Vibration Analysis of an Initially Pre-Stressed Rotating Carbon Nanotube Employing Differential Transform Method

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Received: 13 April 2015, Revised: 19 July 2015, Accepted: 2 August 2015

Abstract: In this paper, nonlocal Euler–Bernoulli beam theory is employed for transverse vibration analysis of an initially pre-stressed size-dependent rotating nanotube. The nonlocal Eringen theory takes into account the effect of small size, which enables the present model to become effective in the analysis and design of nanosensors and nanoactuators. Governing equations are derived through Hamilton's principle and they are solved applying semi analytical differential transform method (DTM). It is demonstrated that the DTM has high precision and computational efficiency in the vibration analysis of nanotubes. The good agreement between the results of this article and those available in literature validated the presented approach. The detailed mathematical derivations are presented and numerical investigations are performed while the emphasis is placed on investigating the effect of the several parameters such as preload stress, hub radius, angular velocity and small scale parameter on vibration behaviour rotating nanotubes in detail. It is explicitly shown that the vibration of a spinning nanotube is significantly influenced by these effects.

Keywords: Differential Transform Method, Nonlocal Elasticity Theory, Spinning Carbon Nanotubem Vibration

Reference: Ebrahimi, F., Shaghaghi, G. R., "Vibration Analysis of an Initially Pre-Stressed Rotating Carbon Nanotube Employing Differential Transform Method", Int J of Advanced Design and Manufacturing Technology, Vol. 8/ No. 4, 2015, pp. 13-21.

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1 INTRODUCTION

In recent years nanostructures have attracted much attention in modern science and technology including communications, machinery, information technology, and biotechnology technologies due to their superior mechanical, electrical and thermal performances. Until now, three main methods were provided to study the mechanical behaviors of nanostructures. These are atomistic model [1], [2], semi-continuum [3] and continuum models [4], [5]. However, both atomistic and semi-continuum models are computationally expensive and are not suitable for analyzing large scale systems. Due to the inherent size effects, at nanoscale, the mechanical characteristics of nanostructures are often significantly different from their behaviour at macroscopic scale. Such effects are essential for nanoscale materials or structures and the influence on nano-instruments is great [6]. Generally, theoretical studies on size effects at nanoscale are done by means of surface effects [7], strain gradients in elasticity [8] and plasticity [9], as well as nonlocal stress field theory [10], [11], etc. Unfortunately, the classical continuum theories are deemed to fail for these nanostructures, because the length dimensions at nano scale are often sufficiently small such that call the applicability of classical continuum theories into the question. Consequently, the classical continuum models need to be extended to consider the nanoscale effects. This can be achieved through the nonlocal elasticity theory proposed by Eringen [10] which considers the size-dependent effect. According to this theory, the stress state at a reference point is considered as a function of strain states of all points in the body. This nonlocal theory is proved to be in accordance with atomic model of lattice dynamics and with experimental observations on phonon dispersion [11]. In nonlocal theory, the nonlocal nanoscale in the constitutive equation could be considered simply as a material-dependent parameter. The ratio of internal characteristic scale (such as lattice parameter, C-C bond length, granular distance, etc.) to external characteristic scale (such as crack length, wave length, etc.) is defined within a nonlocal nanoscale parameter. If the internal characteristic scale is much smaller than the external characteristic scale, the nonlocal nanoscale parameter approaches zero and the classical continuum theory is recovered. At present, the nonlocal elasticity theory has been used extensively to study lattice dispersion of elastic waves, wave propagation in composites, dislocation mechanics, static deflection, damage and fracture mechanics, surface tension fluids, piezoelectric materials, etc. [12-20]. In recent years, nanobeams and carbon nanotubes hold a wide variety of potential applications, such as sensors, actuators, transistors, probes, and resonators in NEMSs. Thus, establishing an accurate model of nanobeams is a key issue for successful NEMS design. As a result, nanotechnological research on free vibration properties of nanobeams is important because such components can be used as design components in nano-sensors and nano-actuators. Understanding the vibration behaviour of nanostructures is a key step for NEMS device design. There have been some studies on the vibration behaviour of beam like nanostructures using continuum model [21], [22].

For example, Wang and Feng [21] investigated the effects of both surface elasticity and residual surface tension on the natural frequency of micro beams Through Laplace-Young equation. Lee and Chang [22] studied the vibration behaviour of a non-uniform nano wire with consideration of surface effects using nonlocal elasticity theory. Miller and Shenoy [23] investigated the size-dependent elastic properties of nanoscale beams and plates by both surface elasticity and atomic simulation. Fu et al., [24] studied the nonlinear static and dynamic behaviour of nanobeams. The nanostructures that undergo rotation are a system with a promise future to be used in nanomachines which include shaft of nanomotor devices such as fullerene gears and carbon nanotube gears [10]. Nowadays, a great effort is devoted to the vibration analysis of nanobeams and CNTS under rotation using the Eringen nonlocal elasticity theory [25-27]. Pradhan and Murmu [25] applied a nonlocal beam model and Differential Quadrature Method (DQM) to investigate the flapwise bending-vibration characteristics of a uniform rotating nanocantilever. Murmu and Adhikari [26] investigated the same problem, but now considering an initially prestressed single-walled carbon to analyze the effect on the initial preload in the vibration characteristics. Narendar and Gopalakrishnan [27] analyzed the wave dispersion behavior of a uniform rotating nanotube modelled as a nonlocal Euler–Bernoulli beam.

Nanostructures undergoing rotation include nanoturbines, nanoscale molecular bearings, shaft and gear, and multiple gear systems. These nanostructure machines are expected to receive considerable attention in the near future. Researchers have thus reported the feasibility of nanoscale rotating structures. Examples include study of molecular gears, fullerene gears, and carbon nanotubes gears.

The motivation of this paper is the implementation of the differential transform method (DTM) to investigate the free vibration of a spinning initially pre-stressed nanotube. The concept of differential transform method was first introduced by Zhou [28] and its main applications therein are to solve both linear and nonlinear initial value problems in electric circuit analysis. The main advantage of this method is that it can be applied directly to partial differential equations

without requiring linearization, discretization, or perturbation. It is a semi-analytical-numerical technique that formulizes Taylor series in a very different manner. By using this method, the governing differential equations can be reduced to recurrence relations and the boundary conditions may be transformed into a set of algebraic equations. It is different from the high-order Taylor series method which requires symbolic computation of the necessary derivatives of the data functions. Another important advantage is that this method reduces the size of computational work while the Taylor series method is computationally time-consuming especially for high order equation.

In perspective of the above discussion, in this paper, we study the nonlocal bending-vibration of an initially pre-stressed carbon nanotube. The carbon nanotube is assumed to be attached to a molecular hub and is undergoing rotation. The other end of carbon nanotube is assumed to be having a tip and simply supported. Nonlocal Euler–Bernoulli beam theory is employed to formulate the governing equations. Differential Transform method is utilized to determine the nonlocal bending frequencies of the rotating system for the first time. The effects of the initial preload on vibration characteristics of rotating carbon nanotube are examined. Influence of nonlocal parameters, angular velocities, hub radii, and higher mode frequencies are studied. It is shown that bending vibration of a rotating carbon nanotube is significantly influenced by the existence of a preload, angular velocity and the nonlocal parameter.

2 DIFFERENTIAL TRANSFORM METHOD

Differential transformation technique is a method for solving ordinary differential equations. It uses a polynomial form that is sufficiently differentiable as the approximation to the exact solution. The technique is based on Taylor series expansion. The main difference between Taylor series method and the differential transformation method is that the former requires computations of higher order derivatives that are quite often formidable, while the latter involves iterative procedures instead. Applying the differential transformation technique in solving free vibration problems generally involves two transformations, namely, differential transformation (DT) and inverse differential transformation (IDT). The definitions of DT and IDT are as following [28]:

$$\mathrm{DT{:}Y(k)}=\frac{1}{\mathrm{k!}}\left[\frac{\mathrm{d^k}}{\mathrm{dx^k}}\mathrm{y(x)}\right]_{\mathrm{x=0}} \tag{1}$$

$$IDT: y(x)=\sum_{k=0}^{\infty}Y_k(x-x_0) \tag{2}$$

Combining Eqs. (1) and (2), one has:

$$y(x)=\sum_{k=0}^{\infty}\frac{x^k}{k!}[\frac{d^k y(x)}{dx^k}]_{x=0} \tag{3.a}$$

Which is the Taylor series of y(x) at x = 0. Eq. (3.a) implies that the concept of differential transformation is derived from the Taylor series expansion. In practical applications, the function y(x) is expressed by a finite series and the IDT is written as:

$$y(x)=\sum_{k=0}^{n}x^k Y[k] \tag{3.b}$$

The number of terms *n* is determined by convergence requirement in practice. In this study lower-case letters are used to represent the original functions and the corresponding upper-case letters to denote the DT transformed functions. Table 1 lists the transformation properties that are useful in the analysis that follows.

Table 1 Some of the transformation rules of the one-dimensional DTM [28]

Original function	Transformed function
$f(x)=g(x)\pm h(x)$	$F(K)=G(K)\pm H(K)$
$f(x)=\lambda g(x)$	$F(K)=\lambda G(K)$
$f(x)=g(x)h(x)$	$F(K)=\sum_{l=0}^{K}G(K-l)H(l)$
$f(x)=\frac{d^n g(x)}{dx^n}$	$F(K)=\frac{(k+n)!}{k!}G(K+n)$
$f(x)=x^n$	$F(K)=\delta(K-n)=\begin{cases}1 & k=n\\ 0 & k\neq n\end{cases}$

Table 2 Transformed boundary conditions (B.C.) based on DTM [28]

X=0		X=L	
Original B.C.	Transformed B.C.	Original B.C.	Transformed B.C.
f(1)=0	$F[0]=0$	f(L)=0	$\sum_{k=0}^{\infty}F[k]=0$
$\frac{df(L)}{dx}=0$	$F[1]=0$	$\frac{df(L)}{dx}=0$	$\sum_{k=0}^{\infty}k\,F[k]=0$
$\frac{d^2f(L)}{dx^2}=0$	$F[2]=0$	$\frac{d^2f(L)}{dx^2}=0$	$\sum_{k=0}^{\infty}k(k-1)F[k]=0$

3 NONLOCAL ELASTICITY THEORY

In the nonlocal elasticity theory, the stress field at a reference point x in an elastic medium is considered to be dependent not only on the strain at that point, but also on the strains at all other points in the domain[10]. This is accounted from the atomic theory of lattice dynamics and experimental observations on phonon dispersion. The basic equations for a linear homogenous nonlocal elastic body neglecting the body force are given as [11]:

$$\begin{aligned}&\sigma_{ij,j}=0,\\&\sigma_{ij}(x)=\int\phi(|x-x'|,\alpha)H_{ijkl}\varepsilon_{ijk}(x')dV(x'),\forall x\in V,\\&\varepsilon_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i})\end{aligned} \tag{4}$$

The terms σ_{ij} ‘ ε_{ij} and H_{ijkl} are the stress, strain, and fourth order elasticity tensors, respectively. $\phi(|x-x'|,\alpha)$ is the nonlocal modulus or attenuation function incorporating into constitutive equations. The nonlocal effects at the reference point *x* produced by local strain at the source x'. $|x-x'|$ represent the Euclidean distance and α is a material constant that depends on the internal (e.g., lattice parameter, granular size, distance between C–C bonds) and external characteristics lengths (e.g., crack length, wavelength). Material constant α is defined as $\alpha=e_0a/l.e_0$ is a constant for calibrating the model with experimental results. e_0 is estimated such that the relations based on nonlocal elasticity model could provide satisfactory approximation of atomic dispersion curves and atomic lattice dynamics results.

Equation (4) is in partial-integral form and generally difficult to solve analytically. Thus, a differential form of nonlocal elasticity equation is often used. According to Eringen, the expression of nonlocal modulus can be given as [10]:

$$\phi(|x|,\alpha)=(2\pi l^2\alpha^2)^{-1}k_0(\sqrt{x.x}/l\alpha) \tag{5}$$

Where K_0 is the modified Bessel function. The equation of motion in terms of nonlocal elasticity can be expressed as:

$$\sigma_{ij,j}+f_i=\rho\ddot{u}_i \tag{6}$$

Where $_i$‘ ρ and u_i are the components of the body forces, mass density, and the displacement vector, respectively. The terms *i*, *j* take up the symbols *x*, *y*, and *z*. Assuming the kernel function ∅ as the Green function, Eringen propose a differential form of the nonlocal constitutive relation as:

$$\sigma_{ij,j}+L(f_i-\rho\ddot{u}_i)=0 \tag{7}$$

Where

$$L=[1-(e_0a)^2\nabla^2] \tag{8}$$

And ∇ is the Laplacian operator. Using Eq. (5), the nonlocal constitutive stress-strain relation can be simplified as [11]:

$$(1-\alpha^2l^2\nabla^2)\sigma_{ij}=t_{ij} \tag{9}$$

4 NONLOCAL MODEL FOR ROTATING CARBON NANOTUBE

The simplest beam/Plate theory is the Euler–Bernoulli theory (EBT), which implies that plane sections that are normal to the mid-plane of the beam/plate, remain straight and normal to the mid-plane after deformation. Thus, the effects of shear deformation and rotational inertia against Timoshenko theory [29-32] are not included in this theory.

Consider a carbon nanotube of length *L* which rotates in a counter clockwise direction at a rotational speed Ω. The free end of the carbon nanotube has tip which is simply supported. The carbon nanotube is prestressed in the axial direction and is idealized as a nonlocal Euler–Bernoulli beam. In one-dimensional form, the constitutive relation (9) can be reduced to [11]:

$$\sigma(x)-(e_0a)^2\frac{\partial^2\sigma(x)}{\partial x^2}=E\,\varepsilon(x) \tag{10}$$

Where *E* is the conventional Young’s modulus of the nanobeam. Value of the nonlocal parameter depends on the boundary conditions, chirality, mode shapes, number of walls, and the nature of motion. In the investigation of the nonlocal parameter effect, it is crucial to determine the magnitude of the parameter e_0 since it has a significant influence on the effect of small length scale. So far, no experiments have been conducted to predict the magnitude of e_0 for CNTs. In the open literature, it is suggested that the value of nonlocal parameter can be determined by using a comparison of dispersion curves from the nonlocal continuum mechanics and molecular dynamics simulation. It should be noted that according to the previous discussions about the values of the nonlocal

parameter in detail, e_0a is usually considered as the single scale coefficient which is smaller than 2.0 nm for nanostructures [33-39].
Let the carbon nanotube be subjected to initial axial stress σ^o. Then the initial axial force is given as:

$$N_x = A\sigma_x^0 \tag{11}$$

The Euler Lagrange equation associated with the nonlocal Euler–Bernoulli beam theory for an preloaded rotating carbon nanotube can be expressed as [26]:

$$\frac{\partial^2 M}{\partial x^2} + \frac{\partial}{\partial x}(T\frac{\partial w}{\partial x}) - N_x\frac{\partial^2 w}{\partial x^2} - \rho A\frac{\partial^2 w}{\partial t^2} = 0 \tag{12}$$

Here, N_x is the preload on the rotating carbon nanotube, A is the cross sectional area of the beam, and T is the centrifugal tension which can be expressed as:

$$T(x) = \int_x^L \rho A\Omega^2(r+x)dx \tag{13}$$

$$\begin{aligned} M &= -EI\frac{\partial^2 w}{\partial^2 x} + (e_0a)^2[-\frac{\partial}{\partial^2 x}(T\frac{\partial w}{\partial x}) \\ &+ N_x\frac{\partial^2 w}{\partial^2 x} + \rho A\frac{\partial^2 w}{\partial^2 x}] \end{aligned} \tag{14}$$

Here, Ω denotes the angular velocity of the carbon nanotube; r is known as the hub radius and is the distance from the origin of rotation to the inner end of the carbon nanotube. By using the nonlocal elastic approach and Eqs. (13) and (14), the equation of motion for the in-plane loaded rotating carbon nanotube can be expressed as [26]:

$$\begin{aligned} &EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} - (e_0a)^2\rho A\frac{\partial^4 w}{\partial x^2\partial t^2} + N_x\frac{\partial^2 w}{\partial x^2} \\ &-(e_0a)^2 N_x\frac{\partial^4 w}{\partial x^4} - \rho A\Omega^2[(-r-x)\frac{\partial w}{\partial x} + (rL + \frac{L^2}{2} \\ &-rx - \frac{x^2}{2})\frac{\partial^2 w}{\partial x^2}] + (e_0a)^2\rho A\Omega^2[-3\frac{\partial^2 w}{\partial x^2} + 3(-r-x) \\ &\frac{\partial^3 w}{\partial x^3} + (rL + \frac{L^2}{2} - rx - \frac{x^2}{2})\frac{\partial^4 w}{\partial x^4}] = 0 \end{aligned} \tag{15}$$

Also boundary condition of carbon nanotube may be expressed as in Table 3.

Table 3 Boundary condition of the carbon nanotube

$w(x=0)=0$	$w(x=L)=0$
$\frac{\partial w}{\partial x}(x=0)=0$	$\frac{\partial^2 w}{\partial x^2}(x=1)=0$

With assumption of harmonic vibration, $w(x,t)$ may be expressed as follows:

$$w(x,t) = W(x)e^{i\omega t} \tag{16}$$

Hence, Eq. (15) can be rewritten in non-dimensional form using the following dimensionless parameters [26]:

$$\begin{aligned} &\xi = \frac{x}{L}, \qquad \delta = \frac{r}{L}, \qquad ,\gamma^2 = \frac{\rho A\Omega^2 L^4}{EI} \\ &\lambda^2 = \frac{\rho A\omega^2 L^4}{EI}, \qquad \overline{N} = \frac{N_x L^2}{EI}, \qquad \psi = \frac{e_0 a}{L} \end{aligned} \tag{17}$$

Substituting dimensionless parameters from Eq. (17) into Eq. (15), linear governing equations of motion may be expressed as:

$$\begin{aligned} &\frac{\partial^4 W}{\partial\xi^4} - \lambda^2 W + \frac{\partial^2 W}{\partial x^2} + \overline{N}\frac{\partial^2 W}{\partial x^2} - \psi^2 N\frac{\partial^4 W}{\partial\xi^4} \\ &-\gamma^2[(-\delta-\xi)\frac{\partial W}{\partial\xi} + (\delta + \frac{1}{2} - \delta\xi - \frac{\xi^2}{2})\frac{\partial^2 W}{\partial\xi^2}] \\ &+\psi^2\gamma^2[-3\frac{\partial^2 W}{\partial\xi^2} + 3(-\delta-\xi)\frac{\partial^3 W}{\partial\xi^3} \\ &+(\delta + \frac{1}{2} - \delta\xi - \frac{\xi^2}{2})\frac{\partial^4 W}{\partial\xi^4}] = 0 \end{aligned} \tag{18}$$

Using the transformation rules described in Table.1 and Eq. 18, the governing equation for prestressed rotating nanotube in DTM form can be expressed as:

$$\begin{aligned} &\frac{(k+4)!}{k!}W(k+4)[1-\psi^2\overline{N} + \psi^2\gamma^2(\delta+.5)] \\ &-\frac{(k+3)!}{k!}W(k+3)[\psi^2\lambda^2(3+\delta k)] - W(k+2) \\ &*[\frac{\psi^2\gamma^2}{2}k(k+1)(k+2)(k-1) + 3\psi^2\gamma^2\frac{(k+2)!}{k!} \\ &+\{\psi^2\lambda^2 + \overline{N} - \gamma^2(\delta+.5)\}\frac{(k+2)!}{k!}] + W(k+1) \\ &*(\gamma^2\delta(k+1)^2) + W(k)[k(k-1)\frac{\gamma^2}{2} - \gamma^2 k - \lambda^2] = 0 \end{aligned} \tag{19}$$

The rules of DTM for defining boundary conditions are given in Table (2).
The boundary condition for the rotating carbon nanotube at the free support using the above parameters can be expressed as:

$$\begin{aligned} &W(\xi=0)=0 \longrightarrow W(0)=0, \\ &\frac{\partial W}{\partial \xi}(\xi=0)=0 \longrightarrow W(1)=0, \\ &W(\xi=1)=0 \longrightarrow W(2)=c1, \\ &\frac{\partial^2 W}{\partial \xi^2}(\xi=1)=0 \longrightarrow W(3)=c2 \end{aligned} \tag{20}$$

Here W(k) are transforms of w(x). By substituting values for k=0,1,2,…, in equation (20), we can evaluate the amounts of and $W(2)$, $W(3)$,… in terms of ω and some constants like c_1,c_2… . The values can be achieved with a computer program and after substituting and in boundary conditions the following equation obtained:

$$M_{j1}^{(n)} c_1 + M_{j2}^{(n)} c_2 = 0,\ j = 1,2,3,\ldots,n \tag{21}$$

Here Ms are polynomials in terms of ω corresponding to nth term. When solving the equation (21) in matrix form, the following eigenvalue equation may be obtained:

$$\begin{vmatrix} M_{11}^{(n)}(\omega) & M_{12}^{(n)}(\omega) \\ M_{21}^{(n)}(\omega) & M_{22}^{(n)}(\omega) \end{vmatrix} \tag{22}$$

The solution of equation (22) gives $\omega^n{}_r$ which is the r^{th} estimated eigenvalue for n^{th} repeat. The number of repeat can be obtained by the equation (23):

$$\left|\omega_i^{(n)} - \omega_i^{(n-1)}\right| < \varepsilon \tag{23}$$

In the present study, δ=0.0001 and it shows the accuracy of calculations. With respect to the differential transformation method and algorithm above, a MATLAB code has been developed in order to determine the vibration characteristics of the nonlocal Euler-Bernoulli nanotubes.

5 RESULT AND DISCUSSION

The nonlocal governing equation is solved utilizing the DT approach. For the present DT analysis, sufficient numbers of increment were considered in the computation. The number of increments is assumed as 40. The computer package MATLAB is used to write code based on DT approach for the expression given by Eq. (19). Using the developed code, the effects of various parameters such nonlocal parameter, initial preload, angular velocity and hub radius on the natural frequencies of the rotating carbon nanotube are investigated in detail and the related graphs are plotted.

For the present study an armchair carbon nanotube with chirality (5, 5) is considered. The radius of each individual carbon nanotube is assumed as 0.34 nm and the Young's modulus, E, is taken as 0.971 TPa. The density and length are taken as 2300 kg/m3 and 100 nm, respectively. It should be noted that the results are depicted in normalized form.

These parameters are used in the computation to keep the versatility as other different geometrical and material properties of nanotubes can be used. As a validation example, the first nondimensional frequency of nonlocal Euler beam undergoing rotation already studied analytically by Wang et al.[40] are re-examined. Table 4 compares the first nondimensional frequency obtained by the present method with the results of Wang et al.[40] for nonlocal nanobeam with δ=0.05. Also table 5 compares the first nondimensional frequency of nonlocal nanobeam undergoing rotation for various scale parameters by the results presented analytically by Eltaher et al., [41]. As it can be seen in Tables (4) and (5) the good agreement and a close correlation among the results validate the proposed method of solution. In addition, the convergence of the differential transformation method is perused. In figure (1) the convergence of the first frequency of a nonlocal Euler–Bernoulli beam is presented. It illustrates that the first frequency converges at 25th iteration. The variation in fundamental frequency parameter (first mode) λ with preload parameter N using both local and nonlocal elastic models is presented in Figure 2. For the case of local model, the nonlocal parameter ($\psi = e_0 a/L$) is assumed to be zero; and is independent of scale effects. While for nonlocal model, the nonlocal parameters ψ are assumed to be 0.1, 0.2, 0.3, and 0.4, for the present numerical study, the preload parameter N *is* assumed to be in the range of -30 to 30. Both of the angular velocity parameter γ of the carbon nanotube and the hub radius δ are assumed to be unity.

Table 4 First non-dimensional frequency of nonlocal Euler beam undergoing rotation with δ=0.05

Ψ	Wang et al. [40]	Present study	Diff.(%)
0	15.3872	15.3342	0.3455
0.1	14.5992	14.5191	0.5518
0.2	-	12.6746	-
0.3	10.7767	10.7158	0.5690
0.4	-	9.0536	-
0.5	7.7835	7.7384	0.5830
0.6	-	6.7111	-
0.7	5.9360	5.9014	0.585

From Fig. 2, it is seen that the fundamental frequencies obtained for a given prestress load of nanotube by nonlocal elastic models are lower than that obtained by local models. Furthermore as the value of nonlocal parameter ψ increases, the value of frequency parameter, λ decreases.

Table 5 First non-dimensional frequency of nonlocal Euler beam undergoing rotation with δ=0.05

Ψ	Eltaher et al. [41]	Present study	Diff.(%)
0	15.3872	15.3342	0.3455
0.01	15.3967	15.3256	0.4639
0.02	15.3335	15.2990	0.2255
0.03	15.1959	15.2552	0.3651
0.04	15.0839	15.2029	0.7820
0.05	15.0249	15.1181	0.6164

Further it is observed that when the preload parameter *N* increases the frequency parameter λ also increases. With the increase in compressive load, (−*N*) the frequencies of the rotating nanotube decreases and as the frequency reaches zero, the nanotube is supposed to have reached the critical buckling state. With increasing the tensile preload *N*, the frequencies of the rotating nanotube increases.

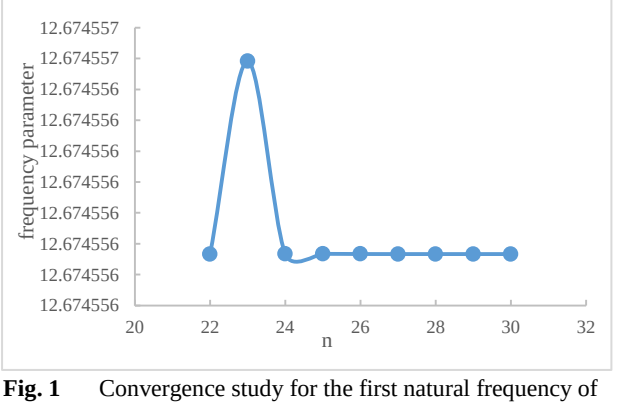

Fig. 1 Convergence study for the first natural frequency of carbon nanotube.

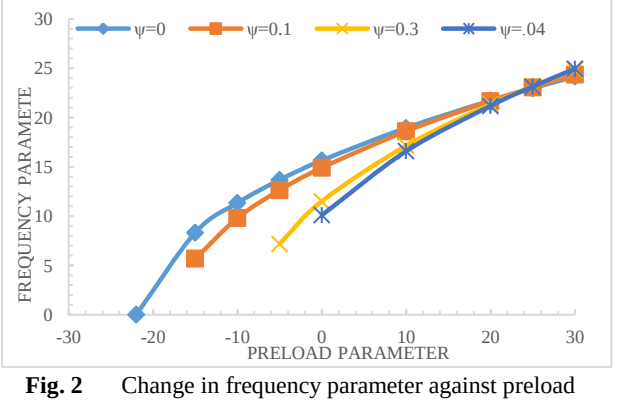

Fig. 2 Change in frequency parameter against preload parameter for various nonlocal parameters

This is attributed to the fact that as the nanotube is put into tension it becomes stiffer and consequently the frequency increases. Further it can be noted from the Fig. 2 that with increasing compressive preload, the vibrating beam tends to buckle. On application of compressive preload, the rotating nanotube with nonlocal parameter (ψ =0.4) is more prone to buckle than rotating nanotube analyzed with nonlocal model (ψ =0.0). This implies that rotating small size nanostructures are more affected by nonlocal parameter. This shows the importance of small-scale parameter in the analysis of rotating nanotube. In order to see the effect of rotation on the frequencies of carbon nanotubes, Figure 3 shows the variation in frequency parameter λ with preload parameter *N* for various values of angular velocities γ. Nonlocal elastic model is used here. We use a value of nonlocal parameter as (ψ =0.1). The angular velocities assumed in the present study are chosen as γ =1, 2, 3, and 4. The hub radius for the nanotube is assumed as δ =1. From Fig. 3, it is observed that the natural fundamental frequency decreases and increases with increasing compressive and tensile preloads, respectively. This trend is noticed for all the values of frequency parameter considered. Further, as the angular velocity γ of the nanotube increase, the fundamental frequency increases for all given axial preload. This is accounted from the fact that the angular rotation makes the nanotube stiffer due to centrifugal force which is directly proportional to the square of the angular velocity and hence the frequency rise. It is also important to notice that nonrotating nanotubes are more prone to reach the buckling state faster than the rotating ones.

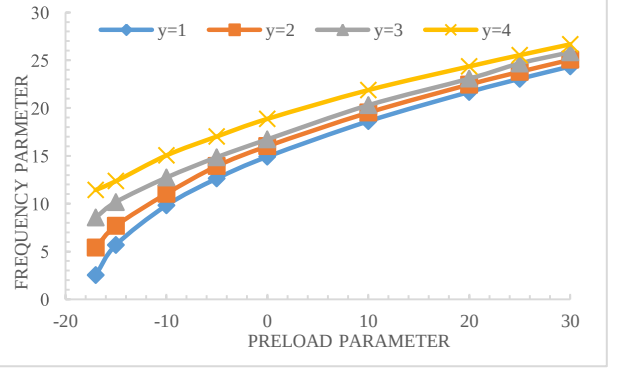

Fig. 3 Change in frequency parameter against preload parameter for various angular rotation speeds.

Figure 4 shows the variation in frequency parameter λ with preload parameter *N* for varies values of hub radius. Nonlocal elastic model is used here with nonlocal parameter assumed (ψ = 0.1). The angular velocities assumed in the present study are γ =1. The hub radii of the nanotube are assumed as δ =0.5, 1.0, 1.5, and 2.0.

From Fig. 4, it is observed that the natural fundamental frequency increases with increasing compressive and tensile preload for all the hub radii, respectively. However, not much variation between the curves with different hub radii is noticed. Though the frequency with higher hub radius δ=2.0 are marginally higher than the frequency with δ=0.5.

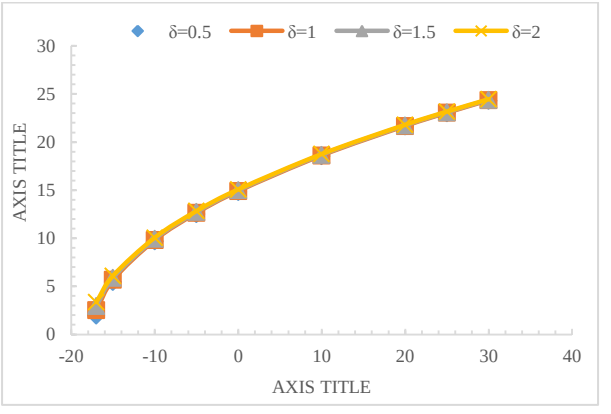

Fig. 4 Change in frequency parameter against preload parameter for various hub radii parameter

Here, we analyze the scale effects in higher modes of vibration. For a preloaded rotating nanotube, Fig. 5 shows the variation in frequency parameter λ with nonlocal parameter for different modes of vibration. Five modes of vibration are analyzed in the article (*m*=1, 2, 3, 4, and 5). Both the hub radius δ and angular velocity γ are assumed as unity, respectively. An axial compressive preload parameter of (*N* = -10) is assumed for the computation. For the rotating nanotube, it is observed that as the nonlocal parameter increases, the frequency decrease. The rate of drop of frequency with nonlocal parameter is magnified for higher modes (*m*=3, 4, and 5).

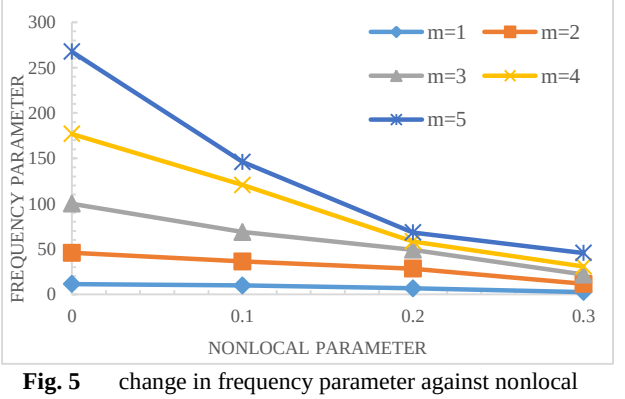

Fig. 5 change in frequency parameter against nonlocal parameter for higher modes of vibration

6 CONCLUSION

A semi-analytical method called differential transformation method is generalized to analyse the vibration characteristics of a rotating carbon nanotube. Frequencies are presented in a dimensionless form for generality. The formulation is based on the assumptions of classical Euler –Bernoulli beam theory and the nonlocal differential constitutive relations of Eringen. Also the effect of axial preload, small scale coefficient, angular velocities and hub radii on vibration characteristics of rotating carbon nanotube are investigated. It is found that that as tensile axial preload increases the fundamental frequency parameter of nanotube also increases. Further with the increase in compressive preload, the frequencies of the rotating nanotube decreases and the buckling state mode of the rotating nanotube is enhanced by the increase in nonlocal parameter. Also as the angular velocity of the nanotube increase, the fundamental frequency increases for all given axial preloads and the effect of hub radius is not prominent in the present nonlocal study. The higher modes frequencies of a given preloaded rotating nanotubes are much affected by nonlocal parameter. It is demonstrated that the DTM has high precision and computational efficiency in the vibration analysis of CNTs and nanobeams.

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