Effects of Geometrical Tolerances on Residual Stresses in a Compound Shrink Fitted Pressure Vessel

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Abstract: Shrink fit process is a useful technique in order to introduce beneficial residual stress in compound pressure vessels. In this paper, the effects of geometrical tolerances on residual stresses have been studied for a compound shrink fitted pressure vessel, practically. Three layers which are designed based on an optimum nominal thickness and overlap dimensions and tolerances, have been fitted by shrink fitting to obtain a multi-layered high pressure vessel with desirable residual stress distribution. But in the manufacturing process, variations of inner and outer diameter of each layer have been observed within the design tolerances. The geometrical tolerances considerably affect the residual stresses. In this work, experimental results for residual stress are obtained from measurements of inner diameter of innermost cylinder due to two stages of shrink fitting. Then, the residual stress distribution is compared with analytical solution and finite element method at the lower limit and upper limit of tolerance domains. It is shown that very small geometrical tolerance could have a significant effect on residual hoop stress distribution. Also, the experimental results have a good agreement with analytical and finite element results.

Keywords: Compound Pressure Vessel, Geometrical Tolerances, Residual Stress, Shrink Fit

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1 INTRODUCTION

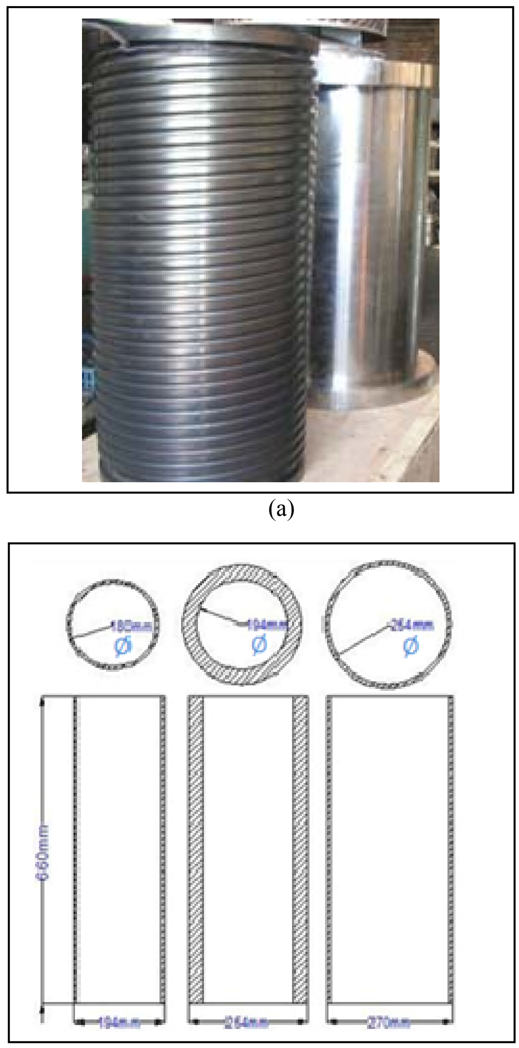
Nowadays application of thick-walled high pressure vessels is considerably developed; in particularthey are commonly used in petrochemical industry, manufacturing and specification improvement of materials, etc. It is necessary to find materials which can tolerate high pressure and temperature in order to increase the vessels efficiency. Shrink fitting is one of the methods recommended to increase strength and fatigue life of thick-walled vessels. In this way, beneficial residual stresses would be produced due to shrink fitting layers and making a compound multilayer vessel. Compression residual stress in the inner layers would lead to increase in the capacity of bearing internal pressure and fatigue life.

In shrink fitting process, the outer diameter of inner cylinder must be a bit larger than the inner diameter of outer cylinder. The inner cylinder is slipped inside the outer one after heating and cooling the outer and the inner cylinder, respectively. When the cylinders are allowed to return to their initial temperatures, a pressure (interface pressure) is created between the cylinders surfaces which are in contact. This pressure introduces compression residual stresses in the inner cylinder and tensile residual stress in the outer cylinder. As a result, the strength of the compoundcylinder subjected to internal pressure is increased. For more than two cylinders this process is repeated for each cylinder that is added to form the compound cylinder [1].

Severn discussed shrink-fit stresses between tubes which have a finite interval contact [2]. First, he studiedthe evaluation of shrinkage stresses when the contact interval between two infinite tubes is finite by using relaxation methods. Then, hediscussedthe problem when both tubes are finite, where, he solved the elastic equations for the stress-functions. An analytical solution is obtained by Gao and Atluri for the axisymmetric shrink fit problem with a thin strain-hardening hub and an elastic solid shaft [3]. The solution is based on the deformation theory of Hencky, the yield criterion of Von Mises, and the assumption of infinitesimal deformation. Jahed et al. proposed an axisymmetric method of elastic–plastic analysis which was capable for predicting residual stress field [4]. They solved inelastic axisymmetric boundary value problems by using linear elastic solution.

Jahed et al. presented a variable material property approach for solving elastic-plastic problems [5]. The method considered the material parameters as field variables. Lee et al. evaluated the residual stress effects on the fatigue life of an externally groovedthick-walled pressure vessel [6]. Fatigue life evaluation was performed based on the local strain approach. The design of shrink-fit precision gear forging dies based on strength considerations by using an analytical approach and the finite element method are compared by kutuk et al. for thick-wall cylinders[7]. Ozel performed a stress and deformation analysis of shrink-fitted joints for various fit forms via finite element method[8], where, he investigated the most appropriate fit type.

Jahed et al. proposed an optimum design for a three-layer vessel under the combined effects of autofrettage and shrink-fit [9]. They employed the Simplex search method for numerical optimization. Pederson showed how relatively simple axisymmetric analysis is possible [10]. He described two points of view, evaluation of classical plane analysis and the design of shrink fit surfaces. Kumar studied optimization of autofrettage-reautofrettage percent and shrink-fit combination for optimum fatigue life in multilayer vessels [11].



(a)

(b)

Fig.1 (a) The assembled cylinder (first, second and third layers), (b) the dimensions of three cylinders

Sedighi et al. investigated residual stresses in thick-walled vessels with combination of autofrettage and wire-winding [12]. They introduced a new wire-winding method based on Direct Method for vessels with nonlinear elastic or plastic behaviour.Shrink fitting is very sensitive to the magnitude of interference. A small variation in magnitude of interference can produce great influence on residual stresses. Therefore, manufacturing tolerances is very significant here. Whereas most of the previous works studied shrink fitting process analytically or numerically, the purpose of this paper isto measurethe experimental effects of geometrical tolerances on residual stresses due to this process.Then,the obtained results will be evaluatedusing analytical and finite element methods. In this work, first, the problem would be defined. Then, experimental results will be presented. Next, the obtained results are evaluated by analytical and finite element method, respectively where, the effects of diametric interference tolerance would be discussed.

2 PROBLEM DEFINITION AND THE GEOMETRY OF THE VESSEL

The cylinders which will be subjected to shrink fit are illustrated in Fig. 1. After cylinders fabrication and finishing process, the diameters of cylinders were measured, as reported in Table 1. Interference, absolute difference of outer diameter of inner cylinder and inner diameter of outer cylinder, may be calculated based on these measurements. Interference values, which introduce residual stresses, are shown in Table 2.

Table 1 Measured diameters of the three layers

	Inner diameter (mm)	Outer diameter (mm)
Inner layer	$180^{+0.04}_{+0.01}$	$194^{+0.34}_{+0.29}$
Middle layer	$194^{+0.00}_{-0.01}$	$254^{+0.06}_{+0.04}$
Outer layer	$254^{+0.04}_{-0.14}$	$270^{+0.10}_{-0.10}$

Table 2 Interference values

	Max interference (mm)	Min interference (mm)
1st shrink fit	0.35	0.29
2nd shrink fit	0.20	0.0

3 EXPERIMENTAL METHOD

Shrink fit process was done in two steps. In each step, inner diameter of the inner cylinder was measured in 16 points at the inner surface (the most critical points), and results were recorded, where the position of these 16 points are shown in Fig. 2.

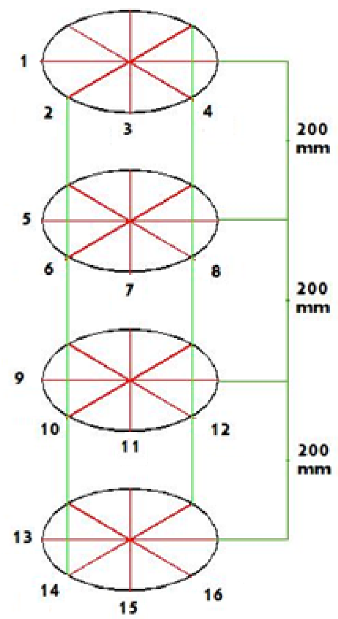


Fig. 2 Positions of the 16 points as indicated on the inner surface where their radial displacement are measured

After measuring redial displacements (considering diameters before and after shrink fitting), residual hoop stresses in the inner radius of inner cylinder could be calculated using the following equations [1]:

$$\varepsilon_\theta = \frac{u}{r} \tag{1}$$

$$\varepsilon_\theta = \frac{1}{E}[\sigma_\theta - \nu(\sigma_r + \sigma_z)] + \alpha\Delta T \tag{2}$$

Where ε_θ is the hoop strain, u and r are the radial displacement and radius at any point of the wall, respectively. Also, α is the thermal expansion coefficient and ΔT is the temperature difference.Since shrink fitting has been done in open end condition($\sigma_z = 0$), and also there is no temperature difference ($\Delta T = 0$) when the cylinders cooled down, so Eqs. (1) and (2) at the inner radius($\sigma_r = 0$) lead to:

$$\sigma_{\theta\ @r=90} = \frac{uE}{r} \tag{3}$$

In the first step, inner cylinder is placed in a combination of dry ice and alcohol and the middle cylinder is heated in order to be prepared for assembling. It should be noted thatthe limitation of allowable temperature must be considered.
After the first shrink fitting, cylinders are allowed to return to their initial equal temperature, before variations of inner diameter forinner cylinder are measured. The results are shown in Table 3.

Table 3 Experimental data for the first shrink fitting

	Diameter reduction (mm)	Residual stress (MPa)
Minimum	-0.24	-278
Maximum	-0.26	-302

In Table 3, Eq. (3) is applied to the measured displacementsin order to determine residual stresses. The resulted compression residual hoop stresses are between 278 and 302 MPa due to minimum and maximum diametric changes, respectively.For the second shrink fitting, the outer cylinder is heated in order to be prepared for assembly.
According to lower magnitude of the second shrink fitting interference, cooling of the first assembled layersis not needed. After assembling the third layer and returning to room temperature, the inner surface diameter of obtained compound vessel is measured. Then, by using Eq. (3), minimum and maximum residual hoop stresses at the inner surface are calculated and reported in Table 4.

Table 4 Experimental data for second shrink fitting

	Diameter reduction (mm)	Residual stress (MPa)
Minimum	0	0
Maximum	-0.05	-58

Total residualhoop stresses for the compound vesselare determined by summation of residual stresses resulted from the two processes (Table 5).

Table 5 Experimental residual hoop stresses and total stresses

	Min. stress (MPa)	Max. stress (MPa)
1st shrink fit	-278	-302
2nd shrink fit	0	-58
total	-278	-360

4 THEORETICAL METHOD

The equations for the elastic stresses in a thick-walled cylinder subjected to internal pressure were developed by Lame and Clapeyron [13]. The interference pressure between the inner and outer layers in a shrink fitted vessel is calculated as follows [14]:

$$P_{if} = \frac{\delta}{D_{if} A}$$
$$A = \frac{1}{E_i}\left(\frac{D_i^2 + D_{if}^2}{D_{if}^2 - D_i^2} - \nu_i\right) + \frac{1}{E_o}\left(\frac{D_o^2 + D_{if}^2}{D_o^2 - D_{if}^2} - \nu_o\right) \tag{4}$$

where P_{if} is the interference pressure between shrink fitted layers, δ is the diametrical interference between inner and outer layers, D_i is the diameter of inside surface of innermost layer, D_o is the diameter of outside surface of outermost layer, D_{if} is the diameter of the interface between layers, E_i and E_o are the elastic modulus of inner and outer layers, respectively. Also, ν_i is the Poisson's ratios of inner layer and ν_o is the Poisson's ratios of outer one. The residual stresses at any point in the inner layer, $D_i<D<D_{if}$, are then calculated from Eqs. (5) and (6) [14]:

$$\sigma_\theta = -\frac{P_{if} Y_i^2}{Y_i^2 - 1}\left(1 + \frac{D_i^2}{D^2}\right) \tag{5}$$

$$\sigma_r = -\frac{P_{if} Y_i^2}{Y_i^2 - 1}\left(1 - \frac{D_i^2}{D^2}\right) \tag{6}$$

And in the outer layer, $D_{if}<D<D_o$, from Eqs. (7) and (8):

$$\sigma_\theta = \frac{P_{if}}{Y_o^2 - 1}\left(1 + \frac{D_o^2}{D^2}\right) \tag{7}$$

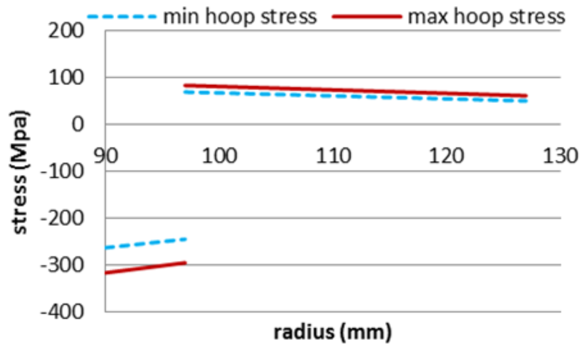
$$\sigma_r = \frac{P_{if}}{Y_o^2 - 1}\left(1 - \frac{D_o^2}{D^2}\right) \tag{8}$$

Where

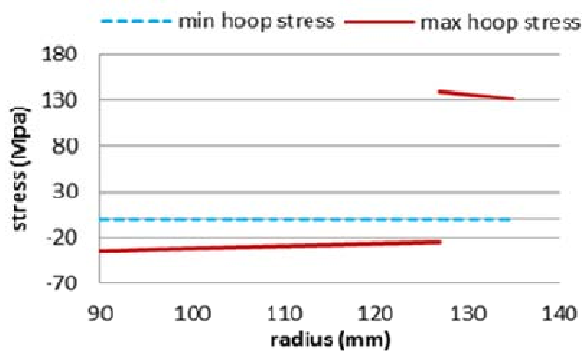
$$Y_i = \frac{D_{if}}{D_i}$$
$$Y_o = \frac{D_o}{D_{if}}$$

Subsequently σ_θ is hoop stress, $\sigma_r$is radial stress and D is diameter at any point. For the case of vessels composed of more than two layers assembled (three layers for this case), interference between first assembly and the next layer should be determined and the resultedresidual stresses should be calculated as if the first two layers were a single layer.

Finally, the stresses calculated in each part should be added together to determine the total residual stress distribution in the final assembly [14].



(a)



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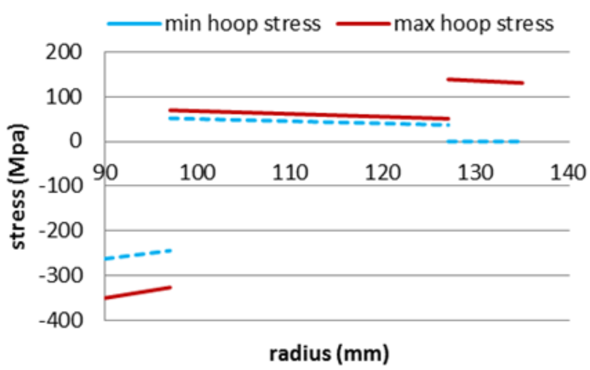


(c)

Fig.3 Residual hoop stresses resulted from theoretical method, a) resulted from the first shrink fit, b)resulted from the second shrink fit, c) summation of a and b

Residual stress distributions are illustrated in Fig. 3 and Fig. 4. Hoop stresses are shown based on upper and lower limit of geometrical tolerances for first assembly in Fig. 3(a). In Fig. 3(b), third layer is added to the assembly, and hoop stresses are determined between upper and lower limits of tolerance domain. Eventually, the stresses at the last two parts are added together to denote the distribution of total residual hoop stress which is shown in Fig. 3(c).

5 FINITE ELEMENT METHOD

Modelling and analysis of shrink fitting of the three layers are prepared by applying Ansys software. Axisymmetric condition is applied to the model, inner and outer layers are created and meshed by beneficial of Plane 42 and Plane 82 elements (Plane 82 elements can model the water channel of second layer more accurately). Residual hoop stresses resulted from minimum interference for the first shrink fitting are illustrated in Fig. 4.

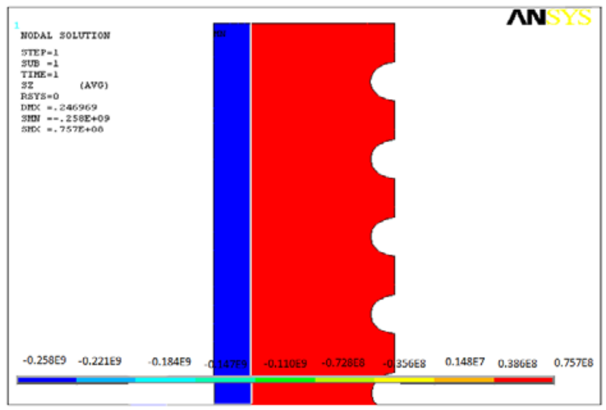
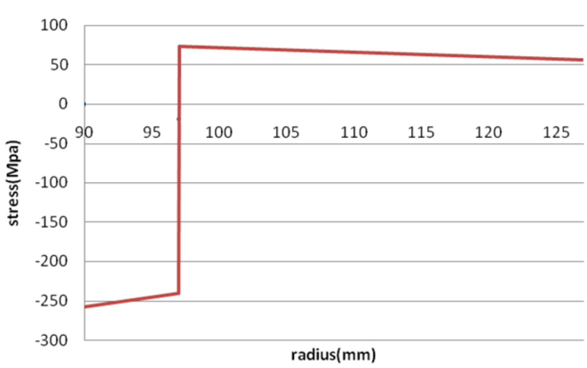
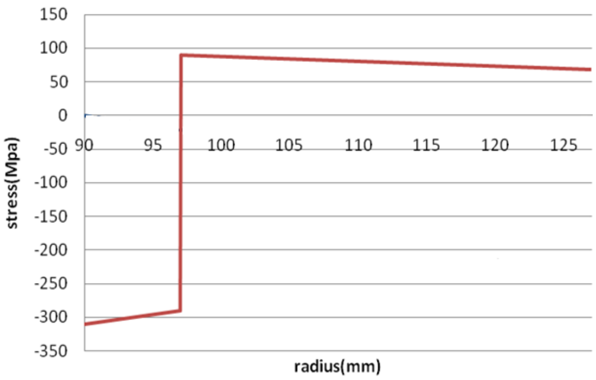


Fig.4 Residual hoop stress resulted from minimum interference for the first shrink fitting by finite element method

Fig. 5 shows calculated stress distribution due to the first shrink fit process. Similarly residual hoop stress resulted from maximum interference for the second shrink fitting are illustrated in Fig. 6. Also Fig. 7 shows the calculatedresidual hoop stress distribution due to the second shrink fitting by finite element method.



(a)



(b)

Fig. 5 Residual hoop stress for the first shrink fitting resulted from finite element method, a) minimum interference, b) maximum interference

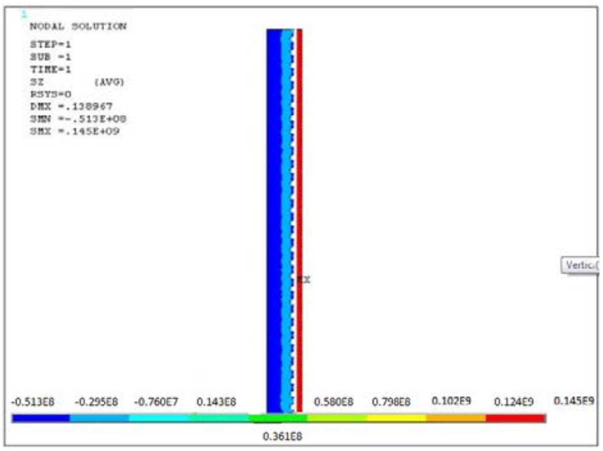


Fig. 6 Residual hoop stress resulted from maximum interference for the second shrink fitting by finite element method

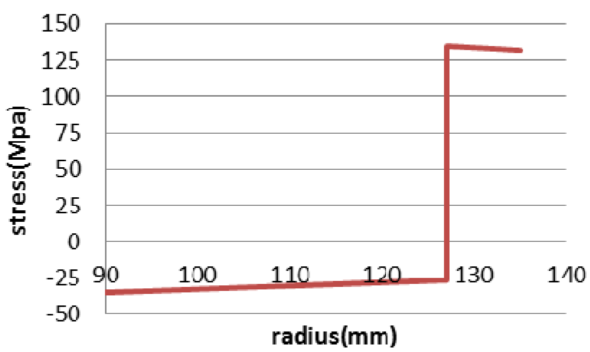


Fig. 7 Residual hoop stress resulted from maximum interference for the second shrink fitting by finite element method

6. COMPARISON OF THE RESULTS

Experimental, analytical and finite element results for residual hoop stress in the most critical points (inner radius) are compared in Table 6. According to Table 6, the theoretical and finite element method results are in a very good agreement and the maximum error (about 2.9%). The small differences between them are related to neglecting the water channel of the middle layer in analytical model. It shows that the water channel have no significant effect on residual stresses. Also, experimental and finite element results are close together which means that variations of diameter are recorded accurately.

Table 6 Comparison of residual hoop stresses from experimental, analytical and finite element methods

		1st shrink fit	Error	2nd shrink fit	Error	Total	Error
	Exp.	-278		0		-278	
Min. stress (MPa)	FEM.	-259	6.8%	0	0.0%	-259	6.8%
			1.2%		0.0%		1.2%
	Theo.	-262		0		-262	
	EXP.	-302		-58		-360	
Max. stress (MPa)	FEM.	-311	3%	-35	39%	-346	3.8%
			1.6%		2.9%		1.1%
	Theo.	-316		-34		-350	

The effects of geometrical tolerances on the hoop stress in the inner surface of vessel are compared between experimental and finite element results in Table 7. It should be noted that very small variation of interference (about 0.06 mm) could causegreat effects on residual hoop stress (about 52 MPa). Since hoop stress is the most important stress in pressure vessel, which has a direct effect on fatigue life, so the interference tolerance should be considered accurate as much as possible.

Table 7 Interference effects on residual stress

	Interference	Residual stress
1st shrink fit-Experiment	0.29 ~ 0.35 mm	-278 ~ -302
1st shrink fit-FEM		-259 ~ -311MPa
2st shrink fit-Experiment	0.00 ~ 0.20 mm	0.0 ~ -58
2nd shrink fit-FEM		0.0 ~ -35MPa

7 CONCLUSION

Geometrical tolerances on contact surfaces directly affect the magnitude of interference, so they have considerable effects on residual stress of multi-layered pressure vessel subjected to shrink fit. In this paper, due to the manufacturing process, the variation of inner diameter of shrink fitted vessel was measured practically for a three-layered shrink fitted vessel. Then, the effect of diametrical interference tolerance on residual hoop stress has been highlighted and it wasshown that very small variation of interference may causegreat effects on residual stress. The obtained results are verified by applying analytical and finite element methods. Therefore, designers are supposed to limit the interference tolerances as much as possible and consider their great disadvantages on residual stresses. Moreover, comparison of analytical and finite element results shows that finite and limited machining such as water channels on the outer surface of layer does not have a considerable effect on residual hoop stress.

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