Development of a New Dynamic Friction Model for Analytical Modeling of Elliptical Vibration Assisted Turning Process

Reza Nosouhi[1, 2]*, Saeed Behbahani[3], Saeid Amini[4], Mohammad Reza Khosrojerdi[4]

[1]Modern Manufacturing Technologies Research Center, Najafabad Branch, Islamic Azad University, Najafabad, Iran
[2]Department of Mechanical Engineering, Najafabad Branch, Islamic Azad University, Najafabad, Iran
[3]Department of Mechanical Engineering, Isfahan University of Technology, Isfahan, Iran
[4]Manufacturing Department, Faculty of Engineering, University of Kashan, Kashan, Iran
*Email of Corresponding author: rezanosuhi@pmc.iaun.ac.ir
Received: September 27, 2016; Accepted: January 6, 2017

Abstract
A new dynamic friction model for modeling of elliptical vibration assisted turning (EVAT) was developed in this research. The periodic change of the friction force direction is known to be one of the most important causes of this phenomenon. In modeling of machining processes (including the EVAT process), static Coulomb friction model was employed by most of the researchers. Because of the periodic change in the friction force direction during the elliptical vibration path, the simple Coulomb friction model could not reflect the dynamic friction phenomena between the chip and the tool such as stick-slip and dynamic breakaway. In this research the LuGre dynamic friction model, which had been successfully employed for modeling of EVAT process, was modified in compliance to the machining process. To modify the LuGre model, the LuGre model coefficients were modified so that the effect of the temperature change of the contacting surfaces (which plays a key role in the friction behavior between the chip and the tool) could be reflected by the model. To determine the coefficients, orthogonal machining tests were performed and the Genetic Algorithm optimization method was carried out to derive the coefficients. Afterward, the results achieved from both dynamic models were compared with experimental results and Coulomb model results. The results from the new modified LuGre model were in better agreements with the experimental results.

1. Introduction
Vibration Assisted Turning process is one of the effective methods for manufacturing of precise machined parts specifically from hard-to-cut materials. After introduction of this process in 1960s [1], this process was successfully employed for machining of Nickel and Titanium based super alloys [2,3], aviation Aluminum alloys [4], hardened steels [5] etc. In this process, a vibration (mostly in ultrasonic frequency) is superimposed with the conventional turning movement of the tool in one dimension or two dimensions, resulting in the separation of the tool and the workpiece in a fraction of the vibration cycle time. According to the report of different researchers, the machining forces were decreased in this process and the tool life was increased [5, 6]. Specifically, in the 2D VAT process, the reduction of the machining forces was more significant than 1D VAT process [7]. One of the most important reasons of the reduction of the machining forces in 2D VAT

is believed to be the change of the tool-chip friction force direction in a fraction of the contacting period between the tool rake face and the chip [7].

A number of researches have been carried out on modeling of 2D VAT. Shamoto et al. modeled the process analytically and compared the results with experimental data [8]. In this research, the friction was modeled based on the static coulomb model. While the agreement of the analytical results with experiment was acceptable, there was still significant error in prediction of machining forces. A dynamic friction model was employed in FE simulation of VAT by Ahmed et al. [9]. However, the model only could calculate an upper bound for the friction force and did not determine the friction force. The change of the friction force direction was extensively discussed in the work of Fan and Miller [10]. Although they showed the change of the friction force direction analytically, the dynamic friction effects were overlooked in their research. Razavi et al. also developed an analytical model for prediction of forces in VAT and compared the results with experimental data [11]. Although the sticking and sliding zones were considered for determination of the tool-chip friction force in this model, their model was still static.

Most of the researchers assumed that the friction phenomenon in VAT process could be analyzed with the static coulomb friction model. Although this assumption is acceptable for conventional machining, since the amount and direction of the relative velocity between the two surfaces (i.e. tool rake face and chip surface) alters in 2D VAT process, some friction dynamic effects have to be taken into account. Specifically the stick-slip phenomenon has a key role in determination of the friction force. There are some friction models developed by different researchers in which the dynamic effects of friction phenomenon including the stick-slip, dynamic breakaway and stribeck effect can be reflected. The most important dynamic friction models are the Karnopp model [12], Dahl model [13], Bristle model [14], reset integrator model [14] and LuGre model [15]. Among all these models, LuGre model has been used in most of the applications successfully. The model input is the relative velocity and not the normal force.

In this research, the LuGre model was modified for the machining applications. In the previous work of the authors, the LuGre friction model was successfully employed for modeling of 2D VAT [16]. It was shown that there were dynamic phenomena in the 2D VAT process which could not be detected by the static friction models such as Coulomb model (which has been widely used by previous researchers [8, 9]), and the implementation of an appropriate dynamic friction model led to better agreement between the analytical and experimental results. It has to be noted that in comparison of the applicability of the Coulomb model and LuGre model in machining, although the results achieved from the analysis the 2D VAT with LuGre model had better agreement with the experimental results, the LuGre model along with most of the other dynamic friction models have been developed for control applications. The machining problem is quite different from a control problem. Specifically, there are three phenomena that is very important in determination of the friction force in machining including the high amounts of chip strain, high strain rates of the chip, and more importantly, high chip-tool contact temperature. These three phenomena change the material properties and the contact conditions and therefore have to be regarded in development of a dynamic friction model which is used for a machining analysis.

In the LuGre model as it will be discussed in the following sections, there are terms related to the strain and strain rate. However, there is no term related to the temperature of the two surfaces. The

authors modified the LuGre model coefficients so that it can reflect the effect of the temperature change in machining. The model coefficients were identified using orthogonal cutting tests along with the genetic algorithm optimization method. The model was then used to calculate the machining forces in 2D VAT. The results were finally compared with the results from LuGre model, Coulomb model and the experimental data. It was shown that the new model results have better agreement with experimental results.

2. The LuGre Friction Model

The contacting surfaces in LuGre model are assumed to be ideally rigid, and there are some deformable bristles in the two surfaces' interface as shown in Figure 1. As a result of the relative movement of the two contacting surfaces, these bristles are deflected. The friction force is the force needed for deflection of these hypothetical bristles. As the force increase, the deflection of the bristles is saturated and new bristles are being bent while some bristles lose their contact with the corresponding opposite side bristle, cause a shock which is very similar to the stick-slip phenomenon.
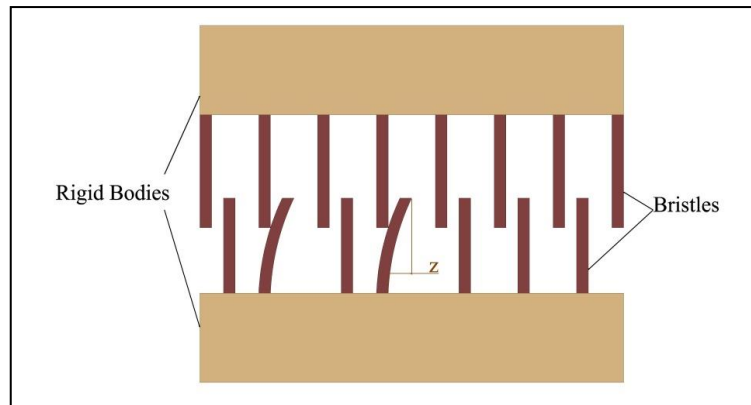

Figure1. The LuGre model contacting surfaces and the bristles

In this model z is the average deflection of the bristles which is a state variable. The friction force could be achieved from the following equation:

$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v \qquad (1)$$

Where σ_1 is the damping coefficient, v_s is the Stribeck velocity, F_C is the level of Coulomb friction, the sticking force level is denoted by F_S, σ_0 is the bristles' stiffness coefficient and $\sigma_2 v$ is the viscose term. The model therefore can be characterized by six parameters: σ_0, σ_1, σ_2, F_C, F_S and v_s [8].

3. The Approach to the New Model

As it was mentioned before, the LuGre model was mostly used for the control applications. To use the model for the machining analysis, some modifications had to be made to the model so that the effective phenomenon in machining problem could be better reflected by the model. In machining problem, the amount of strain is very high which may even exceed 1. The strain rate is also high,

and because of the high plastic strain and strain rate, the temperature increases in machining. The model therefore had to be sensitive to these three major conditions.

According to equations 1 to 4, there are two terms which show that the model is sensitive to the strain and strain rate. As the speed grows up, the amount of *dz/dt* also grows up. Since *z* is the deflection of the bristles, *dz/dt* denotes the rate of deflection, which can be representative of the strain rate sensitivity of the model. Also, with the increase of *z* in equation 4, the friction force also increases. This shows that the model is also sensitive to strain. However, there is no term in the model to indicate the influence of the temperature change on the friction force.

To approach to the problem of modification, the physical effect of temperature change on material properties is investigated firstly. Specifically, the temperature change affects the yield strength of the material and its sensitivity to the strain and strain rate. The objective of this analysis is to find a modified form of the model in which the effects of temperature change is regarded. As well as the LuGre model coefficients, there is no close form relationship for determination of the new coefficients. These coefficients as well as LuGre coefficients are determined using optimization methods.

4. Modification of the LuGre Friction Model

According to the microscopic basis of the LuGre friction model, the stiffness of the material changes according to the temperature variations. This means that σ_0 in the LuGre model is not a constant coefficient anymore and is a function of the temperature ($\sigma_0(T)$). The same modification has to be performed for σ_1, which means that the change of temperature also affects the sensitivity of the material to the strain rate. As a result, equation 4 can be rewritten as follows:

$$F = \sigma_0(T)z + \sigma_1(T)\frac{dz}{dt} + \sigma_2 v \tag{2}$$

Now it is necessary to determine the form of the dependency of σ_0 and σ_1 to the temperature. The temperature in machining is related to the cutting speed by the following relationship:

$$T_m \propto v^\alpha \tag{3}$$

In which T_m is the temperature of the cutting zone, v is the cutting speed and a is a constant which is different for different cutting tool and work-piece materials. The dependency of the two coefficients σ_0 and σ_1 to the temperature was therefore defined as follows:

$$\sigma_0 = \alpha_3 v^{\alpha_4} \tag{4}$$

$$\sigma_1 = \alpha_1 v^{\alpha_2} \tag{5}$$

In which α_1, α_2 and α_3 are positive constants and α_4 is a negative constant (because the sensitivity to the strain decreases as the temperature increases) which has to be determined via experiments. But it should be noted that as the velocity approaches to zero, the amount of the σ_0 approaches infinity. To resolve this problem, σ_0 was defined as follows:

$$\sigma_0 = P - v^Q \tag{6}$$

In which P and Q are constants. The amount of σ_0 decreases as v increases as it can be seen from Equation 10. To ensure that the amount of σ_0 would not be negative, the amount of P and Q should satisfy the following condition:

$$P > v^Q \tag{7}$$

Thus, the new model is formulated as follows:

$$F = (P - v^Q)z + (\alpha_1 v^{\alpha_2})\frac{dz}{dt} + \sigma_2 v \tag{8}$$

And the condition presented in Equation 10 should also be satisfied.

4.1 Identification of the New Model Coefficients

There is no close form equation for identification of the coefficients of the LuGre model and the new model. Therefore as it is proposed in [17], the model coefficients have to be derived from an optimization method. The model coefficients in this research were obtained from the orthogonal cutting tests along with the genetic algorithm optimization method. The orthogonal cutting test setup is schematically presented in Figure 2.
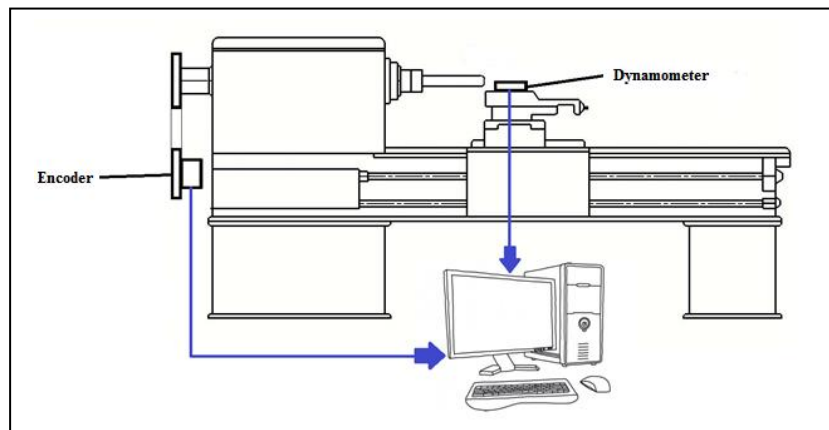

Figure2. The orthogonal cutting test setup for identification of the new model's coefficients

The test specimens' materials were Al6061, Al7075, copper and Inconel 718. Four test specimens in the form of tube were prepared and the orthogonal cutting tests are performed on them. A tool with the rake angle of 0 degree is selected as the cutting tool. In orthogonal cutting condition with 0 degree rake angle, the force in the feed direction would be the friction force. The model identification process was performed in different cutting speeds. To be able to sense the friction phenomena in low speeds, the forces were also read in the acceleration time.

The friction force vs. cutting speed curve was achieved as the result of the orthogonal cutting tests. As it was mentioned previously, there is no close form equation for calculation of the new friction model. These coefficients have to be achieved from an optimization process. The optimization process which was employed in this research was genetic algorithm optimization method. The optimization method changed the eight model parameters in the definite range so that the norm of the difference between the analytical and experimental would be minimized. The fitness function of the genetic algorithm was presented in Equation 12.

$$Error = \frac{\sum_{i=1}^{n} \left| Measured\ Force_i - Calculated\ Force(P, Q, \alpha_1, \alpha_2, \sigma_2, F_C, F_S, v)_i \right|}{\sum_{i=1}^{n} \left| Measured\ Force_i \right|} \quad (9)$$

The results of the optimization were presented in Table 1.

Table1. The results of the optimization algorithm

Material	copper	Al 6061	Al 7075 T6	Inconel 718
P	9966000	350000	962800	482200
Q	0.1033	3.49	0.8944	5.34
σ_2	0.523	0.405	0.0159	0.523
α_1	149	77000	851.5	98333
α_2	0.715	0.02245	0.499	0.0365
F_C	0.1127	9.97	8.97	0.7379
F_S	0.1135	0.1309	7.49	2.56
v_s	0.0199	0.00978	0.00987	0.00468
Error (%)	4.1	4	0.8	1

4.2 Analytical Modeling of EVAT with the New Friction Model

The identified new model was employed for modeling of the EVAT process. The MATLAB software was employed for calculation of the machining forces. The new model equations were used for calculation of the friction forces. The input to the model was the relative velocity between the chip and the tool, which was derived from the kinematic equations of the EVAT process. The output of the model was the friction force between the tool and the chip. This friction force was then employed for calculation of the cutting forces.

The results of the principal and feed forces versus time in a single vibration cycle for copper were presented in the Figurs. 3 and 4.
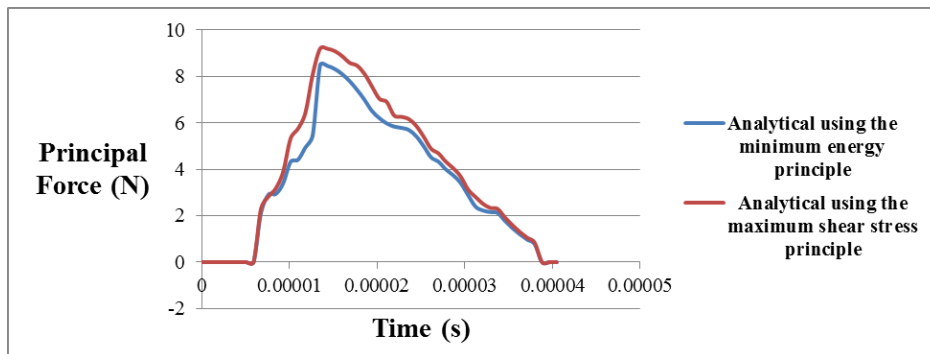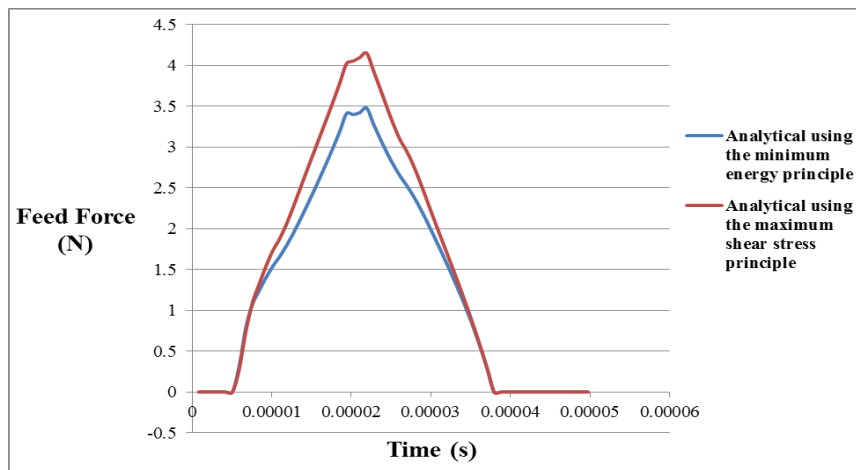

Figure3. The principal force versus time for copper

Figure4. The feed force versus time for copper

5. The Evaluation of the New Model

The model is evaluated by experimental tests. An elliptical vibration tool is designed and manufactured by which the elliptical tool path is achieved by combination of the longitudinal and bending vibrations. The tool was designed so that the resonance in the bending and longitudinal directions occurred at the same frequency. There was a phase shift with the amount of $\pi/2$ between the two vibration modes so that an elliptical tool path could be achieved. The vibration tool is depicted in Figure 5.
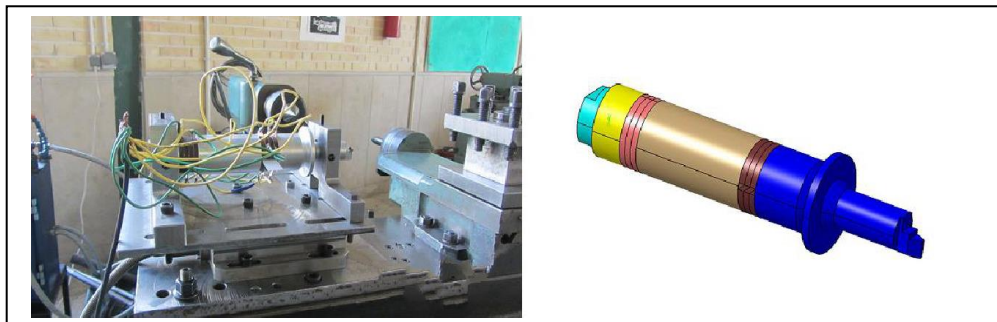

Figure5. The elliptical vibration tool assembled on the lath

The specimens' materials were Al6061, Al7075, copper and Inconel 718. The results of the experimental tests along with the analytical results of the modeling with three friction models (i.e. Coulomb model, LuGre model and the new model) were presented in Table 2.

Table2. The analytical and experimental results

Forces (N)		Experimental results		Analytical, Coulomb friction model		Analytical, LuGre friction model		Analytical, new friction model	
		Mean values of the force	SD of the force peaks	maximum shear stress principal	minimum energy principal	maximum shear stress principal	minimum energy principal	maximum shear stress principal	minimum energy principal
Copper	Principal	10.5	3.2	15.2	18.4	13.3	14.6	9.2	8.5
	Feed	8	2.2	1.9	3	7	7.2	7.9	8.2
Al 6061	Principal	8	1.45	4.3	5.3	9	7.8	10.2	9.7
	Feed	5	0.9	2.5	2	8.3	6.2	6.1	6
Al 7075 T6	Principal	12.2	1.62	16.6	15.7	14	13.8	9.7	12.8
	Feed	3.2	3.4	9.9	10.8	3.4	4.2	3	3.4
Inconel 718	Principal	35	7	46	50	28.7	30.4	38	36.1
	Feed	17.3	1.9	26	25	17.2	22.5	21	22

5.1 Accuracy of the Results

As it can be seen in Table 2, the results achieved from modeling using LuGre model were in better agreement with the experimental results in comparison with the results of the modeling with the Coulomb model. Also, it can be seen that using the new model has excelled the results in comparison with the modeling with LuGre model. In order to compare the three models quantitatively, the accuracy index was defined as follows:

$$Model\ Accuracy\ Index = \sum \left| \frac{(analytical\ result - experimental\ result)}{experimental\ result} \right| \tag{10}$$

The accuracy indexes of the three models were presented in Table 3.

Table3. The accuracy indexes of the three friction models

	Coulomb model	LuGre Model	New model
Model Accuracy Factor	68	32	29

Obviously the dynamic friction models had quiet better accuracy in comparison with the Coulomb static friction model. It was also observed that the new model had better accuracy in comparison with the LuGre model. The reason was that the new model was designed for the machining application.

6. Conclusion

A new friction model for modeling of EVAT process was developed in this process. It was shown in the previous research of the authors [16] that since the friction force direction changes during a vibration cycle in EVAT, static friction models such as Coulomb model cannot satisfactorily reflect the friction phenomenon between the chip and the tool, while a dynamic friction model produces

better results in modeling of EVAT. The new friction model was developed by modification of the LuGre model (which had been successfully employed for modeling of EVAT [16]). The 8 parameters of this new model were identified using orthogonal machining experiments along with genetic algorithm optimization method. The model was then used for modeling of EVAT process and the results were compared with the experimental results. It was shown that the results of the modeling with this new model were more accurate than the results achieved from modeling with Coulomb and LuGre model.

7. References

[1] Isaev, A. and Anokhin, V. 1961. Ultrasonic Vibration of a Metal Cutting Tool. Vest Mashinos.41 (in Russian).

[2] Ezugwu, E. O. 2005. Key Improvements in the Machining of Difficult-to-Cut aerospace Superalloys. International Journal of Machine Tools and Manufacture. 45: 1353-1367.

[3] Maurotto, A., Muhammad, R., Roy, A., et al. 2012. Comparing Machinability of Ti-15-3-3-3 and Ni-625 Alloys in Uat. Procedia CIRP, 1: 330-335.

[4] Adnan, A. S. and Subbiah, S. 2010. Experimental Investigation of Transverse Vibration-Assisted Orthogonal Cutting of AL-2024. International Journal of Machine Tools and Manufacture. 50: 294-302.

[5] Zhang, X., Kumar, A. S., Rahman, M., et al. 2011. Experimental Study on Ultrasonic Elliptical Vibration Cutting of Hardened Steel Using PCD Tools. *Journal* of Materials and Processing Technology. 211: 1701-1709.

[6] Ding, H., Ibrahim, R., Cheng, K., et al. 2010. Experimental Study on Machinability Improvement of Hardened Tool Steel Using Two Dimensional Vibration-Assisted Micro-end-Milling. International Journal of Machine Tools and Manufacture. 50: 1115-1118.

[7] Ma, C., Shamoto, E., Moriwaki, T., et al. 2004. Study of Machining Accuracy in Ultrasonic Elliptical Vibration cutting. International Journal of Machine Tools and Manufacture. 44: 1305-1310.

[8] Shamoto, E., Suzuki, N. and Hino, R. 2007. Simulation of Elliptical Vibration Cutting Process with Thin Shear Plane Model. Proceeding of ASPE. 40: 64–69.

[9] Ahmed, N., Mitrofanov, A.V., Babitsky, V. I., et al. 2007. Analysis of Forces in Ultrasonically Assisted Turning. Journal of Sound and Vibration. 308: 845-854.

[10] Fan, X. and Miller, M.H. 2001. Force Modeling in Vibration Assisted Cutting. Proceeding of the 2001 ASPE Annual Meeting. 92-97.

[11] Razavi, H., Nategh, M.J. and Abdullah, A. 2012. Analytical Modeling and Experimental Investigation of Ultrasonic-Vibration Assisted Oblique Turning, part II: Dynamics Analysis. International Journal of Mechanical Science. 63: 12-25.

[12] Karnopp, D. 1985. Computer Simulation of Slip-Stick Friction in Mechanical Dynamic Systems. Journal of Dynamic Systems and Measurement Control. 107(1): 100-103.

[13] Dahl, P.R. 1968. A Solid Friction Model. Technical Reports. The Aerospace Corporation. El Segundo, California.

[14] Haessig, D.A. and Friedland, B. 1991. On the Modeling and Simulation of Friction. Journal of Dynamic Systems and Measurement Control Transaction of ASME. 113(3): 354-362.

[15] Canudas de Wit, C., Olsson, H., Aström, K. J., et al. 1995. A New Model for Control of Systems with Friction. IEEE Transaction of Automative Control. 40(3): 419-425.

[16] Nosouhi, R., Behbahani, S., Amini, S. and Khosrojerdi, M. R. 2014. Experimental and Analytical Study of the Elliptical Vibration Assisted Turning Process with the Dynamic Friction Model. Journal of engineering manufacture. 228(6): 837-846.

[17] Demosthenis D. R. and Spilios D. F. 2005. Friction Identification based Upon the LuGre and Maxwell Slip models, 16th IAFC world congress.