Constrained Optimization of an Commercial Aircraft Wing Using Non-Dominated Sorting Genetic Algorithms (NSGA)

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Abstract: In this paper, optimization of Boeing 747 wing has accomplished for cruise condition (Mach number=0.85 and flight altitude=35000 ft) and an optimal wing shape have been proposed. Optimization problem has two objectives and is constrained for this research. Objective functions are minimization of wing weight and drag force that as well as confining design parameters, two functional constrains are applied. The first functional constrain is fuel tank volume in the aircraft wing that supply the required fuel. The second functional constrain is lift coefficient that should be equal to initial lift coefficient. Design parameters are root chord, wing span and wing sweep angle. Non-dominating genetic algorithm has been used in optimization process until finding one optimal solution; set of solutions (pareto front) are obtained for two objective functions. Finally a criterion for selecting a best solution for the aircraft on the pareto frontier is addressed.

Keywords: Drag Force, Multi-Objective Genetic Algorithm, Optimization, Pareto Set, Wing Weight

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1 INTRODUCTION

During the last four decades, many algorithms have been developed to find the solution of different engineering optimization problems. Most of these algorithms are based on linear and nonlinear programming methods that require gradient information. These numerical optimization algorithms have made a useful strategy for finding global minimum in simple problems. But many of actual engineering optimization problems are very complex and completely difficult to slove. Moreover if there is more than one local optimal solution, results may become dependant on choosing initial points and optimal solutions may not be global minimum. Moreover gradient search is difficult when objective function and its constraints were multiple [1]. So, traditional calculation systems may are appropriate in precise and exact calculation but have fragile performance. In contrast, emersion of evolutionary calculations creates new source for optimization problem solution.

Evolutionary calculations present a more efficient and robust approach to solve complex problems. Many evolutionary algorithms are proposed such as genetic algorithm, genetic programming and etc. Since these algorithms are evolutionary and stochastic, they have less probability to involve in local minimum. These algorithms are based on individual populations that were created by behaviours where they are similar to biological phenomena.

Among evolutionary algorithms, genetic algorithms are most popular. This algorithm is a stochastic search based on natural selection, heredity science and evolution. Since these algorithms analysis many points in search space simultaneously, they have more probability for finding global solution. Also this algorithm don't require gradient. So this is a global optimization method for problem solution [2].

Genetic algorithms are search methods that mimic natural biology evolution. This algorithm originally was developed by John Holand in 1970 when many developments were applied on by Dejong and Goldberg. Genetic algorithms act on population of innate solutions and apply Principle of Survival of fittest until make better approximation for solutions.

In initial genetic algorithms, parameters are defined as binary but Wright showed in 1991 that using continuous values instead of binary values in genetic algorithms have more advantages such as better performance and less computer memory [3].

One of the most important optimization applications in engineering is optimization and optimal design of aerospace vehicles. Optimal design is an ideal method for aircraft design. At this field many activities have been done in the last thirty decades that focused mostly on best aerodynamic and structural design. Most of researchers have focused on applying multidisciplinary design optimization (MDO) instead of classical method and wing optimization during conceptual and preliminary design stages, respectively [4].

Lee et al., investigated wing preliminary design of an unmanned aerial vehicle by means of a two steps approach. First step is a single objective optimization that aerofoil geometry has been optimized while second step is a structural and aerodynamic optimization (two objectives optimizations).

Applying constrains for aerodynamic are stall speed, maximum speed and rate of climb and for structure optimization are strength and stiffness. Applied optimization algorithm was non-dominated sorting genetic algorithm 2 (NSGAⅡ) where resulted in five pareto optimal points [4].

Ng and Leng have studied airfoil and wing optimization design of an unmanned aircraft. In this research, genetic algorithm and hybrid genetic algorithm have been used as two optimization methods and pareto front set has drawn using these two methods and finally optimization results are compared.

It is worthy to say that considered constrains for this problem is only on parameters [5]. Rajagopal and Ganguli have studied two dimensional wing shape optimization which it was done in presence of ground effect by integration of CFD and multi-objectives genetic algorithm. This study considered lift coefficient, lift to drag ratio and static stability as the objective functions and parameters limit is the only constrain. Obtained solutions are not unique but optimal solutions set (pareto set) has been gained [6]. Lee and Geem investigated Shape optimization of a wing section with ground effects which it was done using CFD and multi-objective genetic algorithms. Main factors are wing aerodynamic characteristics in presence of ground effect, lift force, static stability and lift to drag ratio.

In this study lift coefficient, aerodynamic center and lift to drag ratio are selected as the objective functions and applied constrain is only parameter limitation. Similar to many multi-objective optimizations, instead of one solution, a set of optimal solutions are presented [7]. Park and Lee investigated design optimization of a subsonic wing. This optimization was done for cruise condition and for aerodynamic and structure CFD and finite element has been used respectively. Optimization method is subplex method and paper did not refer to constrains [8].

The aim of this paper is wing optimization of a commercial aircraft. Objective functions are minimization of wing weight and drag force whilst fuel volume and lift coefficient are invariable in cruise condition. With attention to genetic algorithm capability for solving complex problem and their ability to find global solution so considered method is genetic algorithm. Because we face with a multi-objective constrained problem so we use non-dominated sorting genetic algorithm. This method is an upgraded version of ordinary genetic algorithm and are used for multi-objective problem.

This version can make a set of optimal solutions. Preference of this method is that designer can select proper solution among the set of optimal solutions based on other criteria where this flexibility can develop operational area of this method.

The strong point of this study is a criterion for selecting the final optimal solution among the pareto set as well as acceptable modelling of weight and drag forces and volume of fuel tank. This criterion measure the distance between any point of pareto frontier from utopian point and finally each point that has lower distance is choosed as a final solution.

Organization of this paper is as following. In the second section, Non-dominated sorting genetic algorithm is introduced. In third section, optimization formulation of objective functions is extracted. Forth section expresses optimization constrains. In fifth section, a criterion for selecting final solution is introduced. Wing optimization is presented in sixth section and finally, conclusion is expressed in seventh sections.

2 NON-DOMINATED SORTING GENETIC ALGORITHM (NSGA)

Genetic algorithms are stochastic optimization algorithms which are proper for optimization of complex problems with unknown search space. These algorithms are a programing technique that use evolution of genetic as a pattern of problem solution [14]. This algorithm is a global optimization algorithm based on principle of survival of fittest, natural selection mechanism and reproducing. Chromosomes include some strings that represent characteristics of design and generation includes a group of chromosomes.

For solutions evolution, new chromosomes are generated for next generation by genetic operators such as selection, crossover and mutation. Selection introduces direction of evolution for survival of fittest and assigns parents from mating pool. Crossover is a process for reproducing of superior chromosome to global generation evolution.

Since crossover and selection are certain processes and they rearrange genes, it is difficult to create new genes that do not exist in mating pool before. Therefore this drawback causes the algorithm converges to local optimal points. Mutation repairs this drawback and retains balance of genetic algorithm for finding global optimal solutions. The balance between crossover and mutation can effectively converge chromosome to global optimal solutions [6].

One kind of GA is non-dominated sorting genetic algorithm (NSGA) that finds set of optimal solutions (pareto front) by adding a necessary operator to general single objective GA. This operator assigns a preference criterion (rank) based on non-dominated sorting to element of population [13].

A. Domination concept

In a minimization problem with more than one objective function, it is said that X dominates Y if and only if Y doesn't surpass X from any point of view and X strictly surpass Y from minimum one point of view. This concept is expressed mathematically as below [5], [13]:

$$X \le Y(X\ domY) \Leftrightarrow \forall i : X_i \le Y_i \tag{1}$$

B. Non-dominated sorting concept

When we discuss a single objective algorithm, preference criterion of solutions is very simple and evident, because we consider only one objective function and if the problem is a minimization, a solution is desirable when it has minimum of objective function value. But if we use multi-objective algorithm for solving, it means that we consider at least two objectives and therefore we can not judge about the solutions easily. Obtaining solutions often do not have preference to each other completely and can not do comparison among them with dominating concept. So to obtain best solutions, we must arrange them based on a criterion [5], [13].

Multi-objective genetic algorithm starts by finding all non-dominated chromosomes of population and gives them a rank of one. These chromosomes would be eliminated from population and then chromosomes of smaller population are investigated and give a rank of two to its non-dominated chromosomes. This process continues until all the chromosomes have a rank. Largest rank will be less than or equal to size of population. Finally, best solutions, with rank of one, will be selected and ploted [10]. Flowchart of genetic algorithm that generates pareto front with non-dominated sorting concept is shown in (Fig. 1).

3 FORMULATION OF WING OPTIMIZATION PROBLEM

The purposes of optimization in this study are minimization of wing weight and drag force for boeing 747 in cruise condition that as well as parametric constrains, fuel tank volume and lift coefficient are as functional constrains. So it is necessary to extract equations of wing weight force, drag force and fuel tank volume and assign parameters design. Using equations for modelling is empirical equations based on reliable references that have adequate accuracy.

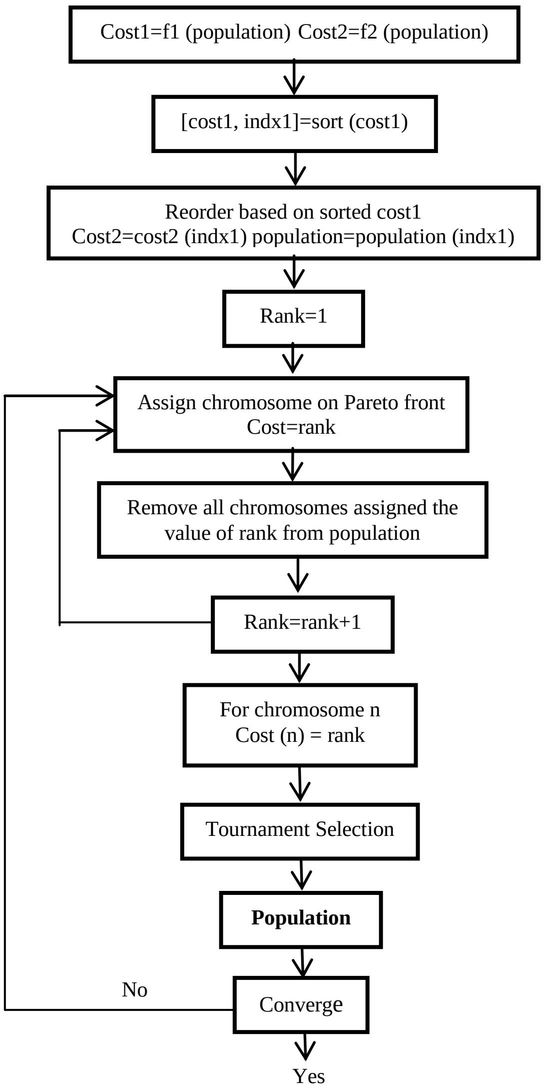

Fig. 1 The Pareto frontier genetic algorithm flowchart Mapping nonlinear data to a higher Mapping nonlinear

Since the purpose of this study is proposing an optimal wing shape (wing optimization) and not wing design and because initial wing design is baseline for our study, we do not need to consider necessary requirements such as stability requirement, controllability requirement, performance requirement and etc. So variable parameters during optimization that are known as design parameters are wing root chord (C_r),wing span (b) and sweepback angle ($\Lambda_{0.5}$) and other parameters such as taper ratio (λ), dihedral angle (Γ) and twist angle (ε) are constant until obtaining solutions are acceptable from practical point of view.

A. Weight

According to reference [13], wing weight of an aircraft calculates as below:

$$W_w = S_w.MAC.(\frac{t}{c})_{\max}\rho_{mat}K_{\rho}.(\frac{AR.n_{ult}}{\cos(\Lambda_{0.5})})^{0.6}\lambda^{0.04}.g \qquad (2)$$

Where, S_w is wing area, MAC is mean aerodynamic chord, $(\frac{t}{c})_{\max}$ is maximum of thickness to chord ratio, ρ_{mat} is density of construction material, K_{ρ} is density factor, AR is aspect ratio, n_{ult} is ultimate load factor and g is gravity constant.

If root chord, taper ratio and span are certain, wing area is calculated as below [14]:

$$S_w = C_r.(1+\lambda).\frac{b}{2} \qquad (3)$$

Taper ratio for initial design is 0.28, so substitute this value in above equation, results in:

$$S_w = 0.64.C_r.b \qquad (4)$$

It is possible to calculate wing mean chord as below [11]:

$$\overline{C} = \frac{2}{3}C_r(1+\frac{\lambda^2}{1+\lambda}) \xrightarrow{\lambda=0.28} \overline{C} = 0.707C_r \qquad (5)$$

In this research, it is assumed that wing aerofoil is NACA2412 for boeing 747 aircraft, so:

$$(\frac{t}{c})_{\max} = 0.12 \qquad (6)$$

Based on information of reference [13], density of construction material and wing density factor for aerospace aluminium alloy are:

$$\rho_{mat} = 2711\frac{kg}{m^3} \,\&\, K_{\rho} = 0.004 \qquad (7)$$

The value of ultimate load factor is [13]:

$$n_{ult} = 1.5\times n_{\max} \xrightarrow{n_{\max}=3} n_{ult} = 4.5 \qquad (8)$$

1.5 is safety factor and it is considered for structural requirements. Equation of aspect ratio with attention to Eq. (4) will be expressed as below:

$$AR = \frac{b^2}{S_w} = \frac{b}{0.64C_r} \rightarrow AR = 1.56\frac{b}{C_r} \tag{9}$$

With substituting the obtained equations in Eq. (2), main equation of wing weight is obtained as a first objective function:

$$W_w = \frac{17.71C_r^{1.4}b^{1.6}}{(\cos\Lambda_{0.5})^{0.6}} \tag{10}$$

B. Drag force

Main equation of drag force expresses as below:

$$D = \bar{q}.S.C_D \tag{11}$$

$\bar{q}$ is dynamic pressure and C_D is drag coefficient that follows from below equation:

$$C_D = C_{D0} + kC_L^2 \tag{12}$$

k In above equation can be calculated as:

$$k = \frac{1}{\pi.AR.e} \quad 13)$$

Therein, AR is aspect ratio and e is Oswald efficiency factor where in this study is considered 0.85. In equation (12), C_{D0} is drag coefficient at zero lift coefficients and can be calculated as:

$$C_{D0} = \frac{f}{S} \tag{14}$$

Where, S is wing area and f is the equivalent parasite area that is calculated as below [14]:

$$\log_{10}^{f} = a + b\log_{10}^{S_{wet}} \tag{15}$$

Since the purpose of this research is wing optimization, it is evident that wing wetted area (S_{wet}) is changing beside the wing area in optimization process and finally f and C_{D0} will change. So for calculation of C_{D0}, an equation for wing wetted area must be assigned at first and then according to Eq. (14) and Eq. (15), zero lift drag coefficient can be calculated. Wing wetted area is shown in Fig. 2.

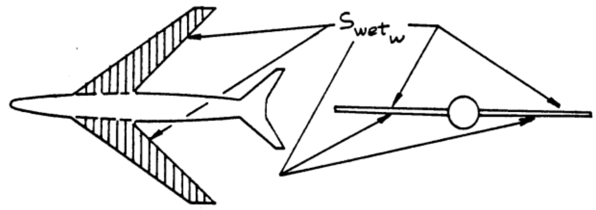

Fig. 2 Definition of wing wetted area [14]

To find an equation for wetted area according to design parameters of optimization problem, it is necessary to earn a chord distribution along the wing span. Using simple geometric equations, finally chord distribution is obtained as below:

$$y = C_r + \left[\frac{2(C_t - C_r)}{b}\right]x \tag{16}$$

To find S_{wet}, chord value in $x = \frac{d_f}{2}$ is the only unknown parameter that can be obtained from Eq. (16). d_f is aircraft fuselage diameter where its value is 6.5 meter for boeing 747 aircraft. Finally main equation is obtained for wetted area according to design parameters:

$$S_{wet} = \left[C_r + C_t + \frac{(C_t - C_r)d_f}{b}\right]\left[b - d_f\right] \tag{17}$$

The equations of equivalent parasite area and zero-lift drag coefficient can be calculated using of Eq. (4), Eq. (14), Eq. (15) and Eq. (17).

$$f = 10^{(-2.5229 + \log_{10}\left[C_r + C_t + \frac{6.5(C_t - C_r)}{b}\right][b - 6.5]} \tag{18}$$

$$C_{D_0} = \frac{10^{(-2.5229 + \log_{10}\left[C_r + C_t + \frac{6.5(C_t - C_r)}{b}\right][b - 6.5]}}{S_W = 0.64.C_r.b} \tag{19}$$

Using table (3-4) in [14], value of *a* and *b* parameters are -2.5229 and 1 respectively. For getting drag coefficient, according to Eq. (12) an equation for lift coefficient must be obtained at first. Lift coefficient is expressed as $C_{L_0} + C_{L_\alpha}.\alpha$ that lift curve slope is calculated as below [14].

$$C_{L_\alpha} = \frac{2\pi AR}{2 + \sqrt{4 + \frac{AR^2\beta^2}{k^2}(1 + \frac{\tan^2\Lambda_{0.5}}{\beta^2})}} \tag{20}$$

k and β are obtained from below equations.

$$\beta=\sqrt{1-M^2} \tag{21}$$

$$k=\frac{C_{l_\alpha}\big|_{atM}}{2\pi} \tag{22}$$

The Mach number is $M=0.85$ in cruise condition of boeing 747 aircraft. Therefor $C_{l_\alpha}\big|_{atM}$ is calculated as below:

$$C_{l_\alpha}\big|_{atM}=\frac{C_{l_\alpha}\big|_{atM=0}}{\sqrt{1-M^2}}$$

$$\xrightarrow{C_{l_\alpha}\big|_{NACA2412}^{M=0}=6.02rad^{-1}} C_{l_\alpha}\big|_{atM=0.85}=11.43rad^{-1} \tag{23}$$

Substituting Eq. (21), Eq. (22), Eq. (23) and aspect ratio in Eq. (20), final equation is obtained for lift curve slope.

$$C_{L_\alpha}=\frac{2\pi b^2/S}{2+\sqrt{4+0.084\frac{b^4}{S^2}(1+3.6\tan^2\Lambda_{0.5})}} \tag{24}$$

Lift coefficient for zero angle of attack (C_{L_0}) similar to drag coefficient at zero lift coefficient, does not have constant value and is changing during optimization process. So next step is extracting an equation for this parameter based on design parameters. Lift coefficient for zero angle of attack is calculated using below equation [14]:

$$C_{L_0}=C_{L_{0wf}}+C_{L_{\alpha h}}\eta_h(\frac{S_h}{S})(i_h-\varepsilon_{0h})+C_{L_{\alpha c}}\eta_c(\frac{S_c}{S})(i_c+\varepsilon_{0c}) \tag{25}$$

Last term in above equation is related to canard that is disregarded in this study. The second term is related to horizontal tail that in comparison of the first term is negligible. Therefor only Lift coefficient for zero angle of attack is calculated for combination of wing body ($C_{L_{0wf}}$) that is obtained as below:

$$C_{L_{0wf}}=(i_w+\alpha_{0LW})C_{L_{\alpha wf}} \tag{26}$$

i_w is incidence angle that the value of this parameter is 2 degree for boeing 747 aircraft. α_{0LW} is wing angle of attack for zero lift and is calculated by below equation.

$$\alpha_{0LW}=\left\{\alpha_{0_l}+\left(\frac{\Delta\alpha_0}{\varepsilon_t}\right)\varepsilon_t\right\}\left\{\frac{\alpha_{0_l}\big|_{atM}}{\alpha_{0_l}\big|_{M=0.3}}\right\} \tag{27}$$

α_{0_l} and $\frac{\Delta\alpha_0}{\varepsilon_t}$ are airfoil zero lift angle of attack and change of wing zero lift angle of attack to wing twist angle respectively. With calculating these parameters based on reference [14] and substituting in above equation, finally value of α_{0LW} is -1.9 degree.

$C_{L_{\alpha wf}}$ is lift curve slop of wing body and is obtained as below:

$$C_{L_{\alpha wf}}=k_{wf}C_{L_{\alpha w}} \tag{28}$$

k_{wf} is wing body interference factor that is obtained from below equation:

$$k_{wf}=1+0.025(\frac{d_f}{b})-0.25(\frac{d_f}{b})^2 \xrightarrow{d_f=6.5} k_{wf}=1+\frac{0.16}{b}-\frac{10.56}{b^2} \tag{29}$$

$C_{L_{\alpha w}}$ is obtained from Eq. (24) so the final equation for $C_{L_{\alpha wf}}$ according to Eq. (24) and Eq. (28) are:

$$C_{L_{\alpha wf}}=(\frac{b^2+0.16b-10.56}{bC_r})(\frac{9.82}{2+\sqrt{4+\frac{0.205b^2}{C_r^2}(1+3.6\tan^2\Lambda_{0.5})}}) \tag{30}$$

According to Eq. (26) and $C_{L_{0wf}}$, finally C_{L_0} is calculated as below.

$$C_{L_0}=(2-1.92)C_{L_{\alpha wf}}=0.08C_{L_{\alpha wf}} \tag{31}$$

Now using Eq. (11) and Eq. (12), final equation is obtained for drag force.

$$D=\bar{q}.(0.64C_rb).\left\{C_{D_0}+\left[\frac{0.25C_r}{b}\right][0.08k_w+\alpha]^2C_{L_\alpha}^2\right\} \tag{32}$$

Dynamic pressure for cruise altitude (35000ft) and angle of attack for cruise condition are $8706.72\frac{kg}{m.s^2}$ and 2.5 deg respectively. By substituting these constant values and Eq. (19), Eq. (24) and Eq. (29) in Eq. (32), main equation is obtained for drag force.

$$D = 8706.72 \times (10^{(-2.5229 + \log_{10}\left[C_r + 0.28C_r - \frac{4.68C_r)}{b}\right][b-6.5]}) \\ + 8.916 \times \left[(1 + \frac{0.16}{b} - \frac{10.56}{b^2}) + (\frac{2.5\pi}{180})\right]^2 \\ \times \left[\frac{9.81^2 b^2}{(2 + \sqrt{4 + \frac{0.205b^2}{C_r^2}(1 + 3.6\tan^2 \Lambda_{0.5})})^2}\right] \quad (33)$$

4 CONSTRAINS OF WING OPTIMIZATION PROBLEM

The considered constrains for this study are functional and parameters constrains. The functional constrains have been divided to two sections that are: 1) fuel tank volume and 2) lift coefficient.

A. Parameters constrains

It is mentioned that design parameters are: 1) root chord 2) wing span 3) half chord sweep angle. Bound values of design parameters are listed in table (1). It must be mentioned that these values are selected based on similar aircraft database.

Table 1 Limits for Design Parameters

Design Parameters	C_r (m)	b (m)	$\Lambda_{0.5}$ (deg)
Upper Limits	20	75	45
Lower Limits	10	55	25

B. Fuel volume constrain

One of the wing rules as well as producing the lift force is supplying a lacuna for fuel tank. Since design parameters vary in optimization process, it is obvious that fuel tank volume will change. But fuel tank in optimized condition must implant similar fuel until aircraft mission does not meet main variations. So the volume between two spars must be calculated and considered as volume constrain.

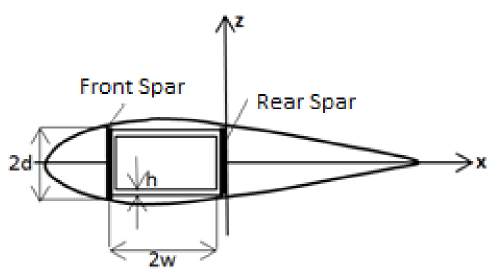

Fig. 3 The wing cross-section and fuel tank parameters

Wing and fuel tank cross-section mid parameters of fuel tank are shown in Fig. 3. Wing in actual condition is taper and its thickness varies from root to tip so fuel tank has been modeled as taper with varying thickness (Fig. 4).

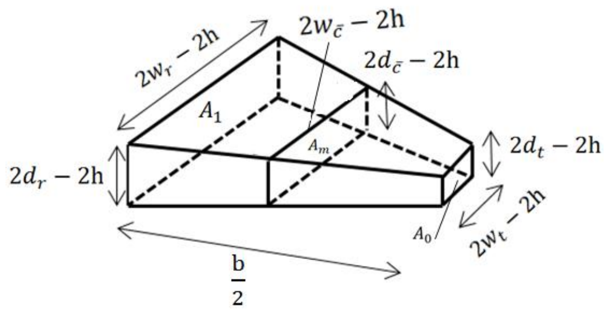

Fig. 4 Fuel tank model

Reference [15] submits a relation for volume of above figure.

$$V = \frac{b/2}{6}(A_0 + A_1 + 4A_m) \quad (34)$$

From Eq. (34) it is obvious that for fuel tank volume calculation, at first 2d, 2w and h must be assigned in root, mean chord and tip. Another significant point is location of spars in along of wing that 2w and 2d are related to these locations. Locations of front and rear spars are:

$$FrontSpar = 0.15C, \mathrm{Re}\,arSpar = 0.7C \quad (35)$$

From above equation, distance of two spars is $0.55C$ so value of 2w that represents the distance of two spars considerng $0.55C$ at different wing sections. The value of 2d and h are selected according to reference [16] and similar works.

$$\frac{2d}{2w} = 0.15, h = 0.02m \quad (36)$$

For final summation, the values of h, 2w and 2d are listed in table 2.

Table 2 Final Values of Fuel Tank

Parameter	$2w$	$2d$
Wing root	$0.55C_r$	$0.15(2w) = 0.15(0.55C_r)$ $= 0.082C_r$
Mean chord	$0.55\overline{C}$	$0.082\overline{C}$
Wing tip	$0.55C_t$	$0.082C_t$

According to Eq. (34), for obtaining fuel tank volume A_0, A_1 and A_m must be calculated:

$$A_1 = (2w_r - 2h)(2d_r - 2h) = (0.55C_r - 0.04)(0.082C_r - 0.04) \rightarrow A_1 = 0.0451C_r^2 - 0.025C_r + 0.0016 \quad (37)$$

$$A_0 = (0.55C_t - 0.04)(0.082C_t - 0.04) \xrightarrow{C_t=\lambda C_r \& \lambda=0.28} A_0 = 0.0035C_r^2 - 0.007C_r + 0.0016 \quad (38)$$

$$A_m = (0.55\overline{C} - 0.04)(0.082\overline{C} - 0.04) \xrightarrow{\overline{C}=\frac{2}{3}C_r(1+\frac{\lambda^2}{1+\lambda})\rightarrow \overline{C}=0.707C_r} A_m = 0.022C_r^2 - 0.018C_r + 0.0016 \quad (39)$$

Therefor final equation for fuel tank is calculated as below.

$$V = \frac{b}{12}(0.137C_r^2 - 0.104C_r + 0.0096) \quad (40)$$

It is stated that in the optimization, fuel tank volume should not have significant variation regarding to baseline volume. Using Eq. (40) and based on baseline root chord and wing span, value of tank volume is $148.8m^3$, so this value is fuel tank volume constrained.

C. Lift coefficient constrain

This constrain is considered as the lift coefficient in optimized and baseline condition to be equal until optimization design has proper condition from aerodynamic performance point of view. For baseline design, lift coefficient is calculated 0.33 so this value is considered as second constrain.

5 CHOISE OF FINAL SOLUTION ON PARETO FRONTIERS

Since solutions do not have preference to each other completely in multi objective optimization problem, we use from dominated concept until best solutions (pareto frontier) is obtained. But selecting a final solution among pareto points is not easy. There are various ways to select a final solution such as design requirements, designer experience and fuzzy logic. In this research another concept is presented for final selecting. This concept is a distance between utopian point and pareto points.

In this method, once the Pareto Frontier is determined all of pareto points are been classified based on this criteria and finally a pareto point that has the best condition is selected. Using this criteria, best compromise is generated among multiple objective functions. As well as, the designer can select more suitable design based on its preferences and requirements. This criterion is calculated for each pareto point as follows:

$$d_{ut} = \sum_{i=1}^{n} \sqrt{\left(\frac{f_i - f_{ut_i}}{f_{ut_i}}\right)^2} \quad (41)$$

Where i is the number of objective function and f_{ut} is the reference value. f_{ut} is an ideal optimum value that is obtained from a single objective optimization for each objective function. In other hand, f_{ut} is a best solution of each objective function. Purpose of this criterion is to find a pareto point that the distance among it and utopian values from single objective optimization is minimum for each objective function.
With single objective optimization for calculating W_{ut} and D_{ut}, the following results were obtained for this study. In the first single objective optimization, drag force is as objective function and in the second optimization weight force is objective function. These results are the best solutions as the minimum values of the objective functions [17].

Table 3 Utopian Values for Drag Force

(m) C_r	(m) b	(deg) $\Lambda_{0.5}$	(N) D_{ult}
14	71.1	40	58181

Table 4 Utopian Values for Weight Force

(m) C_r	(m) b	(deg) $\Lambda_{0.5}$	(N) W_{ult}
15.33	57.99	26	5.7104×10^5

6 WING OPTIMIZATION

For wing optimization of boeing 747 aircraft, since it is a two objectives optimization, non-dominated sorting genetic algorithm has been used with below characteristics.

Generation: 10, population size=300, mutation rate=0.1, selection rate=0.5

Running this algorithm, optimal points are set (pareto frontiers) and all of solutions has been drawn (Fig. 5 and Fig. 6).

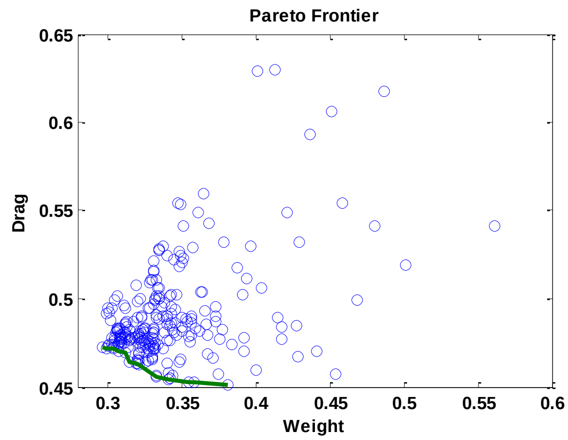

Fig. 5 The optimal and non-optimal solutions set

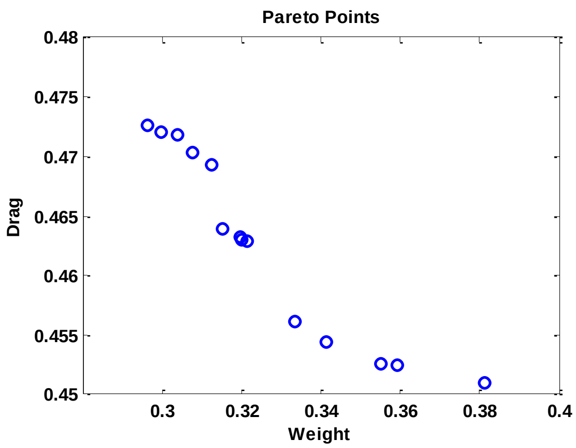

Fig. 6 Optimal points set (pareto front)

Table 5 The pareto points characteristics

Pareto Point	C_r (m)	b (m)	$\Lambda_{0.5}$ (deg)	d_{ut}
1	15.02	60.22	31	0.0653
2	15	60.46	31	0.071
3	14.97	60.2	31	0.0591
4	14.89	60.1	31	0.0472
5	14.83	60.09	31	0.041
6	14.93	61.73	33	0.111
7	14.83	60.09	31	0.041
8	14.83	60.09	31	0.041
9	14.78	61.55	33	0.0925
10	14.97	60.06	31	0.055
11	14.92	60.1	31	0.051
12	14.92	59.38	30	0.023
13	14.7	60.2	30	0.0324

It must be mentioned that each point in Fig. 6 is optimal solution singly. Final selection of optimal solution is done by designer and it is based on other requirements. But in this study, a criterion is proposed which can select best optimal solutions. As mentioned, this criterion measures the distance between pareto and utopian points. Each pareto points that has lower distance, is the best and it means that point is closer to utopian condition. In this study for all pareto points, this distance is calculated as shown in table 5. It is obvious from above table that there are not significance differences among pareto solutions and pareto point with number 12 has lower value of d_{ut}. So it is the best solution among pareto pints. Full specifications of this pareto is shown in the following table.

Table 6 Specification of pareto 12

Parameters	Initial Design	Pareto 12	Difference (%)
C_r (m)	14.63	14.92	+1.98
b (m)	64.44	59.38	-7.8
$\Lambda_{0.5}$ (deg)	35	30	-14.3
C_L	0.33	0.33	-
$V(m^3)$	148	144	-2.5
$D(N)$	59566	58578	-1.7
$W(N)$	6.6994*10^ 5	5.8380* 10^5	-12.8

It can be seen from Table 6 that the final selected optimal solution has wing weight and drag force close to the utopian values while the constrains is satisfied. A significant point is that obtaining volume for this pareto point toward initial volume has a little difference. The reason of this difference is that assigning accurate weights for penalty functions are difficult and is based on try and error. If weights be assigned accurately, these difference will be negligible. The difference of initial wing and proposing optimization wing platform are shown in following figure.

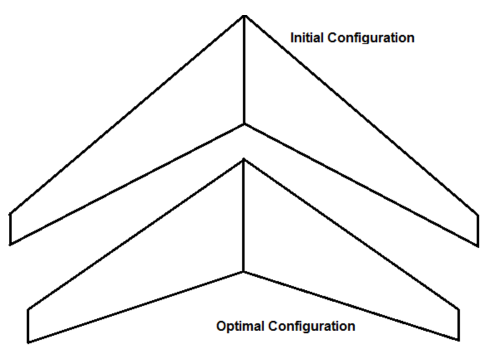

Fig. 7 Top view of initial and optimal design

It is necessary to mention that wing design of an aircraft must perform during aircraft design cycle with considering many requirements. The resulted solutions from optimization algorithm are proposing optimal platforms for wing only and not for entire aircraft. It can be saied that these results are as an initial improved version of baseline wing of boeing 747. In average optimization, results represent 1.2 and 10 percent reduction in drag force and wing weight (Fig. 8 and Fig. 9) that these reduction can submit significant effect in boeaing 747 performance.

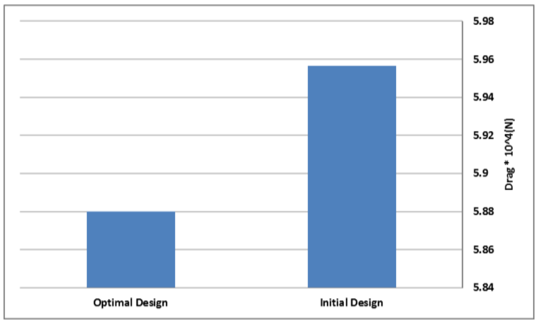

Fig. 8 Comparison of Initial and Optimal Drag Force

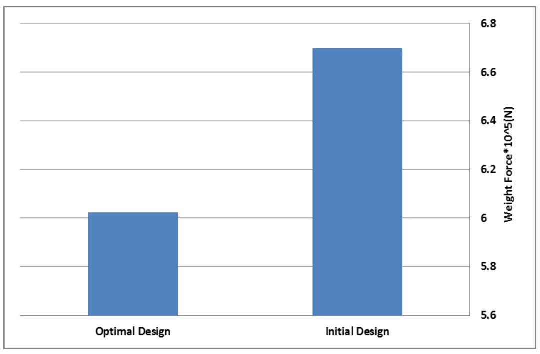

Fig. 9 Comparison of Initial and Optimal Weight Force

7 CONCLUSION

In this study wing optimization of boeing 747 aircraft has been done in cruise condition to minimize wing weight and drag force. Considered constrains are fuel tank volume and lift coefficient. Optimization algorithm with attention to desirable performance of evolutionary optimization algorithms to solve complex problem is genetic algorithm. Since the problem is multi-objective, non-dominated sorting concept is added to ordinary single objective genetic algorithm and multi-objective genetic algorithm is considered until it makes set of optimal solutions as pareto frontier instead of one solution. Each of pareto points is a proposed optimal solution for considered problem but one measurement was introduced and best pareto point was selected among pareto points. The results propose an initial increment in root and tip chords and reduction in sweep angle and wing span during the optimization process but it is necessary to consider the aircraft design cycle for final decision making.

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