A Mathematical Model for a Flow Shop Scheduling Problem with Fuzzy Processing Times

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Abstract

This paper presents a mathematical model for a flow shop scheduling problem consisting of *m* machine and *n* jobs with fuzzy processing times that can be estimated as independent stochastic or fuzzy numbers. In the traditional flow shop scheduling problem, the typical objective is to minimize the makespan). However,, two significant criteria for each schedule in stochastic models are: expectable makespan and the probability of minimizing the makespan. These criteria can be considered for fuzzy problems as well. In this paper, we propose a solution for the fuzzy model by the use of fuzzy logic based on developing the model presented by MacCahon [18].

Keywords: Scheduling, Flo shop, Fuzzy models, Makespan.

1. Introduction

Flow shop scheduling models are effective tools for management that can be utilized for modeling many service and production processes such as continuous production systems. In simple flow shop scheduling problems, each machining center has just one machine. if at least one machining center has more than one machine, each job is started through the first machine and then goes on to the next machine in tandem , ending up with the last machine. In flow shop scheduling problems, all machines are arranged in constant series. The main objective is to find the best schedule and sequence for jobs in order to minimize the makespan**.**

For each machine, there is a one-to-one relationship among job sequences and substitution of jobs indices1, 2,…, *n*. Thus, each machine has *n*! substitutions and *n*! solutions. These *n*! solutions can lead to better results. Interested readers may refer to [2] and [4] for more information. Johnson [13] was pioneer of flow shop sequencing problems in 1954 followed by Baker [2]. The flow shop scheduling problem is known to be NP hard [5], which prompted the development of many heuristics to provide a valuable and quick solution that can be referred to [10], [11], [22], and [23].

For deterministic scheduling models, more investigations have been accomplished in [8, 16 and 6].

Negenman [19] considered local search methods for the multiprocessor flow shop scheduling problem. In this regard, Ying and Liao [26] utilized the ant colony system for a permutation sequencing problem. Gu et al [9] developed special GA to solve such problem.

In industrial systems, there are some haphazard and inevitable events that may damage the process such as machine breakdown, unavailable operator, and probabilistic changes. Therefore, considering a system in stochastic state is more realistic than the crisp one. The stochastic model is used when the processing times of some jobs are not exactly determined, or even cannot be considered as a defined distribution function. Many studies have been conducted on deterministic problems; however, there are a few studies on the stochastic model, in which the most important of which are presented in [4], [20], and [21]. Makino [17] presented a stochastic flow shop problem that minimizes makespan for two machines and two jobs when the processing time of each job on each machine follows the exponential distribution.

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Talwar [24] expanded the MacCahon's rule [18] for n=3 and n=4. Gudda [1] simplified the acquired model of [24]. Ku and Niu [15] showed the relationship between the rules presented in [13] and [24]. Pinedo [20] considered the same problem in the situation where the numbers of jobs are infinite, and proposed a solution for finding the minimum makespan, in which the processing times are independent with similar distribution. Kijima, *et. al.* [14] solved the same problem, whereas there were m identical machines, and proved that the same solution could be obtained for both limited and unlimited buffers. Dodin [3] considered the distribution function of minimizing the makespan. Gourgand, *et. al.* [7] proposed a model for stochastic flow shop sequencing problem in which the processing time of all the jobs on m machine are random variables exponentially distributed with a recursive algorithm based on Markov chain computing the expected makespan. They also presented a discrete event simulation to evaluate the expected makespan and compared the methods together to evaluate their efficiencies.

To solve the fuzzy flow shop problem, several methods have been proposed recently. Ishi, *et al.* [12] constructed the concept of due date, in which a trapezoidal fuzzy function is assigned to each job and showed the satisfactory degree of the job completion time. In this problem, the minimum of satisfaction degree is maximized. Zadeh [25] investigated an approach for incorporating statistics with fuzzy sets in the flow shop sequencing problem. Mackahon and lee [18] considered the fuzzy flow shop sequencing problem with fuzzy processing times.

2. Model description

In this paper, the model will be stochastic, if the processing times of some jobs are not crisp, but are random variables with the known distribution function. It is necessary to define the optimum criterion and thereby optimum solution can be found. Most of the times, obtaining the optimum solution is very difficult. So, we analyze the difficulties associated with finding the optimum solution.

In stochastic model of flow shop problems, the optimum sequence with a single makespan is meaningless because of the stochastic processing times there is no sequence to give a better solution with respect to other sequences. A sequence can optimize criteria such as minimum of mathematical expectation of total processing times or the probability of holding the minimum makespan. In this paper, a flow shop problem with m machine and n jobs is considered, in which job j must be processed on machine i with the given processing times based on independent random variables of the assigned distribution function. The jobs are available for processing on a continuously available machine with unlimited buffers. The objective is to find the best solution with the best optimizing criterion. The value of this criterion depends on the makespan value that equals the completion time of the last job on the last machine.

Definitions of variables used the proposed model are as follows:

T_{ij} Random variable for the processing time of job j on machine i.

$F_{ij}(t)$ Cumulative distribution function.

S Possible n! sequences.

π: One of the sequences of the set S.

$T(\pi)$: Random variable showing the makespan of π^{th} sequence.

T : Random variable showing the minimum makespan among all the sequences of set S (MM).

$F_{\pi}(t)$: Cumulative distribution function of $T(\pi)$

MM stands for a random variable that can be obtained based on the following equation.

$$T = \min_{\pi \in s}\{T(\pi)\} \tag{1}$$

The optimum index of each sequence is described as follows:

$$OI(\pi) = \Pr(T(\pi) \le T(\pi^{/})) \quad \forall \pi^{/} \ne \pi \in S \tag{2}$$

$E(T(\pi))$: Mathematical expectation.

The sequence with the maximum OI is shown as π_0 that could be assigned to the optimum sequence. These two sequences may probably be different from each other. However, both concentrate on the makespan value. Obtaining $OI(\pi)$ is rather difficult. In other words, it is impossible for many distributions, especially if m and n are quite large values.

3. Solution based on fuzzy logic application

Mc Cahon and Lee [18] considered the problem when the processing times are not exactly determined but are just an estimation of processing times based on intervals. They are fuzzy numbers instead of crisp. In this model, the processing times are presented as trapezoidal fuzzy numbers. Thus, in order to obtain the optimum solution, we use the CDS algorithm given in [21]. This algorithm has changed so as to use the fuzzy numbers. To determine A_{il} and B_{il}, more fuzzy numbers are used. To compare these values with each other for purpose of finding the optimum solution, the average of fuzzy numbers is calculated. Therefore, the objective function minimizes the fuzzy numbers related to the makespan. The average of a fuzzy number is calculated by the following equation:

$$m(A)=\frac{\int_S X\mu_A(x)dx}{\int_S \mu_A(x)dx} \quad (3)$$

3.1. A new model for fuzzy flow shop sequencing problem

As mentioned in Section 2, in general there is no possibility for the analysis of stochastic model (i.e., finding a specific distribution function of total processing times). Therefore, the heuristic methods and fuzzifying of the model can also resolve the problem to some extent. In this paper, we model the problem based on fuzzy numbers and propose a solution according to the optimal index criterion as described before.

3.1.1. Comparison of fuzzy sets and fuzzy numbers with crisp sets and numbers.

Take a crisp set S includes x elements, any number, x_i, is either belonging to S, (1), or not belonging to S, (0). Zadeh [27] developed the concept of fuzzy sets and member ship function that is no longer just 0 or 1, but is [0, 1]. This is the main characteristic that makes the fuzzy sets different from crisp sets**.**

$\mu_A(x)$ is the membership degree of x in set A. The closer the value $\mu_A(x)$ is to 1, the more likely it belongs to A. A fuzzy number is a convex set that the member ship function is continuous in a set that its value is greater than zero.

The α Cut of a fuzzy number set, say A, is defined bellow.

$$A_\alpha = \{x \in X \mid \mu_A(x) \geq \alpha\} \quad (4)$$

3.1.2. Comparison of some operations for both fuzzy and stochastic approaches

Considering the α Cut, the algebraic sum of two fuzzy numbers can be stated as follows:

$$\text{If } Y_\alpha = [y_\alpha^L, y_\alpha^R], X_\alpha = [x_\alpha^L, x_\alpha^R] \text{ then,}$$
$$Z_\alpha = X_\alpha \oplus Y_\alpha = [x_\alpha^L + y_\alpha^L, x_\alpha^R + y_\alpha^R] \quad (5)$$

By adding these two random variables, we have the following equation:

$$Z=X+Y$$
$$f_Z(t) = \int_{-\infty}^{+\infty} f_X(u).f_Y(t-u)du \quad (6)$$
$$E(Z)=E(X)+E(Y)$$

Max-min operators: By using the α cut, we can define the maximum of two fuzzy numbers as follows:

$$Z_\alpha = \max(X_\alpha, Y_\alpha) =$$
$$[\max(x_\alpha^L, y_\alpha^L), \max(x_\alpha^R, y_\alpha^R)] \quad (7)$$

For random variables, following equation is defined.

$$Z=\max(X,Y)$$
$$E[\max(X,Y)] \geq \max[E(X),E(Y)] \quad (8)$$
$$F_Z(t) = F_X(t).F_Y(t)$$

The following equation is also given for finding the minimum of two fuzzy numbers.

$$Z_\alpha = \min(X_\alpha, Y_\alpha) =$$
$$[\min(x_\alpha^L, y_\alpha^L), \min(x_\alpha^R, y_\alpha^R)] \quad (9)$$

The fuzzy makspan of the sequence can be obtained only by using the *max* operator and summing up fuzzy numbers

3.1.3. Near-optimal solution in fuzzy model

The following equation is given in the fuzzy flow shop model:

$$ct_{pj} = \max(ct_{p-1,j}, ct_{p,j-1}) + T_{pj} \quad (9)$$

By using Equation (10), the *max* operator, and summing up fuzzy numbers, the makespan of the specific sequence can be obtained in the form of a fuzzy number. MacCahon and Lee [18] obtained m-1 optimum sequence utilizing the CDS algorithm and average of a fuzzy number that can be considered as ($\pi_1, \pi_2, \ldots, \pi_{m-1}$).

As mentioned before, the optimal index of a sequence is the probability of minimization of the total processing time of that sequence among other sequences. When we have fuzzy processing times, we can define the priority of sequences. For each π_i , we have:

$$B_i = \min_{j \neq i}(A_j) \quad (11)$$

A_i is a fuzzy number related to the processing time of i^{th} sequence. B_i is the minimum processing time for other sequences. In fact, B_i illustrates the minimum makespan among $\pi_{j\neq i}$.

$$C_i = B_i - A_i \quad (12)$$

C_i is a fuzzy number achieved by subtracting two fuzzy numbers. The probability of $T(\pi_i)$ for being less than the makespan of other sequences equals $\mu_c(x)$. The considered sequences in this paper are the sequences obtained by the last step of the CDS algorithm. Instead of the optimum index in a stochastic context, we utilize the following equation for the optimal probability.

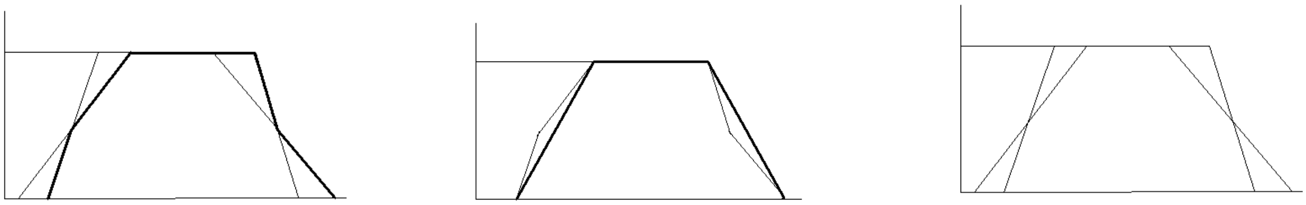
$$\overline{OI}(\pi_j) \equiv K \frac{\int_0^{\infty} \mu_{C_j}(x)dx}{\int_{-\infty}^{\infty} \mu_{C_j}(x)dx} \qquad (13)$$

The constant K is defined in order to obtain 1 for the total probability of optimums for the considered sequences. This method is introduced by developing the example taken from [18].

In this example, a flow shop problem of four machines and four jobs is considered. Processing times of jobs are fuzzy trapezoidal numbers as shown in Table 1. In Figure 1, the results of inserting *max* and *min* on the fuzzy trapezoidal numbers are then estimated for new fuzzy numbers in order to prevent the complexity of calculations in the next steps. It can also be seen in Figure 1.

Table 1
Processing times in fuzzy numbers

Jobs	W_1	W_2	W_3	W_4
1	(4,5,6,7)	(5,5,6,7)	(1,3,4,5)	(2,3,5,6)
2	(2,3,4,6)	(6,7,7,5,8)	(1,3,4,5)	(2,5,5,6,7)
3	(8,9,11,12)	(4,5,6,9)	(3,5,6,7)	(2,4,5,6)
4	(3,4,5,8)	(5,6,8,5,9)	(3,4,5,6)	(1,2,3,4)



Max (A, B) = C' $\simeq$ C

Fig. 1. Fuzzy trapezoidal numbers

The following results are obtained from the last step of the CDS algorithm (i.e., three sequences with fuzzy processing times).

$A_1 = (28,34,43.5,53)$

$A_2 = (32,38,47,55)$

$A_3 = (27,34,44.5,54)$

π_1 : 2, 3 ,1 ,4

π_2 : 3, 2 ,1 ,4

π_3 : 2, 3 ,4 ,1

In the MacCahon method [18], the sequence of μ_1, which contains the minimum average, is selected. With respect to (12) and (13), the following values are considered for the above three sequences.

$B_1 = (27,34,44.5,54)$

$B_2 = (27,34,43.5,53)$

$B_3 = (28,34,43.5,53)$

$C_1 = (-26,-9.5,10.5,26)$

$C_2 = (-28,-13,5.5,21)$

$C_3 = (-26,-10.5,9.5,26)$

$K = 0.72$ $\quad \overline{OI}(\pi_1) \equiv 0.364$

$\overline{OI}(\pi_2) \equiv 0.281$ $\quad \overline{OI}(\pi_3) \equiv 0.354$

Thus, the first sequence with the maximum probability of being optimum is then chosen ($\mu_0 = \mu_1$). The following shows one full numerical example derived from Ref [18]. In this example we have 4 Machines and 4 work pieces (Please see Table 1). Table 2 illustrates 3 sequences and the fuzzy tardiness. Table 3 shows all possible sequences with the delivery tardiness. It has been highlighted that the sequences S7 and S12 are the optimal choices among all 24 options.

4. Conclusion

A flow shop scheduling problem is known to be NP-hardness. Much effort has been made to solve the problem by heuristic and meta-heuristic methods. The stochastic flow shop scheduling problem is more complicated than the crisp one due to the necessity of more precisely determining the criterion. The most significant criteria are the average of total processing times and the probability of minimizing the makspan of the sequence with respect to the other sequences (i.e., optimal index). In general, optimization of a sequence is approximately impossible through analytical methods.

Table 2

The value of fuzzy tardiness evident and the fuzzy mean of the range of production planning

Sequence	M S	E_F	$P_{L(\pi 1)}$
S1: 2314	(28,34,43,5,53)	39.76	0.119
S2: 3214	(32,38,47,55)	43.07	0.235
S3: 2341	(27,34,44.5,54)	39.97	0.158

Table 3:

The criteria values for all sequences regarding due date of different parameters

Sequence	E_F	$P_{L(\pi)}$				
		μ=45 δ=5	μ=55 δ=5	μ=45 δ=10	μ=55 δ=10	μ=55 δ=2
S1 1 2 3 4	39.76	0.714	0.119	0.623	0.258	0.018
S2 1 2 4 3	41.59	0.805	0.195	0.684	0.316	0.080
S3 1 3 2 4	42.76	0.861	0.235	0.720	0.348	0.100
S4 1 3 4 2	44.89	0.927	0.359	0.782	0.423	0.247
S5 1 4 2 3	41.59	0.805	0.195	0.684	0.316	0.080
S6 1 4 3 2	42.89	0.835	0.236	0.709	0.342	0.133
S7 2 1 3 4	39.47	0.714	0.119	0.623	0.258	0.018
S8 2 1 4 3	40.19	0.741	0.165	0.647	0.286	0.066
S9 2 3 1 4	39.75	0.714	0.119	0.623	0.258	0.018
S10 2 3 4 1	39.97	0.770	0.185	0.659	0.290	0.042
S11 2 4 1 3	40.19	0.741	0.165	0.647	0.286	0.066
S12 2 4 3 1	38.68	0.686	0.172	0.611	0.255	0.033
S13 3 1 2 4	43.07	0.861	0.235	0.721	0.348	0.100
S14 3 1 4 2	45.18	0.927	0.359	0.782	0.423	0.271
S15 3 2 1 4	43.07	0.861	0.235	0.721	0.348	0.100
S16 3 2 1 4	43.29	0.898	0.294	0.752	0.385	0.174
S17 3 4 1 2	45.18	0.927	0.359	0.782	0.423	0.271
S18 3 4 2 1	43.39	0.898	0.294	0.752	0.385	0.174
S19 4 1 2 3	40.89	0.773	0.179	0.666	0.300	0.073
S20 4 1 3 2	42.89	0.835	0.236	0.709	0.342	0.133
S21 4 2 1 3	40.89	0.773	0.179	0.666	0.300	0.073
S22 4 2 3 1	39.38	0.72	0.138	0.630	0.268	0.036
S23 4 3 1 2	43.18	0.835	0.236	0.709	0.342	0.133
S24 4 3 2 1	41.38	0.789	0.187	0.675	0.308	0.076

How ever, various methods have been developed by considering special conditions. Thus, most of these studies concentrate on non-exact solutions. In addition, non-deterministic problems can be analyzed by fuzzy logic. Some of other studies on stochastic flow shop scheduling problem are addressed in “Introduction” section. In this paper, we considered the fuzzy flow shop scheduling problem. The optimal probability is defined similar to the definition of the optimal index of the stochastic model. A method is proposed to find a near-optimal solution, which is an expansion of the method presented in [18] for which a fuzzy flow shop model is considered. The criterion of a better solution selection is the possibility of priority of one sequence to other sequences of the last step. In the following section, this research is proposing a method to estimate the distribution function of the makespan by using fuzzy numbers and defining the feasibility of the provided solution through this method.

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