Kinematic Analysis of a Novel 3-CRS/PU Parallel Manipulator

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Abstract: In this research work, a novel parallel manipulator with high positioning and orienting rate is introduced. This mechanism has two rotational and one translational degree of freedom. Kinematics and Jacobian analysis are investigated. Moreover, workspace analysis and optimization has been performed by using genetic algorithm toolbox in Matlab software. Because of reducing moving elements, it is expected much more better dynamic performance with respect to other counterpart mechanisms with the same degrees of freedom. In addition, using couple of cylindrical and revolute joints, increased mechanism ability resulted to have more extended workspace.

Keywords: 3-CRS/PU, Degree of Freedom, Kinematics, Parallel Robot, Workspace

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1 INTRODUCTION

Parallel manipulators have received extensive attention over the last two decades for their potential superior properties, such as low inertia, high stiffness, high precision and high load carrying capacity [1], [2]. Performance indices such as manipulability, condition number, conditioning and dexterity indices are useful to comparison study for ability of different robot structures. Manipulability at first was introduced by Yoshikawa [3] which is determinant of the Jacobian matrix. The Jacobian matrix maps the sphere of unique joint space into an ellipsoid, whose volume is defined as manipulability. This mapping contains initial rotation, extension and a final rotation. The extension term magnifies the rotated spherical space in specified directions by the associate singular values [4].

Salisbury and Craig [5] introduced the ratio between maximum and minimum singular values as the condition number in Euclidean norm. The inverse of the Euclidean condition number is defined as condition index which varies from 0 to 1. Jacobian entries are inhomogeneous in complex degrees of freedom manipulators. Thus, making Jacobian homogeneous is studied by some researchers [6-13]. Rangbaran and Angeles [6] introduced characteristic length to making Jacobian homogeneous. Thus, entries with length dimension will be divided by this factor. Gosselin [9] introduced a method for formulating dimensionally homogeneous Jacobian matrix for a planar mechanism with one rotational and two translational DOF. This Jacobian matrix relates the actuator velocities to the velocities of the *x* -and *y*-coordinates of two points on the end-effector platform.

Kim and Ryu [10] boosted this work by using the velocities of three points on the end-effector platform to develop a dimensionally homogeneous Jacobian matrix. Pond and Corretero [11] advanced this method again by using three independent coordinates of three points on an end-effector platform. Moreover, Angeles [12] introduced engineering characteristic length for a rigid body transformation matrix to make it homogeneous.

In this research work a novel mechanism with high positioning and orienting rate is introduced. Kinematic is studied and Jacobian matrix is derived from these equations. Because of complexity of DOF, Jacobian matrix is homogenized by using weighted factor method [13]. Moreover, as a case study workspace volume and related indices has been studied through the workspace. Using GA method in Matlab software, has led to optimal mechanism structure with maximum workspace volume.

2 3-CRS/PU PARALLEL MANIPULATOR

As depicted in Fig. 1, this mechanism consists of three legs. Each one includes the fixed length link which connects spherical joint attached to moving platform to the revolute joints attached to cylindrical joint slides by a ball screw on a slant link connected to the base platform. Middle links contain passive prismatic joints at bellow and an upper part, which are connected to the moving platform, and are adjoined by a passive universal joint. This universal joint permits the moving platform to have rotational displacement around x and y axis. Moreover lower prismatic joint enables moving platform to have translational motion along z direction.

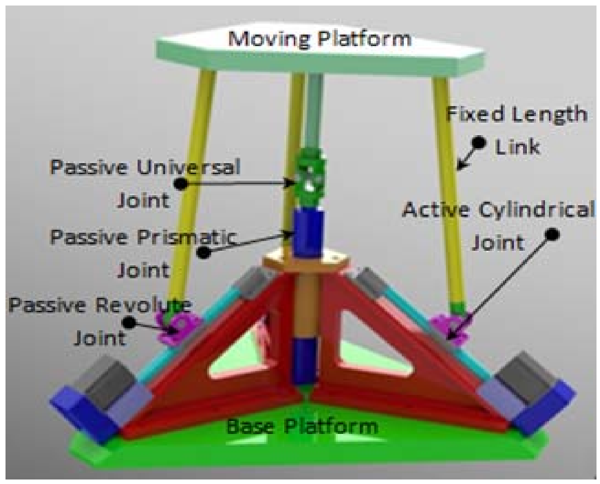

Fig. 1 CAD model of 3-CRS/PU parallel manipulator

3 KINEMATIC ANALYSIS

Geometrical model of the mechanism is illustrated in Fig. 2. Two moving and global frames ({*P* (*uvw*)} and {*O* (*xyz*)}) are attached to the moving and base platforms, respectively. When the moving platform is parallel to the base, the two revolute axes of the passive leg universal joints (x' and y') are parallel to the base frame's axes (*x* and *y*), respectively. The kinematic close loop equation can be written as follow for each leg:

$$c+R(a_i+d)=b_i n_{bi}+l_i n_{li} \tag{1}$$

where 'c' and 'd' are the vectors from '*O*' to '*C*' and '*C*' to '*P*', respectively. Moreover, 'R' is rotation matrix carrying frame {*P*} into an orientation coincident with that of frame {*O*}; 'a_i' is 'i^{th}' spherical joint position vector in moving frame. The connection radius of universal and spherical joint on the base and moving platform are illustrated by b_i and l_i respectively and associated unit vector by 'n_{bi}' and 'n_{li}', as well.

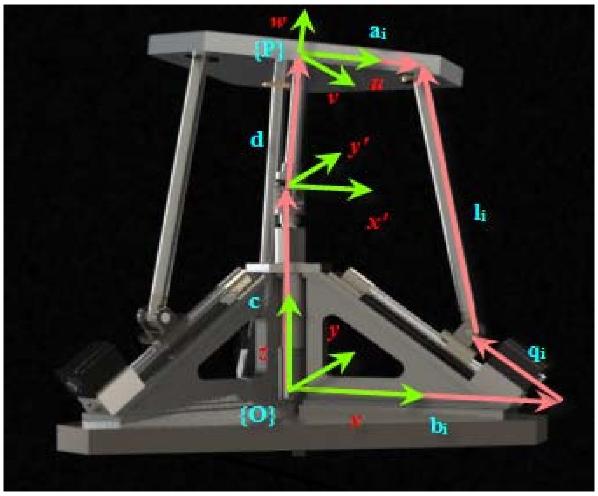

Fig. 2 Geometrical Model of 3-CRS/PU

3.1. Inverse Kinematic

Considering the schematic configuration of the mechanism, it is noteworthy that the end of each fixed length link should be mounted on the sphere surface whose center is 'A_i' and radius of 'l'. Moreover, this point should slide on the slant base whose geometry is identified by the height 'h'. Position vector of 'A_i' can be defined by the following equation.

$$A_i=R(a_i+d)+c \tag{2}$$

Thus, the intersection of slant base and associate sphere determines the revolute joint position and consequently actuator length. Considering spherical and universal joints position vector as $A_i=[x_{ai} \quad y_{ai} \quad z_{ai}]^T$ and $B_i=[x_{bi} \quad y_{bi} \quad 0]^T$, the parametric equation of 'GB_i' can be written as follow.

$$\begin{aligned} x &= -x_{bi}\, t_i + x_{bi} \\ y &= -y_{bi}\, t_i + y_{bi} \\ z &= h t_i \end{aligned} \tag{3}$$

Substituting the above equation in the parametric equation of sphere as the following:

$$(x - x_{ai})^2 + (y - y_{ai})^2 + (z - z_{ai})^2 - l^2 = 0 \tag{4}$$

leads to the following equation

$$m_i t_i^{\,2} - 2n_i t_i + p_i = 0 \tag{5}$$

in which the coefficients are as bellow

$$m_i = (x_{bi}^{\,2} + y_{bi}^{\,2} + h^2) \tag{6}$$

$$n_i = (x_{bi}^{\,2} + y_{bi}^{\,2} - x_{bi}x_{ai} - y_{bi}y_{ai} + hz_{ai}) \tag{7}$$

$$\begin{aligned} p_i = (&x_{bi}^{\,2} + x_{ai}^{\,2} - 2x_{bi}x_{ai} + y_{bi}^{\,2} \\ &+ y_{ai}^{\,2} - 2y_{bi}y_{ai} + z_{ai}^{\,2} - l^2) \end{aligned} \tag{8}$$

Substituting resultant 't_i' from Eq. (5) into Eq. (3) determines the actuator length. However, it is noteworthy that 't_i' limits should vary between 0 to 1.

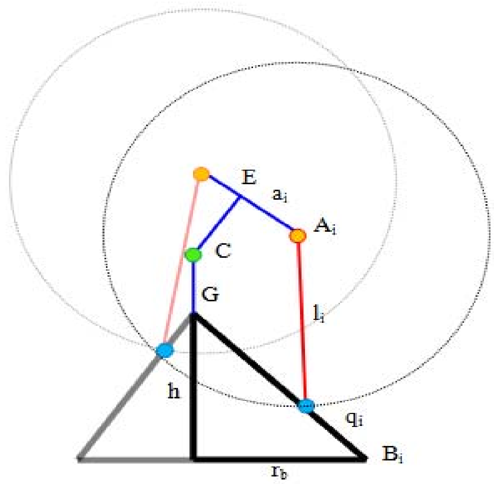

Fig. 3 Schematic configuration of 3-CRS kinematic

Accordingly, following cases may occur.

Case 1) The slant guide way does not intersect associate sphere: thus there is not any root for inverse kinematic equation. Correspondingly, assumed position will be out of reach by the end-effector (EE).

Case 2) The slant guide way tangent to the associate sphere: thus there is one root for the assumed link and with respect to other links conditions, there may be 1, 2 or 4 roots or any root used for kinematic equation.

Case 3) The slant guide way intersects associate sphere: thus there are two roots for the assumed link and with respect to other links conditions, which may be 2 or 4 roots or any root used for kinematic equation.

4 JACOBIAN AND VELOCITY

Taking the first derivative from Eq. (1) leads to the following equation:

$$\dot{c}+\omega_p\times R(a_i+d)=\dot{q}_i n_{qi}+\omega_l\times l n_{li} \quad (9)$$

in which 'ω_l' and 'ω_p' are fixed length link and moving platform angular velocities, respectively. Inner product of the both sides of Eq. (9) by 'n_{li}', and upon simplification leads to:

$$\dot{c}n^T_{li}+\omega_p\times R(a_i+d)n^T_{li}=\dot{q}_i n_{qi} n^T_{li} \quad (10)$$

Equation (10) can be rewritten as bellow

$$n^T_{li}\dot{c}+(n_{li}\times R(a_i+d))^T\,\omega_p=\dot{q}_i\, n^T_{li} n_{qi} \quad (11)$$

The above equation can be written as the following general form

$$A\dot{x}=B\dot{q} \quad (12)$$

in which $\dot{\mathbf{x}}$ and $\dot{\mathbf{q}}$ are end-effector twist array and joint space velocity vector. Moreover, 'A' and 'B' are Two Jacobian matrices which are as Eq. (13) and eq. (14).

$$A=\begin{bmatrix} n_{l1} & (n_{l1}\times R(a_1+d))_x & (n_{l1}\times R(a_1+d))_y \\ n_{l2} & (n_{l2}\times R(a_2+d))_x & (n_{l2}\times R(a_2+d))_y \\ n_{l3} & (n_{l3}\times R(a_3+d))_x & (n_{l3}\times R(a_3+d))_y \end{bmatrix} \quad (13)$$

$$B=\begin{bmatrix} n^T_{l1}n_{q1} & 0 & 0 \\ 0 & n^T_{l2}n_{q2} & 0 \\ 0 & 0 & n^T_{l3}n_{q3} \end{bmatrix} \quad (14)$$

The Jacobian matrix can be determined by Eq. (15).

$$J=B^{-1}A \quad (15)$$

5 SINGULARITY ANALYSIS

Generally, singularity occurs in a condition where there is not any root for Eq. (12) or there are infinity roots. In this condition, robot may miss one or more degrees of freedom or gain more degrees of freedom. This condition may occur with respect to the determinant of two Jacobian matrices, i.e. 'A' and 'B'. Moreover, Zalatanov et. al., [14] illustrated some constraint singularity for mechanisms, too.

For our mechanism, four cases may cause singularity, as follow:

Case 1) Det (A) = 0 or Direct singularity; this condition occurs when the z coordinates of fixed length links to be zero. In this condition all three legs lie in the moving platform plane which is parallel to the base one. Moreover, if any fixed length link locates along the line, connect the middle passive universal joint to associate spherical joint (CA_i) by increasing the actuator length, there are two positions to locate, as depicted in Fig. 4.

Case 2) Det (B)=0 or Inverse singularity; in this condition one of the fixed length link will be perpendicular to the associate linear guide way direction. This case is the same condition which one root may exist.

Case 3) Constraint singularity; this case will occur when the moving platform rotates 90 degrees around x or y axis. In this condition platform will lose another rotation capability.

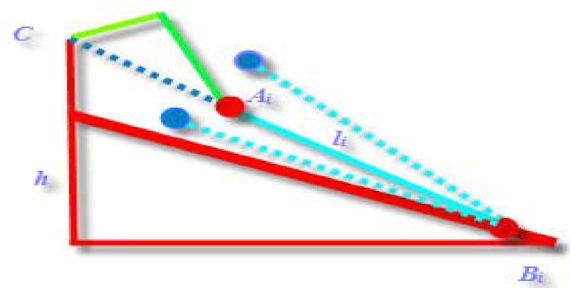

Fig. 4 Schematic of direct singularity

6 WORKSPACE AND OPTIMIZATION

Applying inverse kinematic equation and a search algorithm in different height will determine bound of reachable points [15]. Using genetic algorithm leads to optimized structure, which may cause maximum workspace volume [3], [15].

Hence, design parameters and their limitation should be considered as the optional factors which are the function of workspace volume. With respect to the Fig. 1, design parameters and assumed limitations are cited in Table (1).

Table 1 Geometrical constraint for mechanism

Actuator (mm)	ζ (deg)	d (mm)	l (mm)
400-750	±60	20-200	100-300
λ (deg)		r_b (mm)	r_a (mm)
10-80		300-500	100-300

As a case study, considering the structure using the parameters cited in Table (2) and the constraints according to Table (1), the search algorithm leads to the workspace volume as shown in Fig. 5.

Table 2 The case study design parameters

d (mm)	l (mm)	λ (deg)	r_b (mm)	r_a (mm)
100	250	45	300	100

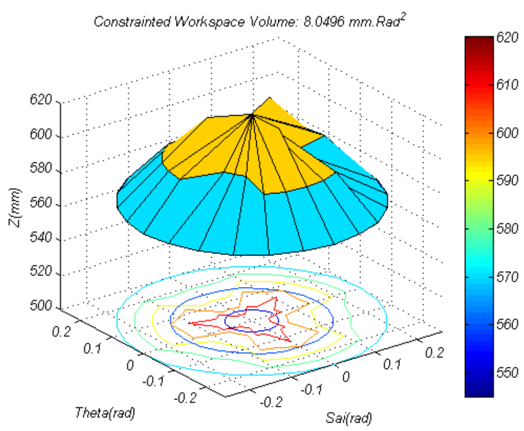

Fig. 5 Case study workspace

As illustrated in Fig. 5, the workspace volume is 8.05 ($mm.rad^2$). Variation of local conditioning index, maximum and minimum singular values in altitude of z=500 mm is depicted in the following figures. It is noteworthy that Jacobian matrix is homogenized by weighted factor as much as 200mm [13].

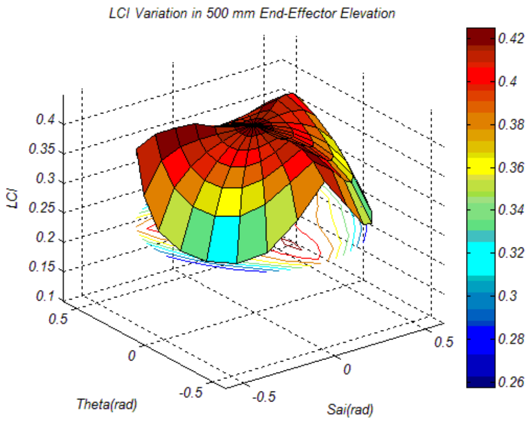

Fig. 6 Local Conditioning Index versus EE orientation in z=500mm

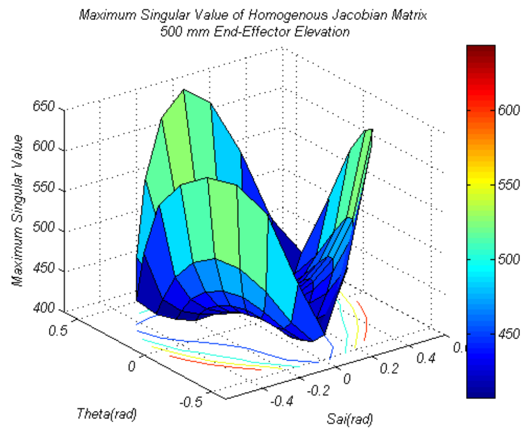

Fig. 7 Maximum singular value of homogenous Jacobian matrix versus EE orientation in z=500mm

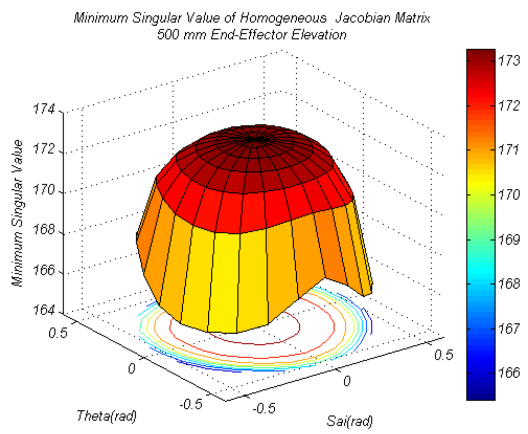

Fig. 8 Minimum singular value of homogenous Jacobian matrix versus EE orientation in z=500mm

Figure 6, shows that how far is the mechanism from the singularity. Moreover, it shows the mechanism isotropic behavior around zero rotations. The amplification factor via unique displacement in actuators is shown by singular values of homogenous Jacobian matrix. Figures 7 and 8, illustrate that these factors are too much. This is very important which identifies compatibility of mechanism in high positioning and orienting tasks.

Using GA in Matlab software, according to the following objective function and constraints, it leads to the structure whose parameters are cited in Table 3.

$$V^{*}=\mathrm{Max}(V(d, r_a, r_b, \lambda, l) \quad (16)$$

Subject to:

1- $100<r_a<300$, $300<r_b<500$, $20<d<200$,
2- $10<\lambda<80$, $100<l<300$,
3- 300<Actuator Length (L)<650
4- $-60<\zeta_i<60$ deg.

Optimization operation leads to design parameters according to Table 3, which indicates that the workspace volume has increased by 19.28 times. Moreover, optimal workspace is illustrated in Fig. 9.

Table 3 Design parameters of optimal workspace

V^{*} (mm.Rad^2)	l (mm)	λ (deg)
155.18	451.292	70.308
d (mm)	r_b (mm)	r_a (mm)
21.547	357.76	211.28

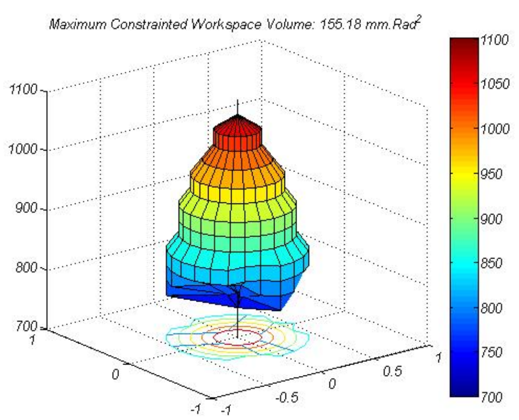

Fig. 9 Optimal workspace for the case study mechanism

7 CONCLUSION

In this research work a novel parallel manipulator with 3-CRS/PU structure is introduced. The mechanism has two translational and one rotational degree of freedom. Inverse kinematic equations with a geometrical approach have been solved and used for workspace evaluation. Jacobian matrix is derived by taking the first time derivative respect to time. Jacobian entries inhomogeneity has resolved by weighted factor approach. Variation of associate singular values through the workspace illustrates the capability of mechanism in high positioning and orienting tasks. Moreover, using genetic algorithm in Matlab software has led to optimal structure with maximal workspace volume.

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