of h over each time interval as shown in Fig. 6 have been compared to the actual value of h considered. As it is shown, there is a tremendous agreement between the actual and estimated results for all the cases investigated. The corresponding heat flux at the convective boundaries could then be computed using the Newton's cooling law and estimated values of h at each time interval, i.e.

$$q = -h(T_s - T_\infty) \tag{10}$$

The heat flux distribution around the boundary at time $t = 80\,s$ is shown in Fig. 7.

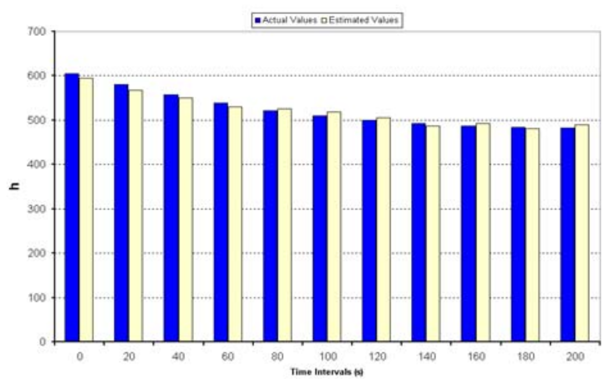

Fig. 6 Comparison of estimated and actual values of h

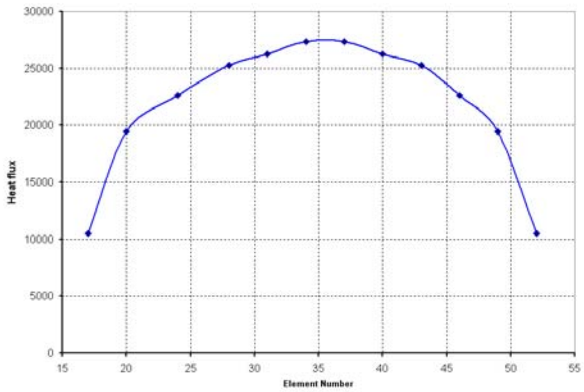

Fig. 7 Heat flux profile on the boundaries at t=80s

6.1. The effect of erroneous measurements on the estimation process

Finally, the effect of inevitable experimental errors on the estimation process is investigated. For simulation of this case, random errors are added to the computed temperatures with exact values of parameters, and these are taken to be the "measured" data. The statistical assumptions regarding the introduced errors are additive, non-correlated, normally distributed and have zero mean and constant variance.

Since the BA algorithm is only a direct search method, small experimental errors do not have much effect on its performance. The unknown parameters are estimated by experimental measurements with 1% up to 5% errors. Since the errors are added to the computed temperatures randomly, for each percent error 10 combinations of different erroneous temperatures are considered. The sample results are presented in table 1.

Table 1 The influence of experimental errors on the estimation process

Percent error	Average % error	Minimum % error	Maximum % error
1 %	2.74	2.18	3.67
2 %	3.53	2.96	4.28
3 %	5.10	4.18	8.18
4 %	6.81	5.11	11.63
5 %	8.26	5.85	14.91

As it is shown in table 1, the third and fourth columns, gives the minimum and maximum percent error. While the percent of random error in measurements increases, the percent error and the value of the coefficient variable also increases. It is observed that a 5% error in experimental measurements, leads to non-realistic values of heat transfer coefficient.

7 CONCLUSION

The combination of the boundary elements method and the Bees algorithm has shown to be an effective method for estimating boundary heat transfer coefficient and heat flux. The computational results show that the Bees algorithm has remarkable robustness in finding best solutions without becoming trapped at local optima. The optimum parameters of the Bees algorithm could be chosen such that the efficacy of the Bess algorithm is enhanced. It is shown that for error free temperature measurements, the exact value of h is estimated. As experimental error increases from 1% to 5% the error percent in the converged value of h also increases.

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