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The impact of Diversification on Risk Reduction: Using a Mix of Merton Model and Random Matrix Approach to Take into Account Non-stationary

Zahra Eskandari[a], Mirfeiz Fallah Shams[a], Gholamreza Zomorodian[a]*

a.Department of Management, Central Tehran Branch, Islamic Azad University, Tehran, Iran.

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Abstract
One of the most important issues in financial markets is the effect of portfolio diversification to reduce the risk. Portfolio diversification has been discussed in many researches and has been proven in different portfolios, including stock portfolios, currency portfolios, etc. In this paper, we are going to investigate the impact of diversification on a credit portfolio risk related to companies listed on Tehran Stock Exchange. To calculate the risk of the companies mentioned, we use structural models of credit risk. In fact, the most important factor for assessing the risks of financial markets is to estimate the loss distribution. On the other hand, the estimation of loss distribution is highly depended on the characteristics of the distribution parameters. One of the characteristics that can affect the loss distribution is non-stationary time series of asset returns. In this research, the data of the adjusted prices of the companies in Tehran Stock Exchange is used during 2011- 2019. The loss distribution of credit portfolio is obtained through Merton's model with regard to non-stationary time series of asset returns and the changes of the asset returns' covariance matrix during the period of 2011-2019. The risk used in this paper is the value at risk. According to the results of the model, at lower confidence levels such as 99% and 99.5%, there is not enough evidence for the impact of diversification in reducing the risk, but at the confidence level of 99.9% and for the type error of 5%, it can be said that diversification has a significant effect on reducing risk.

Keywords: Loss Distribution, Random Matrix, Non-stationary, Value at Risk.

1. Introduction

In recent decades, risk management has become a big challenge for the banks and financial institutions. Due to the occurrence of many crises and the inability of the banks to deal with these crises, the need to create appropriate approach for risk management has become inevitable. Failure to pay attention to this issue can lead to irreparable consequences and even bankrupt of the banks or financial institutions. The most important risk that banks face is credit risk. Credit risk is the probability of the loss that may occur by the failure of any party of a contract to repay a loan or to fulfill contractual obligations.

Banks should have a correct assessment of credit risk. Credit risk assessment at an individual level evaluates the ability of customers to repay loans, but at the portfolio level, this means evaluating possible losses that a bank may incur in the future. The correct assessment of credit risk allows banks to optimize their credit portfolios in time.

At the credit portfolio level, there are several methods for credit risk management. However, the main problem in all methods are the correct estimation of the loss distribution of large credit portfolios. The loss distribution of credit risk has a characteristic shape due to the fundamental properties of the credit contracts. One of the simplest credit contracts is zero coupon bonds. In this contract, the investor buys the bonds from the obligor at the price discounted and a given maturity. The obligor must pay back the nominal value of the bonds to the investor.

In this type of contracts, the nominal value is higher than the discounted price. Therefore, the difference is considered as the investor's profit margin. This process will be true if that the obligor is able to completely pay back the nominal value of the bonds and does not default. In practice, the investor faces the risk of default depending on the obligor's credit rating. In addition, the maximum profit that the investor can achieve is the difference between the nominal value and the discounted value of the bonds, but the maximum loss is losing the total amount paid to the obligor.

[*] Corresponding Author. Email address: mir.fallahshams@iauctb.ac.ir

Typically, the investor (bank) lends money to many institutions and has a portfolio consisting of a large number of credit contracts. Many borrowers will be able to repay their installments, and only a small percentage will default. The loss caused by this low percentage is normal and is compensated by taking risk premium. The real risk lies in the heavy tail of the loss distribution. The rare events that cause large losses, as seen in the 2008 crisis, can affect all investors (banks). The accurate assessment of loss distribution is necessary to determine the risk compensation or to estimate the capital requirements of banks at the time of default (the capital charge to cover all types of risks). Financial institutions often claim that diversification reduces the risk of a portfolio. In fact, reducing risk by diversification is true for portfolios that include stocks, but the accuracy of this information for the portfolio of credit contracts is doubtful. The main reason for such an event is the asymmetry of the loss distribution for the credit portfolio, while in the stock portfolio; the loss distribution can be symmetrical.

In the credit portfolio, if default does not occur, the bank's maximum profit is the interest and risk compensation. However, if the customer defaults, the biggest possible loss of the bank is the complete loss of the money lent. Therefore, it is very important to consider the fluctuation of the correlations to evaluate the tail effects of the loss distribution. Even if there is weak correlation in the credit portfolio, diversification may not reduce the risk. This has been shown in several studies such as (Glasserman & Ruiz-Mata, 2006) and (Schönbucher, 2001) for first passage models and (Chetalova et al., 2015), (Münnix et al., 2012) and (Schmitt, 2014) for Merton model.

The main purpose of this paper is to investigate the impact of diversification in reducing credit portfolio risk. Therefore, we try to evaluate the risk of a credit portfolio consisting of the companies listed. For this purpose, credit risk structural models are used and Merton's model is expanded considering the fluctuations of correlations between asset values.

Because nonstationary prevails in most financial markets, no research estimated the credit loss distribution function in the bank's credit portfolio. In the model presented in this research, the credit risk loss distribution function is used to provide a method for the credit risk assessment of a portfolio consisting of listed companies is presented with the structural branches of credit risk considered as a new method in risk measurement in Iran.

However, there are some weaknesses in Merton model. For example, it assumes the fixed covariance matrix for asset variables during the time horizon. This assumption cannot be valid, especially during crises. To overcome this issue and to have a better credit risk estimation, in this paper, we use random matrix theory to take fluctuations of the correlations matrix into account.

First, we review a correlation averaged multivariate distribution (Chetalova, 2015), (Schmitt et al. 2013) which can describe the multivariate returns and consider the fluctuation correlation matrix. In order to complete the analysis, a covariance matrix with average correlation structure is introduced, which limits the the space to two parameters. These parameters are “an average correlation between asset values” and “a measure for the strength of the fluctuations”. Then, we use this distribution to describe the asset value of the obligor at maturity to model credit risk.

2. Review of Literature

Münnix et al., (2012), conducted an article discussing the financial markets state. They concluded that a correct comprehension of complex systems had become a fundamental issue, because they exist in almost all domains. They considered the financial markets as a complex system and analyzed the financial market data, especially the daily data of Standard and Poor's index over 19 years. In addition, they concluded that considering the correlation structure characteristics are crucial. Non-stationary time series is one of the features studied in this research and is usually one of the inherent assumptions of the models.

Münnix et al., (2014), in the research titled “A random matrix approach to credit risk", integrated the statistical features of credit risk structural methods with an ensemble of random matrices and estimated the credit risk of a portfolio consisting of the companies listed. They explained that if the correlation between asset returns is not zero. Even if the average correlation is zero, the presence of weak correlation would severely limit the impact of diversification.

Schmitt et al., (2013), conducted a study entitled “Non-stationarity in financial time series: Generic features and tail behavior”. This research shows that financial markets are an example of non-stationary systems, and sample parameters such as variance and covariance are highly depended on the time window where the parameters are estimated. This factor mentions the severe limitations of standard approaches in statistical techniques. They also discussed that the time series of asset value is non-stationary and covariance matrix Σ changes during time. This fact can affect the asset distribution in the structural models of credit risk. Therefore, it seems significantly effective to choose a distribution for the asset value consistent with the experimental data and represent the data features.

Gurny et al., (2013), discussed the framework of Merton's model. They first examined the assumptions of Merton’s model and then introduced a new method using the alternative process to overcome the weaknesses of that model. In the new method, a non-Gaussian process was used. They compared the new model and the classic KMV and Merton models and concluded that, in general, Merton model underestimated the probability of default compared to the newly introduced model.

Some financial studies have investigated the existence of correlation in portfolios and the role of diversification in reducing risk. For example, for the first passage models, Schönbucher (2001) and Glasserman & Ruiz-Mata (2006) and for Merton model, Münnix et al. (2012) and Chetalova et al. (2015) and Schmitt, (2014) showed that if there is a weak positive correlation, diversification will not be effective in reducing risk (Mühlbacher & Guhr, 2018).

In structural credit risk models, default and loss given default are both derived from the asset value at maturity. Therefore, the distribution describing the asset values has to be chosen carefully. For this purpose, Schmitt & Schäfer, (2015) introduced a distribution for asset values using the random matrix approach to take into account the non-stationary asset correlations. They considered the average correlation parameter homogeneously and as a result reduced it to two parameters including “average correlation coefficient " and " strength of the fluctuations. They concluded that, under the mentioned approach asset, value distribution can describe well the experimental data. Therefore, in this way, they obtained the average portfolio loss distribution and calculated the value at risk (VaR) and the expected tail loss (ETL) using Monte Carlo simulation approach (Schmitt & Chetalova, 2015).

Sicking et al., (2018), expanded the above topic and considered the problem of Concurrent credit portfolio losses where two non-overlapping credit portfolios are taken into account. They also discussed copulas of homogeneous portfolios. Sandoval & Franca (2012), Sicking et al. (2018) and Mühlbacher & Guhr, (2018) separately analyzed the correlation structure of the stock market and showed that financial market correlation can change during times of crisis partly.

Omar and Prasanna, (2021) studied some weaknesses of Merton model and extended the application of Merton model in six emerging Asian markets to estimate corporate default risk.

Shi et al., (2022), used Machine learning-driven to model credit risk. They systematically reviewed a series of major research contributions (76 papers) over the past eight years using statistical, machine learning and deep learning techniques to address the problems of credit risk. Specifically, they proposed a novel classification methodology for ML-driven credit risk algorithms and their performance ranking using public datasets.

in Iran, one of the leading works in the field of using the KMV model in credit risk modeling is Falah Shams' Ph.D. dissertation and the results were published as an article titled "Credit risk measurement models in banks and credit institutions" in 2014. In this research, he explained the structural models of credit risk and applied these methods to Iran's banking system (Falah Shams, 2014).

Khansari & Shams, (2010), conducted a study entitled "Assessment of KMV structural model application in predicting the default of the companies listed on Tehran Stock Exchange" to predict the bankruptcy of the clients in Iranian bank using the structural model features and to assess the accuracy of the relevant model. The data of their research included a sample of 40 publicly traded companies receiving loans from Iranian banks during 2007-2008. The findings represent that KMV model is capable of predicting default and can distinguish between good and bad customers.

Falahpour & Tadi, (2016), have investigated the relationship between capital structure components and the probability of default of companies listed on the Tehran Stock Exchange in 2013. They selected a sample of 40 companies and used the market data as well as the capital structure of these companies for the purpose of analysis.

In another research presented by Shams Qarneh & Janati, (2011), titled as "Presenting a dynamic model to predict the default rate of companies listed on the Iranian Stock Exchange (case study: metal products manufacturing industry)", the credit risk of selected companies using Merton's dynamic model was evaluated. In this research, 4 companies from the basic metal manufacturing industry were selected during 2001-2011 and their status was examined in terms of the probability of bankruptcy and default.

Falah Shams, (2014), measured the default risk for a sample of 60 companies listed on Tehran Stock Exchange during 2010- 2013 using Black-Scholes-Merton model and analyzed the relationship between corporate governance and default risk in the mentioned companies. They found that among the factors of corporate governance, only the factors related to public and transparent disclosure, at the 95% confidence level; have a significant relationship with the default risk of companies.

In their research, KMV model was firstly used to estimate the default probability of companies and then the panel data method was used to analyze the relationship between capital structure of companies including the variables of company size, asset book value ratio, and leverage, volatility of asset returns, stock return and sensitivity coefficient and their probability of default. The results indicate that there is a significant relationship between the capital structure of companies and their default probability (Azaripanah & Falah Shams, 2013).

Falah Shams et al., (2017), presented an article titled "Measuring default risk using Black-Scholz-Merton model and testing its relationship with corporate governance factors" and predicted default risk (probability of default) in selected Iranian listed companies. By removing the simple assumptions of Merton model, they calculated the annual default probability for the selected companies during 2010-2011 for both models. Finally, they concluded that there is a significant difference between two models in assessing the probability of default.

Considering that the assessment of the credit loss distribution function in the bank's credit portfolio has been discussed, it was not found and therefore it is considered that this paper is a new method of measuring risk in Iran and the theory of knowledge raises a new issue in credit risk management in Iran.

Most of the researches carried out regarding Merton models or structural models including KMV to investigate the default of companies or the relationship between the defaults and other variables. To the best of our knowledge, no research has been done in the area of using random matrix model to consider non-stationary of the time series and estimating risk measures in Iran.

3. Material and methods

The purpose of this research is to investigate the impact of diversification on the risk of a credit portfolio consisting publicly traded companies. In this regard, the value at risk (VaR) is considered as an index of credit portfolio risk measurement. We want to evaluate how much the risk

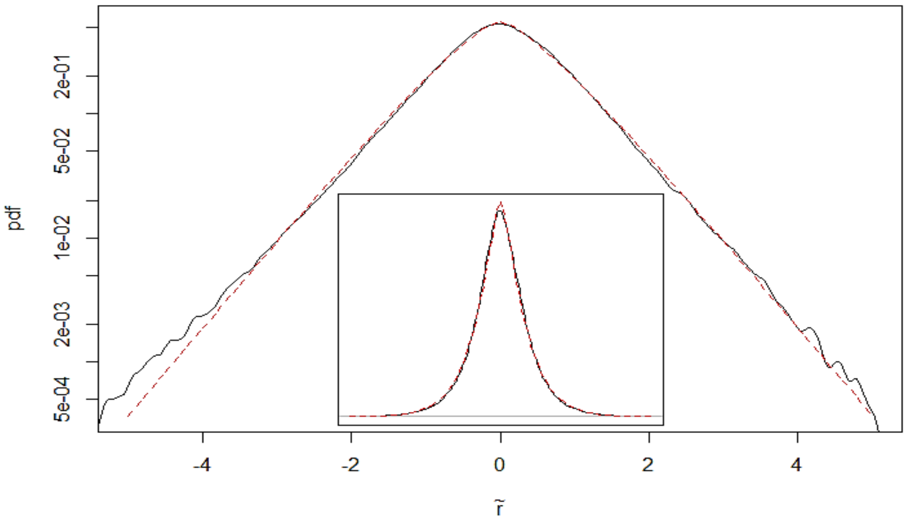

Figure 1. Theoretical and the Empirical Distributions for the Normalized Monthly Returns

Table1. Cramer-Von Mises Test of Goodness-of-Fit

Parameter	Cramer-Von Mises Test statistics	p-value
3	0.070248	0.75

decrease as the number of borrower's increase. However, to calculate the value at risk, the loss distribution of credit risk must be estimated. To obtain the loss distribution, we use Merton model, considering the non-stationary of asset returns and the covariance matrix changes. Merton model assumes that the total value of a company, i.e. $V_k(t)$ at time t can be described by a geometric Brownian motion:

$$dV_k(t) = \mu_k V_k(t)dt + \sigma_k V_k(t)dW(t) \qquad (1)$$

Where, $dW(t)$ is a Wiener process and $V_k(0) > 0$. μ_k is the drift and σ_k is the volatility of the asset value for company k. If at the maturity of the bonds, T, the value of the company's assets is less than the nominal value $V_k(t) < F_k$, the borrower will not be able to fulfill their obligation and will not be able to pay back their debt. Therefore, default will occur. In this case, the investor has the right to take over all assets of the company and liquidate them. If the face value is smaller than the asset value, the company can pay back its obligations and no default or loss occurs.
As mentioned above, in case of the default of any companies, the amount of the bank's loss is equal to $[F - V(T)]$. Let us assume that we have a portfolio of credit contracts consisting of K companies. To extend Merton model, the normalized loss for the k-th contract is as follows:

$$L_k = \frac{F_k - V_k(T)}{F_k}\Theta\big(F_k - V_k(T)\big) \qquad (2)$$

Where, $\Theta(x)$ denotes the Heaviside step function:

$$\Theta(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases} \qquad (3)$$

Heaviside step function $\Theta(F_k - V_k(T))$ is unity only if the face value F_k is larger than the remaining asset value of the obligor $V_k(T)$ otherwise zero. This construction guarantees

that the loss L_k is always equal or greater than zero and equal or lesser than one.
The sum of the individual losses L_k, which are weighted by their fraction F_k of the portfolio, gives the total loss of the portfolio:

$$L = \sum_{k=1}^{K} f_k L_k \qquad f_k = \frac{F_k}{\sum_{k=1}^{K} F_k} \qquad (4)$$

To calculate the loss distribution we need to integrate over the distribution of asset values $g(V|\Sigma)$ at maturity time T with $V = (V_1(T), \ldots, V_K(T))$ and filter for a given loss L using the conditions of equation (4):

$$P(L) = \int_{[0,\infty)^K} d[V] g(V|\Sigma)\delta\left(L - \sum_{k=1}^{K} f_k L_k\right). \qquad (5)$$

However, the time series of the asset value is non-stationary and covariance matrix Σ changes during time. This fact can affect the asset distribution. To take into account non-stationarity and covariance matrices changes, we use random matrix approach and replace the covariance matrix with a random matrix:

$$\Sigma_t \rightarrow \sigma W W' \sigma' \qquad (6)$$

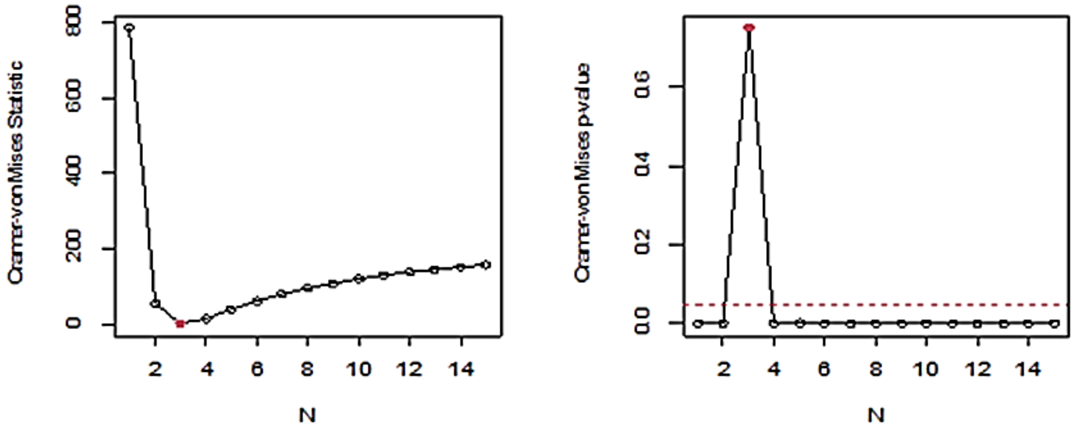

Figure2. Test Results for Different Values of N

Table2. Parameter values based on maturity time

Maturity Time	N	average correlation levels (c)	average volatility σ	average drift μ
Monthly (T=20)	3	0.093	0.076	0.035
Yearly (T=1year)	3	0.277	0.408	0.478

Table3. Regression model for value at risk measure at 99% confidence level

Parameter	Estimate	Std. Error	t-value	p-value
a	0.063783	0.00734	8.69	0.0000
b	-0.00076	0.000408	-1.871	0.0983

Table4. Regression model for value at risk measure at 99.5% confidence level

Parameter	Estimate	Std. Error	t-value	p-value
a	0.075324	0.008582	8.777	0.0000
b	-0.00095	0.000414	-2.292	0.0511

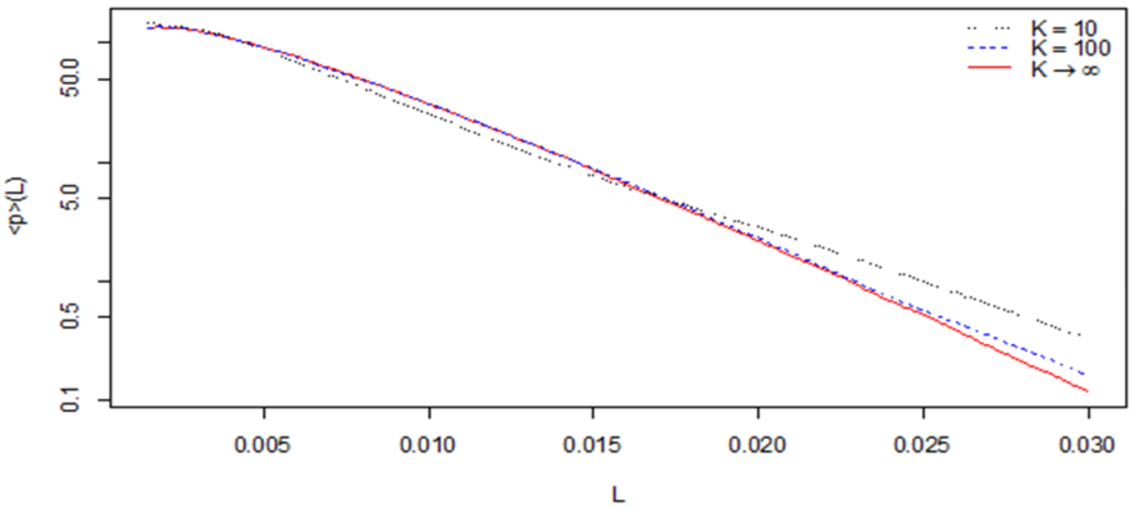

Figure3. Average loss distribution (T = 20 trading days) for different portfolio sizes including 10, 100 and ∞ in logarithmic scale

Where, the element of a $K \times N$ random matrix W is drawn from a multivariate normal distribution and then, WW' follows Wishart distribution:

$$\tilde{w}((WW'|C.N) = \sqrt{\frac{N}{2}}^{KN} \frac{\sqrt{detWW'}^{N-K-1}}{\Gamma_K\left(\frac{N}{2}\right)\sqrt{detC}^{N}} \exp\left(-\frac{N}{2} trW'C^{-1}W\right). \quad (7)$$

Table5. Regression model for value at risk measure at 99.9% confidence level

Parameter	Estimate	Std. Error	t-value	p-value
a	0.096113	0.009867	9.741	0.69%
b	-0.00115	0.000384	-2.989	0.0174

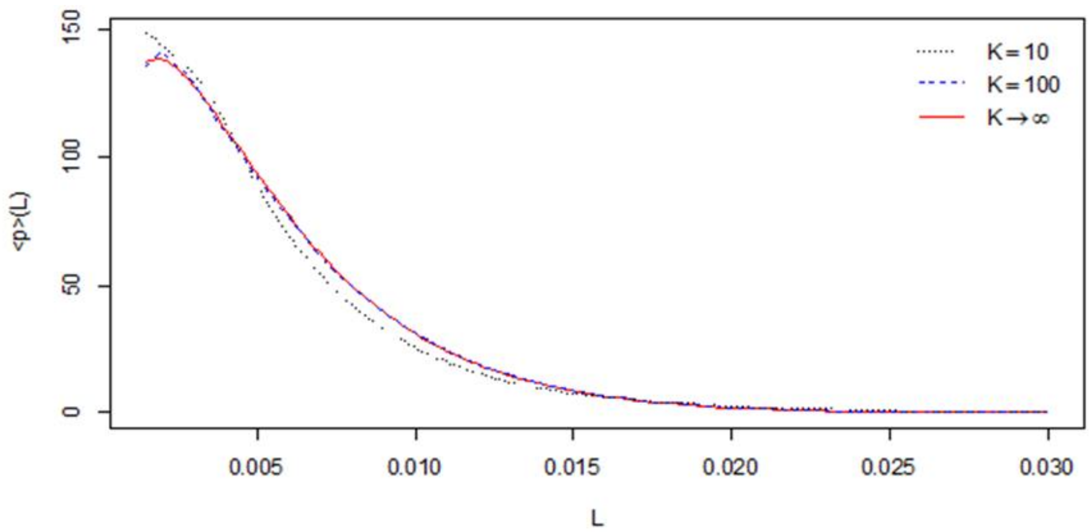

Figure4. Average loss distribution (T = 2020 trading days) for different portfolio sizes including 10, 100 and ∞ on a linear scale

With the multivariate Gamma function:

$$\Gamma_K(\alpha) = \pi^{\frac{K(K-1)}{4}} \prod_{k=1}^{K} \Gamma\left(\alpha + \frac{(1-k)}{2}\right) \qquad (8)$$

This ensemble of Wishart correlation matrix fluctuates around the average correlation matrix C. The variance of WW' is:

$$var(WW')_{kl} = \frac{C_{kl}^2 + 1}{N}. \qquad (9)$$

Where, C the average correlation matrix and N is shows the strength of the fluctuations. Smaller N causes more fluctuations in covariance matrix and larger N can lead to stationary in covariance matrix. (Schmitt et al. 2013), showed that multivariate Gaussian distribution is a good approximation for returns distribution if the covariance matrix is fixed. So following the structure of random matrix distribution (7) and returns distribution, one can construct a correlation-averaged multivariate distribution taking into account the fluctuations of correlations:

$$g(r|\Sigma_0.N) = \frac{1}{2^{\frac{N}{2}+1}\Gamma\left(\frac{N}{2}\right)\sqrt{\det\left(2\frac{\pi\Sigma_0}{N}\right)}} \frac{\mathcal{K}_{\frac{K-N}{2}}\left(\sqrt{Nr'\Sigma_0 r}\right)}{\sqrt{Nr'\Sigma_0 r}^{\frac{K-N}{2}}}. \qquad (10)$$

Where, $\mathcal{K}_\nu$ is the modified Bessel function of the second kind of orderν. After performing a change of variable and using Ito' lemma:

$$r_k \to \ln\left(\frac{V_k(T)}{V_0}\right) - (\mu_k - \frac{\rho_k^2}{2})T. \qquad (11)$$

Where μ and ρ are asset value drift and volatility respectively (Mühlbacher & Guhr, 2018).

Which, m_{jk} is jth moment.
To reduce parameters space, we use a correlation matrix with a simplified structure, *i.e.*, homogeneous correlations between assets. We construct this matrix so that all off-diagonal elements have a value of $C_{k\neq l} = c$
This construction has two advantages. We can simplify the parameter space of the correlation matrix to only one parameter and it will allow us to make analytical progress. Following Mühlbacher & Guhr, (2018), now we can achieve the portfolio loss distribution, which takes into account the fluctuating correlation:

$$\langle p\rangle(L|c.N) = \frac{1}{\sqrt{2\pi}2^{N/2}\Gamma(N/2)} \int_0^{\infty} dz\, z^{N/2-1} e^{-z/2} \sqrt{\frac{N}{2\pi}} \times \int_{-\infty}^{+\infty} du\, \exp\left(-\frac{N}{2}u^2\right) \frac{1}{\sqrt{M_2(z.u)}} \exp\left(-\frac{(L - M_1(z.u))^2}{2M_2(z.u)}\right). \qquad (12)$$

Where,

$$M_1(z.u) = \sum_{k=1}^{K} f_k m_{1\,k}(z.u). \qquad (13)$$

$$M_2(z.u) = \sum_{k=1}^{K} f_k^2 (m_{2k}(z.u) - m_{1\,k}^2(z.u)). \qquad (14)$$

In order to achieve an analytical progress and comparative result, a homogeneous portfolio is used. For a homogeneous portfolio, all contracts have the same face value$F_k = F$, variance $\sigma_k = \sigma$, drift $\mu_k = \mu$ and start asset value$V_{k0} = V_0$. Then, the *k*-dependence is dropped from the average loss distribution. This greatly simplifies the moment functions (13) and (14), which makes the numerical evaluation of the average loss distribution substantially faster (Schmitt & Chetalova, 2015). The *j*-th moment is:

$$m_{jk}(z\text{،}u) = \frac{\sqrt{N}}{\rho_k\sqrt{2\,\pi T(1-c)}} \int_{-\infty}^{\hat{F}_k} d\hat{V}_k \Big(1 - \frac{V_{k0}}{F_k}\exp(\sqrt{z}\hat{V}_k + (\mu_k - {\rho_k}^2)T)\Big)^j \times \exp\left(-\frac{\left(\hat{V}_k + \sqrt{cT}u\rho_k\right)^2}{2\,T(1-c)\,{\rho_k}^2/N}\right) \qquad (15)$$

With the upper limit:

$$\hat{V}_k = \frac{\ln\left(\frac{V(T)}{V_{k0}}\right) - (\mu - \rho^2)T}{\sqrt{z}}$$

$$\hat{F}_k = \frac{1}{\sqrt{z}}\left(\ln\frac{F_k}{V_{k0}} - (\mu_k - {\rho_k}^2)T\right) \qquad (16)$$

Due to the normalization of the weights, the portfolio weights are simply $f_k = \frac{1}{K}$ and all credit contracts have the same weight (Schmitt & Chetalova, 2015). Therefor the functions $M_1(z,u)$ and $M_2(z,u)$ will be simplified to:

$$M_1(z\text{،}u) = m_1(z\text{،}u)$$

$$M_2(z\text{،}u) = \frac{1}{K}\left(m_2(z\text{،}u) - {m_1}^2(z\text{،}u)\right) \qquad (17)$$

4. Results

The data used in this paper are the adjusted returns of all companies listed on Stock Exchange during 2010-2019. The portfolio is comprised only of stocks, which were continuously traded or not stopped trading for more than 20 days per year. For some stocks on some days, there is no price, so we simulate them by Monte Carlo simulation method.

As mentioned before, we are examining the impact of diversification on portfolio risk. For this purpose, we use a credit portfolio consisting of all companies listed, and it is assumed that all listed companies can receive loans from banks. The information related to the daily-adjusted prices of listed companies is extracted from the website of Tehran Stock Exchange (TSE). The basic idea is that the asset value $V_k(t)$ of the company k is the sum of time-independent liabilities F_k and equity $E_k(t)$ i.e.($V_k = F_k + E_k$). According to Merton model V_k is a stochastic process that represents the unobservable assets value. Therefore, we recall the definition of the return as bellow:

$$r_k = \frac{V_k(t+\Delta t) - V_k(t)}{V_k(t)}.$$

For the *k*-th asset and Δt is time to maturity or one year. All parameters defined above can be directly calculated from the data except N. The parameter N is determined by fitting data to formula (10) and confirming by the Crammer von Mises test. As first step, we examine the distribution of data. Using a least squares fit, N will be around 3. The theoretical and the empirical distributions for the normalized monthly returns are shown in figure 1.

In figure 1, the empirical distribution is shown in solid black line, while the theoretical result shown in dotted red line- both of them on a logarithmic scale. In addition, the small box is in linear scale. For monthly returns, the value around 3 is needed for the parameter N to describe the empirical data and the average correlation level is $c =$ %11. We test the result for accuracy by using Crammer von Mises test. The result of the test is shown in table 1.

As it can be seen, the p-value is greater than 0.05, which means that the null hypothesis (N=3) is not rejected. In other words, the null hypothesis of no significant difference between the observed and the theoretical distribution is not rejected. Put differently, the observed values are completely consistent with the theoretical distribution. We test the other values of N and the result is provided in figure 2.

Different values of N against Crammer von Mises statistics and Crammer von Mises p-values are shown in the left and right panel of the figure 2, respectively. Red marks indicate the best value. Both figures verify that the best value of N for fitting the empirical data is around 3.

In this section, the impact of portfolio diversification on the shape of average loss distribution based on homogeneous portfolio is investigated. Average loss distribution $\langle p\rangle(L|c\text{،}N)$ is shown in the figures 3-6 taking into account the correlation fluctuations between asset values. We choose different values for the size of the portfolio including K=10, 100, ∞.

The limit case $K \rightarrow \infty$ is presented in order to examine the portfolio with infinite size K→∞ and it shows how much the risk is reduced if all the companies can receive loansfrom the same market. The remaining parameters are fixed at typical values obtained from empirical data We need to test the impact of diversification on the loss distribution. For this purpose, the same conditions should be considered for all companies.

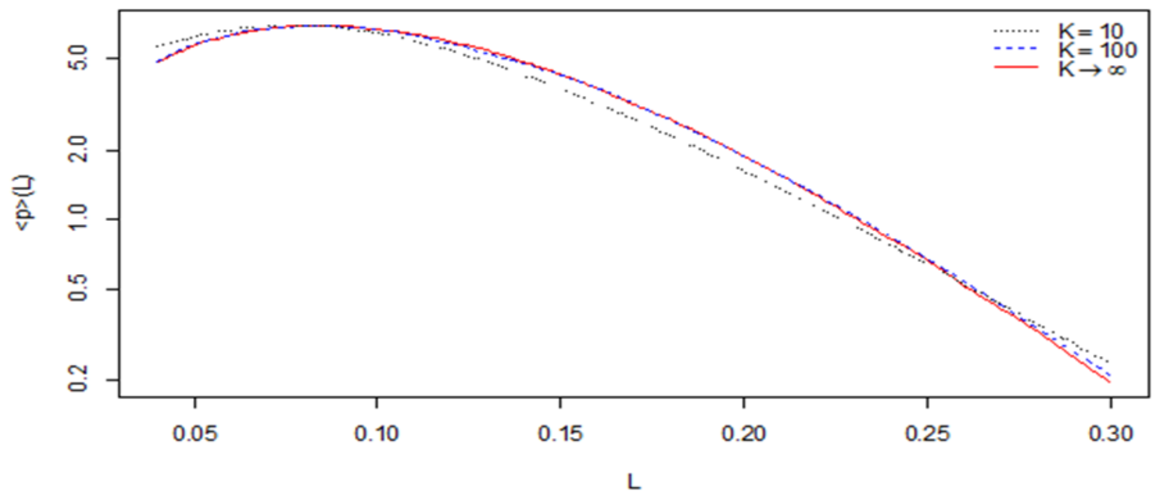

Figure5. Average loss distribution (T= 1 year) for different portfolio sizes including 10, 100 and ∞ on a logarithmic scale

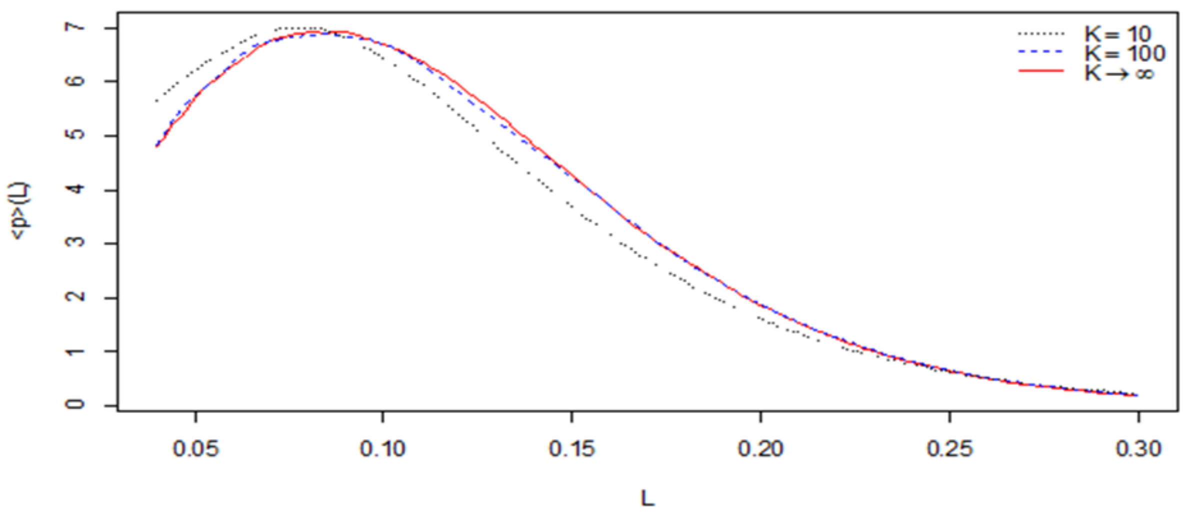

Figure6. Average loss distribution (T = 1 year) for different portfolio sizes including 10, 100 and ∞ on a linear scale

Therefore, for the face

$$t = 0 \text{ is } V0 = 100.$$

Average loss distribution parameters are presented for T=20 trading days and T=1 year in (Table 2). Two figures are drawn for each time horizon, i.e., logarithmic and linear scales. Based on the parameters in table 2, the average loss distribution is as follows. The figure 3 and 4 are related to T=20 trading days (monthly), the figure 5 and 6 are related to one year.

As it can be seen from the graphs, increasing the portfolio size- the number of companies in the credit portfolio- from 10 to 100 contracts leads to a small decrease in risk. However, the advantages of diversification quickly vanish. Here, we achieve a quantitative understanding of why diversification cannot significantly reduce risk even, when there is weak correlation between the assets. In fact, the banks pay less attention to the correlation between customers when paying loans, and so, it may be difficult to reduce risk by diversifying the credit portfolio. Therefore, in the discussion of diversification of the credit portfolio, the bank should pay attention to the correlation of assets and to the costs incurred by the bank for this matter.

To investigate more deeply the impact of increasing the portfolio size (K) or the impact of diversification on risk, for monthly return data, we calculate the value at risk (VaR) for different K. (Figure 7) shows the different values of VaR (vertical axis) against different values of K (horizontal axis) and confidence levels of 99%, 99.5% and 99.9%. According to the figure 7, the increase of K decreases VaR. In other words, by increasing K, the risk decreases slowly, but it seems that the risk reduction is not very significant. Given that, visual analysis may be subject to errors.

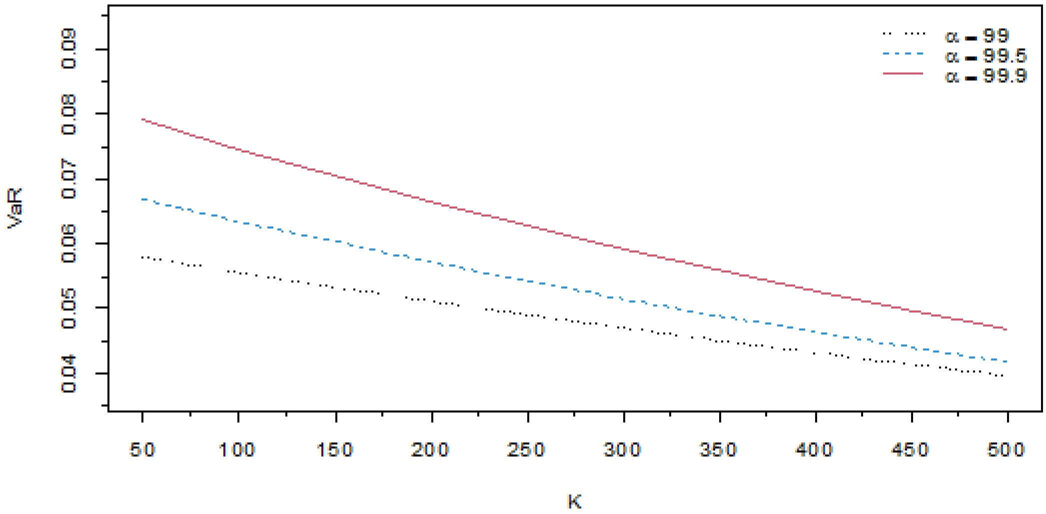

Figure7. Value at risk verses different K

Therefore, to check the significance of risk reduction an exponential regression model is fitted between VaR variable and K as follows The data results of this research showed that the nonstationary in the correlation time series has an effect on the distribution of losses of the bank's credit portfolio using structural methods, and failure to consider this important factor can cause an underestimation in the risk criteria.Many types of research have represented that the fluctuations of financial time series are highly unpredictable and change rapidly (Münnix MC, 2012), (Sandoval L. & Franca, 2012). Changes and nonstationary of fluctuations lead to fundamental challenges for estimating parameters including variance. Variance plays a very important role in financial modelling. In experimental research conducted by (Münnix et al., 2014), it was shown that the covariance matrix is able to describe the different situations of the financial markets (Song D-M, 2011).

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