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# Vehicle Routing Problem with a Time Window and Stochastic Demand and by Assuming a Competitor in Meeting the Customers' Demands

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| CHRONICLE | Abstract |
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| | In this study, a combination of different vehicle routing problems called Vehicle Routing Problem with a Time Window and Stochastic Demand has been examined. The difference between previous papers and this paper is that in our problem, the competitors compete with each other to meet the demands of customers in a specific industry. It seems that these problems occur in real life as well. In most real environmental problems, customers materialize their utmost demand for a vehicle that reaches on time and sooner than other competitors. The target function is to maximize the profit earned through the sale of goods to customers. Solving the research model in several trivial problems has been made precisely by using GAMZ Software. |
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## Introduction

In today's industrial world, transportation plays a great role in the economic development of nations by providing the facility for the consumption of productions in far places from the place where they are produced (Desaulniers et al, 2014). Presenting the final product to a customer requires the transfer of raw materials from suppliers to producers, transfer of incomplete products among factories, and eventually the final product to customers and target markets. Due to the variety of transportation activities, transportation costs include a high percentage of logistic fees (between 30 and 60%) (Belengure et al, 2011). Hence, nowadays, from an economic view, efficient routing for vehicles has been defined as a wide field of study in several academic majors and great stress is attached to this notion accordingly (Ombuki-Berman & Hanshar, 2009).

The general model of the vehicle routing problem (VRP) has been completely defined. In this model, each customer receives complete service just once. Moreover, it is assumed that all vehicles are homogeneous and the start and end of any vehicle are from a specific depot. The main goal of the vehicle routing problem (VRP) is to minimize the entire distance traveled by all vehicles (Contrardo et al, 2013). Upon passage of time and more perception of the transportation industry, different models of VRPs including Capacitated VRP (CVRP), Multiple Depot VRP (MDVRP), Periodic VRP (PVRP) Stochastic VRP (SVRP), VRP with Backhauls (VRPB), VRP with Pick-up & Delivering (VRPPD), VRP with Time Window (VRPTW) have been introduced (Nagata & Gulcin, D.Y, Naveed et al, 2016; Xingyin et al, 2016; Baldacci et al, 2012; Bettinelli et al, 2011; Nihal; Seda et al, 2014 & Braysy). VRP with Time Window (VRPTW) is a specific kind of VRP where

any vehicle starts a route from a depot and after providing the service to some customers (during a specific time span defined by them) ends its rout by the return to the depot. Assuming that we have a set named V including K vehicles in form of V={1,2,…K}, C set is defined consisting of N+1 customers in form of C={0,1,2…N}, a central depot (where a customer is displayed by 0 in C set), and directed network that connects depot and customers. In this problem, each i customer where $i \in C, i \neq 0$ has mi demand and it should be seen just once and by one vehicle (with a limited Q capacity) and then returned to the depot. Thus, the total of demands that are realized during a rout by K vehicle ($k \in V$) should not exceed the capacity of the said vehicle. For each K vehicle, ($k \in V$), one maximum routing time ($R_k$) is defined. It is not allowed to exceed this time. Any vehicle should return to the depot before the set time. Eventually, due to the elements that cause the return of the vehicle to the depot and termination of the path started are maximum routing time and completion of the capacity of the vehicle. It is not allowed to exceed these two items by any means.

It should be noted that $D_{ij}$ distance and $t_{ij}$ travel time for any of the arcs in the network are considered for travel from customer i to j ($i, j \in C$). On the other hand, each i customer ($i, j \in C$) should be serviced in his time window that has been defined in advance in form of [$e_i$, $l_j$] and limited to the soonest time of service start ($I_i$) and the latest time ($e_i$). The vehicles that reach the customer after the latest time of service commencement should be penalized at the commencement of service due today. However, the vehicle that reaches a customer sooner than the soonest time of service, will be charged with additional waiting time.

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Eventually, the goal of this problem to design and to optimize the sets of routes for vehicles where all customers receive the required services during the defined time span and a total time of travel, total distance traveled, and a total of waiting time and delay for vehicles will be minimized not exceeding the capacity and maximum vehicle routing time (Le Bouthillier et al, 2005; Dondo & Cerda, 2007; Lim & Zhang, 2007; Schnieder et al, 2010).

The stochastic vehicle routing problem is another vehicle routing problem. In the basic and classic conditions of the vehicle routing problem, it is assumed that all relevant parameters to the problem such as costs, customer demand, time of travel, and vehicle are finalized. In case one or some of the other parameters are not finalized and they are of random nature, we shall face a stochastic vehicle routing problem. Different types of stochastic routing problems are namely 1) VRP with Stochastic Demand; VRP with Stochastic Travel Time; VRP with Stochastic Customers and VRP with Stochastic Service Time. In VRP with stochastic Demand, it is assumed that the demand for any of the customers is not fixed and it follows a certain stochastic distribution function. In other words, the real demand value of any of the customers is found at the time of visitation with the customer (Hjorring & Holt, 1999; Schneider et al, 2010; Goodson, 2015). In this research customers' demand has followed a steady stochastic distribution function.

As it has been stated, VRP enjoys different assumptions. Thus, this research is willing to indicate the assumptions of limitations with a time window and stochastic demand of customers, competitors for the realization of customers' demands. On the other hand, considering VRP with time window is of NP-Hard type and due to complicated solving methods in VRP with stochastic demand and concerning the fact

that this type of model is not controllable, this research is willing to give some examples in relatively small sixes in GAMZ Software and to solve the presented model in an optimized way accordingly.

## Statement of Problem and formulation (Problem modeling)

In many studies conducted earlier in the field of VRP, generally, there was only one company that has realized the customers' demand. In this paper, customers' demand is realized through a more realistic viewpoint by one of the competitor companies in the industry. In this case, if any of the vehicles reach the customer sooner, the customer shall purchase his entire demand as much as the vehicle has the goods. If any of the competitor vehicles reach a customer simultaneously, the customer, based on designated priority function, decides on which vehicle he will receive the goods. In this research, Priority function or $P_{ivk}$ has been indicated. When cars v and k reach the knot i simultaneously, it is defined as follows: Customer desirability toward car v and car k equals to 0-1. In this regard, formulas 1 and 2 are given as follows:

Customer desirability toward car v and car k equals to 0-1. In this regard, formulas 1 and 2 are given as follows:

$$\frac{\text{Customer desirability to car v}}{\text{customer desirability to car k}} \geq 1 \qquad (1)$$

If formula (1) is established, $P_{ivk}=1$ and car v cover the demand of the customer i.
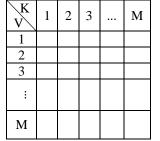
$$\frac{\text{Customer desirability to car v}}{\text{customer desirability to car k}} < 1 \qquad (2)$$

If formula (2) is established, $P_{ivk}= 0$ and car k (competitor car) covers the demand of customer i.

In order to understand the case much better, the desirability of car v to car k has been given in table 1.

**Table 1.** Desirability ratio of car v to car k

| K \ V | 1 | 2 | 3 | ... | M |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| ⋮ | | | | | |
| M | | | | | |

Customer priority, in case desirability of car v is more than that of car k (desirability ratio of car v to cark is > or equals to 1), is given as One. Otherwise, it equals to zero (table 2). It should be noted that in this problem, the time of reaching the car k (competitor car) to knot i is made within the time span of $[c_i \ d_i]$.

**Table 2.** Priority of Car v to Car k

| $P_{ivk}$ | 1 | 2 | 3 | ... | M |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| ⋮ | | | | | |
| m | | | | | |

In this paper, vehicle routes have consisted of profit earned through service provided for customers' demand. Moreover, the following limitations should be taken into consideration:

(1) Each demand should be serviced exactly once by a vehicle.
(2) Load of a vehicle should not exceed its capacity.
(3) Each route starts from the depot and ends at the said depot.
(4) Number of vehicles used should not exceed the number of accessible vehicles.

(5) Total time of each rout (travel time and service) should not exceed a predetermined (certain) limitation (time).
(6) Demand of each customer is determined based on the stochastic steady distribution function.

**Sets and Indices**
I: All customers set
i: customer index (i ∈ I) and i = 1, 2, ..., n
V: vehicles set
V: vehicle index (v ∈ V) and v = 1, 2, ..., m
A: All paths set (Arc) that can be defined from knot i knot j $A = \{((i,j)|i,j \in I)\}$
K: index of competitor vehicle
K: competitor vehicle set: (k ∈ K) and k = 1, 2, ..., m

*Parameters*
$D_{ij}$= distance (length of Arc) between the knots i and j in such a way that it is symmetrical $(d_{ij} = d_{ji})$ and $(i,j) \in A$
$f_v$= fixed cost for use of vehicle v and (v ∈ V)
$g_v$= The demand for customer i that is created stochastically with steady continuous function in the time span of $[a_i, b_i]$.
$t_{ij}$= time of travel between knot i and j, (i,j) ∈ A
$S_{iv}$= time when vehicle v reaches knot i
$e_i$= Changeable service time of each unit per knot i when knot i is inspected.
$T_i$= fixed serving time in knot i when knot i is inspected.
$[c_i, d_i]$: time window in knot i
$C_i$=the soonest start time of servicing at knot i
$D_i$= the latest start time of serving at knot i
$W_i$= waiting time at knot i
$Z_i$= maximum permissible distance for vehicle v

*Scalars*
P: sales price of each unit of product
G: Cost price of each unit of product

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a: Error level

### Decision variables

$X_{ijv}$: When vehicle v moves from knot i to knot j, it equals to 1; otherwise it equals to 0.

$L_{iv}$: When a vehicle covers the demand of knot i, it equals to 1; otherwise it equals to 0.

$P_{ivk}$: When vehicle v is preferred over vehicle k (competitor) to cover the demand at knot i, it equals to one; otherwise, it equals to 0.

### The mathematical model of problem

$$\text{Max } z = \sum_{v \in V} \sum_{i \in I} L_{iv} D_i (P - G) - \left( \sum_{v \in V} f_v \sum_{i \in I} \sum_{j \in I} X_{ijv} + \sum_{v \in V} g_v \sum_{i \in I} \sum_{j \in I} d_{ij} X_{ijv} \right) \quad (3)$$

s.t.

$$\sum_{v \in V} \sum_{j \in I} X_{0jv} \leq m \quad (4)$$

$$\sum_{j \in I} X_{0jv} = \sum_{i \in I} X_{i0v} \leq 1 \quad \forall v \in V \quad (5)$$

$$\sum_{i \in I} X_{irv} - \sum_{j \in I} X_{rjv} = 0 \quad \forall v \in V, \forall r \in I \quad (6)$$

$$\sum_{v \in V} \sum_{i \in I \cup \{0\}, i \neq j} X_{ijv} = 1 \quad \forall j \in I \quad (7)$$

$$\sum_{v \in V} \sum_{\substack{j \in I \cup \{0\} \\ i \neq j}} X_{ijv} = 1 \quad \forall i \in I \quad (8)$$

$$\sum_{i \in I} D_i L_{iv} \leq Q_v \quad \forall v \in V \quad (9)$$

$$Q_v - \sum_{i=1}^{l-1} D_i L_{iv} \geq a_l + (1-\alpha)(a_l - b_l) \quad \forall v \in V \quad \forall l \in I \quad (10)$$

$$\sum_{i \in I} \sum_{j \in I} \sum_{v \in V} X_{ijv} \leq |S| - 1 \quad 2 \leq |S| \leq n \quad (11)$$

$$\sum_{v \in V} \sum_{\substack{i \in I \cup \{0\} \\ i \neq j}} X_{ijv} (S_{iv} + T_i + D_i e_i + t_{ij} + w_i) \leq S_{jv} \quad \forall j \in I \quad (12)$$

$$c_i \leq S_{iv} + w_i \leq d_i \quad \forall v \in V, \forall i \in I \quad (13)$$

$$S_{0v} = w_0 = T_0 = e_0 = 0 \quad (14)$$

$$\sum_{i \in I \cup \{0\}} \sum_{j \in I \cup \{0\}} d_{ij} X_{ijv} \leq z_v \quad \forall v \in V \quad (15)$$

$$S_{iv} \leq S_{ik} + M y_1 \quad \forall v \in V, \forall i \in I, \forall k \in K \quad (16)$$

$$S_{ik} \leq S_{iv} + M y_2 \quad \forall v \in V, \forall i \in I, \forall k \in K \quad (17)$$

$$L_{iv} = (1 - y_1)y_2 + P_{ivk}(1 - y_1 - y_2) \quad \forall v \in V, \forall i \in I, \forall k \in K \quad (18)$$

$$y_1 + y_2 \leq 1 \quad (19)$$

$$y_1, y_2 \in \{0,1\} \quad (20)$$

The target function of the problem consists of profit earned to servicing to customers' demand and fixed and changeable costs of transportation (transfer) (Formulation 3). Limitation (4) indicates that the maximum rout m leaves the depot. It means that the maximum vehicle m leaves the depot. Limitation (5) guarantees the commencement and end of the path of the depot. Limitation 6 shows that any vehicle that enters a knot, it absolutely leaves the said knot. Limitations 7 and 8 indicate that each customer should be inspected by a vehicle. Limitation 9 reveals that a set of demand of knots that are inspected by vehicle v should not exceed the maximum capacity of the vehicle v. Limitation 10 shows the possibility when car v inspects knot i and covers the stochastic demand of knot i that follows a steady continuous function. Limitation 11 has been indicated to prevent the development of sub tour in such a manner that $S \subseteq \{1, 2, …, n\}$.

Limitations 12, 13, and 14 are time window limitations. Limitation 15 indicates that the total distances traveled by vehicle v should be less than the maximum permissible distance traveled. Limitations 16, 17, 18, and 19 indicate if car m reaches sooner than car k (the competitor car), car v covers the demands of knot i. Otherwise, the competitor covers the demand of knot i. Moreover, if the time of cars v and k reach knot i, the car which is preferred by the customer i covers the demand of knot i. Limitations 20 indicates that y1 and y2 are the binary variables (zero and one).

### Results: Solving the problem
For encoding and solving the model, Software GAMS 24.1.2 is used. In this paper, small and medium-sized problems

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have been solved and evaluated by using this software. Particulars of the computer system are given to solve the sample problems including CPU of 2.4 GHz type and Ram 8 and Windows 7 Operating System. According to tables 3 and 4, ten sample problems consisting of five small-sized and five medium-sized problems have been solved and evaluated.

**Table 3.** Solving the problems of small dimensions- the proposed model by GAMS software

| Row sample issue | Kind of problem | The number of iterations to solve | The number of customers (i) | The number of vehicles (v) | The number of vehicles Rivals (k) | The objective function (z) | CPU time (second) | Absol. GAP |
|---|---|---|---|---|---|---|---|---|
| 1 | Small | 8 | 5 | 2 | 3 | 137873 | 0.09 | 287.88 |
| 2 | Small | 12 | 6 | 3 | 3 | 159661 | 0.06 | 366.35 |
| 3 | Small | 12 | 7 | 4 | 3 | 166949 | 0.047 | 310.00 |
| 4 | Small | 16 | 8 | 5 | 3 | 175621 | 0.16 | 328.17 |
| 5 | Small | 20 | 9 | 6 | 4 | 185475 | 0.19 | 449.1 |

As it is indicated in table 3, upon a trivial increase in dimensions of the problem, solving time problems and the value of GAP from an optimized answer has appeared in ascending form. The number of repetitions has almost increased by 2.5 by doubling the number of customers. It

indicates the complexity of this class of problems. As we have expected upon the increase in the number of customers, target function has been ascended. Now for a more precise study, we deal with another group of problems of slightly larger dimensions compared to the previous items and medium-sized problems so that the effect of dimensions of problems at the time of solving and results of operation of the method becomes more clear. Table four has shown five sample problems of medium dimensions and the results are given in Table 4 as well.

**Table 4.** Solving the problems of medium dimensions by using the proposed model with GAMS Software

| Row sample issue | Kind of problem | The number of iterations to solve | The number of customers (i) | The number of vehicles (v) | The number of vehicles Rivals (k) | The objective function (z) | CPU time (second) | Absol. GAP |
|---|---|---|---|---|---|---|---|---|
| 1 | Medium | 50 | 15 | 6 | 4 | 278145 | 0.84 | 527.297 |
| 2 | Medium | 90 | 20 | 6 | 5 | 309678 | 2.82 | 698.22 |
| 3 | Medium | 120 | 25 | 6 | 5 | 335538 | 3.98 | 602.11 |
| 4 | Medium | 170 | 30 | 7 | 7 | 367811 | 5.12 | 725.34 |
| 5 | Medium | 610 | 35 | 7 | 7 | 671231 | 9.87 | 1100.11 |

Table 4 comprises the problems of medium dimensions. The number of customers is given as 15-35 persons. According to the said table, it has been found that the absolute Gap in this group of problems has noticeably increased, and solving the problem has reached 10 seconds. The number of repetitions has exponentially increased. In this group of problems, the number of competitor vehicles has increased by 7 items at most. It is quite clear that upon an increase in the number of competitor vehicles, the complexity of these problems will increase. Concerning the fact that GAMS software is able to solve only small and medium-sized problems in this group of problems, studying and magnifying the dimensions of the problem for 35 customers have been discarded and metaheuristic methods may be used to solve this group of problems.

## Conclusion

In this paper, an integrated model of vehicle routing has been presented by assuming time windows and stochastic demand to maximize the profit earned through the sale of goods to customers when the vehicle reaches the customer sooner. The model presented in this research has been solved by using GAMS software. According to research literature, the said model is of NP=Hard problem type. Thus, upon the increase of dimensions of the problem, the time for solving the problem has exponentially increased. Relying on the results obtained, it can be stated that to solve this model in larger dimensions, metaheuristic methods should be used because the more the dimensions of the problem are increased, the more time will increase to solve the problem by using GAMS software to the extent that there is no possibility to solve the problem in GAMS software.

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