

# A Discrete Hybrid Teaching-Learning-Based Optimization Algorithm for Optimization of Space Trusses

Siyamak Talatahari<sup>\*,a</sup>, Vahid Goodarzimehr<sup>b</sup>

<sup>a</sup>Associate professor of Structural Engineering at University of Tabriz, Iran

<sup>b</sup>Master of science in Structural Engineering from Tabriz

Received 23 July 2018, Accepted 29 November 2018

## Abstract

In this study, to enhance the optimization process, especially in the structural engineering field, two well-known algorithms are merged together in order to achieve an improved hybrid algorithm. These two algorithms are Teaching-Learning Based Optimization (TLBO) and Harmony Search (HS) which have been used by most researchers in varied fields of science. The hybridized algorithm is called A Discrete Hybrid Teaching-Learning Based Optimization (DHTLBO) that is applied to the optimization of truss structures with discrete variables. This new method consists of two parts: in the first part, the TLBO algorithm applied as conventional TLBO for local optimization, in the second stage the HS algorithm is applied to global optimization and exploring all the unknown places in the search space. The new hybrid algorithm is employed to minimize the total weight of structures. Therefore, the objective function consists of the member's weight, which depends on the form of stress and deflection limits. To demonstrate the efficiency and robustness of this new algorithm several truss structures which are optimized by most researchers are presented and then their results are compared to other meta-heuristic algorithm and TLBO and HS standard algorithms.

**Keywords:** Discrete variables, Teaching- learning- based optimization, Harmony search; Size optimization, Truss structures, Structural optimization, Meta-heuristic algorithm.


## 1. Introduction

Recently, many optimization algorithms have been developed to optimize the structural problems. Some of them use mathematical methods and another uses Meta-Heuristic methods to reach the best solution. both of these methods are basically developed for continuous optimization. but, as you know in structural engineering always this is not acceptable. because in most structural problems the design variables are defined as discrete variables.

Due to above-mentioned reason some of the most popular optimization algorithms have been promoted to discrete optimization of structural problems, such as; Genetic Algorithm (GAs) [1-2] is model the process of natural evolution, Steady-state genetic algorithms [3] which are developed by Wu and Chow, Ant Colony Optimization

(ACO) [4] is developed for discrete optimization of space trusses, the HS algorithm [5-6] is based on the Harmony Search algorithm, Big Bang-Big Crunch (BB-BC) [7-8] is developed for both discrete problems, Teaching-Learning-Based

Optimization (TLBO) [9] inspired from a class that consists of some students and a teacher, the teacher tries to teach the students and the students try to share their knowledge with each other in order to promote the class level, Artificial Bee Colony algorithm (ABC) [10] imitated the honey bees' behavior, Charged System Search (CSS) [11] uses Newtonian laws of mechanics. Meshki and Joghataie [17] uses the spherical interpolation of objective function and constraints, Colliding bodies optimization (CBO) [18]. Vibrating Particles System (VPS) [19], krill herd (KH) [20], Whale Optimization Algorithm (WOA) [21], A Hybrid Harmony Search [22], A hybrid algorithm

 \*Corresponding Author: Email address: siamak.talat@gmail.com

based on TLBO[23], TLBO[24], force method and genetic algorithm [25], Water cycle, mine blast and improved mine blast algorithms [26]. These methods have been used in a variety field of science because they are easily simulated engineering problems. Another advantage is that they would implement for any optimization problem and possess a fast speed to achieve the optimum design than previous optimization methods.

In a discrete optimization method, we have to use a set of variables or those determined by the design codes. In practice, due to high computing costs and much consuming time, discrete optimization is difficult. But, using the penalty function which is combined with stress and displacement constraint, initially, the optimization problem will be defined as a continuous problem with the upper and lower bounds and then the discrete design begins.

$$\begin{aligned}
 & \text{Find} && A = [A_1, A_2, \dots, A_n] \\
 & && A \in D_i, D_i = \{d_{i,1}, d_{i,2}, \dots, d_{i,r(i)}\} \\
 & \text{to minimize} && W = \sum_{e=1}^{N_m} \gamma_e \cdot l_e \cdot A_e && (1) \\
 & \text{subject to} && \sigma^l < \sigma_e < \sigma^u \\
 & && \delta^l < \delta_e < \delta^u
 \end{aligned}$$

Where W is the Nm members weight of truss for each member e;  $\gamma_e$  is the unit weight;  $l_e$  is the length of each member;  $A_e$  is the cross-sectional area. This minimum design also has to meet the constraints on each member's stress  $\sigma$  and deflection  $\delta_c$  at each connection.

$$\text{if } \sigma^l < \sigma_e < \sigma^u \text{ then } \varphi_\sigma^e = 0 \tag{2}$$

$$\text{if } \sigma_e < \sigma^l \text{ or } \sigma_e > \sigma^u \text{ then } \varphi_\sigma^e = \left| \frac{\sigma_e - \sigma^{l,u}}{\sigma^{l,u}} \right| \tag{3}$$

The stress penalty  $\phi_\sigma^k$  for a truss design is as follow:

$$\varphi_\sigma^k = \sum_{e=1}^{N_m} \varphi_\sigma^e \tag{4}$$

According to the penalty function, if any of the results obtained do not meet the constraint of stress and displacement, it does not eliminate from the design process, but due to displacement and exploration, it is given an opportunity to generate a new valid optimal design.

In this paper, several discrete truss structures are designed by the DHTLBO algorithm and then its results are compared to standard HS [6] and TLBO [15] and some other meta-heuristic algorithms.

## 2. Truss Structures with a Discrete Variable

In this paper, tries to minimize the cross area of members, but in structural engineering, we faced some limitations such as strength for each member and displacement for each connection. So the objective function is defined as the average weight of the optimal design which can be expressed as:

limits on the cross-sectional area and deflection are given by value at lower L and upper U boundaries.

The stress for each member  $\sigma$  compared with the lower and upper bounds Eq. (10) and the displacement for each connection to lower bound and upper bound Eq. (2).

The formulation of deflection limitation in the X, Y, and Z directions  $\varphi_{\delta_x}^c$ ,  $\varphi_{\delta_y}^c$ , and  $\varphi_{\delta_z}^c$  total deflection penalty function defined as:

$$\text{if } \delta^l \leq \delta_{c(x,y,z)} \leq \delta^u \quad \text{then} \quad \varphi_{\delta_{c(x,y,z)}}^c = 0 \tag{5}$$

$$\text{if } \delta_{c(x,y,z)} < \delta^l \quad \text{or} \quad \delta_{c(x,y,z)} > \delta^u \quad \varphi_{\delta_{c(x,y,z)}}^c = \left| \frac{\delta_{c(x,y,z)} - \delta^{l,u}}{\delta^{l,u}} \right| \tag{6}$$

$$\varphi_{\delta}^k = \sum_{c=1}^{N_m} [\varphi_{\delta_x}^c + \varphi_{\delta_y}^c + \varphi_{\delta_z}^c] \tag{7}$$

The final penalty function  $\psi^k$  for truss composed of stress and deflection penalty as:

$$\psi^k = (1 + \varphi_{\sigma}^k + \varphi_{\delta}^k)^{\epsilon} \tag{8}$$

Where  $\epsilon$  is a positive penalty coefficient. The value of penalized weight defined as:

$$F^k = \psi^k \cdot W^k \tag{9}$$

### 3. Heuristic Teaching-learning-based Optimization and Harmony Search for Truss Structures

#### 3.1. Review of teaching-learning-based optimization algorithm

The TLBO algorithm is inspired by a teacher and its students, which make it able to provide a model for truss optimization. The TLBO algorithm at first proposed by Rao et al [9]. In this method, a classroom consists of one teacher and some students, actually, one of the students who are better than another student is chosen as a teacher. Thus, the teacher increases the knowledge of the class-level by teaching the students. In the student phase, in order to increase the class-level, the student shares their knowledge with each other.

The students represent a population that is considered as design variables in various sciences. Since the cross-section of the members in proportion to the weight of the structure, with minimizing the cross-section the structure weight is also minimized. This procedure will be continued in a repeat process until none of the design constraints violated. Figure 1 shows the optimization process of the TLBO algorithm.

##### 3.1.1. Teacher phase

In this phase, at first, the best student of the class is chosen as the teacher and then the teacher tries to increase the average knowledge of the class-level. This phase can be formulated as [13]:

$$X_{new}^k = X_{old}^k + r(X^{teacher} - T_F * M(j)) \tag{10}$$

$$M(j) = \frac{\sum_{K=1}^N X^K(j)}{\sum_{K=1}^N F^K} \tag{11}$$

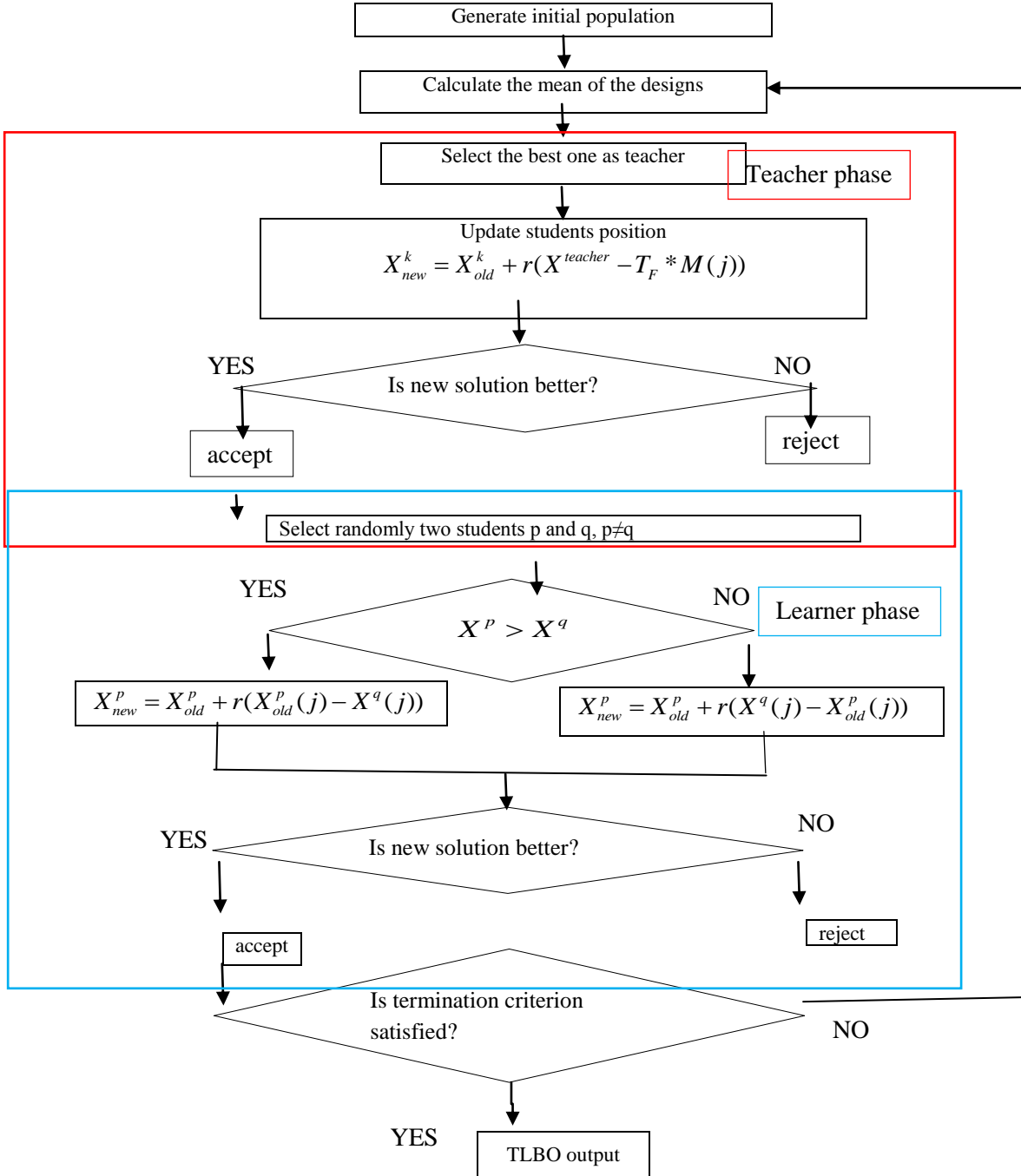


Figure 1. The TLBO flow chart

where the  $X^k(j)$  denotes the  $j$ th design variable,  $T_F$  used as the teaching factor,  $r$  is a random number within the range of  $[0,1]$ ,  $M(j)$  denotes the mean of class.  $F_k$  is the penalized fitness function.

### 3.1.2. Learner phase

In the learner phase, each learner increases their knowledge with the interaction between students

and this procedure will lead to an increase in overall knowledge of the class. The learner phase explained as below:

Randomly select  $p$  and  $q$  students from the class such a way that  $p$  and  $q$  are unequal, the learner phase can be formulated as:

$$\text{If } X_p < X_q$$

$$X_{new}^p = X_{old}^p + r(X_{old}^p(j) - X^q(j)) \quad (12)$$

Otherwise

$$X_{new}^p = X_{old}^p + r(X^q(j) - X_{old}^p(j)) \quad (13)$$

where  $r$  is a random number within the range [0,1].  $X^p(j)$  denotes the  $j$ th design for the  $p$ th design vector.

### 3.2. Review of Harmony search algorithm

The HS algorithm is one of the easiest and most recent meta-heuristic methods applied in the optimization problem, and it is inspired by the process of harmony such as during jazz improvisation. In other word, there is a similarity between finding an optimal point of the optimization problem and the process of Jazz improvisation. This algorithm has lower gradient requirements than other meta-heuristic algorithms and would be adapted to different optimization problems with changes in parameters. In the following, we intend to briefly explain the steps of the HS algorithm, which consist of step1 through 4. Figure 2 shows the optimization procedure of the HS algorithm.

**Step1: initialization:** In the first step, the HS algorithm has several parameters that inquired to be adjusted to solve the optimization problem. Harmony Memory(HM), Harmony Memory Size(HMS), Harmony Memory Consideration Rate(HMCR), and Pitch Adjusting Rate(PAR). In

this section, we must generate a population and store it in the HM.

$$HM = \begin{bmatrix} X^1 \\ \cdot \\ \cdot \\ X^{HMS} \end{bmatrix}_{HMS \times mg} \quad (14)$$

**Step2:** Initialize a new harmony from the HM: The HMCR is within [0,1] and used for considering the HM and choosing a new vector from the previous value. And (1-HMCR) sets the rate of randomly choosing one value from a possible range of values.

**Step3:** Updating the harmony memory: If a new harmony vector is better than the worst harmony in the HM, judged in terms of the objective function value, the new harmony is included in the HM and the existing worst harmony is excluded from the HM [16].

**Step4:** Terminating criterion: Repeat Steps 2 and 3 until the terminating criterion is satisfied [16].

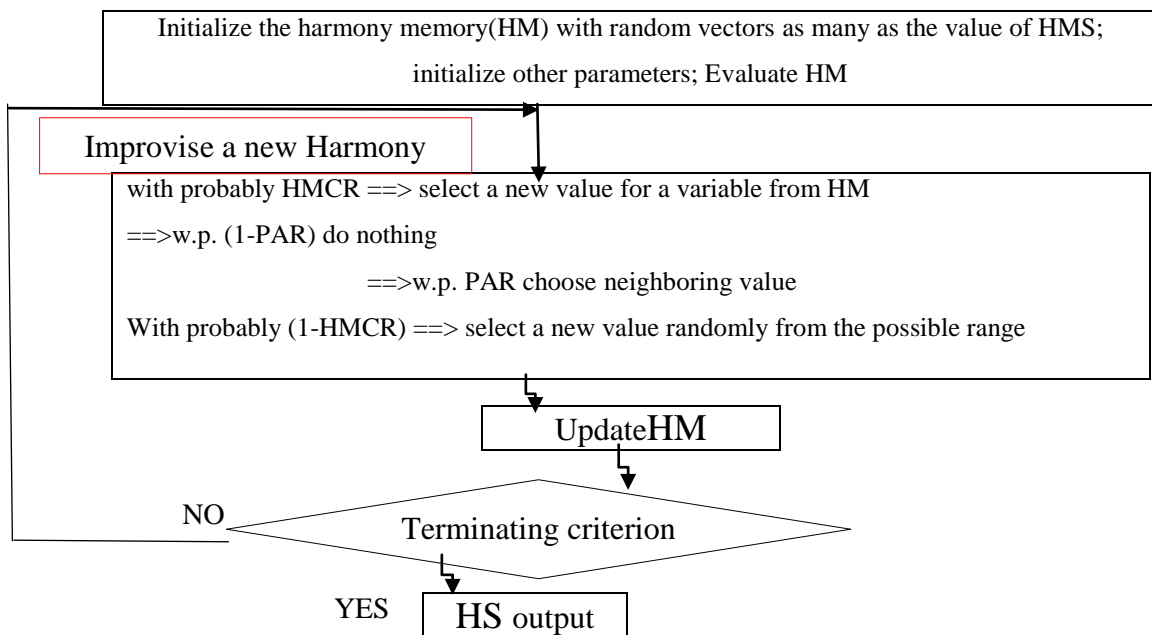


Figure 2. The HS flow chart

$$X_i^k = \begin{cases} \text{select from } \{X_i^1, X_i^2, \dots, X_i^{HMS}\} & w.p. HMCR \\ \text{select from the possible range} & w.p. (1 - HMCR) \end{cases} \quad (15)$$

#### 4. A Discrete Hybrid TLBO and HS algorithm

Camp and Farshchin [15] improved a Modified TLBO algorithm for discrete truss optimization. Lee and Geem [6] proposed the HS algorithm for discrete structural optimization on the basis of the primary continuous HS algorithm. In discrete HS, it's used a rejection strategy for the fitness measure and the optimum solution is approached only from the feasible space. In the last decades, many discrete TLBO and HS algorithms based on conventional TLBO and HS algorithms or relatively utilized their principles to solve the optimization problems.

Eq. (10) shows that the teacher only improves the class level by using the mean of the students and the distance between the teacher and the mean of the students is not considered in the teacher phase. This makes the TLBO algorithm trapping at the local point. In the next stage (learner phase) the optimization procedure may continue with a local point. So, we used an HS-based mechanism to solve this problem.

In this paper, we are presenting a new discrete algorithm called A Discrete Hybrid Teaching-Learning-Based Optimization. The framework of this new algorithm is shown in Figure 3. In fact, optimization based on discrete variables is very difficult, so to improve this problem many different methods for discrete optimization are introduced. Thus, we used the new function to round the values of continuous variables to the nearest discrete variable. According to the given definitions the position of each Teacher and Student are defined as follows:

For Teachers position

$$X_{new}^k = \text{fix}[X_{old}^k + r(X^{teacher} - T_F * M(j))] \quad (16)$$

For Students position

$$X_{new}^p = \text{fix}[X_{old}^p + r(X_{old}^p(j) - X^q(j))] \quad (17)$$

$$X_{new}^p = \text{fix}[X_{old}^p + r(X^q(j) - X_{old}^p(j))] \quad (18)$$

Where  $\text{fix}(X_{ik})$  is a function which rounds each Teacher or Learner of X to nearest permissible discrete value. This process may reduce the exploration algorithm power. Therefore, to eliminate this problem the HS algorithm is used to increase the exploration power. The Hybrid optimization procedure including the following steps:

Step1: As a conventional TLBO algorithm, initialize a population of students; these students are like harmony in the HS algorithm.

Step 2: Calculate the mean of the population using the Eq. (11), because our perception of the class progress is under the improvement of the mean of the class level.

Step 3: Consider individual student's fitness in order to choose the best one as the teacher. The teacher phase will be applying using the Eq. (16).

Step 4: Apply the learner phase using Eqs. (17 and 18).

Step 5: Generate a new student as:

$$X_{i,j} = \begin{cases} w.p. HMCR \implies \text{select a new value from } \text{fix}(X_i^k) \\ \implies w.p. (1 - PAR) \text{ do nothing} \\ \implies w.p. PAR \text{ choose neighboring value} \\ w.p. (1 - HMCR) \implies \text{select a new value randomly} \end{cases} \quad (19)$$

Where  $\text{fix}(X_{ik})$  is a function which rounds the continuous value to nearest discrete value,  $X_{i,j}$  is the jth variable of student i, the HMCR is varied within [0,1] which sets a rate of choosing a value from the historic values stored in the  $X_i^k$ , (1-HMCR) sets the rate of choosing one value from the possible list of values. The pitch adjusting process is performed

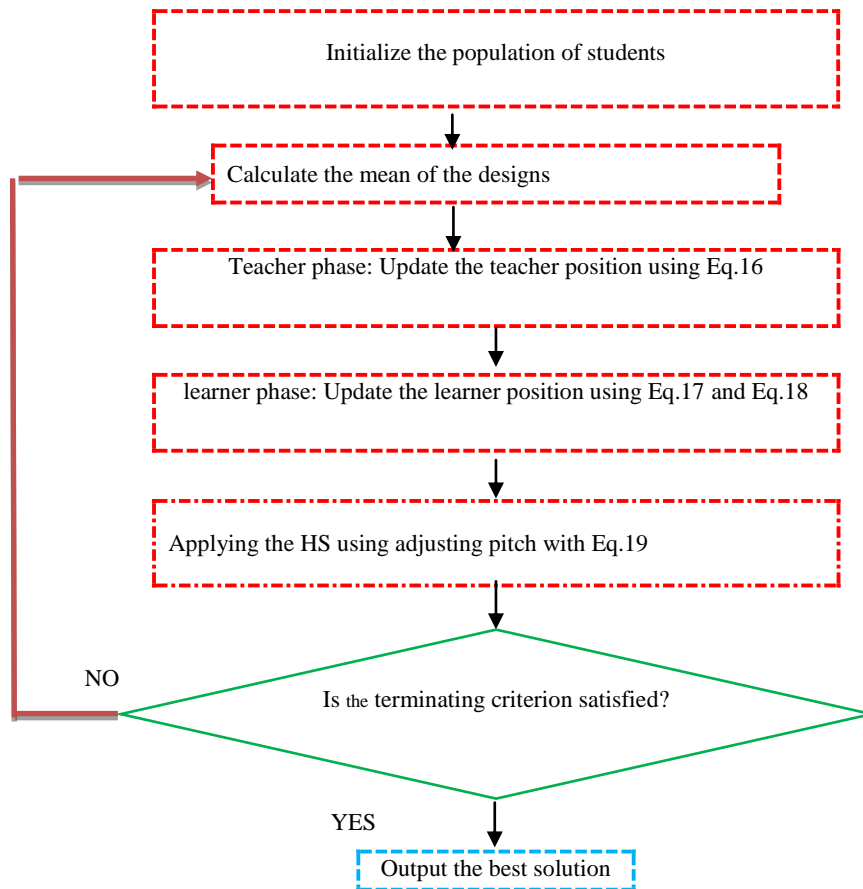


Figure3. The Hybrid TLBO and HS flow chart

only after a value is chosen  $X_i^k$ . the value (1-PAR) sets the rate of doing nothing, A PAR (pitch adjusting rate) of 0.1 indicates that the algorithm will choose a neighboring value with  $10\% \times \text{HMCR}$  probability [27]. The flow chart of the hybrid algorithm is shown in Figure3.

### 5. Numerical examples

In this section, truss structures with discrete variables are presented: 25-bar spatial truss with 8 design variables; 52-bar truss with 12 design variables; 72-bar spatial truss with 16 design variables. In all examples, the results of Hybrid

TLBO and HS are compared to other heuristic-based methods.

#### 5.1. Twenty-five bar spatial truss

The topology of a 25-bar spatial truss is shown in Figure 4. In this example, the structure is subjected to a single load case of Table 1. stress for each member of the structure is the members are subjected to the allowable stress limits of  $\pm 2812.2 \text{ kg/cm}^2$ . all nodes in X, Y, and Z directions are subjected to the allowable displacements  $\pm 0.889 \text{ cm}$ . there are 8 groups of discrete design variables with a range of  $0.064\text{--}21.93 \text{ cm}^2$ , with  $0.64 \text{ cm}^2$  which listed in Table 2. Unit weight  $2.76 \text{ g/cm}^3$ , modulus of elasticity is 107 psi.

The results of the Hybrid TLBO and HS and other Meta-heuristic algorithms are listed in Table 3. As you see the best weight of 25-bar spatial truss designed by the Hybrid TLBO and HS is  $214.12 \text{ kg}$ . with 1,000 searches. the best weight designed by the GA standard [1] is  $242.97 \text{ kg}$  with 800 searches, which more than Hybrid TLBO and HS algorithm also didn't report any information about standard deviation. the best weight of GA [2] algorithm is  $215.84 \text{ kg}$  with 15,000 searches,

also it didn't report any information about standard deviation. The best weight of the ACO [4] algorithm is 215.75kg after 7,700 searches and with a standard deviation of 2.09kg. the best weight of BB-BC [7] is 215.75kg with 6,670 searches, with a standard deviation of 0.27kg. the TLBO [15] algorithm achieved the best weight of 215.75kg after 4,910 searches with a standard deviation of 0.75kg.

However, the result of the Hybrid TLBO and HS is better than other Meta-heuristic algorithms. While the average weight of the Hybrid TLBO and HS is 214.73kg. with standard deviation 0.084kg. also, the Hybrid TLBO and HS have a smaller required number of iteration for convergence. Figure 5 shown the convergence history of the Hybrid TLBO and HS algorithm for the 25-bar spatial truss.

Table 1. Loading conditions for the 25-bar space truss

case	node	$P_x(\text{kg})$	$P_y(\text{kg})$	$P_z(\text{kg})$
1	1	0.445	-4.450	-4.450
	2	0.000	-4.450	-4.450
	3	0.222	0.000	0.000
	6	0.267	0.000	0.000

Note:  $1 \text{ in}^2 = 6.452 \text{ cm}^2$ ;  $1 \text{ lb} = 4.45 \text{ N}$

Table 2. Elements information

Group of elements							
1	2	3	4	5	6	7	8
1(1,2)	2:(1,4)	6:(2,4)	10:(6,3)	12:(3,4)	14:(3,10)	18:(4,7)	22:(6,10)
	3:(2,3)	7:(2,5)	11:(5,4)	13:(6,5)	15:(6,7)	19:(3,8)	23:(3,7)
	4:(1,5)	8:(1,3)			16:(4,9)	20:(5,10)	24:(4,8)
	5:(2,6)	9:(1,6)			17:(5,8)	21:(6,9)	25:(5,9)



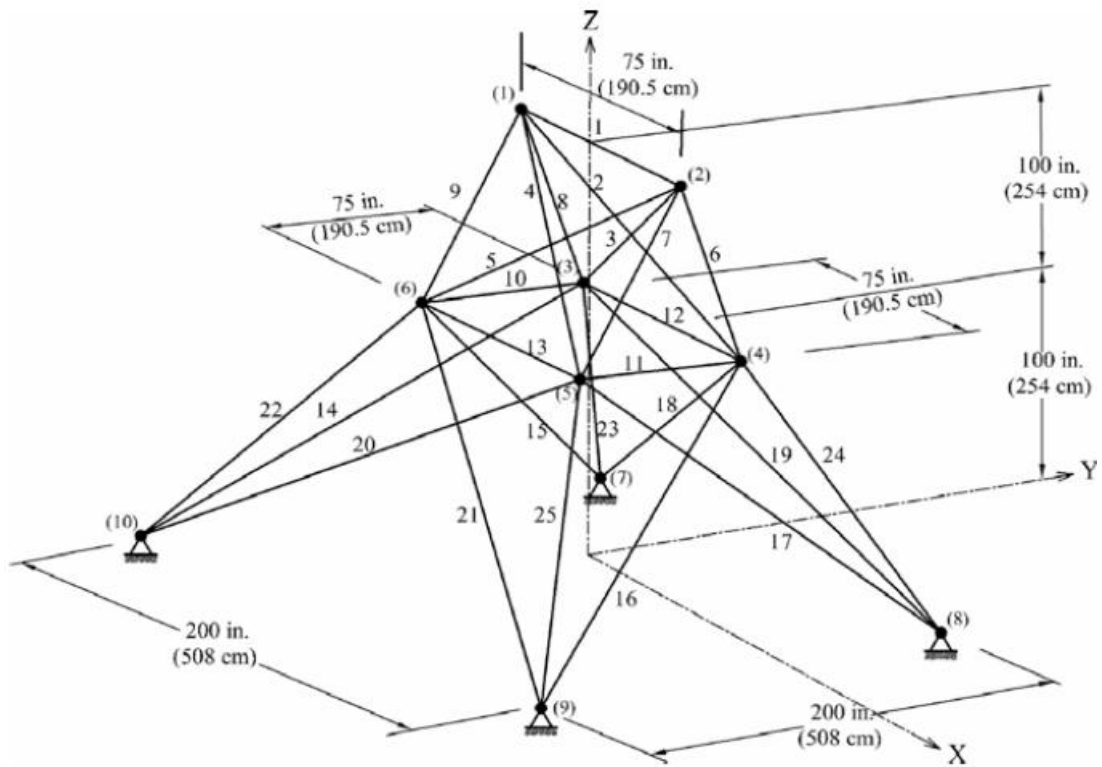


Figure 4. Topology of the 25-bar spatial truss

Table 3. Performance comparison for 25-bar spatial truss with Discrete variables

variables		Cross-sectional area(in <sup>2</sup> )					
Element group	members	GA Rajeev and Krishnamoorthy [1]	GA Cao [2]	ACO Camp and Bichon [4]	BB-BC[7]	TLBO [15]	This work
1	1	0.10	0.10	0.10	0.10	0.10	0.10
2	2,3,4,5	1.80	0.50	0.30	0.30	0.30	0.30
3	6,7,8,9	2.30	3.40	3.40	3.40	3.40	2.30
4	10,11	0.20	0.10	0.10	0.10	0.10	0.10
5	12,13	0.10	1.90	2.10	2.10	2.10	1.50
6	14,15,16,17	0.80	0.90	1.00	1.00	1.00	0.70
7	18,19,20,21	1.80	0.50	0.50	0.50	0.50	0.90
8	22,23,24,25	3.00	3.40	3.40	3.40	3.40	3.10
Weight(kg)		242.97	215.84	215.75	215.75	215.75	214.12
W <sub>avg</sub> (kg)		-	-	216.47	215.91	215.78	214.73
W <sub>stdv</sub> (kg)		-	-	2.09	0.27	0.07	0.08
N <sub>analysis</sub>		800	15,000	7,700	6,670	4,910	1,000

Note: 1 in<sup>2</sup> = 6.452 cm<sup>2</sup>; 1lb = 4.45 N

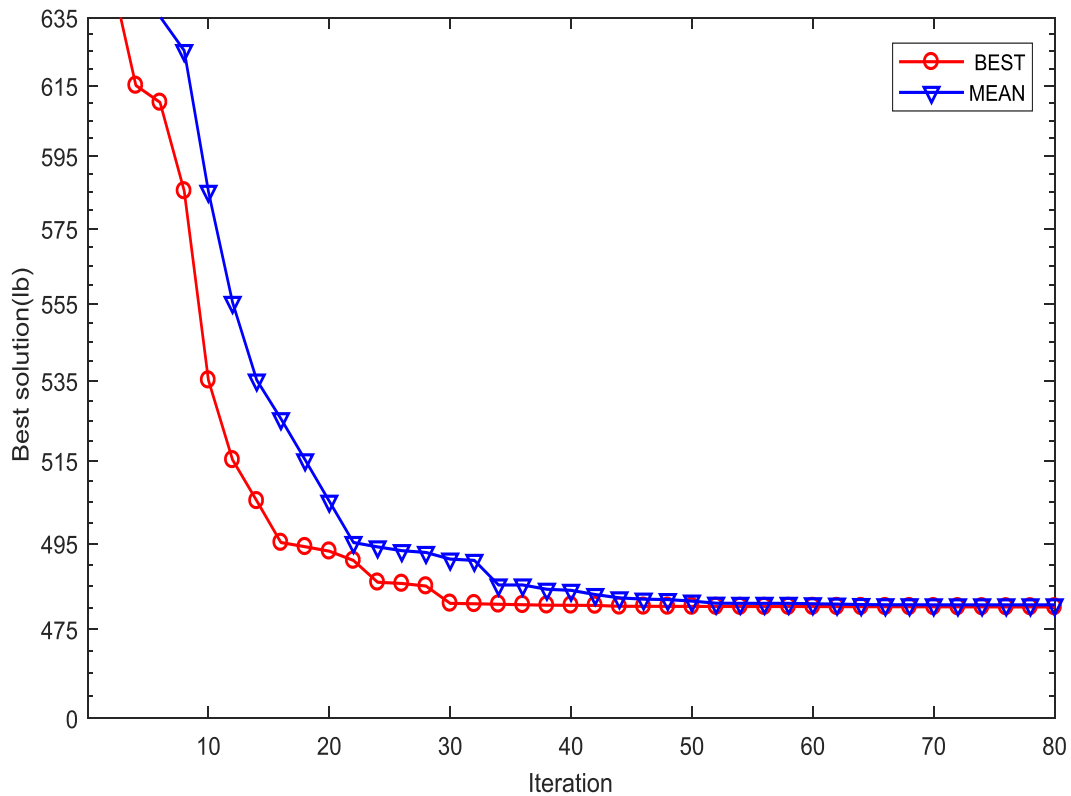


Figure 5. The convergence history of 25-bar spatial truss

## 5.2. Fifty-Two bar planar truss

Figure 6 shows the geometry of the 52-bar planar. This truss structure has been size optimized using other meta-heuristic methods, in this, we are going to optimize this truss structure and then compare the result with other researchers. The modulus of elasticity  $2.09 \times 10^6 \text{ kg/cm}^2$ , the members are subjected to the allowable stress limits of  $\pm 1835.48 \text{ kg/cm}^2$ , the unit weight of the material is  $1 \times 10^{-3} \text{ g/cm}^3$ , the structure is subjected to the load,  $P_x = 100 \text{ KN}$  and  $P_y = 200 \text{ KN}$ . The discrete variables are chosen from Table 4. The design variables are divided into 12 groups as: (A1) 1-4, (A2) 5-10, (A3) 11-13, (A4) 14-17, (A5) 18-23, (A6) 24-26, (A7) 27-30, (A8) 31-36, (A9) 37-39, (A10) 40-43, (A11) 44-49, (A12) 50-52.

The results of the Hybrid TLBO and HS and other Meta-heuristic algorithm are listed in Table 5. As you see the best weight of 52-bar truss designed by the Hybrid TLBO and HS is 1901.10kg. with 4,980 searches. The best weight of GA is

1970.142kg, the best weight of HS 1906.76kg, and the best weight designed by DHPSACO [14] is 1904.83kg, which in all cases are more than Hybrid TLBO and HS algorithm. However, the result of the Hybrid TLBO and HS is better than other Meta-heuristic algorithms. also, they didn't report any information about standard deviation, number of analysis and average weight. Figure 7 shown the convergence history of the Hybrid TLBO and HS algorithm for the 52-bar truss.

## 5.3. Seventy-two bar spatial truss

Figure 8. shows the geometry and more details of a 72-bar truss. The modulus of elasticity is  $1e7 \text{ psi}$ . the unit weight of the material is  $2.76 \text{ g/cm}^3$ . The members are subjected to the allowable stress limits of  $\pm 1757.6 \text{ kg/cm}^2$ , and the maximum displacement of each node is  $\pm 0.635 \text{ cm}$  through X, Y, and Z direction. There are 16 group of design variables with a minimum  $0.6452 \text{ cm}^2$  and

maximum 19.35cm<sup>2</sup>: (A1) 1-4, (A2) 5-12, (A3) 13-16, (A4) 17-18, (A5) 19-22, (A6) 23-30, (A7) 31-34, (A8) 35-36, (A9) 37-40, (A10) 41-48, (A11) 49-52, (A12) 53-54, (A13) 55-58, (A14) 59-66, (A15) 67-70, (A16) 71-72. The structure is designed for two individual cases as:

Case 1: the structure in both cases subjected to multiple loading listed in Table 6. The discrete design variables are chosen from the set {with minimum 0.635 and maximum 20.64, with interval 0.635} (cm<sup>2</sup>).

Case 2: In this case, the design variables are chosen from Table 4.

The result of the meta-heuristic algorithms in the first case is listed in Table 7. The best weight of hybrid TLBO and HS is 171.21kg, which is better than another meta-heuristic algorithm. The best weight of GA [3] is 178.29kg, the best weight designed by HS [6] is 172.63kg, the best weight of Li et al [13] is 173.07kg, and the best weight designed by DHPSACO [14] is 171.56kg, which in all cases are more than the result of the new Hybrid TLBO and HS algorithm. They didn't report any information about standard deviation and average weight.

The result of the meta-heuristic algorithms in the first case is listed in Table 8. As you see the results of previous researches the best weight obtained by DHPSACO [14] is 175.04kg, which is better than other methods. But the result of Hybrid TLBO and

HS is 174.39kg, which is better than DHPSACO [14]. Figure 9 shows the convergence history of the Hybrid TLBO and HS algorithm for the 72-bar spatial truss.

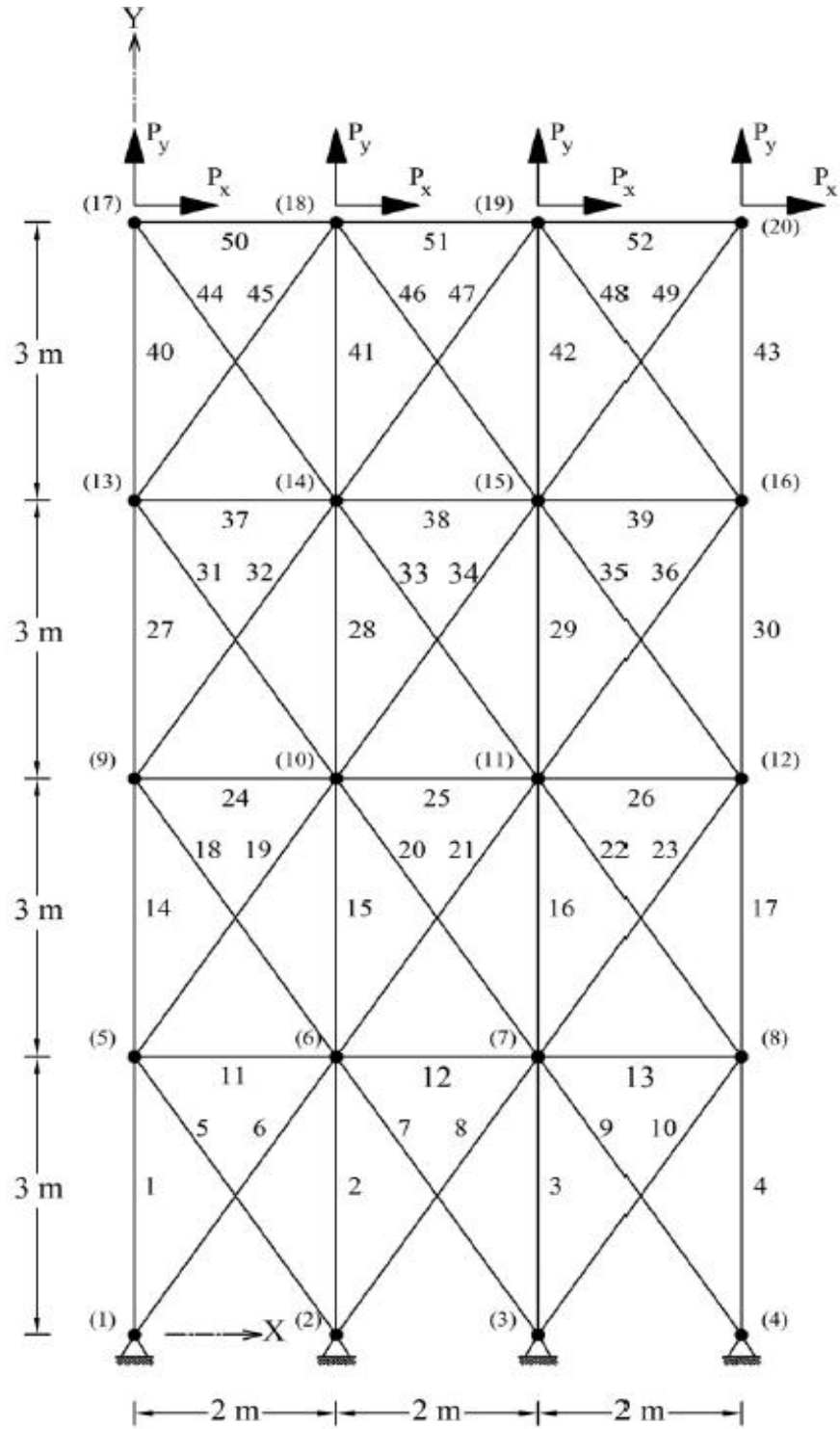


Figure 6. Topology of 52-bar planar truss

Table 4.The available cross-section areas of the AISC code

NO	in <sup>2</sup>	mm	NO	in <sup>2</sup>	mm
1	0.111	71.613	33	3.840	2477.414
2	0.141	90.968	34	3.870	2496.796
3	0.196	126.451	35	3.880	2503.221
4	0.250	161.290	36	4.180	2696.769
5	0.307	198.064	37	4.220	2722.575
6	0.391	252.258	38	4.490	2896.768
7	0.442	285.161	39	4.590	2961.284
8	0.563	363.225	40	4.800	3096.768
9	0.602	388.386	41	4.970	3206.445
10	0.766	494.193	42	5.120	3303.219
11	0.785	506.451	43	5.740	3703.218
12	0.994	641.289	44	7.220	4658.055
13	1.000	645.160	45	7.970	5141.925
14	1.228	792.256	46	8.530	5503.215
15	1.266	816.773	47	9.300	5999.988
16	1.457	393.998	48	10.850	6999.986
17	1.563	1008.385	49	11.500	7419.430
18	1.620	1045.159	50	13500	8709.660
19	1.800	1161.288	51	13.900	8967.724
20	1.990	1283.868	52	14.200	9161.272
21	2.130	1374.191	53	15.500	9999.980
22	2.380	1535.481	54	16.000	10322.560
23	2.620	1690.319	55	16.900	10903.204
24	2.630	1696.771	56	18.800	12129.008
25	2.880	1858.061	57	19.900	12838.684
26	2.930	1890.319	58	22.000	14193.520
27	2.090	1993.544	59	22.900	14774.164
28	1.130	729.031	60	24.500	15806.420
29	3.380	2180.641	61	26.500	17096.740
30	3.470	2238.705	62	28.000	18064.480
31	3.550	2290.318	63	30.000	19354.800
32	3.630	2341.931	64	33.500	21612.860

Table 5. Performance comparison for 52-bar truss with Discrete variables

variables		Cross-sectional area(mm <sup>2</sup> )						
Element group	members	Wu and Chow GA [3]	Lee and Geem HS[6]	Li et al. [13]			DHPSACO [14]	This work
				PSO	PSOPC	HPSO		
1	1-4	4658.055	4658.055	4658.055	5999.988	4658.055	4658.055	4658.055
2	5-10	1161.288	1161.288	1374.190	1008.380	1161.288	1161.288	1161.288
3	11-13	645.160	506.451	1858.060	2696.380	363.225	494.193	363.225
4	14-17	3303.219	3303.219	3206.440	3206.440	3303.219	3303.219	3303.219
5	18-23	1045.159	940.000	1283.870	1161.290	940.000	1008.385	940.000
6	24-26	494.193	494.193	252.260	729.030	494.193	285.161	285.161
7	27-30	2477.414	2290.318	3303.220	2238.710	2238.705	2290.318	2477.414
8	31-36	1045.159	1008.385	1045.160	1008.380	1008.385	1008.385	1045.160
9	37-39	285.161	2290.318	126.450	494.190	388.386	388.386	161.290
10	40-43	1696.771	1535.481	2341.930	1283.870	1283.868	1283.868	1283.868
11	44-49	1045.159	1045.159	1008.380	1161.290	1161.288	1161.288	1161.288
12	50-52	641.289	506.451	1045.160	494.190	729.256	506.451	506.451
<b>Weight(kg)</b>		<b>1970.142</b>	<b>1906.76</b>	<b>2230.16</b>	<b>2146.63</b>	<b>1905.49</b>	<b>1904.83</b>	<b>1901.10</b>

Note: 1 in<sup>2</sup> = 6.452 cm<sup>2</sup> ; 1lb = 4.45 N

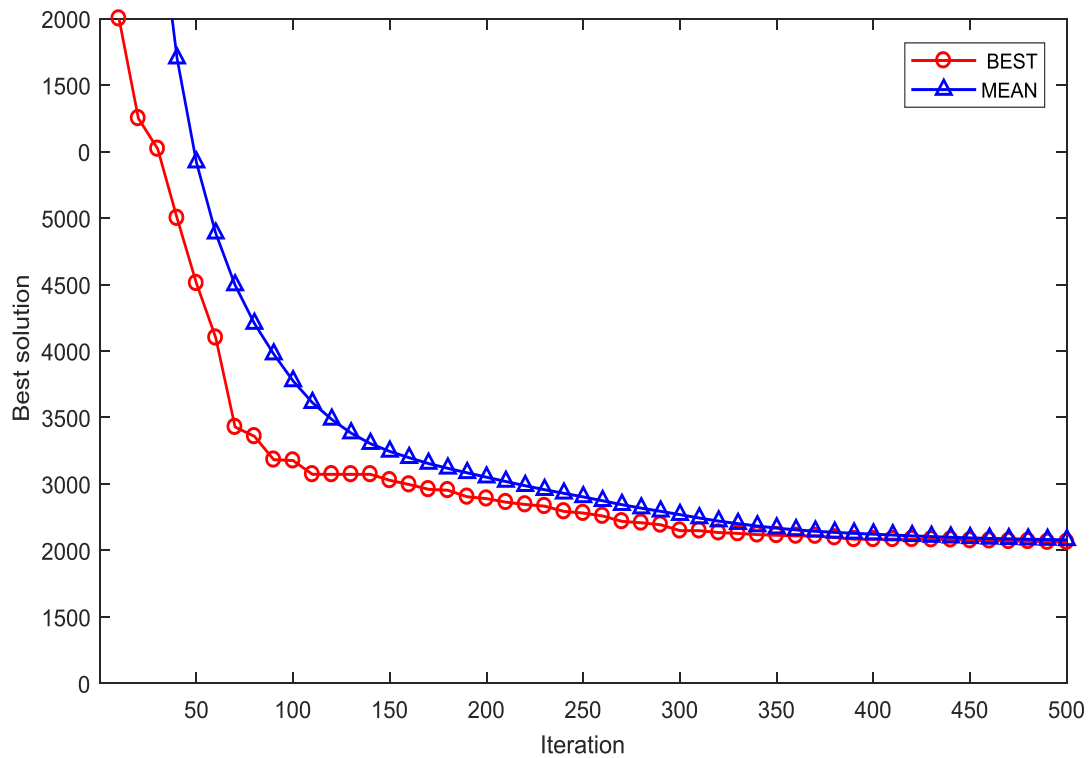


Figure 7. The convergence history of 52-bar truss

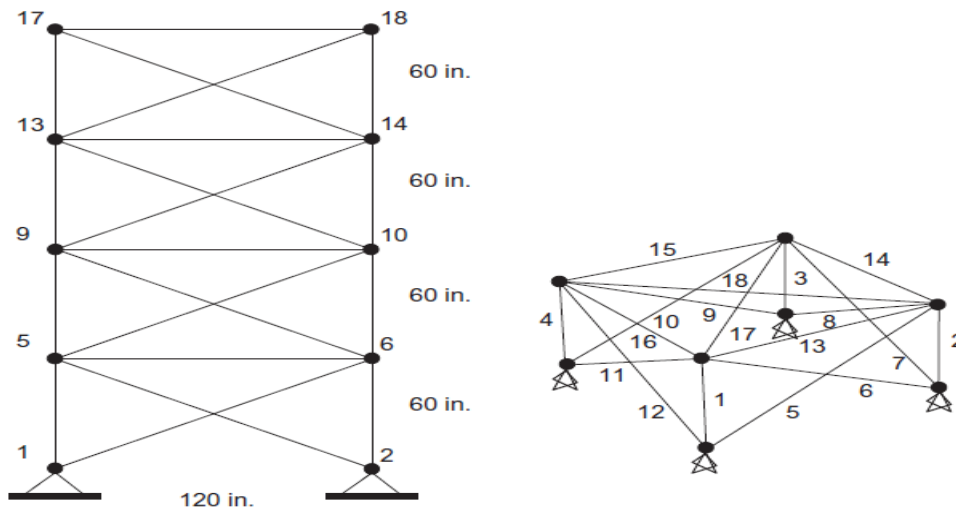


Figure 8. Geometry and elements definition of 72-bar truss;(a) dimension and node numbering; (b) the pattern of element numbering.

Table 6. Multiple loading for the 72-bar truss

case	node	$P_x$ (kg)	$P_y$ (kg)	$P_z$ (kg)
1	17	0.0	0.0	-2.225
	18	0.0	0.0	-2.225
	19	0.0	0.0	-2.225
	20	0.0	0.0	-2.225
2	17	2.225	2.225	-2.225

Note: 1 in<sup>2</sup> = 6.452 cm<sup>2</sup> ; 1lb = 4.45 N

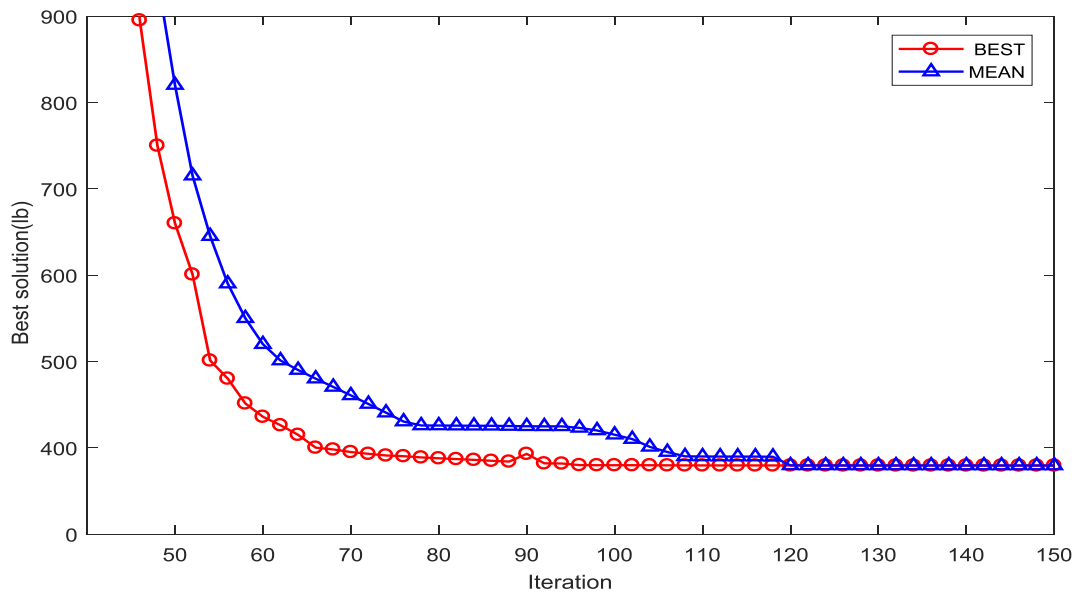


Figure 9. The convergence history of 72-bar spatial truss

Table 7. Performance comparison for 72-bar spatial truss with discrete variables (case 1)

variables		Cross-sectional area(in <sup>2</sup> )						
Element group	members	Wu and Chow GA [3]	Lee and Geem HS[6]	Li et al. [13]			DHPSACO [14]	This work
				PSO	PSOPC	HPSO		
1	1-4	1.5	1.9	2.6	3.0	2.1	1.9	1.5
2	5-12	0.7	0.5	1.5	1.4	0.6	0.5	0.5
3	13-16	0.1	0.1	0.3	0.2	0.1	0.1	0.1
4	17-18	0.1	0.1	0.1	0.1	0.1	0.1	0.1
5	19-22	1.3	1.4	2.1	2.7	1.4	1.3	1.3
6	23-30	0.5	0.6	1.5	1.9	0.5	0.5	0.5
7	31-34	0.2	0.1	0.6	0.7	0.1	0.1	0.2
8	35-36	0.1	0.1	0.3	0.8	0.1	0.1	0.1
9	37-40	0.5	0.6	2.2	1.4	0.5	0.6	0.5
10	41-48	0.5	0.5	1.9	1.2	0.5	0.5	0.5
11	49-52	0.1	0.1	0.2	0.8	0.1	0.1	0.1
12	53-54	0.2	0.1	0.9	0.1	0.1	0.1	0.2
13	55-58	0.2	0.2	0.4	0.4	0.2	0.2	0.2
14	59-66	0.5	0.5	1.9	1.9	0.5	0.6	0.5
15	67-70	0.5	0.4	0.7	0.9	0.3	0.4	0.3
16	71-72	0.7	0.6	1.6	1.3	0.7	0.6	0.6
<b>Weight(kg)</b>		<b>178.29</b>	<b>172.63</b>	<b>484.99</b>	<b>476.05</b>	<b>173.07</b>	<b>171.56</b>	<b>171.21</b>

Note: 1 in<sup>2</sup> = 6.452 cm<sup>2</sup>; 1lb = 4.45 N

Table 8. Performance comparison for 72-bar spatial truss with discrete variables (case 2)

variables		Cross-sectional area(in <sup>2</sup> )					
Element group	members	Wu and Chow GA [3]	Li et al. [13]			DHPSACO[14]	This work
			PSO	PSOPC	HPSO		
1	1-4	0.196	7.220	4.490	4.970	1.800	1.800
2	5-12	0.602	1.800	1.457	1.228	0.442	1.228
3	13-16	0.307	1.130	0.111	0.111	0.141	0.111
4	17-18	0.766	0.196	0.111	0.111	0.111	0.111
5	19-22	0.391	3.090	2.620	1.880	1.228	0.391
6	23-30	0.391	0.785	1.130	1.457	0.563	0.391
7	31-34	0.141	0.563	0.196	0.141	0.111	0.141
8	35-36	0.111	0.785	0.111	0.111	0.111	0.111
9	37-40	1.800	3.090	1.266	1.563	0.563	1.563
10	41-48	0.602	1.228	1.457	1.228	0.563	0.602
11	49-52	0.141	0.111	0.111	0.111	0.111	0.111
12	53-54	0.307	0.563	0.111	0.196	0.250	0.196
13	55-58	1.563	0.990	0.442	0.391	0.196	0.196
14	59-66	0.766	1.620	1.457	1.457	0.563	0.766
15	67-70	0.141	1.563	1.228	0.766	0.442	0.141
16	71-72	0.111	1.266	1.457	1.563	0.563	0.111
<b>Weight(kg)</b>		<b>190.10</b>	<b>538.21</b>	<b>491.10</b>	<b>415.22</b>	<b>175.05</b>	<b>174.39</b>

Note: 1 in<sup>2</sup> = 6.452 cm<sup>2</sup>; 1lb = 4.45 N



## 6. Conclusions

The Hybrid TLBO and HS algorithms are developed based on the standard TLBO and HS algorithms. Thus, we tried to improve the performance of the new algorithm by identifying the merits and demerits of the standard TLBO. As mentioned before the TLBO algorithm consists of two phases, the teacher phase and the student phase, the main disadvantage of TLBO algorithm is in the teacher phase, when the best student selected as teacher and then the teacher tries to improve the average of the class-level there is no control parameter for measuring the distance between the teacher and the average of the class. due to this problem make the algorithm trapping at the local point and then the optimization process will be continuing from this local point.

In order to improve this problem, we used the HS algorithm which is able to explore all the search space and find the global point. So, to demonstrate the efficiency in both performance and convergence, several truss structures have been optimized and as you see the results the best result obtained by the new Hybrid algorithm. According to the high potential of this new algorithm, it can be used to solve the difficult optimization problem in structural engineering.

## References

- [1] Rajeev, S. and Krishnamoorthy, CS., "Discrete optimization of structures using genetic algorithms", J StructEng, 118(5):1233–50, 1992.
- [2] Cao, G. "Optimized design of framed structures using a genetic algorithm. Ph.D.thesis", The University of Memphis, TN; 1996.
- [3] Wu, SJ. And Chow, PT. "Steady-state genetic algorithms for discrete optimization of trusses". Computers and Structures, 56(6), 979-91, 1995.
- [4] Camp, CV. and Bichon, BJ. "Design of space trusses using ant colony optimization". J StructEng, 130(5), 741–51, 2004.
- [5] Lee, KS. And Geem, ZW. "A new structural optimization method based on the harmony search algorithm". Computers & Structures, 82(9–10), 781–98, 2004.
- [6] Lee, KS., Geem, ZW., Lee, SH. and Bae, KW. "The harmony search heuristic algorithm for discrete structural optimization". Engineering Optimization, 37(7), 663-84, 2005.
- [7] Camp, CV. "Design of space trusses using big bang–big crunch optimization". JStructEng, 133(7), 999–1008, 2007.
- [8] Kaveh, A. and Talatahari, S. "Size optimization of space trusses using big bang–big crunch algorithm". Comput Struct, 87(17–18):1129–40, 2009.
- [9] Rao, RV., Savsani, VJ., Vakharia, DP. "Teaching–learning-based optimization: a novel method for constrained mechanical design optimization problems". Computer Aided Design, 43(3):303–15, 2011.
- [10] Sonmez, M. "Artificial bee colony algorithm for optimization of truss structures". Appl Soft Comput, 11(2), 2406–18, 2011.
- [11] Kaveh, A. Talatahari, S. "A novel heuristic optimization method: charged system search", ActaMechanica, 213, 267–289, 2010.
- [12] Kaveh, A. Talatahari, S. "A hybrid particle swarm and ant colony optimization for design of truss structures". Asian Journal of Civil Engineering, 9(4), 329\_48, 2008.
- [13] Li, LJ. Huang, ZB. Liu, F. "A heuristic particle swarm optimization method for truss structures with discrete variables". Computers and Structures, 87(78), 435:43, 2009.
- [14] Kaveh, A. Talatahari, S. "A particle swarm ant colony optimization for truss structures with discrete variables". Journal of Constructional Steel Research, 65, 1558-1568, 2009.
- [15] Camp, CV. Farshchin, M. Design of space trusses using modified teaching-learning based optimization. Computers & Structures, 62–63:87–97, 2014.
- [16] Lee, KS. Sang, Whan. Han. SW. and Geem, ZW. "Discrete size and Discrete continuous configuration optimization methods for truss

- structures using the harmony search algorithm". *Int. J. Optim. Civil Eng*, 1, 107-126, 2011.
- [17] Meshki, H. and Joghataie, A. "Structural optimization by spherical interpolation of objective function and constraints". *ScientiaIranica*, 23(2), 548-557, 2016.
- [18] Kaveh, A. and Ilchi, M. "Computer codes for colliding bodies optimization and its enhanced version". *Int. J. Optim. Civil Eng*, 4(3), 321-339, 2014.
- [19] Kaveh, A. Ilchi, Ghazaan. M. "A new meta-heuristic algorithm: Vibrating particles system, *ScientiaIranica*". *International Journal of Science and Technology*, 24: 551-566, 2017.
- [20] Gandomi, A. H., Alavi, A. H. "Krill herd: A new bio-inspired optimization algorithm". *Communications in Nonlinear Science and Numerical Simulation*, 17, 4831-4845, 2012.
- [21] Mirjalili, S. and Lewis, A. "The whale optimization algorithm", *Advance Engineering Software*, 95: 51-67, 2016.
- [22] Cheng, M-Y, Prayogo, D, Wu, Y-W, MarcellinusLukito M. "A Hybrid Harmony Search algorithm for discrete sizing optimization of truss structure". *Automation in Construction*, 69: 21-33, 2016.
- [23] Tuo, S, Yong, L, Deng, F, Li, Y, Lin Y, Lu, Q. "A hybrid algorithm based on Harmony Search and Teaching-Learning-Based Optimization for complex high-dimensional optimization problems". *Pone*, 10-1371, 2017.
- [24] Ouyang, H. B. Gao, L. Q. Kong, X. Y, Zou D. X, & Li S. "Teaching-learning based optimization with global crossover for global optimization problems". *Applied Mathematics and Computation*, 265, 533-556, 2015.
- [25] Kaveh, A. and Rahami, H. "Analysis, design and optimization of structures using force method and genetic algorithm". *Int. J. Numer. Methods Eng*. 65, pp. 1570-1584, 2016.
- [26] Sadollah, A., Eskandar, H., Bahreininejad, A. and Kim, J.H. "Water cycle, mine blast and improved mine blast algorithms for discrete sizing optimization of truss structures". *Comput. Struct*. 149, pp. 1-16, 2015.
- [27] Kaveh, A. Talatahari, S. "Hybrid Algorithm of Harmony Search, Particle Swarm and Ant Colony for Structural Design Optimization". In: Geem Z.W. (eds) *Harmony Search Algorithms for Structural Design Optimization. Studies in Computational Intelligence*, vol 239. Springer, Berlin, Heidelberg, 2009.