Sensitivity Analysis for Optimal Design of Multibody Systems with Clearance Joint

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Abstract: This paper deals with the sensitivity analysis and optimization of system parameters for a classical slider-crank mechanism as a multibody system which includes a clearance between the joints of coupler and slider. Due to the nonlinearity involved in the dynamics of clearance joints, the base reaction force, exerted on the base from the crank, changes roughly and does not vary as smooth as the case of the mechanism with ideal joint. Variation of the base reaction force can be a measure of the undesired vibrations induced due to the effect of clearance joint. After deriving the equations of motion and modeling the clearance, the direct differentiation method is used to conduct a local sensitivity analysis to assess the sensitivity measure of the base reaction force on some kinematic and contact parameters. The results show that the reaction force is more sensitive to the variation of link lengths and link masses compared to the variation of contact surface characteristics such as Young's modulus, restitution coefficient and contact generalized stiffness in most parts of the motion cycle. On the other hand, the sensitivity of the base reaction force to the clearance size is very higher than its sensitivity to the above-mentioned kinematic and contact properties. Finally, based on the results of the sensitivity analysis, an optimization procedure is used to reduce the amount of the maximum base reaction force by choosing the optimized link lengths.

Keywords: Clearance Joint, Direct Differentiation, Momentum Exchange Approach, Optimization, Sensitivity Analysis

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1 INTRODUCTION

The mechanical joints are not generally ideal in multibody systems and may include clearances between the journal and bearing. This clearance is due to machining tolerances, wear, material deformations, and imperfections, and it can worsen mechanism performance such as precision, dynamic behavior and vibration. Consequently, the mechanical performance is deteriorated due to the relative motion, which in turns, imposes suddenly-applied forces between the linked members. Sudden variations of the base reaction forces can induce undesired vibrations and generate noises specially when the system operates at high speeds. Therefore, finding the main factors contributing to the variations of the base reaction forces and assessment of their influence level is increasingly a demanding task.

Clearance has been the subject of many researches. The impacts in a clearance joint lead to high contact force and consequently high acceleration. Three main types of clearance model could be found in the literature, namely, the massless link approach, the spring–damper approach, and the momentum exchange approach [1]. Among all, the momentum exchange approach for modeling clearance is a realistic model. Not only it does consider the contact forces dependent on the surface elasticity, it also includes the energy loss during contact as well. Furthermore, it considers the clearance as a gap between two colliding bodies. The dynamic response of the system is controlled by the force applied between these bodies. Interested reader can find more information about contact procedures in [2], [3]. Many researchers have studied the dynamic modelling of multibody systems with clearance.

Zhang et al. [4] considered a 3-RRR parallel mechanism with six clearance joints and showed that the clearance joints have a considerable effect on the mechanism dynamic behaviour. Erkaya [5] investigated the effect of joint clearance on the dynamics of six DOF manipulators. Li et al. [6] considered the influence of both joint clearance and link flexibility to analyze the dynamic response of an air rudder transmission mechanism. Ben Abdallah et al. [7] investigated the dynamic response of a planar slider–crank mechanism with flexible coupler including clearance joints using ADAMS. Wang and Liu [8] used an improved nonlinear elastic-damping element to model contact force. Salahshoor et al. [9] tried to obtain an analytical solution for a mechanical system with clearance joint using multiple scales method. Investigating the lubricated joints in mechanical systems can be found for example in [10-11].

A novel transition model for lubricated revolute joints in planar multibody systems was presented by Li et al. in [10]. More recently, Ebrahimi et al. [11] exploited the method of multiple scales for nonlinear vibration analysis of mechanical systems with dry and lubricated clearance joints. In addition to the above studies, wear phenomenon has also been addressed in the literature. Among all, the researches in [12-17] dealt with the wear in imperfect joints. Mukras et al. [12] presented a method to study wear in planar multibody systems. Parametric studies on the dynamic response of planar multibody systems with multiple clearance joints have found great attention during recent years. Flores carried out a parametric investigation and studied how the dynamic response of mechanical systems can be effected by changing the clearance size, crank angular velocity and number of clearance joints [18].

Wang et al. [19] showed the influence of changing the restitution coefficient and material characteristics on the dynamic behavior of planar mechanical systems with revolute clearance joint. Zhang et al. [20] presented a multiobjective optimization problem for multibody systems including clearance. Some other researchers have studied the optimization of the mechanisms with imperfect joints [21-23]. Furthermore, chaos and bifurcation in mechanical systems with imperfect joints are other subjects which were paid attention by some interested researchers [25], [26]. In addition to the above topics, the influence of clearance on the vibrational behavior of mechanical systems has also been investigated [27-30].

Erkaya [28] predicted the vibration characteristics of a slider-crank mechanism due to the effect of imperfect joint by using neural network. Ebrahimi et al. investigated the influence of joint stiffness on the vibration characteristics of a flexible four-bar mechanism with a single clearance joint [29]. Salahshoor et al. [30] comprehensively studied the effect of joint stiffness on the vibration behavior of a typical slider-crank mechanism with a flexible component and joint clearances.

As the above literature survey reveals, there exists a vast amount of studies on the dynamic characterization of multibody systems having clearance joints. However, the parametric studies of these systems based on a sensitivity analysis were rarely taken into account. Fundamentally, in a sensitivity analysis, it is aimed to study how any uncertainty in the system inputs can make effect on its outputs [31]. Innocenti [32] dealt with the kinematic clearance sensitivity analysis to assess the influence of joint clearances on the position of a generic link in spatial structures. Tsai and Lai [33] presented an effective kinematic sensitivity analysis to analyze the transmission performance of linkages that have joint clearance.

Deng [34] studied the sensitivity analysis of two degrees of freedom translational parallel manipulators due to joint clearances and variations in geometric parameters. His analysis aimed to find out how the joint clearances and the variations in geometric parameters affect the pose errors of the robots, and then proposed some strategies for designing robots. Dai et al. [35] proposed a nonlinear equivalent method for investigating clearance effected accuracy and error sensitivity analysis in a spatial parallel robot. Based on statistics, they outlined the sensitivity of orientation error analysis with varying external load and pose.

The above mentioned studies are majorly devoted to the kinematic sensitivity analysis of mechanical systems with joint clearances. To the best knowledge of authors, the sensitivity analysis of the base reaction forces has not been conducted so far in any published study. Variation of the base reaction force can be a measure of the undesired vibrations induced due to the effect of clearance joint. Consequently, finding the main factors contributing to the variations of the base reaction forces and assessment of their influence level can be of great importance. Therefore, in this paper, the direct differentiation method is used to conduct a local sensitivity analysis for a typical slider-crank mechanism having clearance to assess the sensitivity measure of the base reaction force on some kinematic and contact parameters. In other words, the major contribution and novelty of this study is to investigate the dependence of the base reaction forces on the kinematic properties of the mechanism as well as the contact properties of the clearance joint. For this purpose, the sensitivity vector is calculated and plotted to see the amount of dependence of the base reaction force on the kinematic and contact parameters. Finally, based on the results of the sensitivity analysis, an optimization procedure is used to reduce the amount of the maximum base reaction force by choosing the optimized variables.

2 CLEARANCE MODELING

Slider-crank mechanism is one of the most practical mechanisms used in industry. Reciprocating pumps and compressors, sewing machine and internal combustion engines are examples of this mechanism. A slider-crank mechanism having a clearance between the coupler and slider joints is shown in "Fig. 1".

EQUATIONS OF MOTION

A representative demonstration of the imperfect revolute joint is illustrated in "Fig. 2". The amount and orientation of the eccentricity vector $\mathbf{r}$ can be obtained from:

$$r = ((x_4 - L_2c_2 - L_3c_3)^2 + (- L_2s_2 - L_3s_3)^2)^{0.5} \qquad (1)$$

$$\cos(\alpha) = (x_4 - L_2c_2 - L_3c_3)/r, \quad c_i = \cos(\theta_i) \qquad (2)$$

$$\sin(\alpha) = (- L_2s_2 - L_3s_3)/r, \quad s_i = \sin(\theta_i) \qquad (3)$$

Where L_2 and L_3 are the crank and coupler lengths, respectively.

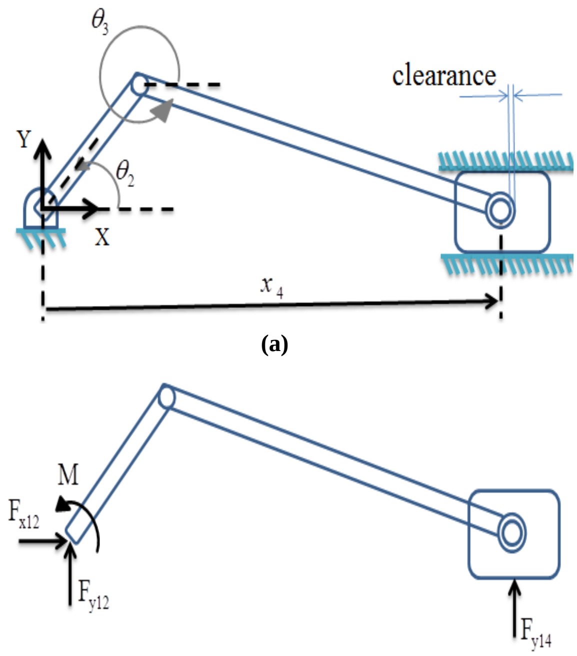


Fig. 1 (a): Slider-crank mechanism including single clearance joint and (b): the base reaction forces and the crank moment.

The equations of motion of the slider-crank mechanism can be derived by the Lagrange approach which is expressed as:

$$\mathrm{d}/\mathrm{dt}\left(\partial L / \partial \dot{q}_k\right) - \partial L / \partial q_k = \sum_{i=2}^{4}\left(\mathbf{F}_i^*.\left(\partial \mathbf{V}_{c,i} / \partial \dot{q}_k\right) + \mathbf{M}_i^*.\left(\partial \boldsymbol{\omega}_i / \partial \dot{q}_k\right)\right) \qquad (4)$$

Where L denotes the Lagrangian, q_k is the generalized coordinate, and $\boldsymbol{\omega}_i$ and $\mathbf{v}_{ci}$ represent the angular and translational velocities of the C.G. of the body i, respectively.

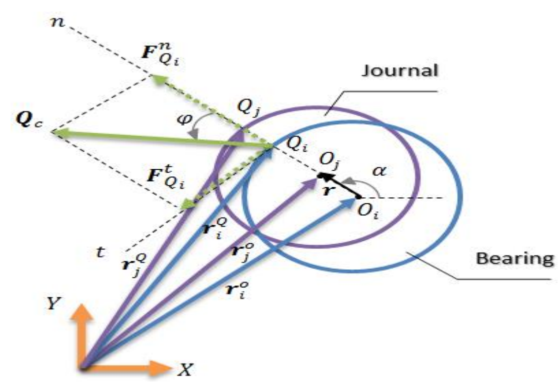


Fig. 2 A sketch of the imperfect revolute joint.

Furthermore, the external force $\mathbf{F}_i^*$ and moment $\mathbf{M}_i^*$ are applied to the C.G. of the body *i*. Parameters θ_2, θ_3 and x_4 are chosen as the generalized coordinates to derive three equations of motion. The crank angle can be obtained from:

$$\theta_2 = \omega_2 t + \theta_{20} \tag{5}$$

where ω_2 is the constant rotational velocity of the crank, and θ_{20} is its initial angle. The equations of motion by considering the clearance joint can be derived from "Eq. (4)" as:

$$\begin{aligned} &\left(J_3 + 0.25 m_3 L_3^2\right)\ddot{\theta}_3 + 0.5 m_3 L_2 L_3 \omega_2^2 \sin\left(\theta_3 - \theta_2\right) \\ &+ 0.5 m_3 g L_3 \cos\theta_3 + L_3 F_{\mathrm{N}} \sin\left(\theta_3 - \alpha\right) = 0 \end{aligned} \tag{6}$$

$$m_4 \ddot{x}_4 + F_{\mathrm{N}} \cos\alpha = 0 \tag{7}$$

$$\begin{aligned} M_2 &= 0.5 m_3 L_2 L_3 \ddot{\theta}_3 \cos\left(\theta_2 - \theta_3\right) \\ &+ 0.5 m_3 L_2 L_3 \dot{\theta}_3^2 \sin\left(\theta_2 - \theta_3\right) + (m_3 + 0.5 m_2) g L_2 \cos\theta_2 \\ &+ L_2 F_{\mathrm{N}} \sin\left(\theta_2 - \alpha\right) \end{aligned} \tag{8}$$

Here, m_i and $J_i (i = 1,2,3)$ are respectively the mass and mass moment of inertia of body *i* about its center of mass, F_N denotes the contact force at clearance joint between the coupler and the slider, and M_2 is the required moment for generating constant ω_2. In deriving "Eqs. (6-8)", the friction at clearance joint is neglected.

CONTACT FORCE MODELING

Lankarani and Nikravesh presented a contact force model which is currently known as one of the most common approaches based on which one can write, see [1]:

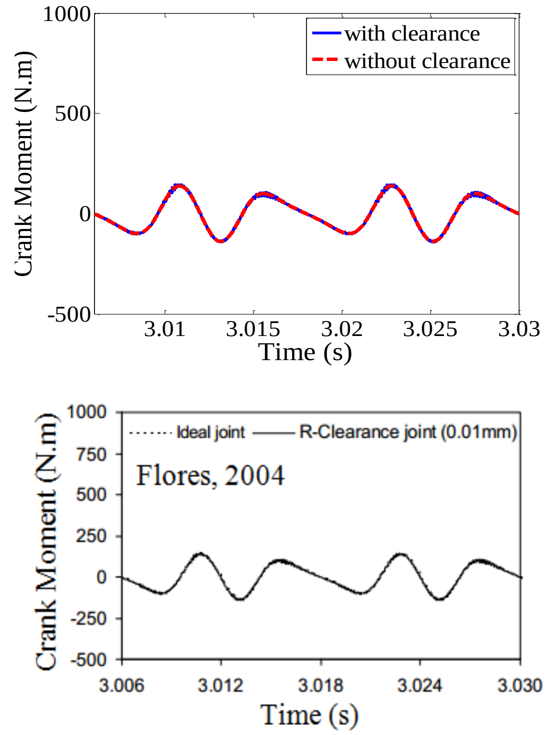
$$\begin{aligned} &F_N = K\delta^{\frac{3}{2}}\left(1 + 0.75(1 - e_r^2)(\dot{\delta} / \dot{\delta}^{(-)})\right) \\ &K = 4 / (3\pi\left(h_i + h_j\right))\left(R_J R_B / \left(R_B - R_J\right)\right)^{0.5} \\ &h_k = \left(1 - \upsilon_k^{\,2}\right) / (\pi E_k) \;\; (k = i, j) \end{aligned} \tag{9}$$

Where E, υ and e_r denote respectively the Young's modulus, Poisson's ratio and restitution coefficient. Parameters R_J and R_B represent the journal and bearing radii. In addition, $\dot{\delta}^{(-)}$ is the impact velocity and, δ and $\dot{\delta}$ are the penetration and penetration velocity which can be given as:

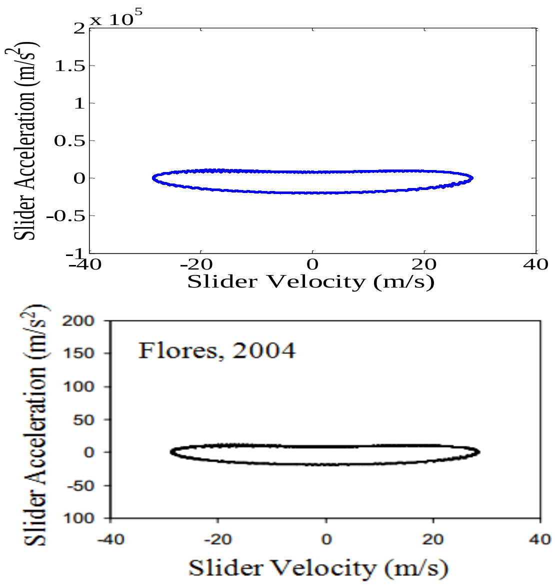
$$\delta = r - (R_B - R_J) \tag{10}$$

$$\dot{\delta} = \dot{r} \tag{11}$$

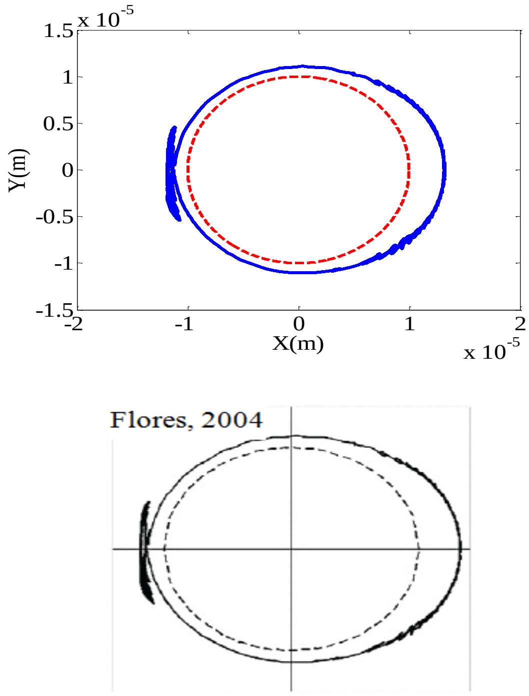
To find the approximate instant of contact for determining the impact velocity, the second approach presented in [36] is employed in this study.



(a)



(b)



(c)

Fig. 3 Comparison of the results with [1]: (a): crank moment, (b): phase space including slider acceleration and slider velocity and (c): Journal center trajectory inside the bearing for clearance size 0.01mm and without friction.

MODEL VALIDATION

To validate the model used later in this work for the sensitivity analysis, a comparison is made between the dynamic response of our approach and the study presented in [1]. For this purpose, the geometrical and mechanical properties of this mechanism are chosen the same [1]. Comparing the obtained results from our analysis with the results of [1], a good agreement between the results can be observed in “Fig. 3”. Therefore, this solution could be used in the sensitivity analysis to achieve the sensitivity vector in the next section.

3 SENSITIVITY ANALYSIS

Measuring the sensitivity of a certain function to the change in values of certain design variables is the objective of a sensitivity analysis. The sensitivity analysis for dynamic systems needs the derivatives of some certain performance measures and the state variables with respect to the design variables [37]. Design sensitivity analysis methods for dynamic systems have been investigated intensively in the literature. Differentiation methods are supposed to be the main methodologies which have widely been used for sensitivity analysis of multibody systems. There are several methods for implementing the important task of finding the derivatives needed in this subject. In multibody design, problems involved in the literature, the sensitivity analysis has been founded on two main analytical methods namely, direct differentiation and adjoint variable methods [38]. The direct differentiation methods may be conceptually the simplest methods in a mathematical point of view. Indeed, the equations are explicitly and directly differentiated with respect to the variables to calculate amounts of state sensitivities. The advantage of employing a direct differentiation method is that sensitivity equations could be determined in a very direct way by differentiating the system equations of motion without presenting extra numerical computations [38]. In this approach, the sensitivity equations lead to an initial value problem and can be integrated simultaneously with the equations of motion [39]. On the contrary, the differential equations are solved backward in time in the adjoint method. Despite the robustness of the theory of the adjoint variable method, its implementation appears to be inconvenient since it needs a large number of input/output operations to store the state variables which are achieved from forward integration of equations of motion. This method is supposed to be computationally error-prone because of the error produced in interpolation [39].

DIRECT DIFFERENTIATION METHOD

The direct differentiation method is similar to the dynamic analysis in procedure and easy to be implemented. Consequently, it is an appropriate approach to be used here due to its solution accuracy. The question that how the amount of accuracy of measurements and data processing of the parameters influences the simulation results could be answered using the sensitivity analysis. Gradients of the objective functions give this information. To achieve the sensitivities, assume a typical objective function to be defined as:

$$J = J(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, t, \mathbf{b}) \tag{12}$$

Where $\mathbf{q}$, $\dot{\mathbf{q}}$, $\ddot{\mathbf{q}}$ denote respectively the positions, velocities and accelerations which are functions of the design parameters $\mathbf{b}$ and time. Therefore, the sensitivity of J with respect to $\mathbf{b}$ is obtained employing the chain rule as:

$$\begin{aligned} \nabla J = \mathrm{d}J / \mathrm{d}\mathbf{b} = \partial J / \partial \mathbf{b} + (\partial J / \partial \mathbf{q})(\partial \mathbf{q} / \partial \mathbf{b}) \\ + (\partial \mathrm{J} / \partial \dot{\mathbf{q}})(\partial \dot{\mathbf{q}} / \partial \mathbf{b}) + (\partial \mathrm{J} / \partial \ddot{\mathbf{q}})(\partial \ddot{\mathbf{q}} / \partial \mathbf{b}) \end{aligned} \tag{13}$$

More concise representation of “Eq. (13)” can be shown as:

$$\nabla J = \mathrm{d}J / \mathrm{d}\mathbf{b} = J_{\mathbf{b}} + J_{\mathbf{q}}\mathbf{q}_{\mathbf{b}} + J_{\dot{\mathbf{q}}}\dot{\mathbf{q}}_{\mathbf{b}} + J_{\ddot{\mathbf{q}}}\ddot{\mathbf{q}}_{\mathbf{b}} \quad (14)$$

Since the design variables and the state variables are explicit, the above terms are simple to be calculated. The sensitivity matrix of "Eq. (14)" can be calculated when the quantities $\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$ are known. To obtain the sensitivity, the direct differentiation method is employed in this paper. Due to the different units of its matrix elements, the sensitivity matrix is difficult to analyze. In order to find an understanding of the effect of the individual design variables on the objective function, "Eq. (14)" is normalized by the respective reference values as [40]:

$$\bar{J} = (\mathrm{d}J/J)/(\mathrm{d}\mathbf{b}/\mathbf{b}_0) = (\mathrm{d}J / \mathrm{d}\mathbf{b})(\mathbf{b}_0 / J) = \nabla J.\mathbf{b}_0 / J \quad (15)$$

Where the subscript of $\mathbf{b}_0$ indicates that the derivative is taken at some fixed point in the space of the input. The differential equations of motion of a general multibody system can be shown as:

$$\mathbf{M}_{n\times n}\ddot{\mathbf{q}}_{n\times 1} = \mathbf{G}_{n\times 1} \quad (16)$$

This equation represents *n* coupled equations associated with *n* DOFs of the system. Furthermore, $\mathbf{M}$, $\ddot{\mathbf{q}}$ and $\mathbf{G}$ denote respectively the generalized mass matrix, generalized acceleration vector, and the generalized force vector associated with the independent coordinates. This type of equations of motion is presented based on the embedding technique. It can be assumed that the generalized mass matrix and force vector have the form $\mathbf{M} = \mathbf{M}(\mathbf{q},\mathbf{b})$ and $\mathbf{G} = \mathbf{G}(\mathbf{q},\dot{\mathbf{q}},t,\mathbf{b})$, respectively. Therefore, differentiating Eq. (16) w.r.t the design parameters yields:

$$\mathbf{M}\ddot{\mathbf{q}}_{\mathbf{b}} = \mathbf{G}_{\mathbf{q}}\mathbf{q}_{\mathbf{b}} + \mathbf{G}_{\dot{\mathbf{q}}}\dot{\mathbf{q}}_{\mathbf{b}} + \mathbf{G}_{\mathbf{b}} - (\mathbf{M}\ddot{\mathbf{q}})_{\mathbf{q}}\mathbf{q}_{\mathbf{b}} - (\mathbf{M}\ddot{\mathbf{q}})_{\mathbf{b}} \quad (17)$$

Which represents a set of second-order ordinary differential equations for the state sensitivity variables $\mathbf{q}_{\mathbf{b}}$. Therefore, "Eqs. (16-17)" can be integrated simultaneously to obtain $\mathbf{q}_{\mathbf{b}}$, $\dot{\mathbf{q}}_{\mathbf{b}}$ and $\ddot{\mathbf{q}}_{\mathbf{b}}$, and then directly substituted into "Eq. (15)" to determine the normalized design sensitivity.

4 OPTIMIZATION

The results obtained from the sensitivity analysis can be used in an optimization process to find out how the desired output is changed when certain parameters change. An optimization problem is defined to find the optimized lengths of the crank and the coupler so that the maximum base reaction force in a full crank rotation is minimized. This problem is solved using the genetic algorithm and is described as:

$$\text{Objective function: } f = \min\{\max(F_{12})\}, \quad (18)$$
$$\text{Design parameters: } L_2, L_3$$

5 RESULTS AND DISCUSSION

SENSITIVITY ANALYSIS RESULTS

In this section, a slider-crank mechanism having a clearance between the coupler and the slider joints is considered. "Tables 1 & 2" represent some mechanism characteristics. Crank and the coupler are considered as uniform thin rods. Base reaction force exerted on the base by the crank is used as an objective function in the sensitivity analysis. Some kinematic and contact properties are considered to see how a change in their values could make effect on the above-mentioned force. The change in the amount of foregoing objective function due to change in the link lengths, coupler mass, slider mass, contact generalized stiffness, Young's modulus of the journal and the bearing, journal radius (which its change leads to a change in the clearance size) and restitution coefficient is investigated. The results are shown in "Figs. 4-6".

Table 1 Lengths and masses of the links

Body No.	Length (m)	Mass (kg)
1	0.05	0.3
2	0.12	0.21
3	-	0.14

Table 2 Clearance joint properties

Bearing Radius	10mm	Young's Modulus	207 Gpa
Poisson's Ratio	0.3	Restitution Coefficient	0.9
Clearance	0.01mm		

After vanishing the transient response, the sensitivity of the base reaction force is considered here for the rest of motion. This could be expectable because of the nearly periodical repetitive behavior of the mechanism with the foregoing conditions. Considering "Fig. 4", the results show that the reaction force is more sensitive to the variation of link lengths and link masses compared to the variation of contact surface characteristics such as Young's modulus, restitution coefficient and contact generalized stiffness in most parts of the motion cycle. Furthermore, the sensitivity to both contact generalized stiffness and Young's modulus of the journal and bearing are exactly the same in the steady state motion.

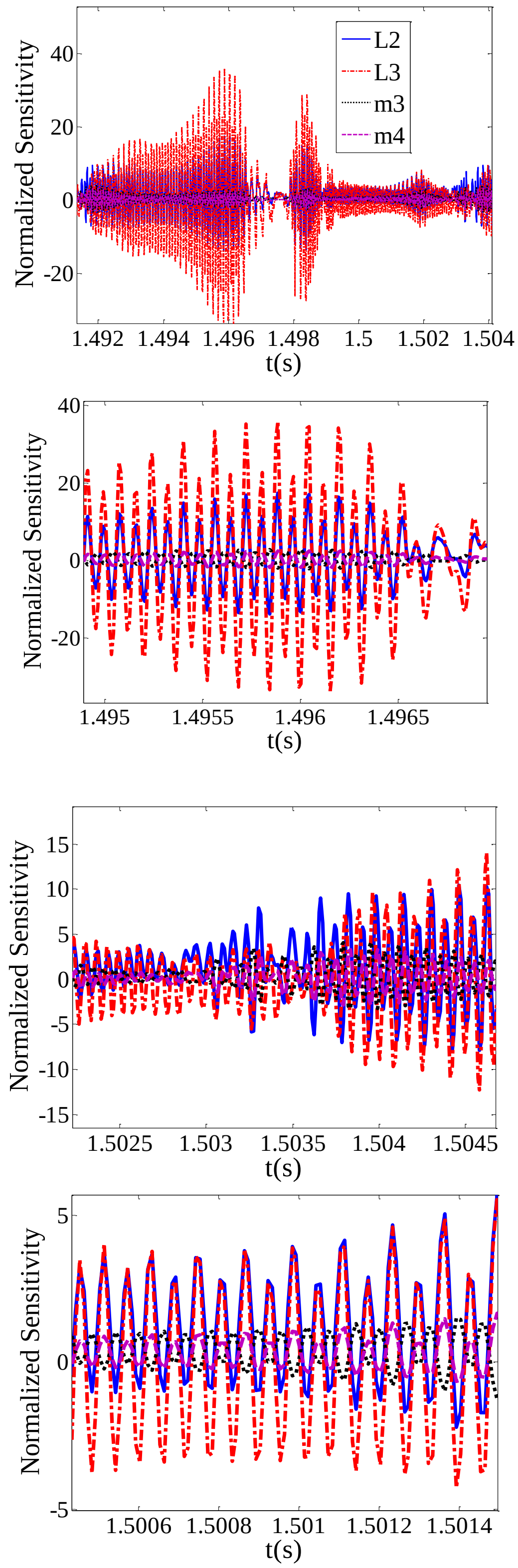


Fig. 4 Sensitivity of the base reaction force exerted on the base by the crank to some kinematic properties, for one cycle of the crank rotation and three zoomed parts of it.

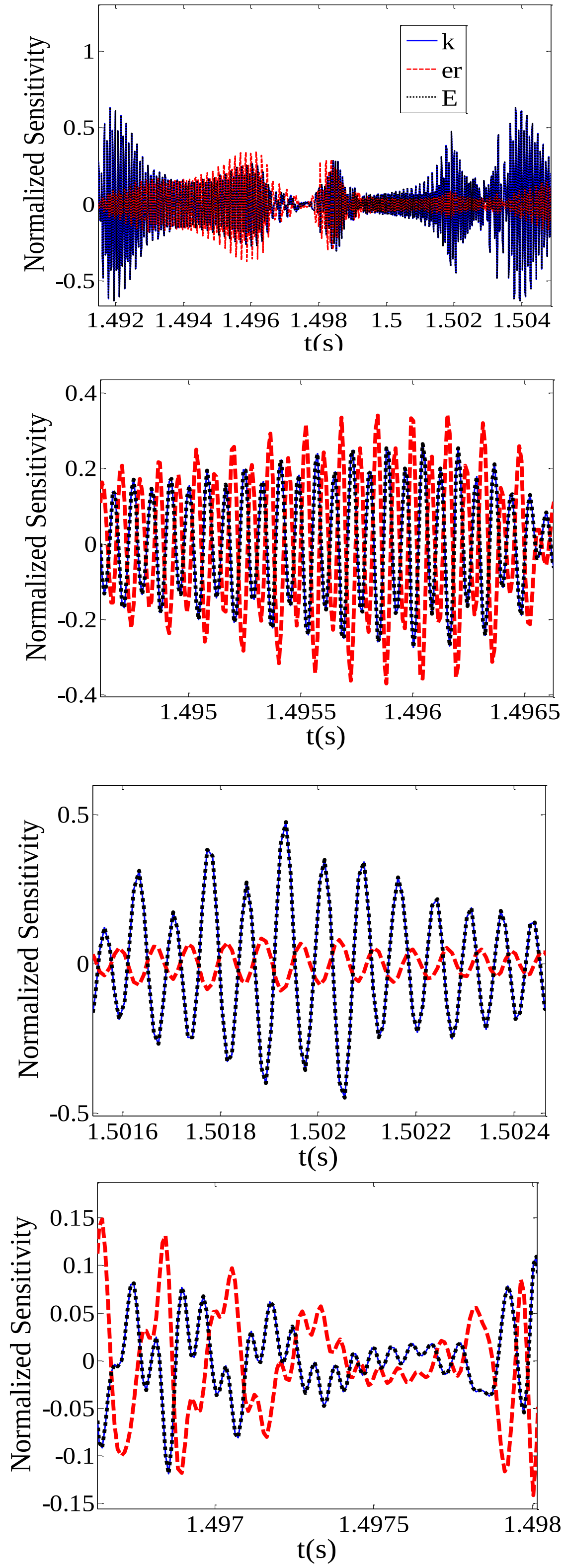


Fig. 5 Sensitivity of the base reaction force exerted on the base by the crank to some contact properties, for one cycle of the crank rotation and three zoomed parts of it.

This is due to this fact that both factors are appeared as the coefficient of the amount of penetration in the contact force model. Consequently, their variations affect the reaction force identically as both journal's and bearing's material are supposed to be the same. By comparing "Fig. 6" with "Figs. 4-5", one concludes that the sensitivity of the base reaction force to the journal radius is generally very higher than its sensitivity to the above-mentioned kinematic and contact properties.

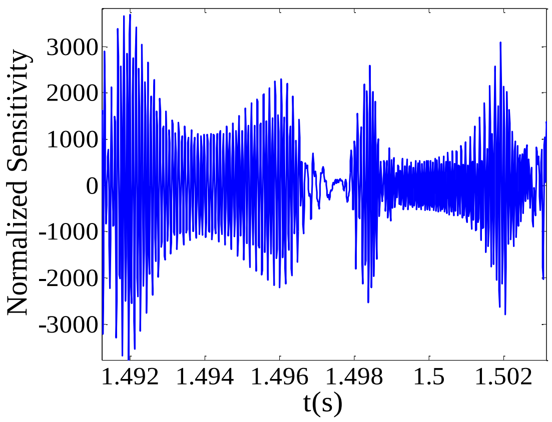


Fig. 6 Sensitivity of the base reaction force exerted on the base by the crank to the journal radius, for one cycle of the crank rotation.

OPTIMIZATION RESULTS

Genetic algorithm has been used by many researchers for optimization problems such as [41-42]. In this section, based on the results of the sensitivity analysis, this optimization procedure is used to reduce the amount of the maximum base reaction force by choosing the optimized link lengths. The clearance size is not changed during the optimization process to prevent the non-periodic nonlinear behavior. The crank and the coupler lengths are varied between 4cm and 6cm, and between 10cm and 14cm, respectively. So, the piston stroke will be at least 8cm and at most 12cm. Other parameters are the same as above. In addition, the maximum base reaction force in the fourth full crank rotation is considered after vanishing the transient response and reaching the steady-state condition. The fitness value versus the number of generations is shown in "Fig. 7a". The base reaction force for the original and optimized lengths of the mechanism links are shown in "Fig. 7b". Furthermore, the original and the optimized lengths are given in "Table 3". As it can be seen, a considerable improvement in the force applied to the base is achieved by modifying the link lengths.

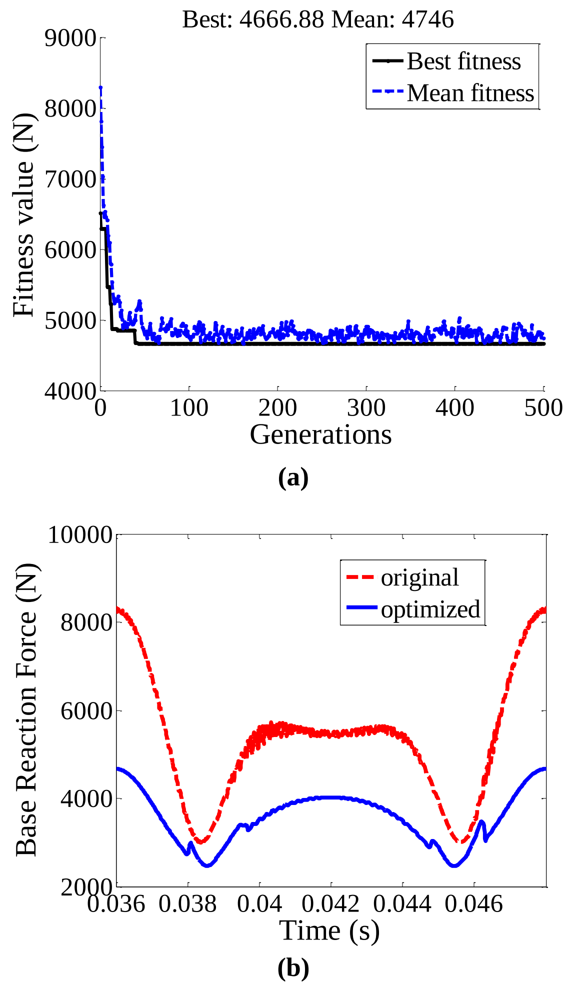


Fig. 7 (a): The fitness value versus the number of generations, (b): The original and obtained optimized base reaction force.

Table 3. The obtained optimized values of the mechanism links

	Coupler Length (cm)	Follower Length (cm)
Original Values	5	12
Optimization Results	4.0733	13.7468

The maximum values of the base reaction force for the original and optimized links are 8311N and 4667N, respectively. This shows a 43.85% reduction of the base reaction force when the optimized link lengths are chosen. In addition, the optimized link lengths are close to the lower (4cm) and upper (14cm) limits of the crank and coupler lengths used in the optimization problem, respectively.

6 CONCLUSION

In this paper, the sensitivity analysis and optimization of system parameters for a classical slider-crank mechanism with a clearance between the joints of coupler and slider was presented. After deriving the equations of motion and modeling the clearance, the direct differentiation method was used to conduct a local sensitivity analysis to assess the sensitivity measure of the base reaction force on some kinematic and contact parameters. According to the results, it was observed that the reaction force is more sensitive to the variation of link lengths and link masses compared to the variation of contact surface characteristics such as Young's modulus, restitution coefficient and contact generalized stiffness in most parts of the motion cycle. On the other hand, the sensitivity of the base reaction force to the clearance size was very higher than its sensitivity to the above-mentioned kinematic and contact properties. Finally, based on the results of the sensitivity analysis, an optimization procedure was used to reduce the amount of the maximum base reaction force by choosing the optimized link lengths. A considerable improvement in the force applied to the base was achieved by modifying the link lengths. The maximum values of the base reaction force for the original and optimized links were 8311N and 4667N, respectively, which showed a 43.85% reduction of the base reaction force when the optimized link lengths were chosen.

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