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Research Paper

Tracking Control of Robots Revisited Based on Taylor Series and Asymptotic Expansion

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Abstract
This paper points out some errors based on the one-dimensional Taylor series for a multi-dimensional function that is used for robots manufacturing. It is argued that the proof of theorem 1 is not mathematically true, and consequently, the obtained results cannot be correct. In addition to this, the stability analysis presented in the paper does not address the saturated area properly. Therein, stability is analyzed separately in saturated and unsaturated operation areas. However, the stability of the closed-loop system may not be guaranteed through these separate analyses, since transitions from saturation area to unsaturated area and vice versa are neglected. This work is an extension of the above paper, based on the revised Taylor series and considering actuator saturation limit in both controller design and stability analysis.

Keywords
Adaptive Uncertainty Estimation, Actuator Saturation, Stone-Weierstrass Theorem, Taylor Series

1. Introduction

During the past several years, function approximation methods such as Taylor series [1], Chebyshev polynomials [2], Fourier series [3,4], neuro-fuzzy systems [5, 6], and Legendre polynomials [7-9] have increasingly been recommended in robust adaptive control of complicated systems. The Stone-Weierstrass theorem from the classical real analysis can be used to show that these architectures possess the universal approximation capability.

Ahamadi and Fateh [10], proposed a new control scheme utilizing the Taylor series as a Universal approximator. They assumed that an n degree of freedom (DOF) actuated robotic system could be decentralized to n double integrators plus lumped uncertainty. The Taylor series expansion can then be utilized to approximate each subsystem (a real-valued nonlinear function of several real variables) to arbitrary accuracy. They claimed that the Taylor series satisfies the conditions of the Stone-Weierstrass theorem. The proof of their main theorem 1, however, is not technically true. In addition to this, the proposed approach in [10] does not give suitable stability analysis for the overall control system. It uses the boundedness of the saturated signal to prove the stability and boundedness of the closed-loop internal signals. It is worth emphasizing that in the saturated area of the control input, the controller operation does not influence the plant since the actuators are driving the system by their maximum value. In this condition, although the tracking error is bounded, it may be unacceptable due to unsatisfactory performance. Nevertheless, the stability analysis presented in [10], does not address the saturated area properly. Another important issue is that in [10] stability is analyzed separately in saturated and unsaturated operation areas. However, the stability of the closed-loop system may not be guaranteed through these separate analyses, since transitions from saturation area to unsaturated area and vice versa are neglected. Therefore, there is a gap in the stability analysis in the above paper. The objective of this paper is to modify the previous results on the controller design and robust stability analysis of the work proposed by [10].

This paper is organized as follows. Section 2 presents some preliminaries about the Heine-Borel and Stone-Weierstrass theorems. Section 3 indicates that the utilized approximation in [10] is not technically correct. Section 4 briefly presents dynamic modeling of the robotic system including the robot manipulator and the permanent magnet DC motors subjected to actuator saturation. In section 5 we'll see how the Taylor series can be used to approximate any continuous function with the Weierstrass approximation theorem. Section 6 presents the Indirect Adaptive Taylor-series-based controller design. Section 7 presents the direct Adaptive Taylor-series-based controller. Finally, concluding remarks are drawn in section 8.

2. Preliminaries

We firstly state the celebrated Heine-Borel and Stone-Weierstrass theorems [11]. Henceforth, we will use the same notation and equation numbers as ref [10].

Theorem 1 (Heine-Borel Theorem). A subset U of $\mathbb{R}^n$ is compact if and only if U is closed (including all of the limited points) and bounded.

As a result of the Heine-Borel Theorem, the sets $\prod_{i=1}^{n}[a_i, b_i]$ are compact $\mathbb{R}^n$.

Theorem 2 (Stone-Weierstrass Theorem). Let U be a compact metric space. Let $Y \subset C(U)$ be an algebra such that

(1) Y contains a non-zero constant function;

(2) Y separates points (i.e., if $x_1, x_2 \in U$, $x_1 \neq x_2$, then there exists $p \in Y$ such that $p(x_1) \neq p(x_2)$).

Then Y is uniformly dense in $C(U)$, the set of continuous real-valued functions on U. In other words, for any $\varepsilon > 0$ and any function g in $C(U)$, there is a function p in Y such that $|p(x) - g(x)| < \varepsilon$ for all $x \in U$.

3. Errors

We will show that the Taylor-series expansion of real-valued functions with one real variable cannot be used to approximate the nonlinear function f_i given by Eq. (6). Some considering to [10], it can be seen that the input universe of discourse U is a compact set in $\mathbb{R}^n$. Furthermore, Theorem 1 of [10] says that

$$\sup_{x \in U} |p_N(x) - g(x)| < \varepsilon \tag{a1}$$

where $p_N(x)=\sum_{j=0}^{N}\frac{h^{(j)}(x_0)}{j!}(x-x_0)^j$. In the above inequality, we note that the domain p_N belongs to $\mathbb{R}$, while the function g is defined on U which is a subset of $\mathbb{R}^n$. So, the above relation holds only on $U \cap \mathbb{R} \subset \mathbb{R}$.

Result 1: The inequality (a1) can be applied in the following forms:
1) Taylor series should be written for *n*-variables: This is complicated, whereas it requires linear parameterization of the Taylor series polynomial including Hessian matrix, etc. Moreover, the availability of the systems' states is another issue in this type of controller design.

2) The domain g is reduced to a subset U of $\mathbb{R}$. This is impossible, unless $p_N(x)$ and $g(x)$ is considered as a function of time, instead of a function of the system's states. In this manner, U is belongs to $\mathbb{R}$. This concept will be utilized for the next controller improvement.

Because of the above discussion, the obtained results in [10], cannot be correct. In the following, we show that the proof of Theorem 1 from [10], Y cannot separates points U by creating **the proposed single variable Taylor series** $p_N(x)$. Toward this end, assume that g be a function of two variables. Suppose that, $x_1=(y,z_1)^T$, and $x_2=(y,z_2)^T$ are two arbitrary points in $U \subset \mathbb{R}^2$. Thus, utilizing the specific Taylor series system described as $p_N(y)=\sum_{j=0}^{N}\frac{h^{(j)}(y_0)}{j!}(y-y_0)^j$ it can easily see that $p_N(x_1)=p_N(x_2)$. As a result, the second condition of the Stone-Weierstrass theorem is not satisfied.

4. Dynamic modeling

Consider an *n*-link manipulator driven by geared permanent magnet DC motors with voltages being inputs to amplifiers. As in [10], the dynamics are described by

$$\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}+\mathbf{g}(\mathbf{q})=\boldsymbol{\tau}_r-\boldsymbol{\tau}_f(\dot{\mathbf{q}}) \tag{a2}$$

$$\mathbf{J}\mathbf{r}^{-1}\ddot{\mathbf{q}}+\mathbf{B}\mathbf{r}^{-1}\dot{\mathbf{q}}+\mathbf{r}\boldsymbol{\tau}_r=\mathbf{K}_m\mathbf{I}_a \tag{a3}$$

$$\mathbf{R}\mathbf{I}_a+\mathbf{L}\dot{\mathbf{I}}_a+\mathbf{K}_b\mathbf{r}^{-1}\dot{\mathbf{q}}+\varphi=\mathbf{v}(t) \tag{a4}$$

where the parameters are defined exactly similarly to [10]. Note that vectors and matrices are represented in bold form for clarity. Now, substitution of (a2) and (a3) into (a4) yields

$$\begin{aligned}\mathbf{R}\mathbf{K}_m^{-1}\mathbf{J}\mathbf{r}^{-1}\ddot{\mathbf{q}}+(\mathbf{R}\mathbf{K}_m^{-1}\mathbf{B}+\mathbf{K}_b)\mathbf{r}^{-1}\dot{\mathbf{q}}+\mathbf{R}\mathbf{K}_m^{-1}\mathbf{r}(\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}+\mathbf{g}(\mathbf{q})\\+\boldsymbol{\tau}_f(\dot{\mathbf{q}}))+\mathbf{L}\dot{\mathbf{I}}_a+\varphi=\mathbf{v}(t)\end{aligned} \tag{a5}$$

For practical situations, the actuator input voltages are subjected to some constraints, called motor saturation limits. This occurs usually between the output of the controller and the PWM module. Following the same notation as in [10], for the development of the controller in this paper, we assume that the relation between the actual actuator's input $(\mathbf{v}(t) \in \Re^n)$ and the control signal produced by the controller $(\mathbf{u}(t) \in \Re^n)$ is given by

$$\mathbf{v}(t) = \mathbf{h}(\mathbf{u}(t)) \tag{a6}$$

$\mathbf{h}(\mathbf{u}(t)) \in \Re^n$ is a continuous nonlinear function representing the saturation nonlinearity or its approximation. The non-implemented control signal of the actuators can be expressed as

$$\Im(\mathbf{u}(t)) = \mathbf{u}(t) - \mathbf{h}(\mathbf{u}(t)) \tag{a7}$$

Now, substituting (a6) into (a5), and using (a7) we have

$$\begin{aligned} &\mathbf{RK}_m^{-1}\mathbf{Jr}^{-1}\ddot{\mathbf{q}} + (\mathbf{RK}_m^{-1}\mathbf{B} + \mathbf{K}_b)\mathbf{r}^{-1}\dot{\mathbf{q}} + \mathbf{RK}_m^{-1}\mathbf{r}(\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \boldsymbol{\tau}_f(\dot{\mathbf{q}})) \\ &+\mathbf{L}\dot{\mathbf{I}}_a + \varphi = \mathbf{u}(t) - \Im(\mathbf{u}(t)) \end{aligned} \tag{a8}$$

To develop our control scheme, assume that Eq. (a8) can be represented by a nonlinear differential equation, called "available model" as

$$\ddot{\mathbf{q}} + \mathbf{F} = \mathbf{u}(t) \tag{a9}$$

where

$$\begin{aligned} \mathbf{F} = {} & (\mathbf{RK}_m^{-1}\mathbf{Jr}^{-1} - \mathbf{I})\ddot{\mathbf{q}} + (\mathbf{RK}_m^{-1}\mathbf{B} + \mathbf{K}_b)\mathbf{r}^{-1}\dot{\mathbf{q}} + \mathbf{L}\dot{\mathbf{I}}_a + \varphi + \Im(\mathbf{u}(t)) \\ &+\mathbf{RK}_m^{-1}\mathbf{r}(\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \boldsymbol{\tau}_f(\dot{\mathbf{q}}))) \in \Re^n \end{aligned} \tag{a10}$$

is referred to as the lumped uncertainty, and $\mathbf{I} \in \Re^{n\times n}$ represents the identity matrix.

Remark 1: The control input given by (a6) indicates that the motor voltage is limited, that is

$$|v(t)| \le u_{\max} \tag{a11}$$

where $v(t)$ stands for the ith entry of vector $\mathbf{v}(t)$, and $u_{\max}$ is a positive constant representing the maximum permitted voltage of the ith motor. As a result, $\mathbf{q} \in \Re^n$, $\dot{\mathbf{q}} \in \Re^n$ and $\mathbf{I}_a \in \Re^n$ are bound. This is a result of BIBO stability.

Remark 2: For using some results of the stability analysis of [10], we present the mentioned Taylor series-based controller in a decentralized form. With this in mind, the controller design will be developed for the following model

$$\ddot{q}_i + f_i = u_i \tag{a12}$$

5. Function approximation using Taylor-series

Uncertainty estimators are not confined to fuzzy systems and neural networks. In the calculus courses, it is well known that, given a function $f(t)$ and a point an in the domain of *f*, suppose the function is *n*-times differentiable at *a*, then we can construct a polynomial

$$f_l(t)=\sum_{p=0}^{l}\frac{f^{(p)}(a)}{p!}(t-a)^p \tag{a13}$$

$f_l(t)$ is called the lth-degree Taylor polynomial approximation of f at a. It is interesting to investigate the capability of the last Equation, Eq. (a13), from a function approximation capability point of view. Herein, we will prove that Eq. (a13) has the universal approximation capability. In the following, we suppose that the input universe of discourse T is a convex set in $\mathbb{R}$.

Proposition1. (Universal Approximation Theorem)

Let $f(t)$ be a continuous real function on the convex set T in $\mathbb{R}$. Then, for each arbitrary $\varepsilon>0$, there exists a function in the form of

$$f_l(t)=\sum_{p=0}^{l}\frac{f^{(p)}(a)}{p!}(t-a)^p \tag{a14}$$

Such that

$$\underset{t\in T}{Sup}\left|\sum_{p=0}^{l}\frac{f^{(p)}(a)}{p!}(t-a)^p-f(t)\right|<\varepsilon \tag{a15}$$

Proof of proposition1: Let o to be a set of a continuous function on T in which T is a Convex set in the form of (a13). Now, suppose $f_{l,1}(t)$ and $f_{l,2}(t)$ are given by

$$\begin{aligned}f_{l,1}(t)&=\sum_{i=0}^{l}\frac{f_1^{(i)}(a)}{i!}(t-a)^i\\ f_{l,2}(t)&=\sum_{j=0}^{l}\frac{f_2^{(j)}(\bar{a})}{j!}(t-\bar{a})^j\end{aligned} \tag{a16}$$

we have

$$f_{l,1}(t)+f_{l,2}(t)=\sum_{i=0}^{l}\frac{f_1^{(i)}(a)}{i!}(t-a)^i+\sum_{j=0}^{l}\frac{f_2^{(j)}(\bar{a})}{j!}(t-\bar{a})^j \tag{a17}$$

$$f_{l,1}(t).f_{l,2}(t)=\left(\sum_{i=0}^{l}\frac{f_1^{(i)}(a)}{i!}(t-a)^i\right).\left(\sum_{j=0}^{l}\frac{f_2^{(j)}(\bar{a})}{j!}(t-\bar{a})^j\right) \tag{a18}$$

Hence, $f_{l,1}(t)+f_{l,2}(t)\in o$ and $f_{l,1}(t).f_{l,2}(t)\in o$. Furthermore, for any arbitrary, $\varsigma\in\mathbb{R}$ we can get

$$\varsigma.f_l(t)=\sum_{i=0}^{l}\varsigma\frac{f^{(i)}(a)}{i!}(t-a)^i \tag{a19}$$

Which that is also in the form of (a13). So, according to (a17) to (a19), we can conclude that o is an algebra. To show that o vanishes at no point of *T*, we simply observe that any function in the form of (a13) with $f(a) \neq 0$ and $f^{(p)}(a) = 0$ for $p = 1,...,l$ has the property of

$$\forall t \in T \quad , \; f_l(t) > 0 \qquad \text{(a20)}$$

Hence, o vanishes at no point of *T*. Now, we show that o separates various points on *T*. Choose the parameters of $f_l(t)$ in (a13) as $l = 1$ and $a = 0$. Since $t_1 \neq t_2$, then $f(0) + f^{(1)}(0)t_1 \neq f(0) + f^{(1)}(0)t_2$, which can be simplified to $t_1 \neq t_2$. Therefore, the second condition is also verified. Therefore, the result follows by Stone-Weierstrass theorem.

6. Indirect Adaptive Taylor series controller

In this section, actuator saturation compensation is considered to achieve satisfactory tracking control of robots as an extended form of [10]. For this purpose, the robust control law is proposed as

$$u_i = \ddot{q}_{mi} + \hat{f}_i + \mathbf{K}_i \mathbf{E}_i + u_{rfi} \qquad \text{(a21)}$$

where the parameters are defined exactly similarly to [10]. Substituting (a21) into (a12) and some simple manipulation lead to

$$\ddot{e}_i + \mathbf{K}_i \mathbf{E}_i = f_i - \hat{f}_i - u_{rfi} \qquad \text{(a22)}$$

$e_i = q_{mi} - q_i$ denotes the joint-space tracking error in a decentralized form. It must be emphasized that the development of the proposed control law is under the assumption that complete information of the actuator and robot dynamic is not available. Furthermore, we don't utilize the Taylor series expression in MIMO form, as mentioned in result 1. With this in mind, a Taylor series expansion will be used to represent the uncertainty term $\hat{f}_i$ as

$$\hat{f}_i(t) = \sum_{k=0}^{l} \frac{\hat{f}_i^{(k)}(a)}{k!}(t-a)^k \qquad \text{(a23)}$$

One can easily represent (a23) as

$$\hat{f}_i = \hat{\boldsymbol{\theta}}_{f_i}^T \boldsymbol{\xi}_{f_i} \qquad \text{(a24)}$$

where $\hat{\boldsymbol{\theta}}_{f_i} \in \Re^{l+1}$ is the vector of Taylor series parameters for $\hat{f}_i$ and $\boldsymbol{\xi}_{f_i}$ is the vector of regressor introduced as

$$\boldsymbol{\xi}_{f_i} = \begin{bmatrix} 1 & (t-a) & \ldots & (t-a)^l \end{bmatrix}^T \qquad \text{(a25)}$$

$$\hat{\boldsymbol{\theta}}_{f_i}^T = \begin{bmatrix} \frac{\hat{f}_i^{(0)}(a)}{0!} & \frac{\hat{f}_i^{(1)}(a)}{1!} & \ldots & \frac{\hat{f}_i^{(l)}(a)}{l!} \end{bmatrix} \qquad \text{(a26)}$$

Suppose that f_i can be modeled as

$$f_i = \boldsymbol{\theta}_{f_i}^T \boldsymbol{\xi}_{f_i} + \varepsilon_{f_i} \tag{a27}$$

ε_{f_i} is the bounded approximation error. The dynamics of tracking error can then be expressed by substituting (a24) and (a27) in (a22) to have

$$\ddot{e}_i + \mathbf{K}_i \mathbf{E}_i = \tilde{\boldsymbol{\theta}}_{f_i}^T \boldsymbol{\xi}_{f_i} + \varepsilon_{f_i} - u_{rfi} \tag{a28}$$

$\tilde{\boldsymbol{\theta}}_{f_i}$ is the parametric estimation error. Using (a28), the state-space representation in the tracking space can be formulated by

$$\dot{\mathbf{E}}_i = \boldsymbol{\Lambda}_i \mathbf{E}_i + \mathbf{b}_i \omega_i \tag{a29}$$

Where

$$\boldsymbol{\Lambda}_i = \begin{bmatrix} 0 & 1 \\ -k_{2i} & -k_{1i} \end{bmatrix}, \quad \mathbf{b}_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{E}_i = \begin{bmatrix} e_i \\ \dot{e}_i \end{bmatrix}, \quad \omega_i = \tilde{\boldsymbol{\theta}}_{f_i}^T \boldsymbol{\xi}_{f_i} + \varepsilon_{f_i} - u_{rfi} \tag{a30}$$

It is noted that the procedure of the stability analysis is the same as the one introduced in the proof of Theorem 3 in [10], considering this fact that $\boldsymbol{\xi}_{f_i}$ is defined by (a26). Therefore, the restatement of stability analysis in this section is not addressed.

7. Direct Adaptive Taylor series controller

It must be noted that the direct Adaptive Taylor series controller and the procedure of stability analysis are the same as the one introduced in section 3.3 in ref [10], considering this fact that $\boldsymbol{\xi}_{f_i}$ is defined by (a26). Therefore, the restatement of the controller design and the stability analysis is not addressed, in this section.

8. Conclusion

To sum up, the article entitled "On the Taylor Series Asymptotic Tracking Control of Robots" comprises a considerable error. Since the second condition of the Stone-Weierstrass theorem cannot be correctly established, thus the results obtained cannot possibly be correct.

9. References

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