Effect of External Pressures on Vibration of Thin-Walled Cylindrical Shell Supported Composed of Functionally Graded Materials

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ABSTRACT

This paper presents the study on influence external pressures on vibration of functionally graded materials thin-walled cylindrical shell supported. The functionally graded materials (FGMs) properties are graded in the thickness direction of the shell. FGMs are advanced composite materials, consisting of different types of materials, in which the properties shift continuously from one material on the one side to another material on the other side with a specific gradient. The FGM cylindrical shell supported equations with external pressure are established based on classical shell theory with beam functions as axial modal function. The governing equations of motion were employed, using energy functional and by applying the Ritz method. The boundary conditions represented by end conditions of the FGM structure which are sliding-sliding, clamped-free and clamped-simply supported being considered. This problem was solved with computer programming using MAPLE package. Comparison results are carried out to verify the validity with published papers. The influence of the external pressures, loop support and effect of the different boundary conditions on natural frequencies of FGM thin-walled cylindrical shell are studied.

1-Introduction

Shells are used as constructional components in engineering application. Shells usually expose more different dynamic behaviors because they can carry applied loads in structures [1]. The dynamic behavior of shell structures has been worked by many researchers. It was first introduce by Love [2]. Liu and Chu [3] used vibration of a thin shell with clamped-free boundary condition.

The cylindrical shell is a type of shell. Cylindrical shells have been used aerospace, civil and mechanical structures [4]. They are used as structures in aircrafts, rockets, missile bodies, etc. Some researchers have worked on vibration of cylindrical shells includes Shen [5] who studied third order shear theory. Liu et al. [6] and Chen et al. [7] studied on circular cylindrical shell. Amongst investigation cylindrical shells, the analysis dynamic is the subject of some of the researches [8, 9].

The study on functionally graded materials (FGMs) has become more popular. FGMs are materials, consisting kinds of different materials, in which the properties shift continuously from one material on one side to another material on other side with a specific gradient. These advance materials have great importance in engineering and other applications due to their mechanical and thermal properties as a thermal barrier in high temperature environments [10, 11].

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The functionally graded materials are the lack of discontinuous across adjoining layers but other materials will fail under high temperature. In functionally graded materials, material repartition is considered by the volume fraction and this repartition leads to continuous change in the combination of the structure as a result in gradient in the mechanical and thermal properties. Japanese scientists in 1987discovered a thermal barrier material and called Functionally Graded Material [12].

FGM shells structures are used as structural components in missiles engine, resistant coatings in space plans, atomic reactors, spacecraft, submarines, turbines and others [13]. Study on the vibration of cylindrical shells made-up of FGM is important in engineering. Vibration characteristics of functionally graded cylindrical shell was reported by Pradhan et al. [14] and natural frequency by constituent volume fractions with simply supported boundary condition was studied. Application of finite element method for vibration of FGM cylindrical shell was worked by Patel et al. [15]. Other researchers studied of vibration of FGM cylindrical shell with effects of radius to span ratio and analysis of wave propagation [16, 17]. Study on natural frequencies of supported FGM cylindrical shells with external pressure is important and research on influence external pressures for vibration functionally graded materials cylindrical shell supported could not be found in the literature. The aims of this study is to developed an analytic method based on the classical shell theory for influence external pressures on vibration of functionally graded materials cylindrical shell supported. The governing equations of motion are derived using energy functional and Ritz technique. The boundary conditions of the functionally graded materials shell considered are the combination of the sliding-sliding (SL-SL), clamped-free (C-F) and clamped-simply supported (C-SS) as defined in the study. The influence of the external pressures, loop support and effect of the different boundary conditions on natural frequencies characteristics of FGM thin-walled cylindrical shell are discussed. The influence of the internal pressure, ring support position and the effect of the considered boundary conditions on the natural frequencies characteristics are discussed.

2- Functionally Graded Materials

Functionally graded materials are made up of different of composition materials and the repartition of each phase of material varies with a specific gradient in the thickness direction, thus the properties of functionally graded material change along this direction.The type of FGM in this study is stainless steel-nickel and stainless steel-nickel consists of stainless steel layer in one side and nickel layer in other side with the different phases. Fig. 1 show the shape of FGM material with the constituent phases graded [18].

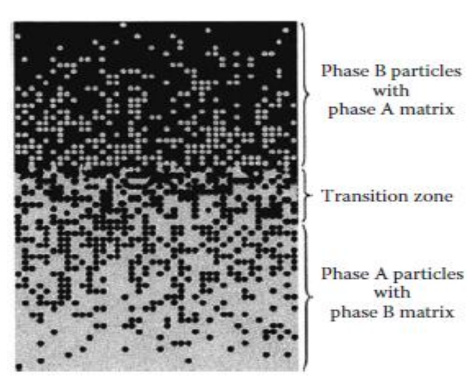

Fig. 1. A functionally grade material with the constituent phases graded

Generally the effective material properties, Q_{fgm} of a FGM depends on its properties and the volume fractions of the constituent materials and it is defined as

$$Q_{fgm}(T,z)=\sum_{j=1}^{k}\overline{Q}_j(T)V_{fj}(z) \tag{1}$$

where $\overline{Q}_j(T)$ is the material property and $V_{fj}(z)$ is the volume fraction for constituent material j. For FGM shell made of two different materials, the volume fractions, $V_{f1}(z)$ and $V_{f2}(z)$ are expressed as [19]

$$V_{f1}(z)=1-\left(\frac{z+h/2}{h}\right)^{N},\ V_{f2}(z)=\left(\frac{z+h/2}{h}\right) \tag{2}$$

$$V_{f1}(z)+V_{f2}(z)=1 \tag{3}$$

where the power-law exponent N is a real value, $0\le N\le\infty$ and z denotes the radial distance measured from mid-surface of the FGM shell, ($-h/2\le z\le h/2$).

In this functionally graded materials shell the material properties are composed of stainless steel and nickel, which the Young's modulus E, Poisson's ratio ν and the mass density ρ are defined as [20]

$$E_{fgm}(T,z)=(E_2(T)-E_1(T))\left(z+\frac{h}{2}/h\right)^{N}+E_1(T) \tag{4}$$

$$\nu_{fgm}(T,z)=(\nu_2(T)-\nu_1(T))\left(z+\frac{h}{2}/h\right)^{N}+\nu_1(T) \tag{5}$$

$$\rho_{fgm}(T,z)=(\rho_2(T)-\rho_1(T))\left(z+\frac{h}{2}/h\right)^{N}+\rho_1(T) \tag{6}$$

3- Classical Shell Theory

Consider a cylindrical shell supported made of FGM subjected to external pressures with the thickness h, radius of the shell R, length L, position of loop supported b, external pressure P, mass density ρ, modulus of elasticity E, and Poisson's ratio ν, as displayed in Fig. 2. The deformation is defined with reference to the coordinate system (x, θ, z) in which x and θ are axial and circumferential directions of the FGM shell and z is in the radial direction to mid-surface. The corresponding displacements on the mid-surface of FGM shell are defined by u, v and w. The constitutive relation stress-strain is given by two-dimensional Hooke's law as

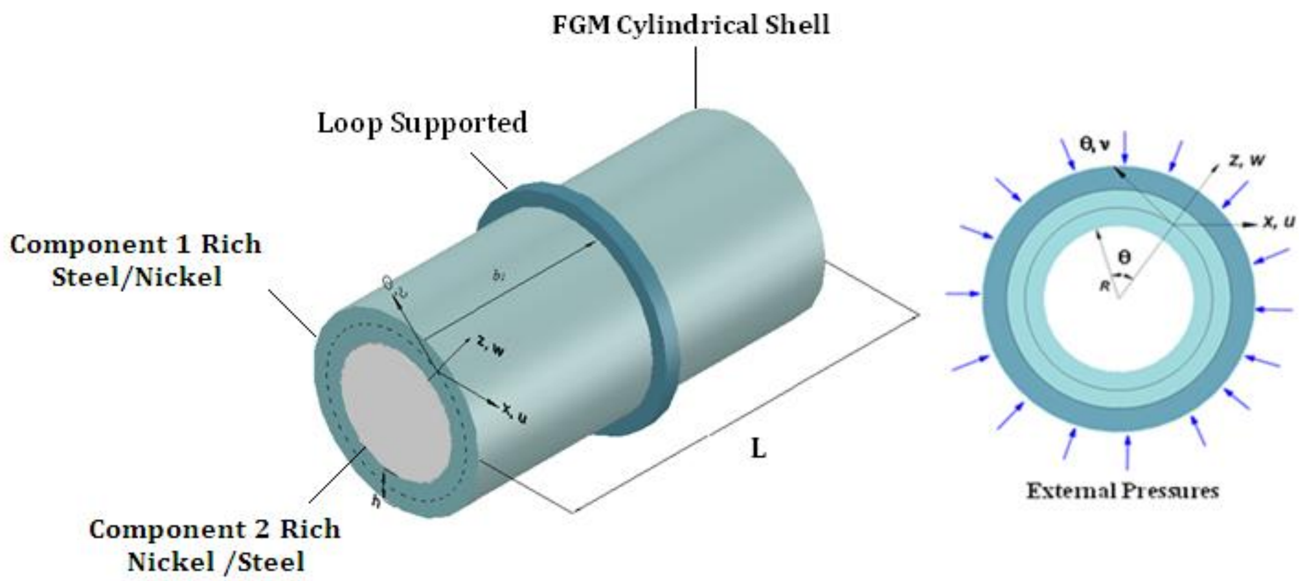

Fig. 2. Shape of FGM cylindrical shell supported with external pressures

$$\{\bar{\sigma}\}=[\bar{Q}]\{\bar{\varepsilon}\} \tag{7}$$

where $\{\bar{\sigma}\}$, $\{\bar{\varepsilon}\}$ are the corresponding stress and strain vectors respectively and $[\bar{Q}]$ is the reduced stiffness matrix expressed as

$$\{\bar{\sigma}\}^T=\{\bar{\sigma}_x \quad \bar{\sigma}_\theta \quad \bar{\sigma}_{x\theta}\},\ \{\bar{\varepsilon}\}^T=\{\bar{\varepsilon}_x \quad \bar{\varepsilon}_\theta \quad \bar{\varepsilon}_{x\theta}\} \tag{8}$$

$$[\bar{Q}]=\begin{pmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 \\ 0 & 0 & \bar{Q}_{66} \end{pmatrix} \tag{9}$$

where, σ_x is the stress in x-direction, σ_θ the stress in

the θ-direction, $\sigma_{x\theta}$ is the shear stress on the xθ-plane, ε_x is the strain in x-direction, ε_θ the strain in the θ -direction and $\varepsilon_{x\theta}$, is the shear strain on the xθ- plane. Then equation (7) can be expressed as

$$\begin{Bmatrix} \bar{\sigma}_x \\ \bar{\sigma}_\theta \\ \bar{\sigma}_{x\theta} \end{Bmatrix}=\begin{pmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 \\ 0 & 0 & \bar{Q}_{66} \end{pmatrix}\begin{Bmatrix} \bar{\varepsilon}_x \\ \bar{\varepsilon}_\theta \\ \bar{\varepsilon}_{x\theta} \end{Bmatrix} \tag{10}$$

The stiffness $\bar{Q}_{ij}$ are defined as

$$\overline{Q}_{11} = \frac{E}{1-\nu^2}, \overline{Q}_{12} = \frac{\nu E}{1-\nu^2}, \overline{Q}_{22} = \frac{E}{1-\nu^2},$$
$$\overline{Q}_{66} = \frac{E}{2(1+\nu)} \quad (11)$$

where modulus of elasticity is E, and Poisson's ratio is ν.

Based on classical shell theory, the strain components are defined as [21]

$$\bar{\varepsilon}_x = \bar{\varepsilon}_1 + z\bar{k}, \bar{\varepsilon}_\theta = \bar{\varepsilon}_2 + z\bar{k}_2, \bar{\varepsilon}_{x\theta} = \bar{\gamma} + 2z\bar{\tau} \quad (12)$$

where $\bar{\varepsilon}_1, \bar{\varepsilon}_2$ and $\bar{\gamma}$ are the surface strains and $\bar{k}_1, \bar{k}_2$ and $\bar{\tau}$ are the surface curvatures and expressed as

$$\bar{\varepsilon}_1 = \frac{\partial u}{\partial x}, \bar{\varepsilon}_2 = \frac{1}{R}\left(\frac{\partial v}{\partial \theta} + w\right), \bar{\varepsilon}_2 = \frac{1}{R}\left(\frac{\partial v}{\partial \theta} + w\right) \quad (13)$$

$$\bar{k}_1 = -\frac{\partial^2 w}{\partial x^2}, \bar{k}_2 = -\frac{1}{R^2}\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right),$$
$$\bar{\tau} = -\frac{1}{R}\left(\frac{\partial^2 w}{\partial x \partial \theta} - \frac{\partial v}{\partial x}\right) \quad (14)$$

The force and moment resultants are defined by

$$\{N_x, N_\theta, N_{x\theta}\} = \int_{-h/2}^{h/2} \{\bar{\sigma}_x, \bar{\sigma}_\theta, \bar{\sigma}_{x\theta}\} \quad (15)$$

$$\{M_x, M_\theta, M_{x\theta}\} = \int_{-h/2}^{h/2} \{\bar{\sigma}_x, \bar{\sigma}_\theta, \bar{\sigma}_{x\theta}\} z\,dz \quad (16)$$

where N_X, N_θ and $N_{X\theta}$ are force components in axial, circumferential and shear directions, respectively and M_X, M_θ and $M_{X\theta}$ are moment components in axial, circumferential and shear directions, respectively. Equations (12), (15) and (16) are combined as

$$\{N\} = [L]\{\bar{\varepsilon}\} \quad (17)$$

where $\{N\}$ and $\{\bar{\varepsilon}\}$ expressed as

$$\{N\}^T = \{N_x, N_\theta, N_{x\theta}, M_x, M_\theta, M_{x\theta}\} \quad (18)$$

$$\{\bar{\varepsilon}\}^T = \{\bar{\varepsilon}_1, \bar{\varepsilon}_2, \bar{\gamma}, \bar{k}_1, \bar{k}_2, 2\bar{\tau}\} \quad (19)$$

For a FGM cylindrical shell $[L]$ is defined as

$$[L] = \begin{pmatrix} X & Y \\ Y & Z \end{pmatrix} \quad (20)$$

in which X_{ij}, Y_{ij}, Z_{ij} are extensional, coupling and bending stiffness expressed as

$$X = \begin{pmatrix} X_{11} & X_{12} & 0 \\ X_{12} & X_{22} & 0 \\ 0 & 0 & X_{66} \end{pmatrix}, Y = \begin{pmatrix} Y_{11} & Y_{12} & 0 \\ Y_{12} & Y_{22} & 0 \\ 0 & 0 & Y_{66} \end{pmatrix},$$
$$Z = \begin{pmatrix} Z_{11} & Z_{12} & 0 \\ Z_{12} & Z_{22} & 0 \\ 0 & 0 & Z_{66} \end{pmatrix} \quad (21)$$

The matrix $[L]$ in terms of $[X]$, $[Y]$ and $[Z]$ can be written as

$$[L]=\begin{pmatrix} X_{11} & X_{12} & 0 & Y_{11} & Y_{12} & 0 \\ X_{12} & X_{22} & 0 & Y_{12} & Y_{22} & 0 \\ 0 & 0 & X_{66} & 0 & 0 & Y_{66} \\ Y_{11} & Y_{12} & 0 & Z_{11} & Z_{12} & 0 \\ Y_{12} & Y_{22} & 0 & Z_{12} & Z_{22} & 0 \\ 0 & 0 & Y_{66} & 0 & 0 & Z_{66} \end{pmatrix} \quad (22)$$

For FGM cylindrical shell supported X_{ij}, Y_{ij} and Z_{ij} are extensional, coupling and bending stiffness and defined as

$$X_{ij}=\int_{-h/2}^{h/2} Q_{ij}\,dz\,, Y_{ij}=\int_{-h/2}^{h/2} Q_{ij}\,z\,d, Z_{ij}=\int_{-h/2}^{h/2} Q_{ij}\,z^2\,dz \quad (23)$$

By substituting equations (18)-(22) into (17) for a FGM cylindrical shell supported, thus

$$\begin{Bmatrix} N_x \\ N_\theta \\ N_{x\theta} \\ M_x \\ M_\theta \\ M_{x\theta} \end{Bmatrix}=\begin{pmatrix} X_{11} & X_{12} & 0 & Y_{11} & Y_{12} & 0 \\ X_{12} & X_{22} & 0 & Y_{12} & Y_{22} & 0 \\ 0 & 0 & X_{66} & 0 & 0 & Y_{66} \\ Y_{11} & Y_{12} & 0 & Z_{11} & Z_{12} & 0 \\ Y_{12} & Y_{22} & 0 & Z_{12} & Z_{22} & 0 \\ 0 & 0 & Y_{66} & 0 & 0 & Z_{66} \end{pmatrix}\begin{Bmatrix} \bar{e}_1 \\ \bar{e}_2 \\ \bar{\gamma} \\ \bar{k}_1 \\ \bar{k}_2 \\ 2\bar{\tau} \end{Bmatrix} \quad (24)$$

4- FGM Shell of Energy Equations

The strain energy, the potential energy of external pressure and the kinetic energy for FGM cylindrical shell supported expressed as

Strain energy:

Based on classical theory the strain energy of the FGM cylindrical shell supported U is expressed as

$$U=\frac{1}{2}\int_0^L\int_0^{2\pi}\{\bar{\varepsilon}\}^T\,[L]\,\{\bar{\varepsilon}\}\,R d\theta\, d \quad (25)$$

Substitution of $\{\bar{\varepsilon}\}^T$, $[L]$ and $\{\bar{\varepsilon}\}$ into the strain energy, thus

$$\begin{aligned} U=\frac{1}{2}\int_0^L\int_0^{2\pi}\{&\bar{\varepsilon}_1^2 X_{11}+\bar{\varepsilon}_1\bar{\varepsilon}_2 X_{12}+\bar{k}_1\bar{\varepsilon}_1 Y_{11}+\bar{k}_2 Y_{12}\bar{\varepsilon}_1 \\ &+\bar{\varepsilon}_1\bar{\varepsilon}_2 X_{12}+\bar{\varepsilon}_2^2 X_{22}+\bar{k}_1\bar{\varepsilon}_2 Y_{12}+\bar{k}_2\bar{\varepsilon}_2 Y_{22}+ \\ &\bar{\gamma}^2 X_{66}+2\bar{\tau}\bar{\gamma}\,Y_{66}+\bar{k}_1\bar{\varepsilon}_1 Y_{11}+\bar{k}_1\bar{\varepsilon}_2 Y_{12}+ \\ &\bar{k}_1^2 Z_{11}+\bar{k}_1\bar{k}_2 Z_{12}+\bar{k}_2\bar{\varepsilon}_1 Y_{12}+\bar{k}_2 Y_{22}\bar{\varepsilon}_2+ \\ &\bar{k}_1\bar{k}_2 Z_{12}+\bar{k}_2^2 Z_{22}+2\bar{\tau}\bar{\gamma}\,Y_{66}+ \\ &4\bar{\tau}^2 Z_{66}\}\,R\,d\theta\,dx \end{aligned} \quad (26)$$

Kinetic energy

Based on classical theory the kinetic energy for FGM cylindrical shell supported is given by [22]

$$T=\frac{1}{2}\int_0^L\int_0^{2\pi}\rho_T\left[(\frac{\partial u}{\partial t})^2+(\frac{\partial v}{\partial t})^2+(\frac{\partial w}{\partial t})^2\right]R\,d\theta\, d \quad (27)$$

External pressure

The potential energy of the external pressure $P_{External}$ for FGM cylindrical shell supported with classical theory, is

$$E_{External}=\frac{P_{External}}{4}\int_0^L\int_0^{2\pi}\left(\frac{\partial w_0(x,\theta)}{\partial x}\right)^2 R^2\,d\theta dx \quad (28)$$

Therefore, the energy functional for vibration of FGM cylindrical shell supported with external pressure can be written as

$$F=U-T+E_{External} \quad (29)$$

5-FGM Shell Displacement

The displacement field for vibration of FGM cylindrical shell supported with external pressure can be expressed as [23]

$$u = \bar{E}_1 \frac{\partial \Omega(x)}{\partial x} \cos(n\theta)\cos(\omega t)$$

$$v = \bar{E}_2 \Omega(x) \sin(n\theta)\cos(\omega t)$$

$$w = \bar{E}_3 \Omega(x) \prod_{i=1}^{H} (x - b_i)^{\mu_i} \cos(n\theta) \cos(\omega t) \tag{30}$$

where $\bar{E}_1$, $\bar{E}_2$ and $\bar{E}_3$ are constants denoting the vibrational amplitude. $\Omega(x)$, is the axial function that satisfies boundary conditions, b_i is loop position, H is the number of supported, μ_i is a parameter having a value of 1 when there is one supported, n is the circumferential waves number and ω is the natural frequency. The axial modal function $\Omega(x)$ is selected as the beam function is given by [24]

$$\Omega(x) = \Psi_1 \cosh(\frac{\Phi_m x}{L}) + \Psi_2 \cos(\frac{\Phi_m x}{L}) - \mu_m (\Psi_3 \sinh(\frac{\Phi_m x}{L}) + \Psi_4 \sin(\frac{\Phi_m x}{L})) \tag{31}$$

The boundary conditions for sliding-sliding (SL-SL), clamped-free (C-F) and clamped-simply supported (C-SS) that satisfy, x=0 and x=L and the values of Ψ_i $(i = 1,...,4)$, Φ_m and μ_m are given in Table 1.

6- Solution Method

Ritz method is commonly used as an approximation method for a solution of problems in mechanics. This method is based on variational principles. The energy method developed by Ritz. To determine the natural frequency of vibration for FGM cylindrical shell supported with external pressures, the Ritz technique is used. The energy functional F defined by the Lagrangian function as

$$F = U_{max} - T_{max} + E_{External} \tag{32}$$

Table 1 Values of Ψ_i, Φ_m and μ_m for boundary conditions

Boundary Conditions	Ψ_i $(i = 1,...,4)$	Φ_m	μ_m
Sliding-Sliding (SL-SL)	$\Psi_1 = 0$, $\Psi_2 = 0$ $\Psi_3 = 0$, $\Psi_4 = -1$	$m\pi$	1
Clamped-Free (C-F)	$\Psi_1 = 1$, $\Psi_2 = -1$ $\Psi_3 = 1$, $\Psi_4 = -1$	$(2m-1)\pi/2$	$\frac{\sinh\Phi_m - \sin\Phi_m}{\cosh\Phi_m + \cos\Phi_m}$
Clamped-Simply supported (C-SS)	$\Psi_1 = 1$, $\Psi_2 = -1$ $\Psi_3 = 1$, $\Psi_4 = -1$	$(4m+1)\pi/4$	$\frac{\cosh\Phi_m - \cos\Phi_m}{\sinh\Phi_m - \sin\Phi_m}$

Substituting Eq. (30) into Eqs. (26), (27) and (28) and applying Ritz technique with minimizing the energy functional *F* as

$$\left.\begin{aligned}\frac{\partial(U_{\max}-T_{\max}+E_{External})}{\partial\bar{E}_1}=0\\ \frac{\partial(U_{\max}-T_{\max}+E_{External})}{\partial\bar{E}_2}=0\\ \frac{\partial(U_{\max}-T_{\max}+E_{External})}{\partial\bar{E}_3}=0\end{aligned}\right\} \quad (33)$$

There are three equations of motion in Eq.(33) characterizing the vibration characteristic of FGM cylindrical shell supported with external pressures. Therefore, the governing eigenvalue equation can be written in a matrix form as

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{12} & C_{22} & C_{23} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}\begin{Bmatrix}\bar{E}_1\\ \bar{E}_2\\ \bar{E}_3\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\end{Bmatrix} \quad (34)$$

The solution is obtained by setting the determinant of matrix *C* equals to zero:

$$\left|C_{ij}\right|=0 \qquad (i,j=1,2,3) \quad (35)$$

The solution for equation (35) is obtained a characteristic of the FGM cylindrical shell supported with external pressures is expressed in the power of ω as

$$\beta_\circ\omega^6+\beta_1\omega^4+\beta_2\omega^2+\beta_3=0 \quad (36)$$

The solution of equation (36) consists of six roots and the three positive roots are the natural frequencies. The smallest positive root is applied in the present work. The functional graded material composed of stainless steel and nickel and the constituent materials considered with properties reported in Table 2.

7- Comparison of Research

In order to validate the accuracy of the present analysis, the results for FGM cylindrical shell without supported and external pressures are compared with the results available in open literature.

Table 2 Mechanical properties of constituent materials for FGM cylindrical shell [14]

Coefficients of temperature	Stainless Steel			Nickel		
	$E(Nm^{-2})$	v	$\rho(kgm^{-3})$	$E(Nm^{-2})$	v	$\rho(kgm^{-3})$
Q^0	201.04×10^9	0.3262	8166	223.95×10^9	0.3100	8900
Q^{-1}	0	0	0	0	0	0
Q^1	3.079×10^{-4}	-2.002×10^{-4}	0	-2.794×10^{-4}	0	0
Q^2	-6.534×10^{-7}	3.797×10^{-7}	0	-3.998×10^{-9}	0	0
Q^3	0	0	0	0	0	0
Q	2.07788×10^{11}	0.317756	8166	2.05098×10^{11}	0.3100	8900

Table 3 Comparisons of the frequency parameter, $\Gamma = \omega R\sqrt{((1-\nu^2)\rho / E}$ for a cylindrical shell without external pressures and supported

Boundary conditions	*n*	*m*	*L/R*	*h/R*	Chung [25]	Present
C-F	2	1	1.14	0.05	0.3076	0.3132
C-F	2	2	2.88	0.05	0.3081	0.3178
C-F	2	3	5.07	0.05	0.3079	0.3143
SL-SL	4	1	10	0.002	0.0150	0.0269

Table 4 Comparison of the natural frequency for FGM cylindrical shell without external pressure and supported (L/R = 20, R = 1, N = 1)

h/R	*m*	*n*	Natural frequency (Hz)	
			Loy *et al.* [26]	Present
0.002	1	1	4.156	4.063
	1	2	4.480	4.021
	1	3	7.038	7.105
	1	4	11.24	11.56
	1	5	13.21	13.69
	1	6	16.45	16.22
	1	7	22.63	22.30
	1	8	29.77	30.11
	1	9	37.86	37.56
	1	10	46.90	46.39

Table 3 shows the comparison of frequency parameter $\Gamma = \omega R\sqrt{((1-\nu^2)\rho / E}$ of cylindrical shells without supported and external pressures with different *h/R* and *L/R* ratios. The comparisons presented in Tables 3, show good agreeable results with published works. Table 4 shows the variation of the natural frequency with the circumferential waves numbers for FGM cylindrical shell without external pressure and supported with the h/R ratios. The comparisons presented in Table 4, shows good agreeable results with published works.

8- Results and discussion

Tables 5-7 show natural frequency of response FGM cylindrical shell with external pressures and without supported for different circumferential wave numbers (n) for the different boundary conditions is analysed. The analyses are conducted by assuming external pressures equal to 400 and 600 kPa. For all the boundary conditions when the external pressure is zero, the natural frequency initially decreases and then increases. When FGM cylindrical shell is subjected to external pressures without supported, for all boundary conditions the natural frequencies of response increase as the circumferential wave number n is increased. The results show that external pressures have effect on the natural frequency of a FGM cylindrical shell and cause the natural frequency to increase. When the value of the external pressures is large, the natural frequency is higher. The results obtained also show the natural frequency of a FGM cylindrical shell with and without external pressures are different for different boundary conditions. Figure 3-5 show the variation of natural frequency of FGM cylindrical shell supported under external pressures with the circumferential wave numbers n at loop position of b = 0.3 L for the different boundary conditions. In these figures the boundary conditions represented by end conditions of the FGM cylindrical shell supported under external pressures which are sliding-sliding, clamped-free and clamped-simply supported being considered. Simulation results were found to yield similar trends for all positions. For illustration b = 0.3L is presented here. In FGM cylindrical shell with supported under external pressures, the natural frequencies for the all different boundary conditions increase as the circumferential wave number n is increased. The results show that supported has effect on the natural frequency of a FGM cylindrical shell with external pressures and causes the natural frequency to increase. The results obtained also show that the natural frequency characteristics of a FGM cylindrical shell with supported under external pressures are different for different boundary conditions.

Table 5 Natural frequency of FGM cylindrical shell without supported with and without external pressures for SL-SL boundary conditions. (h/R = 0.002, L/R = 20, R = 1)

Circumferential Wave Number (n)	Without External Pressure (P=0)	With External Pressure (P=400 KPa)	With External Pressure (P=600 KPa)
1	13.209	21.075	21.075
2	4.476	38.311	46.658
3	4.156	65.528	80.179
4	7.044	91.816	112.335
5	11.254	117.583	143.786
6	16.475	143.135	174.914
7	22.664	168.641	205.917
8	29.811	194.203	236.911
9	37.912	219.892	267.972
10	46.967	245.761	299.155

Table 6 Natural frequency of FGM cylindrical shell without supported with and without external pressures for C-F boundary conditions. (h/R = 0.002, L/R = 20, R = 1)

Circumferential Wave Number (n)	Without External Pressure (P=0)	With External Pressure (P=400 KPa)	With External Pressure (P=600 KPa)
1	20.198	6.118	6.118
2	6.849	37.807	46.273
3	4.836	65.48	80.151
4	7.187	91.81	112.334
5	11.292	117.582	143.788
6	16.489	143.134	174.915
7	22.67	168.639	205.917
8	29.814	194.201	236.91
9	37.914	219.889	267.97
10	46.969	245.758	299.153

Table 7 Natural frequency of FGM cylindrical shell without supported with and without external pressures for C-SS boundary conditions. (h/R = 0.002, L/R = 20, R = 1)

Circumferential Wave Number (n)	Without External Pressure (P=0)	With External Pressure (P=400 KPa)	With External Pressure (P=600 KPa)
1	6.118	20.198	20.198
2	2.345	38.321	46.682
3	3.737	65.539	80.195
4	6.96	91.821	112.342
5	11.226	117.586	143.79
6	16.461	143.136	174.916
7	22.654	168.64	205.918
8	29.801	194.202	236.911
9	37.903	219.89	267.971
10	46.959	245.759	299.154

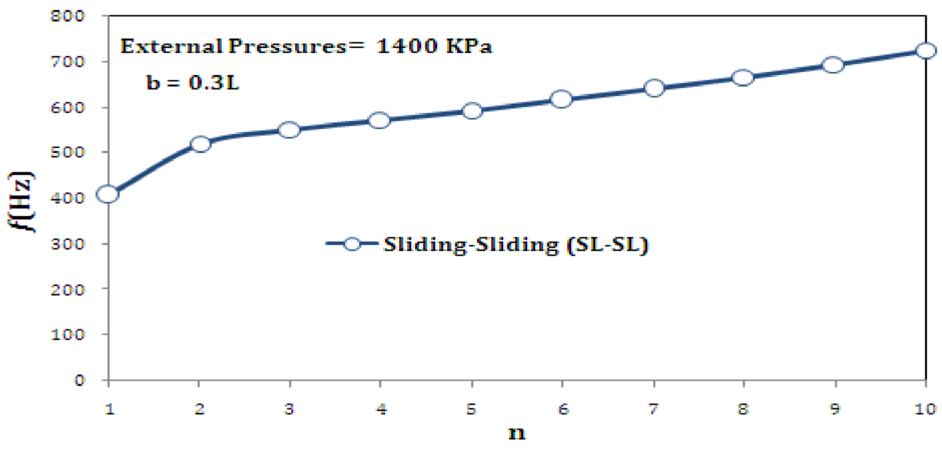

Fig. 3. Natural frequency of FGM cylindrical shell with supported and external pressures for SL-SL boundary conditions (P = 1400 KPa, h/R = 0.002, L/R = 20, b= 0.3L, R = 1)

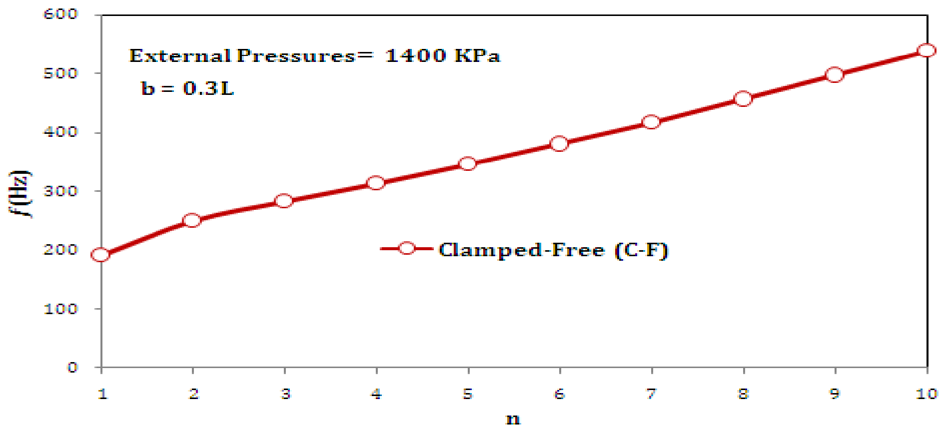

Fig. 4. Natural frequency of FGM cylindrical shell with supported and external pressures for C-F boundary conditions
(P = 1400 KPa, h/R = 0.002, L/R = 20, b= 0.3L, R = 1)

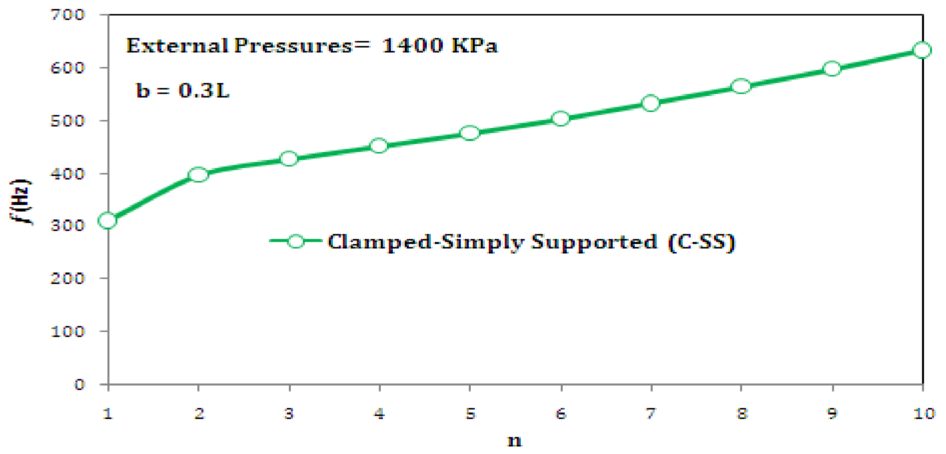

Fig. 5. Natural frequency of FGM cylindrical shell with supported and external pressures for C-SS boundary conditions (P = 1400 KPa, h/R = 0.002, L/R = 20, b= 0.3L, R = 1)

9- Conclusions

In this study, the natural frequency response and influence external pressures on vibration of functionally graded materials thin-walled cylindrical shell supported for different boundary conditions was investigated. The functionally graded materials (FGMs) properties are graded in the thickness direction of the shell. The classical shell theory is employed and the governing equations of motion were derived, using energy functional applied to the Ritz method. The boundary conditions represented by the end conditions are sliding-sliding (SL-SL), clamped-free (C-F) and clamped-simply support (C-SS). The influence of the external pressures, loop support and effect of the different boundary conditions on natural frequencies of FGM thin-walled cylindrical shell are discussed. This study shows that supported and external pressures have effect on the natural frequency of FGM cylindrical shell and cause the natural frequency to increase. When the value of the external pressures is large, the natural frequency is higher. Another point deduced here is that the natural frequency characteristics of FGM cylindrical shell with and without external pressures and supported are different for the three different boundary conditions.

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