Nonlinear Vibration and Instability of Embedded Viscose-Fluid-Conveying Pipes using DQM

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Abstract: In this paper, nonlinear vibration and instability response of an embedded pipe conveying viscose fluid is investigated. The pipe is considered as a Timoshenko beam embedded on an elastic foundation which is simulated by spring constant of the Winkler-model and the shear constant of the Pasternak-model. The external flow force, acting on the beam in the direction of the flexural displacement is described by Navier-Stokes equation. The corresponding governing equations are obtained using Hamilton's principle considering nonlinear strains and first shear deformation theory. In order to obtain the nonlinear frequency and critical fluid velocity for clamped supported mechanical boundary condition at two ends of the pipe, Differential Quadrature Method (DQM) is used in conjunction with a program being written in MATLAB. The effect of dimensionless parameters such as aspect ratios of length to radius of the pipe, Winkler and Pasternak modules, fluid velocity and viscosity as well as the material type of the pipe on the frequencies and instability of pipe are investigated. Results indicate that the internal moving fluid plays an important role in the instability of the pipe. Furthermore, the nonlinear frequency and instability increases as the values of the elastic medium constants and viscosity of fluid increases.

Keywords: DQM, Fluid, Instability, Nonlinear Vibration, Winkler-Model

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1 INTRODUCTION

Pipes conveying fluid have become one of the most important structures widely used in engineering, such as those employed in nuclear reactor, ocean mining, heat exchanger, drug delivery, microfluidic and nanofluidic devices [1-6]. In such applications, one of the most important issues is to accurately measure the vibration characteristics, such as natural frequency, stability and critical flow velocity of the fluid-conveying systems. It is not surprising, therefore, that the study on this topic has been constantly expanding in the past decades. In fact, the vibration and stability of pipes conveying fluid have been studied for more than six decades, both theoretically and experimentally. A good review of the related literature was provided by Païdoussis and Li [1]. As for the literature published so far, it is noted that several methods have been used to solve the vibration problem of such structures both in linear and nonlinear dynamics, such as Galer kinmethod [7-13], DQM [14-16], finite element method (FEM) [17-20], power series expansion and D-decomposition method [21].

In this study, the differential transformation method (DTM) is employed to investigate the free vibration of pipes conveying fluid with different boundary conditions. The DTM was first proposed by Zhou [22] for solving linear and non-linear initial value problems in electrical circuit analysis. Based on Taylor's series expansion, the DTM provides an effective and simple means of solving linear and non-linear differential equations. By using DTM, Chen and Ho [23] investigated the eigen values of Strum-Liouville problem. They also used this method to study the transverse vibration of rotating twisted Timoshenko beams under axial loading [24]. Mei [25] utilized the DTM to analyze the free vibration of a centrifugally stiffened beam. Muge and Metin [26] adopted the DTM to analyze the vibration of an elastic beam supported on elastic soil.

Chenand Chen [27] studied the free vibration of a conservative oscillator with inertia and static cubicnon-linearity and pointed out that the DTM has the inherent ability to deal with non-linear problems and it can be employed for the solutions of both ordinary and partial differential equations. In a latest literature, Odibat et al. [28] proposed a reliable new algorithm of DTM, namely multi-step DTM, which will increase the interval of convergence for the series solution, and applied the multi-step DTM to study the non-chaotic and chaotic dynamics of Lotka-Volterra, Chen and Lorenz systems. A generalized differential transform method (GDTM), i.e., the differential transform-Padé technique, mixed by DTM and Padé approximation, was proposed and used to solve differential-difference equation successfully by Zou et al. [29]. Very recently, Chenet al. [30] studied the natural frequencies and mode shapes of marine risers with different boundary conditions by using the DTM.

From a mathematical point of view, in fact, the DTM displays its advantage in many computational problems governed by differential equations. As an example, Chen and Chen [31] investigated the free behavior of a strongly non-linear oscillator with fifth-order non-linearities and showed that the DTM is a powerful tool for solving non-linear problems. They also pointed out that the DTM provides an accurate and efficient way for solving differential equations with high-order non-linearities. Kurnaz et al. [32] generalized the DTM to n-dimensional cases for solving partial differential equations (PDEs) with n-variables. It was shown that the DTM is a feasible tool to solve n-dimensional linear or nonlinear PDEs.

In this paper, nonlinear vibration and instability response of an embedded pipe conveying viscose fluid is investigated based on Timoshenko beam. The effect of dimensionless parameters such as aspect ratios of length to radius of the pipe, Winkler and Pasternak modules, fluid velocity and viscosity as well as the material type of the pipe on the frequencies and instability of pipe are investigated.

2 FORMULATION

Fig. 1 shows the pipes modeled as a Timoshenko beam with length L, inner radius r_1, outer radius r_2 and equal thickness h embedded in an elastic medium. The surrounding medium is described by the Winkler foundation model with spring constant k and Pasternak foundation model with shear constant G. Based on the Timoshenko beam theory, the displacements of an arbitrary point in the beam along the x- and z-axes, denoted by U and W respectively, take the form of Eq. (1) [6].

$$\begin{aligned} \tilde{U}(x,z,t) &= U(x,t) + z\psi(x,t) \\ \tilde{W}(x,z,t) &= W(x,t) \end{aligned} \tag{1}$$

Where U(x, t) and W(x, t) are displacement components in the mid-plane, ψ is the rotation of beam cross-section and t is time. The Von Karman type nonlinear strain–displacement relations are given by:

$$\begin{aligned} \varepsilon_{xx} &= \frac{\partial U}{\partial x} + z\frac{\partial \psi}{\partial x} + \frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2 \\ \gamma_{xz} &= \frac{\partial W}{\partial x} + \psi \end{aligned} \tag{2}$$

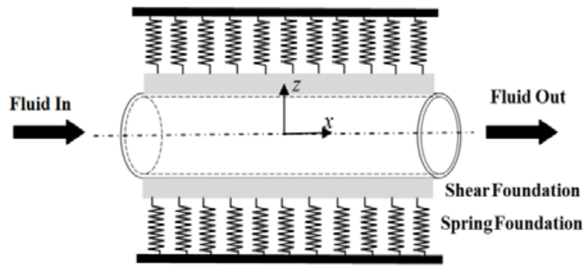

Fig. 1 Geometry of the pipe modeled as the nonlocal Timoshenko beam

For a beam structure, the constitutive relation scan be approximated to one-dimensional form as:

$$\sigma_{xx} = C_{11}\varepsilon_{xx}$$
$$\sigma_{xz} = C_{55}\left[\frac{\partial W}{\partial x} + \psi\right] \tag{3}$$

Where E and G are Young's modulus and shear modulus, respectively. The strain energy V of the pipes embedded in an elastic medium can be calculated from:

$$U = \frac{1}{2}\int_0^L \int_{Ai} (\sigma_{xx}\varepsilon_{xx} + \sigma_{xz}\gamma_{xz})\; dA_i dx \tag{4}$$

Submitting Eq. (2) into Eq. (4) gives:

$$U = \frac{1}{2}\int_0^l \left\{ N_x \frac{\partial U}{\partial x} + M_x \frac{\partial \psi}{\partial x} + \frac{1}{2} N_x \left(\frac{\partial W}{\partial x}\right)^2 + Q_x \frac{\partial W}{\partial x} + Q_x \psi \right\} dx \tag{5}$$

The normal resultant force N_x, bending moment M_x, and transverse shear force Q_x are defined as:

$$N_x = \int_A \sigma_{xx}\; dA,\; M_x = \int_A \sigma_{xx} z\; dA,\; Q_x = \int_A \sigma_{xz}\; dA \tag{6}$$

The work done by the elastic medium is denoted by:

$$\Omega = \frac{1}{2}\int_0^L \left(-K_w W + G_p \nabla^2 W\right) W dx \tag{7}$$

The work done by the viscose fluid is denoted by:

$$m_f \left[\frac{\partial}{\partial t} + U_f \frac{\partial}{\partial x}\right]\left[\frac{\partial W_1}{\partial t} - U_f \sin\theta\right] =$$
$$-\nabla P.A_f + \mu A_f \frac{\partial^2}{\partial x^2}\left[\frac{\partial W_1}{\partial t} - U_f \sin\theta\right]$$
$$m_f \frac{\partial^2 W_1}{\partial t^2} + m_f U_f \frac{\partial^2 W_1}{\partial x \partial t}\cos\theta + m_f U_f \frac{\partial^2 W_1}{\partial x \partial t}$$
$$+ m_f U_f^{\,2} \frac{\partial^2 W_1}{\partial x^2}\cos\theta = -\nabla P.A_f + \mu A_f \frac{\partial^3 W_1}{\partial x^2 \partial t} \tag{8}$$
$$+ U_f \mu A_f \frac{\partial^3 W_1}{\partial x^3}\cos\theta + U_f \mu A_f \left(\frac{\partial^2 W_1}{\partial x^2}\right)^2 \sin\theta$$

$$\Omega_{fluid} = \int (F_{fluid}) w dz, \tag{9}$$

The kinetic energy T is given by:

$$T_{tube} = \frac{1}{2}\rho_t \int_0^l \int_A \left[\left(\frac{\partial U}{\partial t} + z\frac{\partial \psi}{\partial t}\right)^2 + \left(\frac{\partial W}{\partial t}\right)^2\right] dA\, dx \tag{10}$$

The equations of motion of the fluid-conveying pipes embedded in an elastic medium can be derived from the Hamilton principle:

$$\int_{t_0}^{t_1} \left[\delta U - \left(\delta T + \delta\Omega\right)\right] = 0 \tag{11}$$

Introducing the following dimensionless quantities:

$$\xi = \frac{x}{L} \quad (w,u) = \frac{(W,U)}{r} \quad \eta = \frac{L}{r} \quad \overline{\mu} = \frac{\mu}{r\sqrt{E\rho_f}}$$
$$\bar{I} = \frac{\rho I}{\rho A r^2} \quad \tau = \frac{t}{L}\sqrt{\frac{E}{\rho_t}} \quad u_f = \sqrt{\frac{\rho_f}{E}} U_f \quad \psi = \overline{\psi}$$
$$f = \frac{EA_f}{EA} \quad \overline{\rho} = \frac{\rho_f}{\rho_t} \quad \overline{C} = \frac{Cl^2}{EA} \tag{12}$$
$$\overline{K}_w = \frac{K_w L^2}{EA} \quad \overline{G}_p = \frac{G_p}{EA} \quad \beta = \frac{K_s GA}{EA}$$

Substituting Eqs. (4), (7), (9) and (10) into Eq. (11), integrating by parts and setting the coefficients of δU, δW and $\delta\psi$ to zero lead to the dimensionless equations of motion as:

$$-\frac{1}{1-\upsilon^2}\frac{\partial^2 u_1}{\partial \xi^2}-\frac{1}{1-\upsilon^2}\frac{1}{\eta_1}\frac{\partial^2 w_1}{\partial \xi^2}\frac{\partial w_1}{\partial \xi}+\left(1+\bar{\rho} f_1\right)\frac{\partial^2 u_1}{\partial \tau^2}$$
$$-\sqrt{\bar{\rho}}\, f_1 u_f \frac{1}{\eta_1}\frac{\partial^2 w_1}{\partial \xi \partial \tau}\frac{\partial w_1}{\partial \xi}-f_1 u_f^2\frac{1}{\eta_1}\frac{\partial^2 w_1}{\partial \xi^2}\frac{\partial w_1}{\partial \xi} \qquad (13)$$
$$-\bar{\mu}\sqrt{\bar{\rho}}\, f_1 \frac{1}{\eta_1}\frac{\partial^3 u_1}{\partial \xi^2 \partial \tau}+f_1\bar{\mu} u_f\left(\frac{1}{\eta_1}\right)^2\frac{\partial^3 w_1}{\partial \xi^3}\frac{\partial w_1}{\partial \xi}=0$$

$$-\beta_1\frac{\partial^2 w}{\partial \zeta^2}-\eta\beta_1\frac{\partial \bar{\psi}}{\partial \zeta}-(1+\bar{\rho} f_1)\frac{1}{\eta_1}\frac{\partial^2 u}{\partial \tau^2}\frac{\partial w}{\partial \zeta}+\sqrt{\bar{\rho}}\, f_1 u_f\left(\frac{1}{\eta}\right)^2\frac{\partial^2 w}{\partial \zeta \partial \tau}\left(\frac{\partial w}{\partial \zeta}\right)^2$$
$$+f_1 u_f^2\left(\frac{1}{\eta}\right)^2\frac{\partial^2 w}{\partial \zeta^2}\left(\frac{\partial w}{\partial \zeta}\right)^2-\frac{1}{1-\upsilon^2}\frac{1}{\eta}\frac{\partial u}{\partial \zeta}\frac{\partial^2 w}{\partial \zeta^2}-\frac{1}{2}\frac{1}{1-\upsilon^2}\left(\frac{1}{\eta}\right)^2\left(\frac{\partial w}{\partial \zeta}\right)^2\frac{\partial^2 w}{\partial \zeta}$$
$$+(1+f_1\bar{\rho})\frac{\partial^2 w}{\partial \tau^2}+2\sqrt{\bar{\rho}}\, f_1 u_f\frac{\partial^2 w}{\partial \zeta \partial \tau}-\sqrt{\bar{\rho}}\, f_1 u_f\left(\frac{1}{\eta_1}\right)^2\frac{\partial^2 w}{\partial \zeta^2}\frac{\partial w}{\partial \zeta}\frac{\partial w}{\partial \tau} \qquad (14)$$
$$-\sqrt{\bar{\rho}}\, f_1 u_f\frac{1}{\eta}\frac{\partial^2 w}{\partial \zeta^2}\frac{\partial u}{\partial \tau}-\sqrt{\bar{\rho}}\, f_1 u_f\frac{1}{\eta_1}\frac{\partial^2 u}{\partial \zeta \partial \tau}\frac{\partial w}{\partial \zeta}+f_1 u_f^2\frac{\partial^2 w}{\partial \zeta^2}$$
$$-\sqrt{\bar{\rho}}\, f_1\bar{\mu}\frac{1}{\eta}\frac{\partial^3 w}{\partial \zeta^2 \partial \tau}+\bar{\mu}\sqrt{\bar{\rho}}\, f_1\left(\frac{1}{\eta}\right)^2\frac{\partial^3 u}{\partial \zeta^2 \partial \tau}\frac{\partial w}{\partial \zeta}^2-f_1 u_f\bar{\mu}\left(\frac{1}{\eta}\right)^3\frac{\partial^3 w}{\partial \zeta^3}\left(\frac{\partial w}{\partial \zeta}\right.$$
$$-f_1 u_f\bar{\mu}\left(\frac{1}{\eta}\right)^3\left(\frac{\partial^2 w}{\partial \zeta^2}\right)^2\frac{\partial w}{\partial \zeta}-f_1\bar{\mu} u_f\left(\frac{1}{\eta}\right)\frac{\partial^3 w}{\partial \zeta^3}+f_1\bar{\mu} u_f\left(\frac{1}{\eta}\right)^3\left(\frac{\partial^2 w}{\partial \zeta^2}\right)^2\frac{\partial w}{\partial \zeta}=$$

$$\frac{E}{1-\upsilon^2}\bar{I}\left(\frac{1}{\eta}\right)^2\frac{\partial^2 \bar{\psi}}{\partial \zeta^2}+\beta_1\frac{1}{\eta_1}\frac{\partial w}{\partial \zeta} \qquad (15)$$
$$+\beta_1\bar{\psi}+(\bar{I}+\bar{\rho}_f \bar{I}_f)\left(\frac{1}{\eta}\right)^2\frac{\partial^2 \bar{\psi}}{\partial \tau^2}=0$$

The associated boundary conditions can be expressed as:

$$w=v=u=0 \qquad @ \quad x=0,L$$
$$\frac{\partial w}{\partial x}=0 \qquad @ \quad x=0,L \qquad (16)$$

3 DIFFERENTIAL QUADRATURE METHOD

The differential quadrature (DQ) method is used to solve the nonlinear Eqs. (13)-(15) and the associated boundary conditions to determine the nonlinear free vibration frequencies of the pipes. The main idea of the differential quadrature (DQ) method is that the derivative of a function at a sample point can be approximated as a weighted linear summation of the function value at all of the sample points in the domain. The functions f={u, w, ψ } and their kth derivatives with respect to x can be approximated as [7]:

$$\frac{d^n f(x_i)}{dx^n}=\sum_{j=1}^{N} C_{ij}^{(n)} f(x_j) \qquad n=1,...,N-1. \qquad (17)$$

Where N is the total number of nodes distributed along the x-axis and C_{ij} is the weighting coefficients, the recursive formula for which can be found in [8]. The cosine pattern is used to generate the DQ point system.

$$X_i=\frac{L}{2}\left[1-\cos\left(\frac{i-1}{N_x-1}\right)\pi\right] \qquad i=1,...,N \qquad (18)$$

Using DQM, Eqs. (13) to (15) can be expressed in matrix form as:

$$\left(\left[\underbrace{K_L+K_{NL}}_{K}\right]+\Omega[C]+\Omega^2[M]\right)\begin{Bmatrix}\{d_b\}\\ \{d_d\}\end{Bmatrix}=0, \qquad (19)$$

Where M is the 'mass' matrix, KL is the linear 'stiffness' matrix and KNL is the nonlinear stiffness matrix. However, the frequencies obtained from the solution of Eq. (19) are complex due to the damping existed in the presence of the viscous fluid flow. Hence, the results are containing two real and imaginary parts. The real part is corresponding to the system damping, and the imaginary part representing the system natural frequencies.

5 NUMERICAL RESULTS AND DISCUSSION

The final converged solutions using the numerical procedure outlined in section *B* above are illustrated as nonlinear frequency and critical fluid velocity in Figs. 2-6 below. In the following subsections, the effects of aspect ratios of length to radius of the pipe, Winkler and Pasternak modules, fluid velocity and viscosity as well as the material type of the pipe on the frequencies and instability of pipe are studied and discussed in details.

Figs. 2 and 3 illustrate the effects of aspect ratio (L/R) on the dimensionless frequency versus fluid velocity and nonlinear frequency ratio against maximum amplitude, respectively. It is evident that an increase in the aspect ratio increase dimensionless frequency and critical fluid velocity. Also, with increasing L/R, nonlinear frequency ratio increases. This is because increasing L/R leads to softer pipe.

Figs. 4 and 5 illustrate the influence of the normalized Pasternak shear modulus ' K_g ' on dimensionless frequency versus fluid velocity and nonlinear frequency ratio (i.e. the dimensionless nonlinear to linear frequency (Ω_{NL}/Ω_L)) versus maximum amplitude ' $w_{\max}$ ', respectively. The results indicate that Ω_{NL}/Ω_L decreases substantially as harder elastic medium is employed. Hence, with increasing Pasternak shear modulus, Ω_{NL}/Ω_L decreases. Furthermore, as K_g increases, the critical fluid velocity and nonlinear frequency increase.

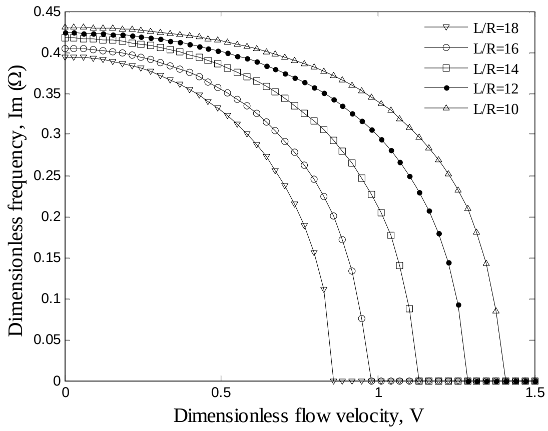

Fig. 2 The effect of geometrical parameter on nonlinear frequency versus Fluid velocity

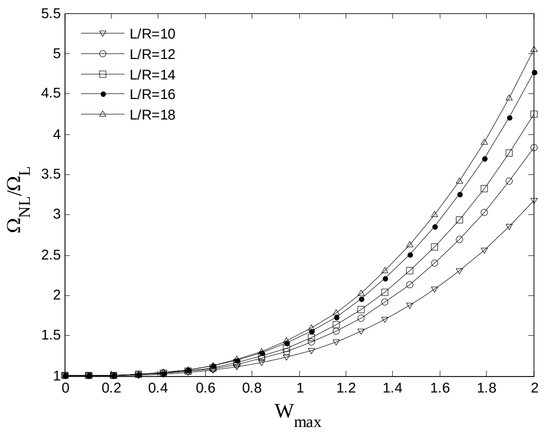

Fig. 3 The effect of geometrical parameter on nonlinear frequency ratio versus maximum amplitude

Fig. 6 illustrates the effect of fluid viscosity on the dimensionless frequency versus fluid velocity. The results indicate that viscous fluid increases natural frequency very little. However, during the flow of a fluid through a pipe as a Timoshenko beam, the effect of fluid viscosity on the vibration and instability of pipes may be ignored.

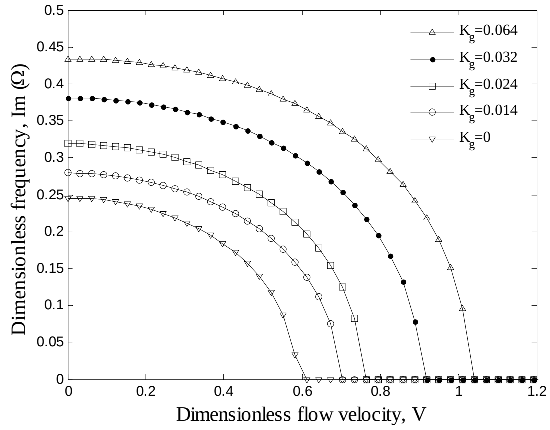

Fig. 4 The effect of Pasternak foundation on nonlinear frequency versus Fluid velocity

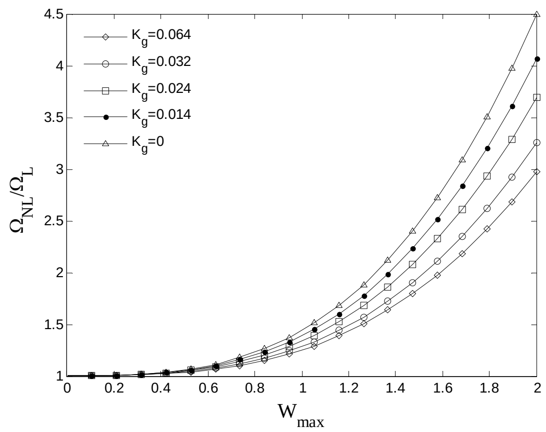

Fig. 5 The effect of Pasternak foundation on nonlinear frequency ratio versus maximum amplitude

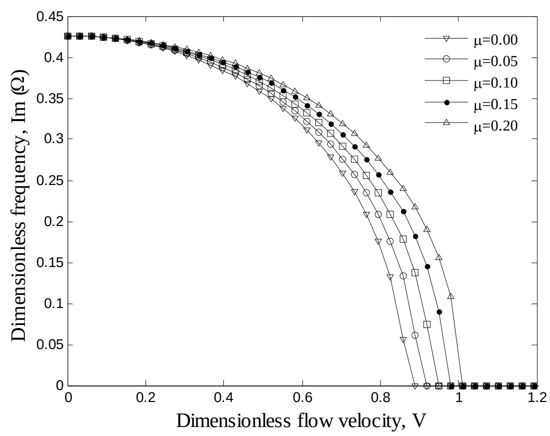

Fig. 6 The effect of fluid viscosity on nonlinear frequency versus Fluid velocity

6 CONCLUSION

This paper investigates the nonlinear free vibration and instability of oil pipes based on Von Karaman geometric nonlinearity and Timoshenko beam theory. The differential quadrature (DQ) method and a direct iterative approach are employed to obtain the nonlinear vibration frequencies and critical fluid velocity of pipe with clamped supported. The effects of aspect ratios, Winkler and Pasternak modules, fluid velocity and viscosity as well as the material type of the pipe on the frequencies and instability of pipe are investigated.

Results indicate that the internal moving fluid plays an important role in the instability of the pipe. Furthermore, the nonlinear frequency and instability increases as the values of the elastic medium constants and viscosity of fluid increases. Furthermore, during the flow of a fluid through a pipe as a Timoshenko beam, the effect of fluid viscosity on the vibration and instability of pipes may be ignored. Finally, it is hoped that the results presented in this paper would be helpful for study and design of oil pipes

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