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Sustainable Development Goals and Homelessness

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Abstract. The United Nation's Sustainable Development Goals encourage countries to solve many social problems. One of these problems is homelessness. We consider those goals which are most pertinent to homelessness according to [13]. We rank countries with respect to the achievement of these goals. We use fuzzy similarity measures to determine the degree of similarity between these rankings. We use three methods to rank the counties, namely, the Analytic Hierarchy Process, the Guiasu method, and the Yen method. Overall scores of categories in some basic research papers pertaining to Sustainable Development Goals were obtained by using multiplication of the scores of the category's targets. Multiplication was used to agree with the philosophy that in order for a high score to be obtained, all targets must have a high score. To support this philosophy in the decision process, we use the t-norms bounded difference, algebraic product, and standard intersection as experts. We also suggest a way the techniques used here can be extended to nonstandard analysis.

AMS Subject Classification 2020: 94D05; 03E72
Keywords and Phrases: Homelessness, Sustainable development goals, Analytic hierarchy process, Fuzzy similarity measures, Country rankings.

1 Introduction

The United Nation's Sustainable Development Goals provide a mechanism for encouraging nations to make progress towards shared goals. They generate collaboration, funding, definition, targeting, and measurement for many social problems such as poverty and sanitation for all, [15]. However, homelessness is not explicitly mentioned in the Sustainable Development Goals, [1]. The United Nations Human Settlement Program estimates that 1.6 billion people live in inadequate housing, and the best data available suggest that more than 100 million people have no housing at all. Related works can be seen in [2], [3] and [14].

In this paper, we consider four SDGs as seen by [13] as pertinent to homelessness. We rank countries with respect to their achievement of these goals. We then use fuzzy similarity measures to determine the degree of similarity between these rankings and the ranking of countries with respect to the number of people, per 10,000 who are homeless, [5]. We determine measures of similarity of these rankings using the techniques of fuzzy similarity relations developed in [8]. For the similarity measure M, if the value is between 0 and 0.2, we say the similarity is very low, between 0.2 and 0.4, we say the similarity is low, between 0.4 and 0.6 medium, between 0.6 and 0.8 high, between 0.8 and 1 very high. We find that the similarity of the four rankings is medium. A similar interpretation can be made for the similarity relation S. The rankings and similarity measures are done for various regions of the world. We find that the similarity measures are very high. The results can be found in detail in Sections 4, 5, and 6. We also determine the similarity measure between a

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ranking of a country's number of homelessness and the ranking of countries according to their achievement of the SDGs. We found that similarity ranged from medium to high depending on the region involved.

We use three methods to rank countries with respect to their achievement of the SDGs pertinent to homelessness. The Analytic Hierarchy Process (AHP) is a multicriteria decision method introduced in [11] and [12]. We consider a factor to be studied by the examination of subfactors of the factor. In our case, each expert $E_j, j = 1, ..., n$, assigns a number w_{ij} to each subfactor, $i = 1, ..., m$, of the factor, as to its importance with respect to the overarching goal. The row average, w_i, of each row of the matrix $[w_{ij}]$ is determined to form a matrix R whose ij-th element is w_i/w_j. The columns of R are then normalized in order to form the $m \times n$ matrix N whose ij-th element is $(w_i/w_j)/\sum_{i=1}^{m} w_i/w_j = w_i/\sum_{i=1}^{m} w_i, i = 1, ..., m$. The row vector yields the weights for the subfactors for the linear equation of the overarching goal, the dependent variable, in terms of the subfactors, the independent variables.

If the matrix W already has its columns normalized, then $w_i = \sum_{j=1}^{n} w_{ij}/n, i = 1, ..., m$. Since $\sum_{=1}^{m} w_{ij} = 1, j = 1, ..., n$, it follows that $\sum_{i=1}^{m} w_i = 1$. Hence $w_i/\sum_{i=1}^{m} w_i = w_i$, i.e., w_i is the weight for the i-th subfactor in the linear equation, $i = 1, ..., m$. It thus follows that if the columns of W are already normal, then the Guiasu method (with probabilistic assignments) and the analytic hierarchy process yield the same weights. However, in general, the Guiasu weights and the AHP weights can have quite different weights [9].

Yen's method addresses the issue of managing imprecise and vague information in evidential reasoning by combining the Dempster-Shafer theory with fuzzy set theory, [16]. Several researchers have extended the Dempster-Shafer theory to deal with vague information, but their extensions did not preserve an important principle that the belief and plausibility measures are lower and upper probabilities. Yen's method preserves this principle. Nevertheless, we use various measures of subsethood to determine belief functions. We do this to compare the results of the beliefs with Yen's method.

Yen's method is developed under the assumption that the focal elements are normalized. If the focal elements are not normal, he normalizes them.

We let $\mathbb{N}$ denote the positive integers. If X is a set, we let $\mathcal{FP}(X)$ denote the set of all fuzzy subsets of X. We let $\vee$ denote supremum or maximum and $\wedge$ denote infimum or minimum.

2 Preliminary Results

Proposition 2.1. *Let T denote an $m \times n$ matrix whose entries are from the closed interval $[0, 1]$. Let C_j denote the sum of the entries from column $j, j = 1, ..., n$. If $C_1 = ... = C_n$, then the AHP and the Guiasu weights are the same.*

Proof. Let $C = C_1 = ... = C_n$. Let R_i denote the sum of the elements in row $i, i = 1, ..., m$. Then in the AHP matrix, the row averages are $R_i/n, i = 1, ..., m$. Hence the coefficients for the AHP equation are $(R_i/n)/(R_1 + ... + R_m)/n = R_i/(R_1 + ... + R_m), i = 1, ..., m$. The Guiasu matrix is obtained from the AHP matrix by dividing each entry in its column by that column sum which by assumption is C. Thus the row average of the i-th row is $R_i/nC, i = 1, ..., m$. Hence the coefficients of the Guiasu equation is are $(R_i/nC)/(R_1 + ... + R_m)/nC) = R_i/(R_1 + ... + R_n), i = 1, ..., m$. □

Proposition 2.2. *Let M denote the $m \times n$ Guiasu matrix. Let m_j^* denote the maximum entry in column $j, j = 1, ..., n$. Suppose there exists m^* such that $m_1^* = ... = m_n^* = m^*$. Then the Guiasu and the Yen weights are the same.*

Proof. The entries of the columns of M add to 1. It follows that the row average column entries are $\frac{1}{n}R_i, i = 1, ..., m$, and so the Guiasu weights are $\frac{R_i}{R_1+...+R_m}, i = 1, ..., m$. The entries of the Yen matrix are $\frac{a_{ij}}{m^*} i = i, ..., m; j = 1, ..., n$. Hence the entries of the Yen row average column are $\frac{1}{n}\frac{R_i}{m^*}, i = 1, ..., n$. Hence the Yen weights are $(\frac{1}{n}\frac{R_i}{m^*})/(\frac{1}{n}\frac{R_1+...+R_m}{m^*}) = \frac{R_i}{R_1+...+R_n}, i = 1, ..., m$. □

Proposition 2.1 suggests that if the column sums are nearly equal, then the AHP and Guiasu weights will be nearly equal. We examine this in a nonstandard analysis setting. This examination suggests a possible extension of the paper to nonstandard analysis, [7]. First, we review some basic concepts from nonstandard analysis. Let $\mathbb{R}$ denote the real numbers. Let $\mathbb{R}^*$ denote the field of hyperreals which includes infinitesimal numbers and infinite numbers. Let $\mathbb{R}_{fin}$ denote the set of those elements of $\mathbb{R}^*$ which are not infinite. Then $\mathbb{R}_{fin}$ is a local ring with unique maximal ideal M, where M denotes the set of all infinitesimal elements, [7]. It follows that the relation $\approx$ defined on $\mathbb{R}_{fin}$ by for all $x, y \in \mathbb{R}_{fin}, x \approx y$ if and only if $x - y \in M$ is an equivalence relation.

Proposition 2.3. *Let* $a, c \in \mathbb{R}_{fin} \backslash M$ *(set difference) and* $b, d \in \mathbb{R}_{fin}$ *be such that* $a \approx b$ *and* $c \approx d$. *Then* $\frac{a}{c} \approx \frac{b}{d}$.

Proof. Since $a \approx b$ and $c \approx d$, there exists $m, m' \in M$ such that $b = a + m$ and $d = c + m'$. Thus $a(c + m') - c(a + m) = m' - m \in M$. Since $a, c \notin M$ and $\mathbb{R}_{fin} is$ a local ring,$\frac{1}{c} \in \mathbb{R}_{fin}$. Since M is an ideal in $\mathbb{R}_{fin}$, $\frac{a}{c}(c + m') - (a + m) \in M$. Now $\frac{1}{c+m'} \in \mathbb{R}_{fin}$ since $\mathbb{R}_{fin}$ is a local ring. Thus $\frac{a}{c} - \frac{a+m}{c+m'} \in M$. Hence $\frac{a}{c} - \frac{b}{d} \in M$. That is, $\frac{a}{c} \approx \frac{b}{d}$. $\square$

To see how this applies to our situation, consider the situation where the $m \times n$ matrix has entries a_{ij} from $\mathbb{R}_{fin}$ and are positive. Let C_j denote the sum of the a_{ij} in column $j, j = 1, ..., n$. Suppose there exists $C \in R_{fin}$ and $\in_j \in M$, such that $C_j = C + \in_j, j = 1, ..., n$. Then the weights of the AHP equation are $\sum_{j=1}^{n} a_{ij} / \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij}$. The weights of the corresponding Guiasu equation are $(\sum_{j=1}^{n} a_{ij}/(C+ \in_j))/(\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij}/(C+ \in_j)) \approx (\sum_{j=1}^{n} a_{ij}/(C))/(\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij}/(C)) = \sum_{j=1}^{n} a_{ij} / \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij}$., where we have $\approx$ holding by Proposition 2.3 and by noting that $C+ \in_j \approx C$.

Similar comments concerning Proposition 2.2 can be made.

3 SDGs and Homelessness

In the following table, the G_i denote a particular Sustainable Development Goal. Here G_1 denotes End poverty in all its forms everywhere, G_8 denotes Promote sustained, inclusive and sustainable economic growth, full and productive employment and decent work for all, G_{10} denotes Reduce inequality within and among countries, and G_{11} denotes Make cities and human settlements inclusive, safe, resilient, and sustainable. The scores of the assessors were used to obtain an average for each category. Then these category averages were multiplied to obtain an overall average score for each target. Multiplication was used to agree with the philosophy that in order for a high score, all categories must have a high score. To support this philosophy, we use the t-norms bounded difference, algebraic product, and standard intersection. These t-norms are considered as experts when we apply the methods known as AHP, Guiasu and Yen. The entries of the Target values are taken from [10] and then divided by 2 so that the values will be in the closed interval $[0, 1]$. The Goal values are obtained by averaging the Target values. Applicability: In the opinion of the assessor is the target relevant, suitable and/or appropriate to developed countries; Implementable: In the opinion of the assessor will a reasonable allocation of resources result in the achievement of the goal/target in developed countries; Transformationalism: In the opinion of the assessor will the achievement of the goal/target require significant and additional policy action beyond what is currently in place and/or planned.

Table 1: t-norms as Decision Makers

Goal/Target	Applicable	Implementable	Transformative
G_1	0.575	0.85	0.325
1.4	0.5	0.85	0.15
1.5	0.65	0.85	0.5

Table 1: t-norms as Decision Makers (cont.)

Goal/Target	Applicable	Implementable	Transformative
G_8	0.85	0.85	0.65
8.5	0.85	0.85	0.65
G_{10}	0.667	0.9	0.617
10.2	0.5	0.85	0.5
10.3	0.5	0.85	0.5
10.4	1.0	1.0	0.85
G_{11}	0.5	0.85	0.5
11.1	0.5	0.85	0.5

The equations determined below are used to determine how well countries are doing in achieving the SDGs pertinent to homelessness. The entries in Table 2 below are obtained from Table 1. Recall that bounded difference is defined as $0 \vee (a + b - 1)$ for all $a, b \in [0, 1]$, see [4]. Consider G_1. For Bounded Difference, we get $0 \vee (0.575 + 0.85 - 1) = 0.425$ and $0 \vee (0.425 + 0.325 - 1) = 0$ or equivalently $0 \vee (0.425 + 0.85 + 0.325 - 2) = 0$.

Table 2: AHP Method

AHP	Bounded Difference	Algebraic Product	Standard Intersection	Row Average
G_1	0	0.159	0.325	0.161
G_8	0.350	0.470	0.650	0.490
G_{10}	0.184	0.370	0.617	0.390
G_{11}	0	0.213	0.500	0.238
Col Sum	0.534	1.212	2.092	1.279

$H_1 = 0.126G_1 + 0.383G_8 + 0.305G_{10} + 0.186G_{11}$.

Table 3: Guiasu Method

Guiasu	Bounded Difference	Algebraic Product	Standard Intersection	Row Average
G_1	0	0.130	0.155	0.095
G_8	0.655	0.388	0.311	0.451
G_{10}	0.345	0.306	0.295	0.315
G_{11}	0	0.176	0.239	0.138
Col Sum				0.999

$H_2 = 0.095G_1 + 0.451G_8 + 0.315G_{10} + 0.138G_{11}$.

Table 4 below is determined from Table 3 by dividing each entry in the column by the maximum entry of that column.

Table 4: Yen Method

Yen	Bounded Difference	Algebraic Product	Standard Intersection	Row Average
G_1	0	0.335	0.498	0.278

Table 4: Yen Method(cont.)

Yen	Bounded Difference	Algebraic Product	Standard Intersection	Row Average
G_8	1.000	1.000	1.000	1.000
G_{10}	0.527	0.789	0.949	0.755
G_{11}	0	0.454	0.768	0.407
Col Sum				2.440

$H_3 = 0.114G_1 + 0.410G_8 + 0.309G_{10} + 0.167G_{11}$.

4 Country Rankings

The values that state how well a country is achieving the *SDG*s are given in [15]. We do not present them here. These values are substituted into the variables $G_1, G_8, G_{10,}$ and G_{11} in the above equations to determine the values provided in Tables 5-10.

OECD

Table 5: OECD Ranks

Country	AHP / rank	Guiasu / rank	Yen / rank
Australia	0.820 / 24	0.814 / 25	0.818 / 25
Austria	0.865 / 13	0.859 / 13	0.863 / 13
Belgium	0.875 / 10	0.870 / 10	0.873 / 10
Canada	0.837 / 23	0.833 / 19	0.835 / 20
Chile	0.667 / 33	0.656 / 33	0.663 / 34
Czech Rep.	0.899 / 7	0.893 / 7	0.897 / 7
Denmark	0.909 / 4	0.902 / 4	0.906 / 4
Estonia	0.839 / 20	0.830 / 21	0.835 / 21
Finland	0.905 / 5	0.899 / 5	0.902 / 5
France	0.847 / 15	0.837 / 17	0.843 / 15
Germany	0.872 / 11	0.864 / 12	0.869 / 11
Greece	0.671 / 32	0.650 / 35	0.663 / 33
Hungary	0.830 / 23	0.822 / 23	0.827 / 24
Iceland	0.913 / 2	0.906 / 3	0.911 / 3
Ireland	0.877 / 9	0.875 / 9	0.876 / 9
Israel	0.753 / 29	0.747 / 30	0.750 / 30
Italy	0.775 / 26	0.770 / 27	0.773 / 27
Japan	0.838 / 21	0.840 / 15	0.839 / 17
Korea Rep.	0.868 / 12	0.867 / 11	0.868 / 12
Latvia	0.837 / 22	0.830 / 20	0.835 / 22
Lithuania	0.738 / 31	0.720 / 32	0.734 / 32
Luxembourg	0.839 / 19	0.820 / 24	0.831 / 23
Mexico	0.585 / 36	0.571 / 36	0.580 / 36
Netherlands	0.902 / 6	0.894 / 6	0.899 / 6
N. Zealand	0.841 / 17	0.839 / 16	0.840 / 16
Norway	0.891 / 8	0.883 / 8	0.888 / 8
Poland	0.759 / 28	0.754 / 29	0.757 / 29

Table 5: OECD Ranks (cont.)

Country	AHP / rank	Guiasu / rank	Yen / rank
Portugal	0.771 / 27	0.763 / 28	0.768 / 28
Slovak Rep.	0.840 / 18	0.834 / 18	0.838 / 19
Slovenia	0.913 / 3	0.911 / 2	0.913 / 2
Spain	0.788 / 25	0.774 / 26	0.783 / 26
Sweden	0.918 / 1	0.911 / 1	0.915 / 1
Switzerland	0.858 / 14	0.843 / 14	0.852 / 14
Turkey	0.665 / 35	0.655 / 34	0.661 / 35
U. K.	0.843 / 16	0.830 / 22	0.838 / 18
U. S.	0.750 / 30	0.743 / 31	0.747 / 31

Some countries in the following are not ranked due to insufficient data.

East and South Asia

Table 6: East and South Asia Ranks

Country	AHP / rank	Guiasu / rank	Yen / rank
Bangladesh	0.698 / 11	0.716 / 8	0.705 / 11
Bhutan	0.745 / 6	0.735 / 6	0.742 / 6
Brunei Dar			
Cambodia	0.770 / 4	0.757 / 5	0.765 / 4
China	0.779 / 3	0.779 / 3	0.779 / 3
India	0.653 / 14	0.669 / 13	0.659 / 14
Indonesia	0.616 / 16	0.616 / 16	0.616/ 16
Korean Dem. Rep.			
Lao PDR	0.709 / 9	0.713 / 9	0.710 / 9
Malaysia	0.717 / 8	0.706 / 11	0.713 / 8
Maldives	0.809 / 1	0.796 / 1	0.804 / 1
Mongolia	0.725 / 7	0.732 / 7	0.727 / 7
Myanmar	0.708 / 10	0.706 / 12	0.708 / 10
Nepal	0.695 / 12	0.712 / 10	0.702 / 12
Pakistan	0.621 / 15	0.623 / 15	0.622 / 15
Philippines	0.614 / 17	0.610 / 17	0.612 / 17
Singapore			
Sri Lanka	0.693 / 13	0.687 / 14	0.691 / 13
Thailand	0.767 / 5	0.758 / 4	0.763 / 5
Timor Leste			
Vietnam	0.787 / 2	0.780 / 2	0.784 / 2

Eastern Europe and Central Asia

Table 7: Eastern Europe and Central Asia Ranks

Country	AHP / rank	Guiasu / rank	Yen / rank
Afghanistan			
Albania	0.689 / 17	0.670 / 17	0.682 / 17

Table 7: Eastern Europe and Central Asia Ranks(cont.)

Country	AHP / rank	Guiasu / rank	Yen / rank
Andorra			
Armenia	0.635 / 21	0.623 / 21	0.630 / 21
Azerbaijan	0.750 / 12	0.733 / 13	0.743 / 12
Belarus	0.834 / 3	0.827 / 3	0.831 / 3
Bosnia & Herzegovina	0.748 / 13	0.734 / 12	0.743 / 13
Bulgaria	0.770 / 8	0.762 / 8	0.767 / 8
Croatia	0.778 / 6	0.762 / 7	0.775 / 6
Cyprus	0.792 / 5	0.783 / 5	0.787 / 5
Georgia	0.646 / 19	0.632 / 19	0.640 / 19
Kazakhstan	0.755 / 10	0.745 / 10	0.751 / 10
Kyrgz Rep.	0.778 / 7	0.766 / 6	0.773 / 7
Liecheristan			
Malta	0.903 / 1	0.902 / 1	0.903 / 1
Moldova	0.840 / 2	0.831 / 2	0.836 / 2
Monaco			
Montenegro	0.701 / 16	0.690 / 16	0.697 / 16
North Macedonia	0.643 / 20	0.629 / 20	0.638 / 20
Romania	0.675 / 18	0.664 / 18	0.67 / 18
Russian Federation	0.733 / 14	0.720 / 15	0.728 / 14
San Marino			
Serbia	0.753 / 11	0.745 / 11	0.750 / 11
Tajikistan	0.730 / 15	0.720 / 14	0.726 / 15
Turkmenistan			
Ukraine	0.831 / 4	0.821 / 4	0.827 / 4
Uzbekistan	0.770 / 9	0.762 / 9	0.767 / 9

Latin America and the Caribbean

Table 8: Latin America and Caribbean Ranks

Country	AHP / rank	Guiasu / rank	Yen / rank
Antigua & Barbuda			
Argentina	0.675 / 7	0.659 / 10	0.669 / 7
Bahamas			
Barbados			
Belize			
Bolivia	0.713 / 2	0.732 / 1	0.710 / 2
Brazil	0.610 / 13	0.599 / 13	0.606 / 13
Columbia	0.601 / 14	0.587 / 15	0.596 / 14
Costa Rica	0.695 / 4	0.679 / 5	0.689 / 4
Cuba			
Dominica			
Dominican Rep.	0.670 / 9	0.659 / 9	0.666 / 9
Ecuador	0.676 / 6	0.661 / 6	0.670 / 6
El Salvador	0.668 / 10	0.650 / 11	0.661 / 11

Table 8: Latin America and Caribbean Ranks(cont.)

Country	AHP / rank	Guiasu / rank	Yen / rank
Grenada			
Guatemala	0.599 / 15	0.589 / 14	0.595 / 15
Guyana			
Haiti	0.540 / 18	0.555 / 18	0.546 / 18
Honduras	0.584 / 16	0.580 / 16	0.582 / 16
Jamacia	0.708 / 3	0.695 / 3	0.703 / 3
Nicaragua	0.670 / 8	0.661 / 7	0.666 / 8
Panama	0.657 / 12	0.641 / 12	0.651 / 12
Paraguay	0.690 / 5	0.682 / 4	0.687 / 5
Peru	0.666 / 11	0.660 / 8	0.664 / 10
St Kitts and Nevis			
St. Lucia			
St Vincent and the Grenadines			
Suriname			
Uruguay	0.734 / 1	0.721 / 2	0.729 / 1
Venezuela	0.541 / 17	0.556 / 17	0.547 / 17

Middle East and North Africa

Table 9: Middle East and North Africa Ranks

Country	AHP / rank	Guiasu / rank	Yen / rank
Algeria	0.785 / 1	0.779 / 1	0.783 / 1
Bahrain			
Egypt	0.583 / 7	0.573 / 7	0.579 / 7
Iran	0.722 / 3	0.710 / 3	0.717 / 3
Iraq	0.740 / 2	0.738 / 2	0.739 / 2
Jordan	0.659 / 6	0.645 / 6	0.654 / 6
Kuwait			
Lebanon	0.707 / 4	0.701 / 4	0.705 / 4
Libya			
Morocco	0.700 / 5	0.688 / 5	0.695 / 5
Oman			
Qatar			
Saudi Arabia			
Syria			
Tunisia			
UAE			
Yemen			

Sub-Saharan Africa

Table 10: Sub-Saharan Africa Ranks

Country	AHP / rank	Guiasu / rank	Yen / rank
Angola	0.546 / 20	0.557 / 20	0.551 / 20

Table 10: Sub-Saharan Africa Ranks(cont.)

Country	AHP / rank	Guiasu / rank	Yen / rank
Benin	0.502 / 25	0.523 / 25	0.510 / 25
Botswana	0.468 / 33	0.455 / 35	0.463 / 33
Burkino Faso	0.641 / 5	0.662 / 4	0.649 / 5
Burundi	0.482 / 30	0.491 / 31	0.485 / 31
Cabo Verde	0.612 / 9	0.611 / 12	0.612 / 10
Cameroon	0.526 / 23	0.543 / 22	0.533 / 23
Central African Rep.	0.254 / 42	0.269 / 42	0.260 / 42
Chad	0.473 / 32	0.490 / 32	0.480 / 32
Comoros	0.544 / 21	0.530 / 24	0.538 / 21
Congo Dem. Rep.	0.494 / 28	0.517 / 26	0.503 / 26
Congo Rep.	0.427 / 38	0.438 / 38	0.431 / 38
Cote d'lvoire	0.594 / 14	0.609 / 13	0.560 / 19
Djibouti	0.601 / 12	0.599 / 17	0.600 / 12
Equatorial Guinea			
Eritrea			
Eswatini	0.357 / 40	0.342 / 41	0.351 / 40
Ethiopia	0.632 / 6	0.649 / 6	0.639 / 6
Gabon	0.592 / 15	0.588 / 19	0.591 / 17
Gambia	0.599 / 13	0.601 / 16	0.600 / 13
Ghana	0.652 / 3	0.665 / 5	0.657 / 3
Guinea	0.651 / 4	0.667 / 3	0.657 / 4
Guinea-Bissau			
Kenya	0.533 / 22	0.546 / 21	0.538 / 22
Lesotho	0.345 / 41	0.344 / 40	0.345 / 41
Liberia	0.584 / 18	0.617 / 10	0.597 / 14
Madagascar	0.453 / 35	0.470 / 33	0.460 / 34
Malawi	0.496 / 26	0.512 / 28	0.502 / 27
Mali	0.624 / 7	0.642 / 7	0.631 / 7
Mauritania	0.609 / 10	0.606 / 14	0.608 / 11
Mauritius	0.700 / 2	0.682 / 2	0.693 / 2
Mozambique	0.495 / 27	0.501 / 29	0.497 / 29
Namibia	0.460 / 34	0.450 / 36	0.456 / 35
Niger	0.606 / 11	0.630 / 9	0.616 / 9
Nigeria	0.358 / 39	0.382 / 39	0.367 / 39
Rwanda	0.481 / 31	0.498 / 30	0.488 / 30
Sao Tome & Principe	0.736 / 1	0.740 / 1	0.738 / 1
Senegal	0.586 / 17	0.604 / 15	0.593 / 16
Seychelles			
Sierra Leone	0.568 / 19	0.588 / 18	0.576 / 18
Somalia			
South Africa	0.442 / 37	0.431 / 37	0.438 / 37
South Sudan			
Sudan	0.523 / 24	0.535 / 23	0.528 / 24
Tanzania	0.616 / 8	0.635 / 8	0.624 / 8
Togo	0.490 / 29	0.519 / 27	0.501 / 28

Table 10: Sub-Saharan Africa Ranks(cont.)

Country	AHP / rank	Guiasu / rank	Yen / rank
Uganda	0.587 / 16	0.612 / 11	0.597 / 15
Zambia	0.443 / 36	0.456 / 34	0.448 / 36
Zimbabwe			

5 Fuzzy Similarity Measures and Conclusions

In this section, we briefly consider the fuzzy similarity measures we will be using.

Definition 5.1. *Let S be a function of $\mathcal{FP}(X) \times \mathcal{FP}(X)$ into $[0,1]$. Then S is called a* ***fuzzy similarity measure*** *on $\mathcal{FP}(X)$ if the following properties hold $\forall \mu, \nu, \rho \in \mathcal{FP}(X)$:*
(1) $S(\mu,\nu) = S(\nu,\mu)$;
(2) $S(\mu,\nu) = 1$ *if and only if* $\mu = \nu$;
(3) *If* $\mu \subseteq \nu \subseteq \rho$, *then* $S(\mu,\rho) \leq S(\mu,\nu) \wedge S(\nu,\rho)$;
(4) *If* $S(\mu,\nu) = 0$, *then* $\forall x \in X, \mu(x) \wedge \nu(x) = 0$.

We apply fuzzy similarity measures to rankings of a finite set. Suppose that X is a finite set with n elements. Let A be a one-to-0ne function of X into $\{1, 2, ..., n\}$. Then A is called a ranking of X. Define the fuzzy subset μ_A of X as follows: $\forall x \in X, \mu_A(x) = A(x)/n$. We wish to consider the similarity of two rankings of X by using fuzzy similarity measures. We use the two fuzzy similarity measures provided in the following Example.

Example 5.2. Let μ_A and μ_B be the fuzzy subsets of X associated with two rankings A and B, respectively. Then M and S below are fuzzy similarity measures.

$$M(\mu_A,\mu_B) = \frac{\sum_{x\in X} \mu_A(x) \wedge \mu_B(x)}{\sum_{x\in X} \mu_A(x) \vee \mu_B(x)};$$
$$S(\mu_A,\mu_B) = 1 - \frac{\sum_{x\in X} |\mu_A(x) - \mu_B(x)|}{\sum_{x\in X}(\mu_A(x) + \mu_B(x))}.$$

Theorem 5.3. (See [6]) *Let $n \in \mathbb{N}$ and*
(1) *Let n be even. Then the smallest value $M(\mu_A,\mu_B)$ can be is $\frac{n+2}{3n+2}$.*
(2) *Let n be odd. Then the smallest value $M(\mu_A,\mu_B)$ can be is $\frac{n+1}{3n-1}$.*
(3) *Let n be even. Then the smallest value $S(\mu_A,\mu_B)$ can be is $\frac{n/2+1}{n+1}$.*
(4) *Let n be odd. Then the smallest value $S(\mu_A,\mu_B)$ can be is $\frac{1}{2} + \frac{1}{2n}$.*

It follows that the quantity, the value of M minus the smallest value it can be, divided by the quantity 1 minus the smallest value M can be, is the percentage of the way M is from 0 to 1.

Let $n \in \mathbb{N}, n \geq 2$, and let X be a set. Let $\mathcal{FP}^n(X) = \{(\mu_1, ..., \mu_n) | \mu_i \in \mathcal{FP}(X), i = 1, ..., n\}$.

Definition 5.4. (See [8]) *Let $\widehat{S}$ be a function of $\mathcal{FP}^n(X)$ into $[0,1]$. Then $\widehat{S}$ is called an n-**dimensional fuzzy similarity measure** on $\mathcal{FP}(X)$ if the following properties hold:*
(1) $\widehat{S}(\mu_1, ..., \mu_n) = \widehat{S}(\mu_{\pi(1)}, ...\mu_{\pi(n)})$ *for any permutation π of* $\{1, ..., n\}$;
(2) $\widehat{S}(\mu_1, ..., \mu_n) = 1$ *if and only if* $\mu_1 = ... = \mu_n$;
(3) *If* $\mu_{i_1} \subseteq \mu_{i_2} \subseteq \mu_{i_3}$, *then* $\widehat{S}(..., \mu_{i_1}, ..., \mu_{i_3}, ...) \leq \widehat{S}(..., \mu_{i_1}, ..., \mu_{i_2}, ...) \wedge \widehat{S}(..., \mu_{i_2}, ..., \mu_{i_3}, ...)$;
(4) *If* $\widehat{S}(\mu_1, ..., \mu_n) = 0$, *then for all* $x \in X$, *there exists* $i \in \{1, ..., n\}$ *such that* $\mu_i(x) = 0$.

Example 5.5. (See [8]) Let $\mu_1, ..., \mu_n$ be fuzzy subsets of X. Then $\widehat{M}$ and $\widehat{S}$ are n-similarity fuzzy similarity measures, where

$$\begin{aligned}\widehat{M}(\mu_1, ..., \mu_n) &= \frac{\sum_{x\in X}\mu_1(x)\wedge ... \wedge \mu_n(x)}{\sum_{x\in X}\mu_1(x)\vee ... \vee \mu_n(x)};\\ \widehat{S}(\mu_1, ..., \mu_n) &= 1-\frac{\sum_{x\in X}(\vee\{\mu_j(x)|j=1,...,n\}-\wedge\{\mu_j(x)|j=1,...,n\})}{\sum_{x\in X}(\vee\{\mu_j(x)|j=1,...,n\}+\wedge\{\mu_j(x)|j=1,...,n\})}.\end{aligned}$$

Suppose we consider n elements and that they have been ranked twice 1 through n with no ties. We wish to consider their rankings using the above similarity operations. We can accomplish this by mapping the elements to their rank divided by n. For example, let X denote a set of n elements and if x is ranked i, then we define the fuzzy subset μ of X by $\mu(x) = \frac{i}{n}$. Let μ and ν be two such fuzzy subsets of X. Then

$$\widehat{M}(\mu,\nu) = \frac{\sum \mu(x_i)\wedge \nu(x_i)}{\sum \mu(x_i)\vee \nu(x_i)} = \frac{\sum n\mu(x_i)\wedge n\nu(x_i)}{\sum n\mu(x_i)\vee n\nu(x_i)}.$$

Consequently, there is no loss in generality in assuming that we are measuring the similarity of two rankings using the integers, $1, ..., n$. The notion can be extended from 2 rankings to any finite number of rankings.

Let m and n be positive integers such that $2 \leq m \leq n$. Then there exist positive integers q and r such that $n = qm + r$, where $0 \leq r < m$.

Theorem 5.6. (See [8]) *The smallest value $\widehat{M}$ can be is* $\frac{m(\frac{(q+1)q}{2})+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)}$.

Theorem 5.7. (See [8]) $\widehat{S} = \frac{2\widehat{M}}{1+\widehat{M}}$.

Corollary 5.8. (See [8]) *The smallest value $\widehat{S}$ can be is $\frac{2a}{1+a}$, where a is the smallest value $\widehat{M}$ can be.*

Let $\widehat{m} = 3$. It is shown in [8] that the values for $\widehat{M}$ and $\widehat{S}$ can be converted to the case where $m = 2$ by the following formulas

$$\begin{aligned}M &= \frac{5}{6}\widehat{M} + \frac{1}{6},\\ S &= \frac{3}{4}\widehat{S} + \frac{1}{4}.\end{aligned}$$

We next provide the similarity measures for the regions.μ_1, μ_2,and μ_3 denote AHP, Guiasu, and Yen, respectively.

For OECD, $\widehat{M}(\mu_1, \mu_2, \mu_3) = \frac{639}{686} = 0.931$ and $\widehat{S}(\mu_1, \mu_2, \mu_3) = 1 - \frac{47}{1325} = 0.965$. Here $n = 36, m = 3, q = 12$, and $r = 0$. The smallest $\widehat{M}$ can be is $[\frac{m(q+1)q}{2} + r(q+1)]/[\frac{m(2qn+q-q^2)}{2} + r(n-q)] = \frac{(13)(12)}{2(12)(36)+12-144} = \frac{156}{732} = 0.213$. The smallest $\widehat{S}$ can be is $\frac{2(0.213)}{1+0.213} = 0.351$. Now $\frac{\widehat{M}-0.213}{1-0.213} = \frac{0.931-0.213}{1-0.213} = \frac{0.718}{0.787} = 0.912$ and $\frac{\widehat{S}-0.351}{1-0.351} = \frac{0.965-0.351}{1-0.351} = \frac{0.614}{0.649} = 0.946$.

For East and South Asia, $\widehat{M}(\mu_1, \mu_2, \mu_3) = \frac{146}{160} = 0.9125$ and $\widehat{S}(\mu_1, \mu_2, \mu_3) = 1 - \frac{14}{306} = 0.954$. Here $n = 17, m = 3, q = 5$, and $r = 2$. The smallest $\widehat{M}$ can be is $[\frac{m(q+1)q}{2} + r(q+1)]/[\frac{m(2qn+q-q^2)}{2} + r(n-q)] = \frac{45+12}{225+24} = 0.229$. The smallest $\widehat{S}$ can be is $\frac{2(0.229)}{1+0.229} = 0.373$. Now $\frac{\widehat{M}-0.229}{1-0.229} = \frac{0.912-0.229}{1-0.229} = \frac{0.683}{0.771} = 0.886$ and $\frac{\widehat{S}-0.373}{1-0.373} = \frac{0.954-0.373}{1-0.373} = \frac{0.581}{0.627} = 0.927$.

For Eastern Europe and Central Asia, $\widehat{M}(\mu_1,\mu_2,\mu_3)=\frac{228}{234}=0.974$ and $\widehat{S}(\mu_1,\mu_2,\mu_3)=1-\frac{6}{462}=0.987$. Here $n=21, m=3, q=7$, and $r=0$. The smallest $\widehat{M}$ can be is $[\frac{m(q+1)q}{2}+r(q+1)]/[\frac{m(2qn+q-q^2)}{2}+r(n-q)]=\frac{8(7)}{14(21)+7-49}=\frac{56}{252}=0.222$. The smallest $\widehat{S}$ can be is $\frac{2(0.222)}{1+0.222}=0.363$. Now $\frac{\widehat{M}-0.222}{1-0.222}=\frac{0.974-0.222}{1-0.222}=\frac{0.752}{0.778}=0.967$ and $\frac{\widehat{S}-0.363}{1-0.363}=\frac{0.987-0.363}{1-0.363}=\frac{0.624}{0.637}=0.980$.

For Latin America and the Caribbean, $\widehat{M}(\mu_1,\mu_2,\mu_3)=\frac{164}{178}=0.921$ and $\widehat{S}(\mu_1,\mu_2,\mu_3)=1-\frac{14}{342}=0.959$. Here $n=18, m=3, q=6$, and $r=0$. The smallest $\widehat{M}$ can be is $[\frac{m(q+1)q}{2}+r(q+1)]/[\frac{m(2qn+q-q^2)}{2}+r(n-q)]=\frac{7(6)}{216-30}=0.226$. The smallest $\widehat{S}$ can be is $\frac{2(0.226)}{1+0.226}=0.367$. Now $\frac{\widehat{M}-0.226}{1-0.266}=\frac{0.921-0.226}{1-0.226}=\frac{0.695}{0.774}=0.898$ and $\frac{\widehat{S}-0.367}{1-0.367}=\frac{0.959-0.367}{1-0.367}=\frac{0.592}{0.633}=0.935$.

For Middle East and North Africa, there wasn't sufficient data available.

For Sub-Sahran Africa, $\widehat{M}(\mu_1,\mu_2,\mu_3)=\frac{869}{942}=0.923$ and $\widehat{S}(\mu_1,\mu_2,\mu_3)=1-\frac{73}{1811}=0.960$. Here $n=42, m=3, q=13$, and $r=0$. The smallest $\widehat{M}$ can be is $[\frac{m(q+1)q}{2}+r(q+1)]/[\frac{m(2qn+q-q^2)}{2}+r(n-q)]=\frac{15(14)}{1176-182}=\frac{210}{994}=0.211$. The smallest $\widehat{S}$ can be is $\frac{2(0.211)}{1+0.221}=0.346$. Now $\frac{\widehat{M}-0.211}{1-0.211}=\frac{0.923-0.211}{1-0.211}=\frac{0.712}{0.789}=0.902$ and $\frac{\widehat{S}-0.346}{1-0.346}=\frac{0.960-0.346}{1-0.346}=\frac{0.614}{0.654}=0.939$.

6 SDG Achievement vs Number of Homeless

In [5], the number of homeless people per country was given. We ranked the countries according to homeless per $10,000$. The fewer the homeless the higher the rank. We do not present the rankings here. We then found the similarity between this ranking and the ranking of countries according to their achievement of the SDGs given in the above tables.

For OECD, $M(SDG,H)=\frac{398}{724}=0.550$ and $S(SDG,H)=1-\frac{328}{1122}=0.708$. Here $n=33$. The smallest M can be is $\frac{n+1}{3n-1}=\frac{34}{98}=0.347$ and the smallest S can be is $\frac{1}{2}+\frac{1}{2n}=\frac{1}{2}+\frac{1}{66}=0.515$. Now $\frac{M-0.347}{1-0.347}=\frac{0.550-0.347}{1-0.347}=\frac{0.203}{0.653}=0.311$ and $\frac{S-0.515}{1-0515}=\frac{0.708-0.515}{1-0.515}=\frac{0.193}{0.485}=0.398$.

For East and South Asia, $M(SDG,H)=\frac{34}{56}=0.607$ and $S(SDG,H)=1-\frac{22}{90}=0.756$. Here $n=9$. The smallest M can be is $\frac{n+1}{3n-1}=\frac{10}{28}=0.357$ and the smallest S can be is $\frac{1}{2}+\frac{1}{2n}=\frac{1}{2}+\frac{1}{18}=0.556$. Now $\frac{M-0.357}{1-0.357}=\frac{0.607-0.357}{1-0.357}=\frac{0.250}{0.643}=0.389$ and $\frac{S-0.556}{1-0.556}=\frac{0.756-0.556}{1-0.556}=\frac{0.200}{0.444}=0.450$.

For Eastern Europe and Central Asia, $M(SDG,H)=\frac{26}{46}=0.565$ and $S(SDG,H)=1-\frac{18}{72}=0.750$. Here $n=8$. The smallest M can be is $\frac{n+2}{3n+2}=\frac{10}{26}=0.385$ and the smallest S can be is $\frac{n/2+1}{n+1}=\frac{5}{9}=0.556$. Now $\frac{M-0.385}{1-0.385}=\frac{0.565-0.385}{1-0.385}=\frac{0.180}{0.615}=0.293$ and $\frac{S-0.556}{1-0.556}=\frac{0.750-556}{1-0.556}=\frac{0.194}{0.444}=0.437$.

For Latin America and the Caribbean, $M(SDG,H)=\frac{19}{23}=0.828$ and $S(SDG,H)=1-\frac{4}{42}=0.901$. Here $n=6$. The smallest M can be is $\frac{n+2}{3n+2}=\frac{8}{20}=0.400$ and the smallest S can be is $\frac{n/2+1}{n+1}=\frac{4}{7}=0.571$. Now $\frac{M-0.400}{1-0.400}=\frac{0.828-0.400}{1-0.400}=\frac{0.428}{0.600}=0.713$ and $\frac{S-0.571}{1-0.571}=\frac{0.901-0.571}{1-0.571}=\frac{0.330}{0.430}=0.767$.

For the Middle East and North Africa, there wasn't sufficient data available.

For Sub-Saharan Africa, $M(SDG,H)=\frac{106}{166}=0.639$ and $S(SDG,H)=1-\frac{60}{272}=0.779$. Here $n=16$. The smallest M can be is $\frac{n+2}{3n+2}=\frac{18}{50}=0.360$ and the smallest S can be is $\frac{n/2+1}{n+1}=\frac{9}{17}=0.529$. Now $\frac{M-0.360}{1-0.360}=\frac{0.639-0.360}{1-0.360}=\frac{0.279}{0.640}=0.436$ and $\frac{S-0.529}{1-0.529}=\frac{0.779-0.529}{1-0.529}=\frac{0.250}{0.471}=0.531$.

7 Conclusion

In this paper, we considered those Sustainable Development Goals which are most pertinent to homelessness. We ranked countries with respect to the achievement of these goals. We used fuzzy similarity measures to determine the degree of similarity between these rankings. We used three methods to rank the counties, namely, the Analytic Hierarchy Process, the Guiasu method, and the Yen method. We found that the similarity measures were very high. We also determined the similarity measure between a ranking of a country's number of homelessness and the ranking of countries according to their achievement of the SDGs. We found that similarity ranged from medium to high depending on the region involved.

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