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Optimization of Composite Structures in Vibration View

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Abstract

Two dimensional analysis of fluid flow and heat transfer through the trapezoidal microchannel has been done. The energy and Nervier stokes equations have been solved considering the wall slip velocity and temperature jump using the Lattice Boltzmann method (LBM). The relation between relaxation times and Knudsen number has been derive in LBM. The effects of Reynolds number, Nusslelt number, temperature jump and velocity slip on different Knudsen number ($0.001<Kn<0.1$) and different aspect ratios ($0.2<AR<1.2$) were investigated. The good agreement between results and earlier studies were found. Results show the important effect of AR and Knudsen number on Nusselt number in trapezoidal microchannel. At low Reynolds number the significant influence on Nusselt number has been seen.

***Keywords*: **Genetic Algorithm, optimization, composite laminate, vibration

1. Introduction

The important properties of composite materials, including low weight and high strength, have led to significant progress in its use in various industries. The first people who used these materials were the ancient Egyptians who used straw to build their buildings. The Palestinians also used reeds and mats to make bricks. In five hundred years before Christ, polymer was used to coat boats with bitumen. became common. Composite material engineering has grown a lot in recent years, even genetic algorithm science is used to improve these materials. But why is the genetic algorithm used in the optimization of composite materials? The genetic algorithm is a very strong and powerful tool for solving complex problems and can be used as a suitable substitute for expensive optimization tools. In recent years, great research works have been done using genetic algorithm and this case is always growing. All of them have a purposeful idea and that is to reduce computing costs. Optimizing and choosing the right materials in practical structures is of great use today. Although the ratio of high strength to weight properties of composite materials is very significant, but their biggest advantages are that designers can increase the strength and hardness of a composite material; Apply a lot of environmental loads on the structure. For layered composite structures, each layer has the greatest properties of hardness and strength in the direction of the longitudinal axis of the fibers. By changing the direction of the fibers of each layer at different angles based on different environmental loads, a special design can be made. However, from the negative point of view, composite structures have a very high cost for construction, and so far there is less information about useful life-repair or maintenance instructions. Compared to other optimization methods, the genetic algorithm is an ideal method for solving these types of problems.

2. classical lamination Theory of Composite Materials

Classical Laminate theory has been extensively used to describe the behavior of composite materials under mechanical, thermal loading conditions. Many references are available where classical lamination theory is utilized to describe composite material behavior. In Classical Lamination Theory, the plate is assumed to have infinite dimensions and the whole panel undergoes the same thermal gradients. In this process however the heat is applied locally, and the rest of the panel effectively acts as a barrier to curvature growth. At the completion of every layer or cycle, every point on the panels surface experiences the same thermal history at different times. As a result, the thermal signature of this process is experienced everywhere but at an offset time corresponding to the time difference between each point on the surface. Since Classical Lamination Theory is time independent, this model can be used as a method for determining the stress fields through the thickness as long as the panel is sufficiently large. One important difference however is that since the heated region is local, the solid boundary acts as a mechanism for constraint. This unique boundary is included in the model as a partial constraint and the method by which it is applied to the model is explained later. The effect of partial constraint is not as significant in the simplified residual stress model as the heat input is restricted to the surface ply and the remaining preconsolidated material acts as a similar partial constraint. In the second-generation model, the effect of partial constraint becomes a more critical mechanism for limiting the curvature growth as the actual heat input is applied through the thickness of the laminate. This increases the difference in absolute temperatures between the cooler solid boundary and the temperatures within the heat-affected zone.

Material Properties

3.**Material Properties**

In addition to the stacking sequence of the laminate, the material properties of the composite material must be defined. The following properties must be defined: - Mechanical Elasticity (E_{11}, E_{22}, G_{12} and υ_{12}) - Environmental Elasticity (α_{11}, α22, β_{11}, β_{22}) which represent thermal and moisture expansion, respectively.

4.Mechanical and Environmental Loads

Finally, the mechanical and Environmental loads must be defined: - Normal Forces (Nxx, Nyy, Nxy) - Moment (twisting) forces (Mxx, Myy, Mxy) - Environmental (ΔT and ΔM in Celsius and % Moisture, respectively)

5.The ABD Matrix

The ABD matrix is a 6x6 matrix that serves as a connection between the applied loads and the associated strains in the laminate. It essentially defines the elastic properties of the entire laminate. To assemble the ABD matrix, follow these steps:

5.1. Calculate reduced stiffness matrix Q_{ij} for each material used in the laminate (if a laminate uses only one type of composite material, there will be only 1 stiffness matrix). The stiffness matrix describes the elastic behavior of the ply in plane loading

$$Q_{ij} = \begin{vmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{vmatrix} \qquad (1)$$

Where

$$Q_{11}=\frac{E_{11}^2}{(E_{11}-\vartheta_{12}.E_{22})} \qquad (2)$$

$$Q_{12}=\frac{\vartheta_{12}.E_{11}.E_{22}}{(E_{11}-\vartheta_{12}^2.E_{22})} \qquad (3)$$

$$Q_{22}=\frac{E_{11}.E_{22}}{(E_{11}-\vartheta_{12}^2.E_{22})} \quad (4)$$

$$Q_{66}=G_{12} \qquad (5)$$

5.2. Calculate the transformed reduced stiffness matrix Q_{ij} for each ply based on the reduced stiffness matrix and fiber angle.

Where

$$\overline{Q_{11}} = Q_{11}\cos(\theta)^4 + 2(Q_{12}+2Q_{66})\cos(\theta)^2.\sin(\theta)^2 + Q_{22}\sin(\theta)^4 \qquad (6)$$

$$\overline{Q_{12}} = \overline{Q_{21}} = Q_{12}(\cos(\theta)^4+\sin(\theta)^4) + (Q_{11}+Q_{22}-4Q_{66})\cos(\theta)^2\sin(\theta)^2 \qquad (7)$$

$$\overline{Q_{16}} = \overline{Q_{61}} = (Q_{11}-Q_{12}-2Q_{66})\cos(\theta)^3\sin\theta\text{-}(Q_{22}-Q_{12}-2Q_{66})\cos(\theta)\sin(\theta)^3 \qquad (8)$$

$$\overline{Q_{22}} = Q_{11}\sin(\theta)^4 + 2(Q_{12}+Q_{66})\cos(\theta)^2\sin(\theta)^2 + Q_{22}\cos(\theta)^4 \qquad (9)$$

$$\overline{Q_{26}} = \overline{Q_{62}} = (Q_{11} - Q_{12} - 2Q_{66}) \cos\theta \sin(\theta)^3 \text{-} (Q_{22} - Q_{12} - 2Q_{66}) \cos(\theta)^3 \sin\theta \quad (10)$$
$$\overline{Q_{66}} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) \cos(\theta)^2 \sin(\theta)^2 + Q_{66(} \cos(\theta)^4 + \sin(\theta)^4) \quad (11)$$

$$\overline{Q_{ij}} = \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} \end{bmatrix} \quad (12)$$

5.3. Calculate the A_{ij}, B_{ij}, D_{ij} matrices using the following equations where z represents the vertical position in the ply from the midplane measured in meters

$$A_{ij} = \sum_{k=1}^{n} \{Q_{ij}\}_n (Z_k - Z_{k-1}) \ (13)$$
$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} \{Q_{ij}\}_n (Z_k^2 - Z_{k-1}^2) (14)$$
$$C_{ij} = \frac{1}{3} \sum_{k=1}^{n} \{Q_{ij}\}_n (Z_k^3 - Z_{k-1}^3) (15)$$

5.4. **Assemble ABD**:

$$\boldsymbol{ABD} = \begin{bmatrix} A & B \\ B & C \end{bmatrix} \quad (16)$$

6. Calculate inverse of ABD:

abd= $[ABD]^{-1} = \begin{bmatrix} A & B \\ B & C \end{bmatrix}^{-1}$ (17)

Calculate thermal and moisture expansion coefficients for each ply: Calculate the effective thermal and moisture expansion coefficients for each ply:

$$\alpha_{xx} = \alpha_{11} \cos(\theta)^2 + \alpha_{22} \sin(\theta)^2 \quad (18)$$
$$\alpha_{yy} = \alpha_{11} \sin(\theta)^2 + \alpha_{22} \cos(\theta)^2 \quad (19)$$
$$\alpha_{xy} = 2 \cos\theta \sin\theta \, (\alpha_{11} - \alpha_{22}) \quad (20)$$
$$\beta_{xx} = \beta_{11} \cos(\theta)^2 + \beta_{22} \sin(\theta)^2 \quad (21)$$
$$\beta_{yy} = \beta_{11} \sin(\theta)^2 + \beta_{22} \cos(\theta)^2 \quad (22)$$
$$\beta_{xy} = 2 \cos\theta \sin\theta \, (\beta_{11} - \beta_{22}) \quad (23)$$

Calculate thermal and moisture stress and moment resultants:

Thermal Resultants:

$$N_{xx}^T = \Delta T \ \sum_{k=1}^{n} \left\{ \left[\overline{Q_{11}} \alpha_{xx} + \overline{Q_{12}} \alpha_{yy} + \overline{Q_{16}} \alpha_{xy} \right]_k [z_k - z_{k-1}] \right\} \quad (24)$$
$$N_{yy}^T = \Delta T \ \sum_{k=1}^{n} \left\{ \left[\overline{Q_{12}} \alpha_{xx} + \overline{Q_{22}} \alpha_{yy} + \overline{Q_{26}} \alpha_{xy} \right]_k [z_k - z_{k-1}] \right\} \quad (25)$$
$$N_{xy}^T = \Delta T \ \sum_{k=1}^{n} \left\{ \left[\overline{Q_{16}} \alpha_{xx} + \overline{Q_{26}} \alpha_{yy} + \overline{Q_{66}} \alpha_{xy} \right]_k [z_k - z_{k-1}] \right\} \quad (26)$$
$$M_{xx}^T = \frac{\Delta T}{2} \sum_{k=1}^{n} \left\{ \left[\overline{Q_{11}} \alpha_{xx} + \overline{Q_{12}} \alpha_{yy} + \overline{Q_{16}} \alpha_{xy} \right]_k [z_k^2 - z_{k-1}^2] \right\} \quad (27)$$
$$M_{yy}^T = \frac{\Delta T}{2} \sum_{k=1}^{n} \left\{ \left[\overline{Q_{12}} \alpha_{xx} + \overline{Q_{22}} \alpha_{yy} + \overline{Q_{26}} \alpha_{xy} \right]_k [z_k^2 - z_{k-1}^2] \right\} \quad (28)$$
$$M_{xy}^T = \frac{\Delta T}{2} \sum_{k=1}^{n} \left\{ \left[\overline{Q_{16}} \alpha_{xx} + \overline{Q_{26}} \alpha_{yy} + \overline{Q_{66}} \alpha_{xy} \right]_k [z_k^2 - z_{k-1}^2] \right\} \quad (29)$$

6.1. Calculate midplane strains and curvatures induced in the laminate. These represent the deflections of the laminate

$$\begin{bmatrix}\varepsilon_{xx}^0\\ \varepsilon_{yy}^0\\ \varepsilon_{xy}^0\\ \kappa_{xx}^0\\ \kappa_{yy}^0\\ \kappa_{xy}^0\end{bmatrix}=\begin{bmatrix}a_{11}&a_{12}&a_{16}&b_{11}&b_{12}&b_{16}\\ a_{12}&a_{22}&a_{26}&b_{12}&b_{22}&b_{26}\\ a_{16}&a_{26}&a_{66}&b_{16}&b_{26}&b_{66}\\ b_{11}&b_{12}&b_{16}&d_{11}&d_{12}&d_{16}\\ b_{12}&b_{22}&b_{26}&d_{12}&d_{22}&d_{26}\\ b_{16}&b_{26}&b_{66}&d_{16}&d_{26}&d_{66}\end{bmatrix}.\begin{bmatrix}N_{xx}+N_{xx}^T+N_{xx}^M\\ N_{yy}+N_{yy}^T+N_{yy}^M\\ N_{xy}+N_{xy}^T+N_{xy}^M\\ M_{xx}+M_{xx}^T+M_{xx}^M\\ M_{yy}+M_{yy}^T+M_{yy}^M\\ M_{xy}+M_{xy}^T+M_{xy}^M\end{bmatrix} \quad (30)$$

6.2. For each ply

a. Calculate ply strains in the x-y coordinate system

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\epsilon_{xx}^0\\ \epsilon_{yy}^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}\kappa_{xx}\\ \kappa_{yy}\\ \kappa_{xy}\end{Bmatrix} \quad (31)$$

b. Calculate ply stresses in the x-y coordinate system

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix}=\begin{bmatrix}\overline{Q_{11}}&\overline{Q_{12}}&\overline{Q_{16}}\\ \overline{Q_{12}}&\overline{Q_{22}}&\overline{Q_{26}}\\ \overline{Q_{16}}&\overline{Q_{26}}&\overline{Q_{66}}\end{bmatrix}.\begin{Bmatrix}\varepsilon_{xx}-\Delta T\alpha_{xx}-\Delta M\beta_{xx}\\ \varepsilon_{yy}-\Delta T\alpha_{yy}-\Delta M\beta_{yy}\\ \varepsilon_{xy}-\Delta T\alpha_{xy}-\Delta M\beta_{xy}\end{Bmatrix} \quad (32)$$

7. Genetic Algorithm(GA)

The idea of genetic algorithm was thought to have been proposed by John Holland at the University of Michigan in the 1970s. Holland was interested in using the laws of natural selection to develop artificial systems rather than systems based on some reasoning process. These artificial systems can be built using computer software and applied in various disciplines that emphasize design, optimization, and machine learning.

Basic Structure of Genetic Algorithms

GAs are probabilistic algorithms that mimic natural selection processes by the concept of survival testing. The main element of a GA is an organism that is usually composed of a certain number of chromosomes. In turn, each chromosome may consist of one or more genes. Typically, each gene of a chromosome is encoded using a binary alphabet. , which indicates whether a gene is present or not. Active (indicated by 1) or inactive (indicated by 0). Other representations have used gene alphabets with many more elements or multiple gene alphabets for different types of genes. The complexity of an organism can be controlled by the length and number of chromosomes and gene strands and the size and number of gene alphabets. A genetic algorithm usually consists of a group of organisms, usually known as a subpopulation or population of organisms. If there is more than one group of organisms, then each group is called a subpopulation. A group of sub-populations is called a population. Such terms are often used when discussing parallel genetic algorithms. A parallel GA invokes several subpopulations at once (usually implemented on a parallel computer) and allows each to migrate toward an optimal solution. In some methods, the subpopulations are allowed to interact with One is the other to improve GA performance. If there is only one group of organisms, the terms subpopulation and population are synonymous. The research done in this work is only with one group of organisms and henceforth it is called population. Here is a schematic of a typical GA structure

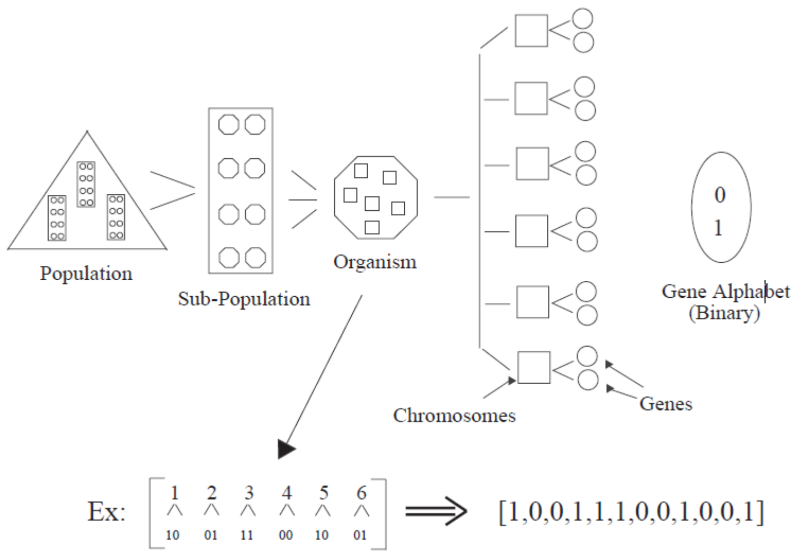

Figure 1. Multi-Objective Optimization: Simultaneous Cost and Weight Minimization of a Simply Supported CompositePlate

The weight is calculated as

$$W=ab[\rho_1 t_1 N_1+ \rho_2 t_2 N_2] \quad (33)$$

where a and b are the dimensions of the plate, ρ is the material density, t is t thicknesses, and Nis the number of plies for each material of glass-epoxy and graphite-epoxy in the laminate stacking sequence, respectively. The cost of a laminate is based on two quantities: material cost and lay-up cost. The material cost for a laminate, C_m, is determined by multiplying the weight of each material in a laminate by its corresponding cost factor (C_f) given in

$$C_m=ab[C_1\rho_1 t_1 N_1+ C_2\rho_2 t_2 N_2] \quad (34)$$

Lay-up cost, Cl, is based on the amount of time required by the lay-up machine to construct eachlaminate. Data was obtained from a standardized manufacturing process relating ply orientationangle and plate dimensions. However, since the dimensions of the plate are the same for eachlaminate, lay-up cost becomes a function of ply orientation angle only. The analysis procedure used to compute the layup cost cannot be revealed in this document because they are company proprietary information., but the total cost for a laminate can now be determined by adding the correspondingmaterial and lay-up costs:

$$CT = Cm + Cl \quad (35)$$

8. Results

All the investigations that have been done so far are based on how to increase the vibration frequency of a plate as much as possible by using the genetic algorithm and taking into account the geometry of the object and the mechanical properties.
In the following, we discuss the vibration analysis of a plate with specific geometric and mechanical characteristics.

- The geometric characteristics of the sheet, including length, width, and height, remain constant throughout the discussion
- The number of layers is fixed
- The material of the sheet is specified
- The dimensions of both layers are fixed

Plate specification:

Table 1. The effect of changing the angle between the longitudinal direction of the fibers and the direction of force application on the vibration frequency

	Glass-epoxy	**Graphite- epoxy**
E_1(N/mm^2)	54000	207000
E_2(N/mm^20	18000	5000
υ_{12}	0.25	0.25
υ_{21}	0.08333	0.006038647
G_{12} (N/mm^20	9000	2600
a (mm)	900	900
b (mm)	750	750
t (mm)	1	1
ρ (Kg/mm^3)	3.0E-08	5.0E-08
θ (Deg.)	45	45
Q_{11}	5.515E+04	2.073E+05
Q_{12}	4.596E+03	1.252E+03
Q_{22}	1.838E+04	5.008E+03
Q_{66}	9.00E+03	2.600E+03
$\overline{Q_{11}}$	4.064E+04	5.631E+04
$\overline{Q_{12}}$	1.147E+04	5.111E+04
$\overline{Q_{22}}$	3.309E+4	5.631E+04
$\overline{Q_{66}}$	1.431E+4	5.245E+04

Our design problem was the minimization of the weight and the cost of a rectangularlaminated plate of length a =900 mmin and width $b = 750\ mm$, subject to a constraint on thefirst natural frequency. The first natural frequency f of a rectangular plate is given bythe following expression

$$f = \frac{\pi}{2\sqrt{\rho h}}\sqrt{\frac{D_{11}}{a^4} + 2\frac{D_{12}+2D_{66}}{a^2 b^2} + \frac{D_{22}}{b^4}} \quad (36)$$

where D is the flexural stiffness matrix calculated using the Classical Lamination. Theory according the table 1.,ρ is the average density and h the total thickness of the plate.

Table 2. flexural stiffness matrix

D_{11}	3.23E+04
D_{12}	2.09E+04
D_{22}	2.98E+04
D_{66}	2.23E+04

A two-material composite made of graphite-epoxy and glass-epoxy was considered. The stiffness-to-weight ratio of graphite-epoxy is about four times higher than that of glass-epoxy, with $E1/\ \rho= 345$ against $E1/\ \rho= 87.5$.
However, it is also more expensive, with a cost per pound that is 8 times higher than that of glass-epoxy. If the first priority is weight, then graphite-epoxy will be preferred; while if cost is paramount the optimum laminate will obviously contain glass-epoxy plies.

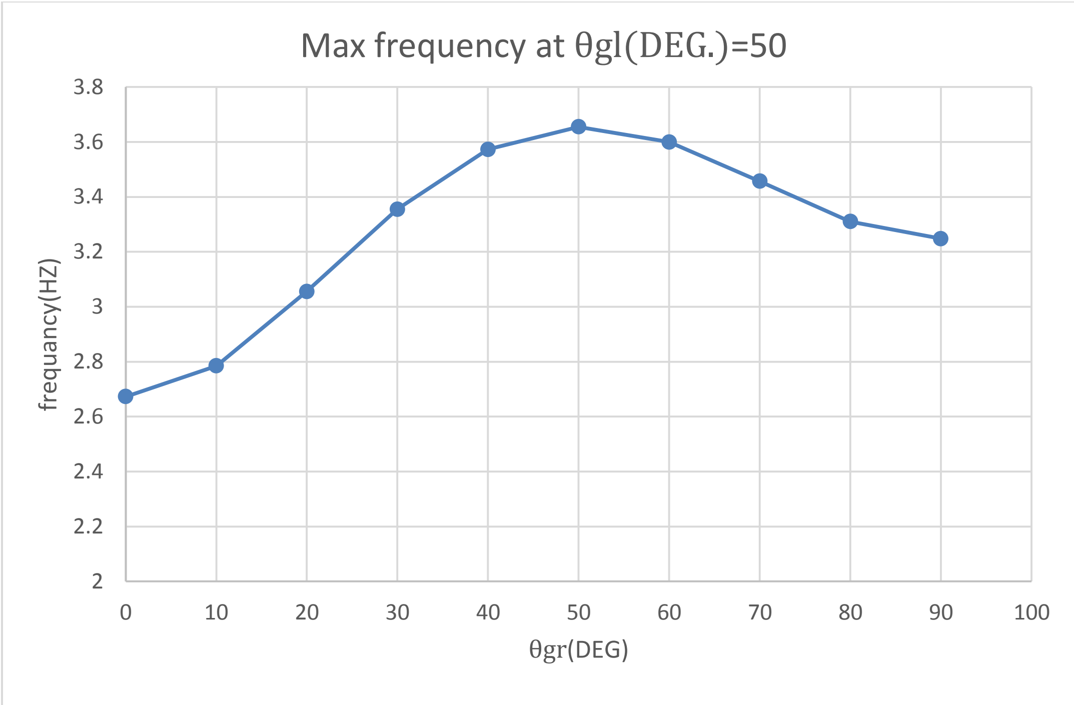

Figure2. The maximum frequency according to the change of the angle of the fibers and the direction of the force

The relationship between sheet dimensions and frequency: If we represent the ratio of length to width of the sheet with R(R=a/b)

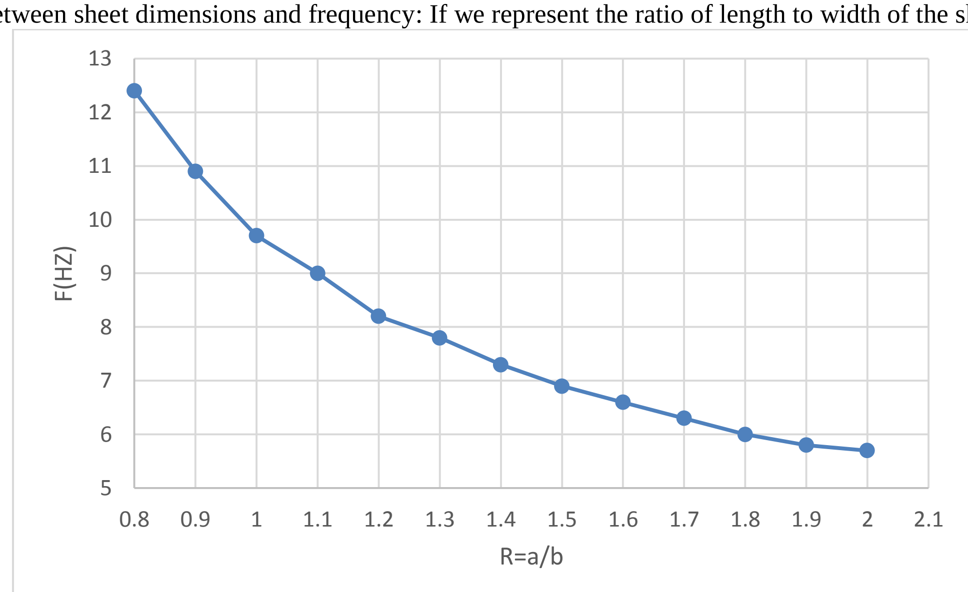

Fig. 3. The relationship between frequency and R

9. Conclusion

From the totality of all investigations and studies, and considering that the goal is to increase the forced frequency of a sheet with known characteristics, the ratio of plate length to width (R=a/b) is effective in changing the maximum frequency. As per the figure3. the best value of R is 1.2 to 1.4. The highest frequency is when the angle between the longitudinal direction of the fiber and the direction of force application is about 45 degrees. According to the frequency relation in symmetrical sheets with respect to the middle plane, there is no relationship between the frequency value and the axial stiffness matrix, and the frequency is only a function of the bending stiffness.

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