# A Heuristic Approach for Optimization of Gearbox Dimension

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### Abstract

A powerful optimization method is proposed in this study for the minimal dimensional design problem of gearbox. It is a general model that is suitable to use for any series of gear drives system and can extract both dimensional and layout of components-limited optimization design together. The objective function in this study has many local extremes so for avoiding this situation, various constraints have been determined Then, Particle swarm optimization algorithm has been implemented to speed up the convergence of optimization and elitist particles searched in problem space to find optimum value of goal function until all of them converge to the similar set of values. At the end, Results have been presented in the utilitarian diagrams to obtain optimal parameters from useful diagrams. The results display that the proposed method in this study is better than other reported in last works and it shows optimum volume of gearbox being related to a decrease of not just space but costs, material used to make gearbox component, etc.

#### **Keywords**

Reduce Volume, Dimensional Optimization, Particle Swarm Optimization (PSO), Gearbox

## **1. Introduction**

Weight/Volume optimization of gearbox has been more attractive for researchers. The volume is depending on configuration of the affected parameters such as, location of gears, number of gears, and number of teeth and so on. To achieve the best parameter for gearbox many researchers used different method for optimization. Chong and Lee used genetic algorithm for design gear trains to achieve the automate preliminary [1]. Gologlu and Zeyveli by using GA worked on optimization of helical gear with parallel axis gearbox to approach minimizing volume [2]. They optimized the number of teeth, module and width of teeth for gear and pinion. Panda et al. researched on weight optimization for single-stage gearbox consists of spur gear [3]. They used different evolution algorithm to achieve optimum weight of spur gear set in single-stage. Results were compared with other modern algorithm and proved to achieve better results than other heuristic method. Zolfaghari et al. worked on volume optimization of straight bevel gears by employing evolutionary algorithm [4]. To achieve this purpose, they used two optimization techniques include Genetic Algorithm and simulated annealing algorithm (SA). Miler et al. utilized Genetic algorithm to optimize weight of gear pair and studied on design spur gear with considered profile shift [5]. Alexandru et al. studied on the steering gearbox design and simulation with variable transmission ratio [6]. They focused on important objectives consist of mathematical model in theoretical bases, determined geometrical parameters and simulated the ability of the gearbox. Tudose et al. studied on two-stage reducer

consists of helical gear for automate process of optimum design by means of evolutionary algorithm [7]. Li et al. presented how to use Genetic Algorithm to solve the multi-objective gear reducer design problems in optimization process [8]. Kang et al. presented optimization method for obtaining the optimum helix angle of gears [9]. They presented the relation between the transmission error and contact ratio. Abderazek et al. worked on spur gear and introduced a method for achieving the optimal tooth profile for gears [10]. Yokota and Gen studied on weight design of gears and used genetic algorithm to achieve a solution method for optimum weight [11]. Savsani et al. utilized simulated annealing and particle swarm algorithms to achieve optimum weight of a gear train in multi stage [12]. Swantner and Campbell worked on optimized gear trains with a method that automates the design of gear trains and consists of various type of gear such as bevel, worm, simple and compound gears [13]. Marjanovic et al. studied on optimization of spur gear trains [14]. They studied on position of shaft axes in gear train for reducing the volume. Their strategy to select optimal parameters has three stages: optimal materials, gear ratios and position of shaft axes. They presented gear trains with 22% reduction in volume. Chong et al. proposed an optimization algorithm with four important stages [15]. In the first stage, the user selects number of reduction stages. Next, gear ratios are specified for each stage by using the random search. Third, basic parameters for gear design are generated by using test methods. At the end, simulated annealing algorithm specified shafts position and other design parameters for minimizing the gearbox volume is presented. Pomrehn and Papalambros worked on discrete optimum design model in gearbox that used spur gear pairs [16]. Thompson et al. studied on optimal volume design for spur gear reduction units [17]. They presented optimal design formulation which is applicable to two-stage and three stages gearbox of arbitrary complexity. Mendi et al. carried out an genetic algorithm for optimization of rolling bearing, shaft diameter and module [18]. They compared genetic algorithm with analytic method and the results showed that the genetic algorithm is better than the analytic method to achieve optimal gear volume. Zarefar and Muthukrishnan used random-search methodology for helical gear optimization [19]. Salomon et al. worked on optimization of gearbox design by using active robust considering requirements of uncertain load [20]. Ciavarella and Demelio worked on optimization of fatigue life of gears, specific sliding and stress concentration by using numerical methods [21]. Wang et al. studied on optimum design of tooth profile and spur gear [22, 23]. Golabi et al. worked on design optimization of single and multistage gearbox based on minimum volume/weight [24]. They used fmincon method to perform optimization and consider different values for gear ratio, input power and hardness of material to draw practical curves. They presented the design parameters with some graphs such as number of stages, modules, shafts diameter and face width of gears, but location of gears is considered to change in two directions (height and length). The minimum weight for a gearbox occurs if the location of gears is changed in three directions as locating in height, length and width directions.

In this paper the optimum volume/weight of a gearbox is investigated that the location of gears is varied in 3-stage dimensional direction. In this point of view, the presented gearbox has the minimum weight/volume of possible gearbox. To optimize the problem, particle swarm optimization (PSO) algorithm is used. The algorithm optimized (minimize) the weight of gearbox and presented the location of gears in gearbox, number of teeth, module, width of teeth and helical angel for each gears, etc. The optimum parameters for gear box are presented as practical graphs for

use. At the end, an example is presented to show how to use the diagrams and obtain the best parameter for each gearbox. The presented results are validated by comparing with those reported in previous papers.

	. Nomenclatures
Nomenclatures	
a' Gap as technical clearance for gear and shell on all sides	$L_{out}$ Length of output shaft
b face width	$M_a$ Periodical bending moment defined in shaft
<i>c</i> ' Insurance constant	$N^g$ Number of teeth in gear
<i>d</i> Shaft diameter	$N^p$ Number of teeth in pinion
$d_{wl}$ Operational pitch diameter of pinion	<i>O</i> Origin coordinate center
(mm)	$O_{g}$ Center point of gear
$d_i^{s}$ diameter of gear	$O_p$ Center point of pinion
$d_i^p$ Diameter of pinion	$R_e$ Total reduction ratio of gearbox
$m^{g}$ Module of gear (mm)	<i>R'</i> Radius of wheel
$m_t$ Transverse module (mm)	$S_e$ Fatigue strength of shaft
$m^p$ Module of pinion (mm)	$S_{f}$ Safety Factor-bending
<i>n</i> Number of shafts in gearbox	$S_{fs}$ Safety factor for shaft design
o Center state origin coordinate	$S_H$ Safety factor-pitting
q Tangential distance between pinion and gear	$S_{y}$ Yield strength of shaft
<ul><li><i>r</i> Radius of gears</li><li><i>r'</i> Radius of pinion</li></ul>	
<i>s</i> Number of gearbox stages	$T_m$ Torque of shaft
<i>t</i> Thickness of shell of gearbox	$Y_J$ Geometry coefficient for bending stress
$u_i$ Partial reduction ratio of any stages of gearbox	$Y_N$ Life coefficient for bending stress
(for example $i = 1, 2, 3$ )	$Z_I$ Geometry coefficient for pitting resistance
<i>c</i> Center distance between gear pair	$Y_V$ Temperature coefficient
$F_t$ Tangential load (N)	$Z_N$ Life coefficient for pitting resistance
H Height of gearbox	$Z_R$ Surface condition coefficient for pitting resistance
$K_H$ Load distribution coefficient	$Z_R$ Burlace condition coefficient for pitting resistance $Z_W$ Hardness ratio coefficient for pitting resistance
$K_B$ Rim thickness coefficient	$\theta_i$ Position angle (degree)
$K_s$ Size coefficient	$\delta$ Helix angle
$K_{V}$ Dynamic coefficient	$\sigma_{HP}$ Permissible contact stress (N/mm <sup>2</sup> )
<i>L</i> Length of gearbox	$\sigma_{FP}$ Permissible bending stress (N/mm <sup>2</sup> )

#### Table1. Nomenclatures

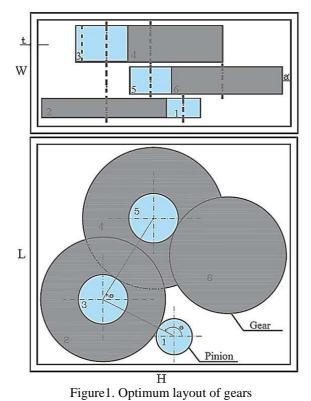
## **2. Problem Definition**

Working on gearbox design and optimization has been more attractive for researchers. In gearbox, the position of shafts is located in parallel plane, but the location of gears in gearbox can affect the minimum volume of gearbox effectively. The gearbox volume is the outcome of multiplying length

A Heuristic Approach for Optimization of Gearbox Dimension, pp. 17-39

(L), width (W) and height (H) of gearbox. In this paper, the location of gears can be changed along the three directions in gearbox and so. After implementing the optimization algorithm the minimum possible volume for gearbox is identified.

It should be considered that the volume of gearbox depends on the layout of the gears so a suitable layout provides compact gearbox. Figure 1 shows optimum layout of gears in gearbox.



## 2.1 Mathematical Model-Minimum Volume

To find the optimum volume of gearbox, the fitness function and constrains which consist of geometrical, design and control parameter constraints must be specified. The parameters that are relative to the gear pairs and shafts are named design variables. The volume of material of gearbox is considered as fitness function for optimization algorithm. This volume of materials is formulated in Equation (1) which is sum of the volume of shell, shafts and gears.

Volume of materials = 
$$m_{shaft} + m_{gear} + m_{shell}$$
 (1)

And the considering volumes are presented as:

$$m_{shell} = (w.l.h) - [(w - 2t).(l - 2t).(h - 2t)]$$
(2)

$$m_{\text{shaft}} = \frac{\pi . d_1^2}{4} \times (w + L_{\text{in}}) + \frac{\pi . d_n^2}{4} \times (w + L_{\text{out}}) + \sum_{i=2}^{n-1} (\frac{\pi . d_i^2}{4} \times w)$$
(3)

$$m_{\text{gear}} = \sum_{i=1}^{2s} \frac{\pi . D_i^2}{4} \times b_i - \sum_{i=2}^{n=1} \frac{\pi . d_i^2}{4} \times (b_{2i-2} + b_{2i-1}) - \frac{\pi . D_i^2}{4} \times b_1 - \frac{\pi . d_n^2}{4} \times b_{2s}$$
(4)

Also the center distance between gear pairs is presented as:

$$C = \left(r_{2i-1} + r_{2i}\right) \tag{5}$$

## 2.2 Calculating the Width of the Gearbox (W)

According to Figure 1, the width of gearbox is calculated by using Equation (6):

$$W = \frac{b_{first gear}}{2} + \frac{b_{end gear}}{2} + 2a' + 2t + Z$$
(6)

Where "Z" is the distance between the center of first and end gear in the z-direction as shown in Figure 1.

## 2.3 Calculate Height of Gearbox "H"

The height of gearbox as shown in Figure 1 can be obtained as:

$$H = diff_{H} + 2a' + 2t \tag{7}$$

Where,  $diff_H$  is the difference between the top point of the gears and the lowest point of the gears in gearbox as:

$$diff_{H} = \max(P_{h}) - \min(P_{h}) \tag{8}$$

And P<sub>h</sub> for all gears is obtained from:

$$P_{h}(2i) = \sum_{i=2}^{s} (r_{2i-3} + r_{2i-2}) Cos(\theta_{i}) + a_{i} \pm (C_{h}(2i)) =$$

$$\sum_{i=2}^{s} (r_{2i-3} + r_{2i-2}) Cos(\theta_{i}) + (r_{2i-1} + r_{2i}) Cos(\theta_{i}) \pm (C_{h}(2i)) \qquad (i = 1, 2, ..., s)$$

$$P_{h}(2i-1) = \sum_{i=2}^{s} (r_{2i-3} + r_{2i-2}) Cos(\theta_{i}) + a_{i} \pm (C_{h}(2i-1)) = \sum_{i=2}^{s} (r_{2i-3} + r_{2i-2}) Cos(\theta_{i}) + (r_{2i-1} + r_{2i}) Cos(\theta_{i}) \pm (C_{h}(2i-1)) \qquad (i = 1, 2, ..., s)$$

$$(10)$$

In Equations 9 and 10, the term  $C_h$  is the edge of each gear in "x" direction according to Figure 2.

A Heuristic Approach for Optimization of Gearbox Dimension, pp. 17-39

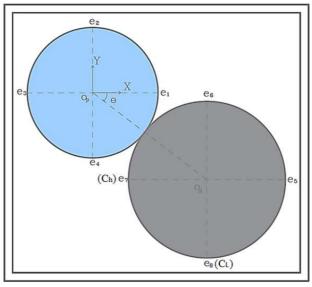


Figure2.Display gear pair to show edge-points

## 2.4 Calculate Length of Gearbox "L"

The length of gearbox is obtained:

$$L = diff_1 + 2a' + 2t \tag{11}$$

Where, "diff  $_{L}$ " is the difference between the first point of the first gear and end point of the end gear along the "y" direction and is presented as:

$$diff_{l} = \max(P_{l}) - \min(P_{l}) \tag{12}$$

And, P<sub>1</sub> for all gears in gearbox is obtained from:

$$z_{i} = (r_{2i-1} + r_{2i})\sin(\theta_{i}) \qquad (i = 1, 2, ..., s)$$
(13)

$$P_{l}(2i) = \sum_{i=2}^{s} (r_{2i-3} + r_{2i-2}) Sin(\theta_{i}) + z_{i} \pm (C_{l}(2i)) =$$

$$\sum_{i=2}^{s} (r_{2i-3} + r_{2i-2}) Sin(\theta_{i}) + (r_{2i-1} + r_{2i}) Sin(\theta_{i}) \pm (C_{l}(2i)) \quad (i = 1, 2, ..., s)$$
(14)

$$P_{l}(2i-1) = \sum_{i=2}^{s} (r_{2i-3} + r_{2i-2}) Sin(\theta_{i}) + z_{i} \pm (C_{l}(2i-1)) =$$

$$\sum_{i=2}^{s} (r_{2i-3} + r_{2i-2}) Sin(\theta_{i}) + (r_{2i-1} + r_{2i}) Sin(\theta_{i}) \pm (C_{l}(2i-1)) \quad (i = 1, 2, ..., s)$$
(15)

Where in Equations 14 and 15, "Cl" is the edge of each gear in "y" direction as shown in Figure 2.

### 3. Mathematical Model - Constraints

The results of optimization process propose a lot of possible solutions; therefore, different constrains must be defined in order to investigate and determine the feasible design variables to attain the optimum weight/volume of the gearbox. To this end, these constrains should be converted to the mathematical model, so these constrains will be divided into three pivotal categories including geometrical constraint, design constrain and control parameter constrain.

## 3.1 Geometrical Constraints

A geometrical constraint is defined to avoid three main types of clashes. At first, the geometrical constraint should control the area that there is the possibility of the clash between the gear and the next shaft in each stage as shown in Figures 3 and 4. The minimum possible distance for all of the gears in each stage in separate planes can be written as:

$$r_{2i}Cos(\theta) < r_{2i+2}Cos(\theta) + r_{2i+1} + a'$$
(16)

Where, minimal possible distance is showed by "a'".

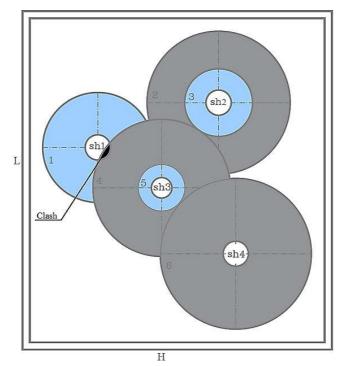


Figure3. Possible interface (clash) between gear and shaft in the separate plane (3D optimization)

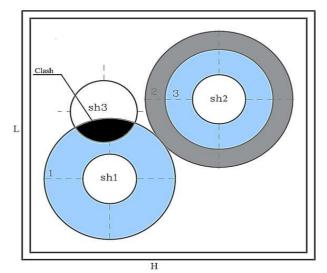


Figure 4. Possible interface (clash) between gear and shaft in the separate plane (3D optimization)

A Heuristic Approach for Optimization of Gearbox Dimension, pp. 17-39

The second geometrical constraint is presented to achieve the best choice to position each gear in two types of layout in all stages: initial arrangement for placing the gear in the same planes (2D optimization) that is showed in Figure 5 and the optimized location of gears in separated planes (3D optimization) that is presented in Figure 7. In other words, the second geometrical constraint has been proposed to ensure that there is no clash or interface in initial and optimized model among non-paired elements as shown in Equation (17) and (18) respectively.

$$C_{ij} > \frac{d_{i-1}^{p}}{2} + \frac{d_{j-1}^{s}}{2}$$
(17)

Where, "i" and "j" are paired gear.

$$r_{2i+3} + r_{2i} \cos(\theta) < r_{2i+2} \cos(\theta) + r_{2i+1} + C$$
(18)

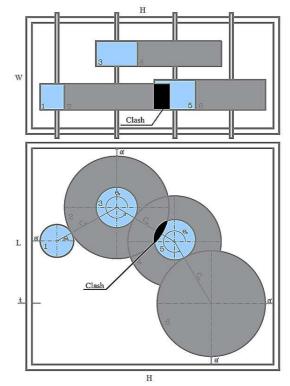


Figure 5. Possible interface (clash) between non-paired in the same plane (2D optimization)

Part a) Figure 6 expressed the state that the Equation (17) is satisfying and if Equation (17) is not satisfying, the position of gear with initial optimization was changed. It is showed in part b) in Figure 6.

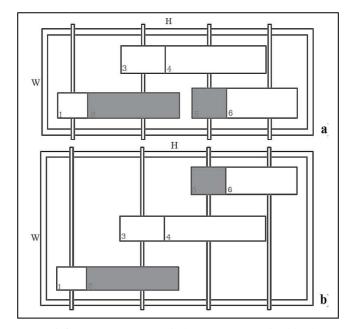


Figure6. State a) Satisfy geometry constrain between non-paired in the same plane and b) Common arranged gear position in gearbox in the same plane

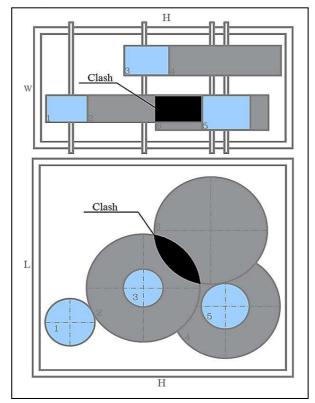


Figure 7. Possible interface (clash) between non-paired in the separate plane (3D optimization)

## 3.2 Design Constraints

The design constraints for gearbox have been divided into three parts including bending strength and pitting resistance for each gear in all of the stages of gearbox by the strength of shafts that has been presented in maximum shear stress theory. All of the design constrains are indicated in Equations (19) to (21) [25].

	Table2. Design Constraints	
Diameter constrain	$d \ge \left\{ \left[ \left(\frac{M_a}{S_e}\right)^2 + \left(\frac{T_m}{S_y}\right)^2 \right]^{\frac{1}{2}} \cdot \frac{32S_{FS}}{\pi} \right\}^{\frac{1}{3}}$	(19)
$\sigma_{_{Bending}} \leq \sigma_{_{Allowable} (Bending)}$	$K_V.F_t.K_s.K_o.\frac{K_B.K_H}{Y_J.m_t.b} < \frac{Y_N}{Y_z.Y_\theta}.\frac{\sigma_{FP}}{S_F}$	(20)
$\sigma_{\scriptscriptstyle Contact} \leq \sigma_{\scriptscriptstyle Allowable (Contact)}$	$\left(K_{V}.F_{t}.K_{s}.K_{o}.\frac{Z_{R}}{Z_{I}}.\frac{K_{H}}{b.d_{wl}}\right)^{\frac{1}{2}}.Z_{E} < \frac{Z_{W}.Z_{N}.\sigma_{HP}}{Y_{z}.Y_{\theta}.S_{H}}$	(21)

## 3.3 Control Parameter Constraints

Control parameter constraints are final constraints which have been indicated in Equations (22) to (27) in Table 3. For each pinion the minimum number of tooth is equal by [26]:

	Table3. Control Parameter Constraints	
Minimum teeth	$N^{p} \geq \frac{2 \operatorname{Cos}(\delta_{2i-1})}{\left(2\left(\frac{N^{g}}{N^{p}}\right)+1\right)\operatorname{Sin}^{2}(\phi_{t})}\left(\sqrt{\left(2\left(\frac{N^{g}}{N^{p}}\right)+1\right)\operatorname{Sin}^{2}(\phi_{t})+\left(\frac{N^{g}}{N^{p}}\right)^{2}}+\frac{N^{g}}{N^{p}}\right)}$	(22)
Ratio check point	$R_e = \prod_{i=1}^{s-1} \frac{M_{2i-1}}{M_{2i}}$	(23)
reduction the ratio of gearbox	$N^P \leq N^g$	(24)
Gear face width	$3.\pi.m' \le F \le 5.\pi.m'$	(25)
Modulus constant number	$1 \le m^p \le 50$	(26)
Constrain for modulus of each pair	$m^{g} = m^{p}$	(27)

The speed ratio is one of the main check points to achieve the speed ratio of gearbox." $R_e$ ", the speed ratio of each stage should be multiplied as it is presented in Equation (23), shown Table 3.

## 4. Methods of Solution

## 4.1 Particle Swarm Optimization (PSO algorithm)

The PSO algorithm was introduced by Eberhart and Kenney [27]. Kulkarni et al. worked on application of PSO method to mechanical engineering [28]. PSO method is the pivotal entry into a computation technique that used meta-heuristic according to stochastic optimization that used

behavior of population. PSO studied on the social behavior of a bird or fish group and it used random search in nature. In PSO, each bird wanders in the problem space, called Particle. They are potential solutions and assumed position of particles, Velocity and final fitness function. For initial steps all of particles have random position and zero velocity. Each particle being random searched to find piece of food in problem space and they have same question that where the target or food is but in iteration the particles just know how far the targets or foods are in space. One of the particles that is nearest to the target or food, is effective to follow. So P<sub>best</sub> and G<sub>best</sub> are the best values for govern to optimum target each particles updating generations. that achieved by particle position update in every iteration. Each particle finds fitness value by evaluating and it is also collected. The best previous position, indeed the best fitness value in each iteration is called P<sub>best</sub> that all particles save and remember. The historical best value that is the highest value or maximum food source or value of fitness function obtained so far by each N particle in whole swarm is named G<sub>best</sub>. Two best values (P<sub>best</sub> and G<sub>best</sub>) are used for updating velocity and positions vectors for any of the N particles in population. Particle velocity is obtained from the way that each N particles move all over in problem space. That is consisting of three terms: in the first, the decrypted inertia or momentum prohibits the particle to extremely changing direction. The second, called the self (individual) intelligence that is tendency of particles toward their own best locations in each particle's memory. At the end, named the social (group) intelligence and denotes the particle steers to move towards the general (global) best situation (location) of the whole population. Velocity and position of particle j in the i<sup>th</sup> iteration are obtained and updated from Equations (28) and (29) respectively [29].

$$V_{i}^{(j)} = V_{i}^{(j-1)} + y_{1} \times R_{1} \times [P_{best, j} - X_{j}^{(i-1)}] + y_{2} \times R_{2} \times [G_{best} - X_{j}^{(i-1)}] \qquad j = 1, 2, ..., N$$
(28)

$$X_{i}^{(i)} = X_{i}^{(i-1)} + V_{i}^{(j)} (29)$$

Where individual and social intelligence factor are called  $y_1$  and  $y_2$ , respectively and usually  $y_1 = y_2 = 2$  and  $R_1$  and  $R_2$  are random numbers that are chosen in the range 0 and 1. Calculate the fitness function values (target) corresponding to the particles as  $f(X_1^{(i)})$ ,  $f(X_2^{(i)})$ , ...  $f(X_N^{(i)})$ . PSO method is converged when the locations of whole particles (birds) converge to the same set of values. If the convergence current solution is not satisfied, position and velocity would be repeated by updating the number of iteration as i=i+1, and by calculating the new values of P<sub>best,j</sub> and G<sub>best</sub> [29].

The basic flowchart of operation the PSO method and implementation of the optimization process for achieving a global optimum is illustrated in Figure 8.

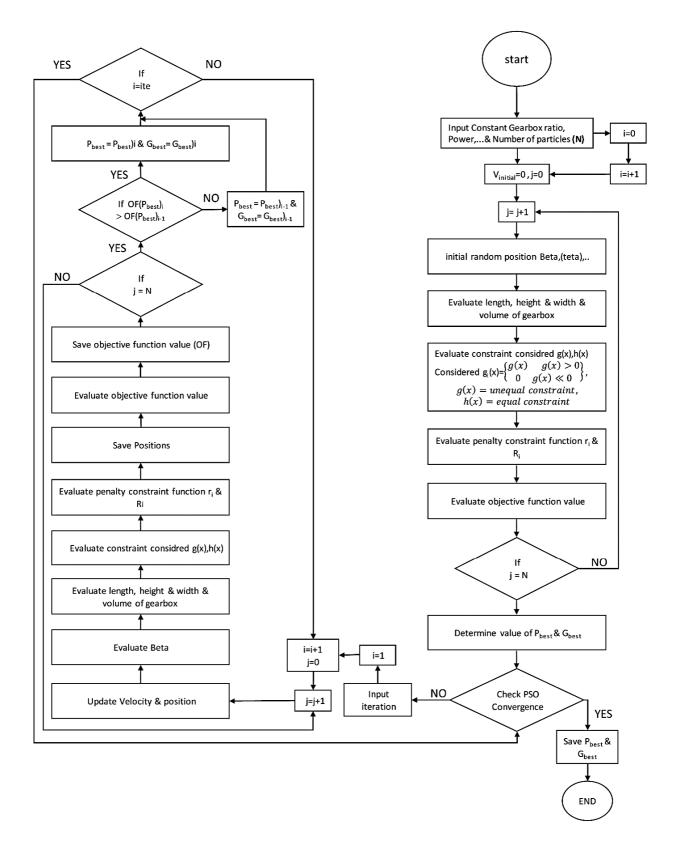


Figure8. Implementation of PSO flowchart

#### 5. Results and Example

In order to obtain the optimum results and present them, it should be considered that the value of optimized fitness function needs to satisfy the constraints in Equations (16) to (27). The parameters of optimum design gearbox in multistage gearbox can be obtained from the graphs such as modules, shaft diameter, and the number of stages and face width of gears. To this end, and to obtain the optimum results, a lot of problems have been solved using Particle Swarm Optimization which is one of the powerful optimization methods. Useful diagrams are extracted from the results of the program calculations. Table 4 showed the candidate for input design gearbox parameter values such as hardness, gearbox ratio and power. In order to use the results of optimization process, the required design parameters are converted to applicable curves, using the flowchart in Figure 8. The ratio of gearbox conversion specifies the number of gearbox stages extracted from Figure 9. According to this figure the mass of materials has been plotted based on the ratio conversion of gearbox.

	Table4. Elected S	Specific input data	
Input parameters	Transmission power (hp)	Hardness of material (BHN)	Gearbox ratio
Elected Specific values	2, 5, 10, 20, 30, 50, 80,	200, 300, 400	1.5, 2, 3, 5, 8, 10, 15
Elected Specific values	100, 150, 200		20, 40, 50

The flowchart shown in Figure 8 has expressed the steps for using the graphs. Firstly, the number of stages is extracted considering the transmission power and total ratio of gearbox conversion as shown in Figure 9, in which the volume of gearbox has been plotted considering the range of transmission power. Hence, the lowest volume of gearbox determines the optimal number of stages. Secondly, the ratio of trivial conversions in each stage will be obtained from Figure 10. The overall ratio of gearbox conversion is generated by multiplying the speed ratio of trivial conversions in each stage (partial ratios). Hence, Figures 11 to 16 show the optimum of design parameters in 3-stage of gearbox based on Table 4. As it was mentioned above, in order to explain how to use the curves, an example of selecting the design parameters of gearbox has been presented in order to reach optimum weight based on the curves presented in this paper. In order to compare the acquired results of this study with that of Golabi et al. the inputs are the same as it is indicated in Table 5 [24].

Table5. input data for applicable example consideration with Golabi et al. [24]

Input parameters Example	Transmission power (hp)	Hardness of material (BHN)	Gearbox ratio
Example input data	150	400	15

As it was mentioned above, the number of stages of gearbox as optimal state is extracted from Figure 9. Considering the transmission power and total ratio of gearbox, Figure 9 has been chosen as the optimal for 3-stage gearbox. Then, the optimal of the trivial conversion ratio has been used from Figure 10, considering the transmission power and total ratio. Finally, Figures 11 to 16 respectively have been used to achieve optimal design parameters.

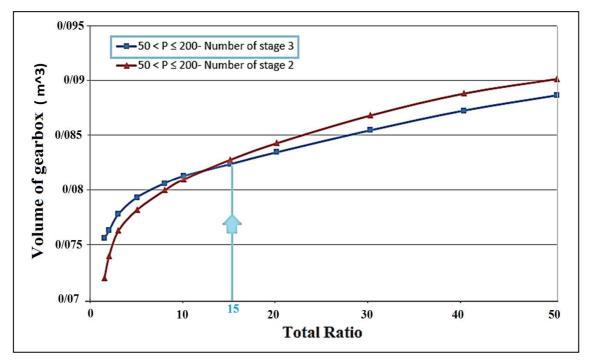


Figure 9. Optimum number of stages, comparison between the second and third stages parameters (50 hp $\leq$  Power  $\leq$  200 hp)

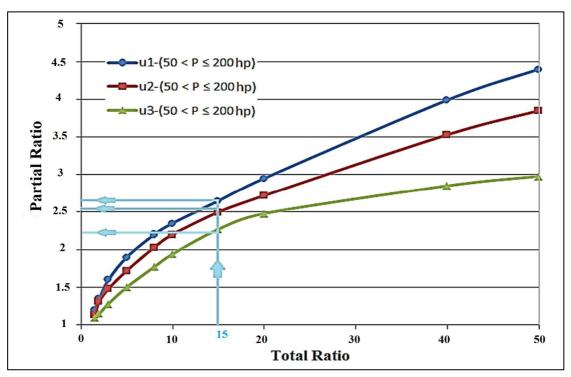


Figure 10. Optimum partial ratio of third-stage (50 hp  $\leq$  Power  $\leq$  200 hp)

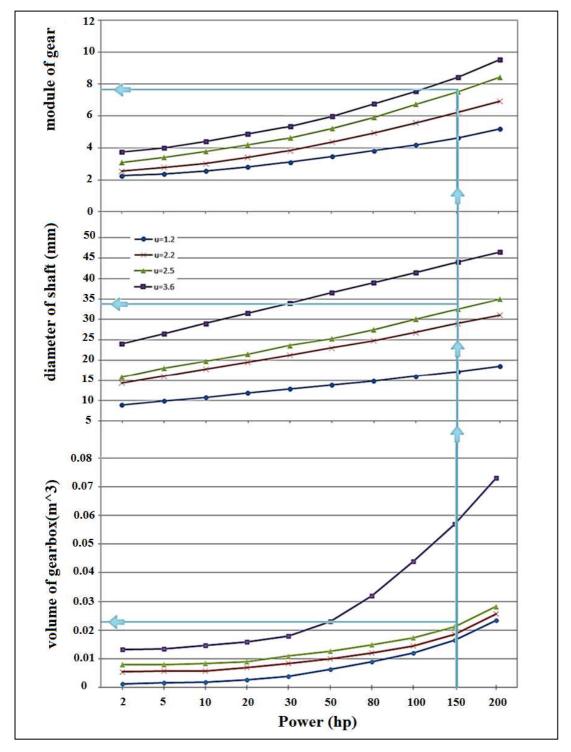


Figure 11. Volume, shaft diameter and module of 3-stage gearbox - BHN = 400-(stage one)

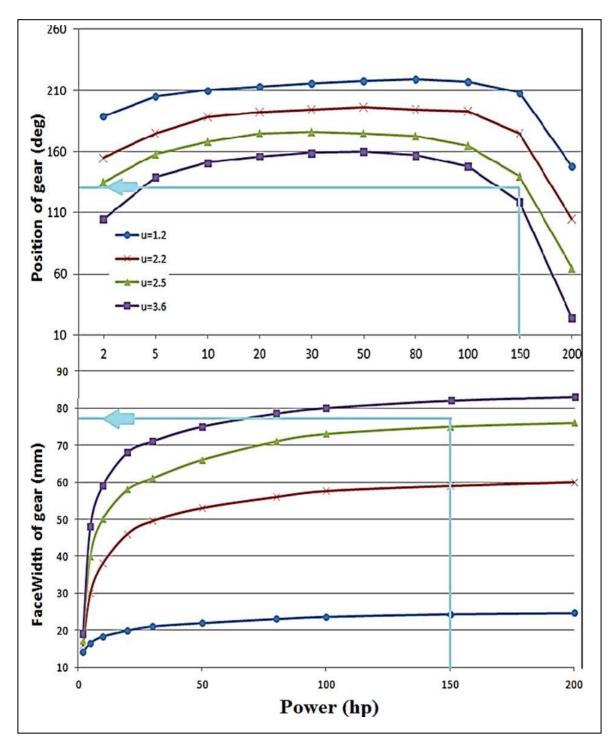


Figure 12. Face width and position of gear of 3-stage gearbox BHN = 400 - (stage one)

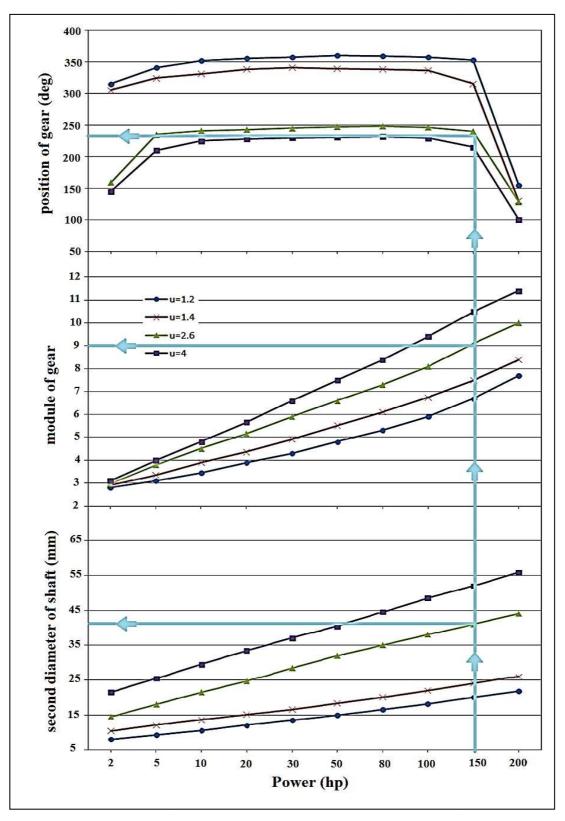


Figure 13. Shaft diameter, module and position of gear of 3-stage gearbox BHN = 400 - (stage two)

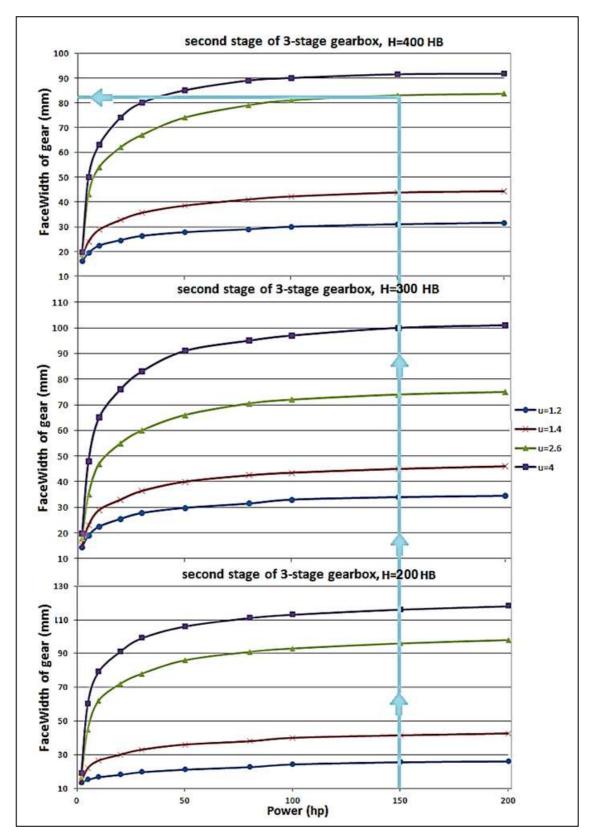


Figure 14. Face width of gear of 3-stage gearbox BHN =200, 300 and 400 – (stage two)

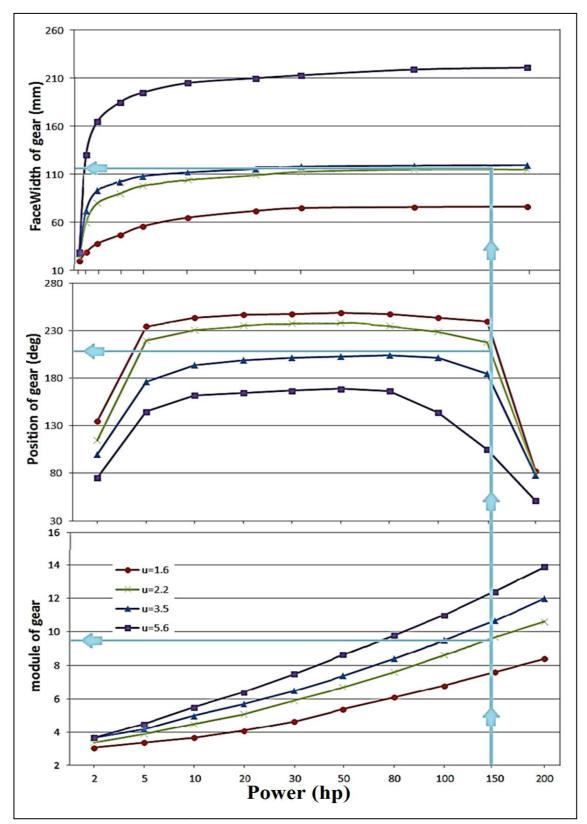


Figure 15. Module, position and face width of gear of 3-stage gearbox BHN = 400 - (stage three)

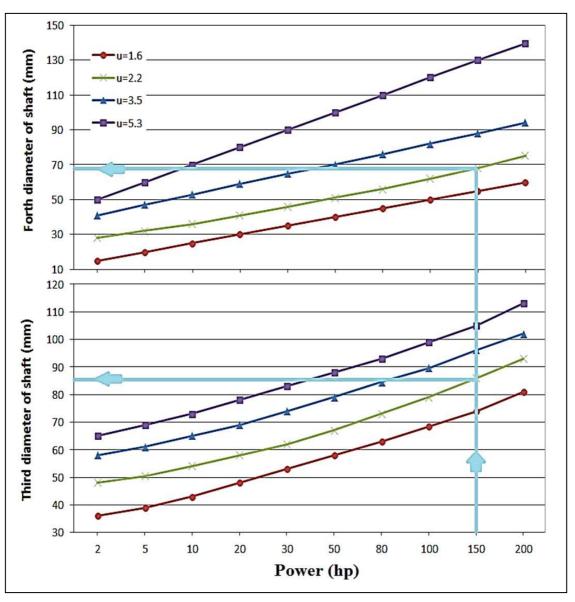


Figure16. Shaft diameters of 3-stage gearbox BHN= 400- (stage three)

## 6. Results Validation

Results obtained from this paper are compared with Golabi et al. to confirm the results [24]. Golabi et al. worked on optimization weight/volume in single and multistage gearbox, so for illustrating the results, the different input parameters for gearbox are considered [24]. The range of input data is the same as Golabi et al. that is presented in Table 4 [24]. Particle swarm optimization algorithm has been implemented by using specific parameters for the gearbox. Then, and finds optimum values for all components of the gearbox and all optimum values are presented as useful diagrams. As it was mentioned above, in order to explain how to use the curves, an example of selecting the design parameters of gearbox has been presented in order to obtain optimum weight based on the curves presented in this paper. In order to compare the acquired results of this study with that of Golabi et al. the inputs are the same as it is indicated in Table 5 [24]. Finally, Figures 9 to 16 respectively

have been used to achieve optimal design parameters and at the end, result of this paper shows that they get to optimum volume about 15% less than volume obtained from Golabi and et al. that is illustrated in Table 6 [24].

Descrip	tion	Ref [24]	Presented Research
u1	First Stage	2.6	2.7
u2	Second Stage	2.6	2.6
u3	Third stage	2.2	2.2
	First Stage	5	6
Module	Second Stage	8	9
	Third stage	10	8
	First Stage	51	77
Face Width (mm)	Second Stage	82	81
	Third stage	170	112
	First Stage	35	33
Shaft Diameter (mm)	Second Stage	43	41
	Third stage	125, 72	87, 68
	First Stage	-	130
Gear Position Angle	Second Stage	-	240
	Third stage	-	210
Volume (n	nm^3)	2.6 e7	2.2 e7
Differe	nce		-15 %

Table6. Comparison between results obtained from this paper and previous publication by Golabi et al. [24] Total Ratio=15 Power=150 hp\_hardness of material=400 BHN

### 7. Conclusion

In this paper, particle swarm optimization method is employed for dimensions and layout optimization process of 3-stage of gearbox to obtain minimum weight/volume design. To achieve the optimal weight/volume of gearbox, it is mathematically formulated as fitness function and defined design constrains as conditions that must be satisfied. For avoiding local extremes that are reported as possibilities solution in optimization process, three pivotal types of constrains which include geometrical constrain, including design constrain and control parameter constrain are defined. Optimization process implemented three-stage gearbox by selecting different input data include gear ratio, power and hardness of material. Utilitarian diagrams are obtained from optimization's results for achieving the minimal weight/volume of gears. Value of optimum weight/volume, all the necessary design parameters of gearbox and layout of gears are obtained from the utilitarian diagrams such as position angle, face width of gears, number of stage, shaft diameter and module. Verification of model is presented by comparing other reports in the previous published works. At the end, an example was elaborated to display how utilize utilitarian diagrams.

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