# (1, T) policy for a Two-echelon Inventory System with Perishable-on-the-Shelf Items

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**Abstract**

This paper deals with an inventory system with one central warehouse and a number of identical retailers. We consider perishable-on-the-shelf items; that is, all items have a fixed shelf life and start to age on their arrival at the retailers. Each retailer faces Poisson demand and employs (1, T) inventory policy. Although demand not met at a retailer is lost, the unsatisfied demand at the central warehouse is backordered. In this study, the long-run system total cost rate is derived. Moreover, a proposition is proved to define a domain for the optimal solution. Also, a search algorithm is presented to obtain this solution. Further, we extend the existent paper of the well-known (S-1, S) policy to cope with our considered model. Finally, in a numerical study, we compare (1, T) policy with (S-1, S) policy in terms of system total cost. The results reveal that when transportation time from the central warehouse to the retailers is long, the (1, T) policy outperforms the (S-1, S) policy.

*Keywords:* (1, T) policy, (S-1, S) policy, Two-echelon inventory system, Perishable items.

## 1. Introduction

In this paper, a two-echelon inventory system consisting of one central warehouse and a number of identical retailers is considered. Each retailer faces Poisson demand, and replenishes its stock from the central warehouse. The central warehouse, in turn, replenishes its stock from an external supplier which has an infinite supply. After joining the stock at a retailer, an item has a constant shelflife beyond which it is no longer usable. Demand that cannot be met immediately at the retailers is lost. The fixed ordering cost is negligible from which it is justified to employ (1, T) or (S-1, S) policies.

The (1, T) policy is to order one unit at each fixed time period T. This policy was introduced by Haji and Haji (2007) for a single installation inventory system with nonperishable items. When (1, T) policy is employed in the first level of a supply chain, it prevents expanding the demand uncertainty for other levels and makes their demand deterministic, one unit every T units of time. Therefore, this policy enjoys a number of advantages. For example, the cost of holding the safety stock in the upstream locations is eliminated. Also, this policy is very easy to apply and leads to simplify the inventory control and production planning. Following Haji and Haji (2007), Haji et al. (2009) applied (1, T) policy to a two-echelon inventory system with nonperishable items. Mahmoodi et al. (2014a) considered (1, T) policy for a single stage inventory system with perishable items. All of these papers, in their numerical experiments, compare (1, T) policy with the well-known (S-1, S) policy. They conclude that when the lead time is high, (1, T) policy is better than (S-1, S) policy in terms of total cost rate.

To our limited knowledge, there are very few papers concerning perishable items in multi echelon inventory systems. Abdel-Malek and Ziegler (1988) analyzed the optimal replenishment policies for a single perishable item, two-echelon inventory system assuming deterministic demand. Under demand uncertainty, Fujiwara et al. (1977) considered the problem of periodic review ordering and issuing policies in a two-echelon inventory system assuming separate lifetime for each echelon. Kanchanasuntorn and Techanitisawad (2006) investigated the effect of product perishability on system total cost in a two-echelon inventory system. They assume Normal demand for retailers and develop an approximate inventory model under periodic review policies. Olsson (2010) developed an approximate technique for evaluation of base stock policies in a continuous review, two-echelon serial inventory system with perishable items. He assumes that the downstream location faces Poisson demand, and unsatisfied demand is backordered.

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Mahmoodi et al. (2014b) consider a two-echelon serial inventory system with perishable-on-the-shelf items and lost sales. They employ (S-1, S) policy to control the inventory of both echelons. Using METRIC approach (Sherbrooke 1968), they approximate the system total cost rate, and present two procedures to obtain the optimal or near optimal solutions.

In this paper, we apply (1, T) inventory policy to a two-echelon distribution system with identical retailers. All items start to age as they arrive at a retailer. Further, we extend the model of Mahmoodi et al. (2014b) to deal with a distribution system with identical retailers. Finally, we compare (1, T) and (S-1, S) policies in terms of system total cost rate in a numerical experiment. The results show that when both the shortage unit cost and the transportation time from the central warehouse to the retailers are high and the perishing unit cost is low, (1,T) policy outperforms (S-1, S) policy.

The proceeding parts of this paper are organized as follows. In section 2, the considered model is described. In section 3, considering (1, T) policy the system total cost rate is derived. Also a solution procedure is presented. Then in section 4, we extend (S-1, S) policy to deal with a distribution system with identical retailers. Furthermore, in section 5, a numerical study is carried out to compare (1, T) policy with (S-1, S) policy. Finally, the conclusion and future research are presented in section 6.

## 2. Model Description

We consider a single item, two-echelon distribution system consisting of a central warehouse and a number of identical retailers. The items have a constant shelflife and start to age on their arrival at a retailer. All retailers face Poisson demand. The fixed ordering cost in all locations is negligible. Although demand not met at a retailer is lost, the unsatisfied demand at the central warehouse is backordered. All transportation times are fixed. All satisfied demands are met based on FIFO (First in first out) policy. A fixed penalty cost per lost sale and a fixed penalty cost per perished item are incurred at the retailers. Items held in stock both at the central warehouse and at a retailer incur holding costs per unit per time unit. Also the central warehouse pays a fixed purchase cost per item. Therefore, Shortage, perishing and holding costs are incurred at the retailers, and purchasing and holding costs are incurred at the central warehouse.

Two scenarios are considered to deal with the inventory control of presented model. The first one is to apply (1, *T*) policy at the retailers and (*N*, *T*) policy at the central warehouse, whereas the second is to employ $(S_0-1, S_0)$ policy at the central warehouse and $(S_1-1, S_1)$ policy at the retailers, in which $S_0$ and $S_1$ represent inventory position at their corresponding location. The (*N*, *T*) policy is to order *N* units at each fixed time period *T*. The objective is to find optimal *T* in the first scenario and optimal inventory positions in the second. The following notations are used in subsequent parts of the paper.

*N*: The number of retailers

$\mu_1$: The customer demand rate at a retailer

$\pi$: Cost of a lost sale

$h_0$: Holding cost per unit per time unit at the central warehouse

$h_1$: Holding cost per unit per time unit at a retailer

*p*: Cost of a perished item

*c*: Purchase cost per unit at the central warehouse

$\alpha$: The probability of perishing for an item

$\alpha_1$ : The rate of perishing at a retailer

$\tau_0$: Transportation time from the external supplier to the central warehouse

$\tau_1$: Transportation time from the central warehouse to a retailer

*m*: Items' shelflife at the retailers

$I_0$: Average on-hand inventory at the central warehouse

$I_1$: Average on-hand inventory at a retailer

$B_0$:The average number of backorders at the central warehouse

$S_0$: Inventory position at the central warehouse

$S_1$: Inventory position at a retailer

$H_0$: Average total holding cost per time unit at the central warehouse

$H_1$: Average total holding cost per time unit at a retailer

$\Pi$: Average total shortage cost per time unit at a retailer

*OC*: Average total perishing cost per time unit at a retailer

*PC*: Average total purchasing cost per time unit at the central warehouse

$C_0$: Total cost rate at the central warehouse

$C_1$: Total cost rate at a retailer

$TC^T$ : System total cost rate for (1,*T*) inventory system

$TC^S$ : System total cost rate for (*S*-1,*S*) inventory system

## 3. (1, T) Policy

### *3.1. Preliminaries – Single installation model*

Mahmoodi et al. (2014a) consider a single perishable item, single location inventory system operating under (1, *T*) policy with lost sales, Poisson demand, perishing costs, purchase costs, and per unit per period holding costs. The results of their model are used to obtain the cost function of the retailers in our model. Therefore, some key results of their model are presented in follows. Let *m* represent the shelflife, and $\mu$ denote the Poisson demand rate. They present the following formula for the percent of perished items, $\alpha$.

$$\alpha = \frac{e^{-\mu m}}{1 + \sum_{i=1}^{N_1} (-\mu)^i e^{-i\mu T} \frac{[m - iT]^i}{i!}} \quad (1)$$

Where $N_1 = floor(\frac{m}{T}) = \left[\frac{m}{T}\right]$.

They also show that the proportion of time that the system is out of stock, $P_0$, could be obtained from

$$P_0 = 1 - \frac{1-\alpha}{\mu T} \quad (2)$$

Furthermore, they obtain the expected on hand inventory as follow.

$$I_1 = \frac{1}{T}(m\alpha + \Theta) \quad (3)$$

In which

$$\Theta = \int_0^m y h(y) dy\,, \quad (4)$$

Where $h(y)$ is as in Eq. (5).

*3.2. Central Warehouse cost*

Let $T^*$ represent the optimal $T$. Since the retailers are identical, $T^*$ for all of them is the same. Thus the central warehouse faces a deterministic demand of $N$ units per $T^*$ time units. The ordering cost is negligible, and all transportation times are fixed. Therefore, the optimal policy for the central warehouse is to order $N$ units at each fixed time period $T^*$. Since the transportation times are fixed, the central warehouse can place an order for $N$ units to the external supplier in such a way that these items arrive at the time of which the retailers' orders are placed. Consequently, the central warehouse receives its orders from the external supplier and sends them to the retailers at the same time. Accordingly, the holding cost at the central warehouse is zero. Also the purchase cost rate is $PC = \frac{Nc}{T}$. Hence, the central warehouse total cost rate is:

$$C_0 = \frac{Nc}{T} \quad (6)$$

*3.3. Retailers' cost*

Since the central warehouse policy is to dispatch an item at the same time as a retailer places its order, the leadtime at the retailer is deterministic and equals to the fixed transportation time from the central warehouse to the retailer.

Thus the retailers' model is quite similar to the single installation model of Mahmoodi et al. (2014a). Therefore,

$$\left[\left[\left[ h(y) = \begin{cases} \alpha e^{\mu(m-y)} \left( \mu \sum_{i=0}^{N_1} \frac{(-\mu)^i}{i!} e^{-i\mu T} (m - y - iT)^i + \sum_{i=1}^{N_1} \frac{(-\mu)^i}{(i-1)!} e^{-i\mu T} (m - y - iT)^{i-1} \right) & ; 0 \le y < m - N_1 T \\ \alpha e^{\mu(m-y)} \left( \mu \sum_{i=0}^{n} \frac{(-\mu)^i}{i!} e^{-i\mu T} (m - y - iT)^i + \sum_{i=1}^{n} \frac{(-\mu)^i}{(i-1)!} e^{-i\mu T} (m - y - iT)^{i-1} \right) & ; m - (n+1)T \le y < m - nT \; ; n = 1, \ldots, N_1 \\ \alpha \mu e^{\mu(m-y)} & ; m - T \le y \end{cases} \right]\right]\right] \quad (5)$$

since the proportion of items that is perished is $\alpha/T$, from (1) the perished cost rate at a retailer is obtained as:

$$OC = \frac{p\alpha}{T} = \frac{pe^{-\mu_1 m}}{T\left(1+\sum_{i=1}^{N_1}(-\mu_1)^i e^{-i\mu_1 T}\frac{[m-iT]^i}{i!}\right)} \quad (7)$$

Furthermore, since $P_0$ is the proportion of time that the system is out of stock, the proportion of demand lost at a retailer is $\mu_1 P_0$. Therefore, from (2) the shortage cost rate at a retailer is:

$$\Pi = \pi\mu_1 P_0 = \pi\mu_1 - \frac{\pi(1-\alpha)}{T} \quad (8)$$

In addition, the holding cost rate at a retailer could be obtained from (3) as follow.

$$H_1 = h_1 I_1 = \frac{h_1}{T}(m\alpha + \Theta) \quad (9)$$

Finally, one can obtain the total cost rate at a retailer from (7), (8) and (9) as in Eq (10)..

$$C_1 = OC + \Pi + H_1 = \frac{pe^{-\mu_1 m}}{T\left(1+\sum_{i=1}^{N_1}(-\mu_1)^i e^{-i\mu_1 T}\frac{[m-iT]^i}{i!}\right)} + \pi\mu_1 - \frac{\pi(1-\alpha)}{T} + \frac{h_1}{T}(m\alpha+\Theta) \quad (10)$$

Consequently, from (6) and (10) the system total cost rate for (1, *T*) policy is

$$TC^T = C_0 + NC_1 = N\left(\frac{c}{T} + \frac{pe^{-\mu_1 m}}{T\left(1+\sum_{i=1}^{N_1}(-\mu_1)^i e^{-i\mu_1 T}\frac{[m-iT]^i}{i!}\right)} + \pi\mu_1 - \frac{\pi(1-\alpha)}{T} + \frac{h_1}{T}(m\alpha+\Theta)\right) \quad (11)$$

From (11), one can conclude that the system total cost rate is independent of lead time which is an interesting characteristic of (1, *T*) Policy.

### *3.4. Solution procedure*

Due to the complex form of (11), we cannot prove the convexity of system total cost rate. However, we can prove that if the optimal *T* is not infinitive, it always would be in the interval of (0, *m*]. This is proved in the following proposition.

**Proposition 1**: If the establishment of the inventory system is affordable, say $T^* \neq \infty$, then $T^*$ would be in the interval of (0, *m*].

**Proof:** see Appendix A.

Since proposition 1 defines a domain for the optimal solution when the establishment of the inventory system is affordable ($T^* \neq \infty$), one can use a simple search algorithm to obtain the optimal solution. If $T^* = \infty$, then total lost sales is the optimal solution and the system total cost rate is $N\pi\mu_1$. Therefore, by using a search algorithm to find the best amount of *T* in (0, *m*] and compare its total cost rate with $N\pi\mu_1$, the optimal solution of (1, *T*) inventory system could be obtained. Let $\varepsilon$ be a small value, the optimal solution of (1, *T*) could be found with the accuracy of $\varepsilon$ using the following procedure.

**Procedure 1**

Step 1: Set $T = \varepsilon$. Calculate $TC^T$ using (11). Set $T^* = T$ and $C^* = TC^T$.

Step 2: If $T + \varepsilon \leq m$, set $T = T + \varepsilon$, calculate $TC^T$ using (11), and go to step 3. Otherwise, go to step 4.

Step 3: If $TC^T < C^*$, set $T^* = T$ and $C^* = TC^T$. Go to step 2.

Step 4: If $N\pi\mu_1 < C^*$, set $T^* = \infty$ and $C^* = N\pi\mu_1$. Go to step 5.

Step 5: The algorithm is finished and $(T^*, C^*)$ is the best solution of (1, *T*) inventory system.

## 4. (S-1, S) Policy

In this scenario the central warehouse operates under $(S_0 - 1, S_0)$ policy, and the retailers operate under $(S_1 - 1, S_1)$ policy. The objective is to obtain $S_0$ and $S_1$ in such a way that the total cost rate is minimized. Mahmoodi et al. (2014b) consider a two-echelon serial inventory system with similar assumptions. They approximate the central warehouse demand with a Poisson process, and approximate the retailer leadtime using the well-known METRIC approach of Sherbrook (1968). Their model could easily be extended to a distribution system with identical retailers. Thus in this paper, we modify their approach to cope with our considered inventory system. The only required modification is that the demand of the central warehouse

is the summation of $N$ identical retailers' demand. Therefore, the demand of the central warehouse, $\mu_0$, is

$$\mu_0 = N\mu_1(1-P_0) + N\alpha_1 \tag{12}$$

Furthermore, adapting Equation (4) and (7) of Mahmoodi et al. (2014b), $\alpha_1$ and $P_0$ at a retailer is obtained as follows.

$$\alpha_1 = \frac{K'e^{-\mu_1(\tau_1'+m)}(\tau_1'+m)^{S_1-1}}{(S_1-1)!} \tag{13}$$

$$P_0 = \frac{K'e^{-\mu_1\tau_1'}\tau_1'^{S_1}}{S_1!} \tag{14}$$

In which $K' = \left( \frac{e^{-\mu_1\tau_1'}\tau_1'^{S_1}}{S_1!} + \int_{\tau_1'}^{\tau_1'+m} \frac{x^{S_1-1}e^{-\mu_1 x}}{(S_1-1)!}dx \right)^{-1}$, and

$$\tau_1' = \tau_1 + \frac{B_0}{\mu_0} \tag{15}$$

Where $B_0 = \sum_{j=S_0+1}^{\infty} (j-S_0)\frac{(\mu_0\tau_0)^j}{j!}e^{-\mu_0\tau_0}$ (16)

Consequently, adapting Equation (11) of Mahmoodi et al. (2014b), the total cost rate of a retailer is approximated as

$$C_1 = (p-h_1\tau_1')\frac{K'e^{-\mu_1(\tau_1'+m)}(\tau_1'+m)^{S_1-1}}{(S_1-1)!} + (\pi+h_1\tau_1')\mu_1\frac{K'e^{-\mu_1\tau_1'}\tau_1'^{S_1}}{S_1!} + h_1S_1 - h_1\tau_1'\mu_1 \tag{17}$$

Also, by substitution of $\mu_0$ in Equation (16) of Mahmoodi et al. (2014b), the approximated total cost rate of the central warehouse is

$$C_0 = c\mu_0 + h_0\sum_{j=0}^{S_0}(S_0-j)\frac{(\mu_0\tau_0)^j}{j!}e^{-\mu_0\tau_0} \tag{18}$$

Therefore, from (17) and (18) the system total cost rate is

$$TC^S = C_0 + NC_1 = c\mu_0 + h_0\sum_{j=0}^{S_0}(S_0-j)\frac{(\mu_0\tau_0)^j}{j!}e^{-\mu_0\tau_0} + N(p-h_1\tau_1')\frac{K'e^{-\mu_1(\tau_1'+m)}(\tau_1'+m)^{S_1-1}}{(S_1-1)!} + N(\pi+h_1\tau_1')\mu_1\frac{K'e^{-\mu_1\tau_1'}\tau_1'^{S_1}}{S_1!} + Nh_1S_1 - Nh_1\tau_1'\mu_1 \tag{19}$$

Mahmoodi et al. (2014b) present two procedures to obtain the optimal solution of the system. In their first procedure, they present a heuristic to find the optimal solution using the approximated total cost rate. However, the optimal solution of the approximated model is not necessarily the optimal solution of the considered system. Therefore, they present the second procedure which is a simulation-based neighborhood search heuristic. The obtained solution of the first procedure is used as the starting point of the second procedure. In second procedure, for a given point $S=(S_0,S_1)$, eight neighborhoods including $S=(S_0-1,S_1)$, $S=(S_0+1,S_1)$, $S=(S_0,S_1-1)$, $S=(S_0,S_1+1)$, $S=(S_0-1,S_1-1)$, $S=(S_0-1,S_1+1)$, $S=(S_0+1,S_1-1)$ and $S=(S_0+1,S_1+1)$ are considered. Then for each of them 3 simulations with 10000 time units is executed. The average cost rate obtained from these 3 simulations is assigned to the corresponding neighborhood. If at least one improving solution has been determined, the neighborhood search is restarted with respect to the best of them. If no superior solution was found, the procedure stops and returns the best found solution. Mahmoodi et al (2014b), in a numerical experiment show that their second procedure obtains the optimal solution of the system almost in all considered problems. We adapt their procedures for our considered model as follows.

**Procedure 2.** (Adapted from Procedure 1 of Mahmoodi et al. (2014b))

STEP 0: Set $S_0 := 0$, $TC^{\min} := N\pi\mu_1$, $S_0^{opt} := 0$ and $S_1^{opt} := 0$.

STEP 1: Set $s := 0$, $S_1 := 1$, $\tau_1' := \tau_1$, $TCS0^{\min} := N\pi\mu_1$ and $TCS1^{\min} := \pi\mu_1$. Calculate $\alpha_1$ from (13), and $P_0$ from (14).

STEP 2: While $(TCS1^{\min} > p\alpha_1)$, do {

Calculate $\mu_0 = N\mu_1(1-P_0) + N\alpha_1$, $\tau_1'$ from (15), and $C_1(S_1,\tau_1')$ from (17).

Calculate $C_0(S_0,\mu_0)$ from (18) and set

$TC(S_0) := C_0(S_0, \mu_0) + NC_1(S_1, \tau_1')$.

If $TC(S_0) < TCS0^{\min}$, then set $TCS0^{\min} := TC(S_0)$, $\mu := \mu_0$ and $s := S_1$.

If $C_1(S_1) < TCS1^{\min}$, then set $TCS1^{\min} := C_1(S_1)$.

Set $S_1 := S_1 + 1$.

Calculate $\alpha_1$ from (13), and $P_0$ from (14).

}

STEP 3: Calculate $C_0(S_0, \mu)$ from (18),

If $TCS0^{\min} < TC^{\min}$ then $TC^{\min} := TCS0^{\min}$ and let $S_0^{opt} := S_0$ and $S_1^{opt} := s$.

If $TC^{\min} < C_0(S_0, \mu)$ then set $S^* := (S_0^{opt}, S_1^{opt})$, return $S^*$ and $TC^{\min}$, and stop the procedure. Otherwise, set $S_0 := S_0 + 1$ and go to STEP 1.

**Procedure 3.** (Adapted from Procedure 2 of Mahmoodi et al. (2014b))

Step 0: Set $j := 1$ and $J_{\max} := 50$. Execute Procedure 2. Let $S^1$ be the best policy found by Procedure 2.

Step 1: Execute 3 simulations with 10000 time units for $S^j$ and set $TC^j$ to the average total cost rate obtained by these simulations.

Step 2: Execute 3 simulations with 10000 time units for all neighborhoods of $S^j$ which are not simulated before. Set $S^{j+1}$ to the policy with the minimum average total cost rate among all neighborhoods. Also set $TC^{j+1}$ to this total cost rate.

Step 3: If $TC^{j+1} < TC^j$ and $j \le J_{\max}$, set $j := j + 1$, and go to step 2.

Otherwise, return $S^j$ as the best policy and $TC^j$ as its total cost rate. Stop the Procedure.

The interested reader is referred to Mahmoodi et al. (2014b) for more details about convergence, effectiveness and accuracy of these procedures.

## 5. Numerical Study

This section is devoted to compare the performance of (1, *T*) policy (Scenario 1) with (*S*-1, *S*) policy (Scenario 2) in terms of system total cost rate. To do this, for considered problems, $\%\Delta C = \frac{TC^S - TC^T}{TC^S}$ is calculated. Where $\%\Delta C$ shows how much in percent (1, *T*) policy performs better than (*S*-1, *S*) policy.

Apart from comparison between two considered policies, the effect of varying the values of $\pi$, $p$, $m$ and $\tau_1$ on the optimal solution for both policies is studied. In all considered test problems, we have $N = 5$, $c = 5$, $\mu_1 = 1$, $h_0 = 2$, $h_1 = 1$, $\tau_0 = 0.5$, $\tau_1 \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$ and $\varepsilon = 0.01$.

To obtain the best solution of Scenario 1 and Scenario 2, we utilize Procedure 1 and Procedure 3, respectively. Matlab software is used to coding these procedures. In table 1, the effect of varying items' shelflife and transportation time from the central warehouse to the retailers on both policies is studied. In this table, the results for $p = 10$, $\pi = 40$ and $m \in \{0.5, 1, 2\}$ are presented. As it could be seen, $T^*$ increases as $m$ increases. This observation is intuitively expected, since increasing of $m$ leads to decreasing of perishing rate. Total cost rate of (1, *T*) policy is independent of transportation times, but total cost rate of (*S*-1, *S*) policy increases as $\tau_1$ increases. Therefore, for small values of $\tau_1$, $\%\Delta C$ is negative, and by increasing $\tau_1$, $\%\Delta C$ becomes positive. The positive amount of $\%\Delta C$ means that the (1, *T*) policy outperforms the (*S*-1, *S*) policy. This behavior is illustrated in Figure 1 for $m = 1$. According to Table 1, the optimal inventory positions are not a monotonic function of *m*, as one might expect. However, as expected, increasing in *m* leads to decreasing in total cost rate of both policies. Although $S_1^*$ is an increasing function of $\tau_1$, $S_0^*$ decreases as $\tau_1$ increases. This appears due to that we consider a fixed transportation time from the external supplier to the central warehouse, then with increasing of $\tau_1$ the delay in the central warehouse due to stockouts would be a smaller percent of retailers' leadtime.

Table 2 presents the results for $m = 1$, $\pi_1 = 40$, $p \in \{5, 10, 20\}$. As can be seen, when $\tau_1$ increases, $\%\Delta C$ increases which means that for high values of $\tau_1$ the (1, *T*) policy outperforms the (*S*-1, *S*) policy. This result is similar to that obtained from Table 1. Moreover, as $p$ increases the threshold value of $\tau_1$, for which (1, *T*) is better than (*S*-1, *S*), increases. Most of results are what one intuitively expects. For example, as $p$ increases the total cost rate of both policies increases. Furthermore,

increasing in $p$ leads to decreasing of $T^*$ and increasing of $S_1^*$, in order to decrease the total number of perished items.

The results for $m=1$, $p=10$ and $\pi \in \{20,40,60\}$ are presented in Table 3. For the comparison of two policies, similar results can be seen. That is, for enough long $\tau_1$, (1, $T$) policy is better than ($S$-1, $S$) policy. Moreover, as $\pi$ increases the threshold value of $\tau_1$, for which (1, $T$) outperforms ($S$-1, $S$), decreases. Other results of this table are the same as what expected. For example, as $\pi$ increases, the total cost rate of both policies does increase, $T^*$ increases, and $S_1^*$ decreases.

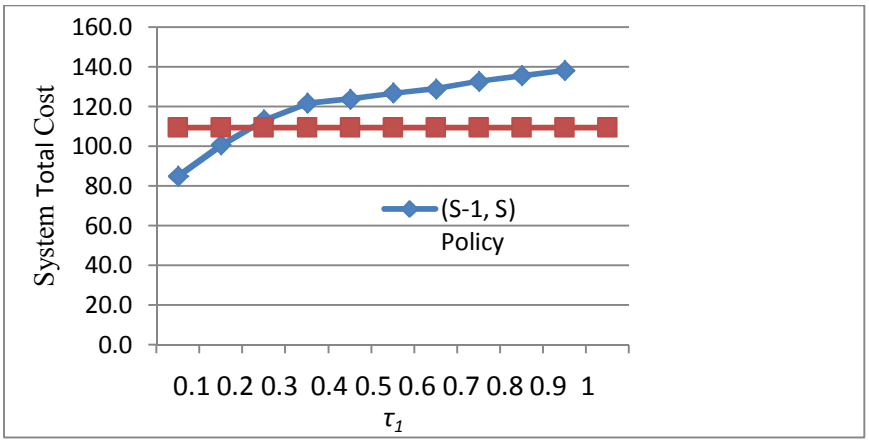


**Fig. 1.** System total cost of the policies with respect to $\tau_1$

Table 1
Numerical results for $p = 10$ and $\pi = 40$

| $m$ | 0.5 | | | | 1 | | | | 2 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T^*$ | 0.50 | | | | 0.68 | | | | 0.81 | | | |
| $TC^I(T^*)$ | 157.2 | | | | 109.4 | | | | 78.3 | | | |
| $\tau_1$ | $S_0^*$ | $S_1^*$ | $TC^S(S^*)$ | $\%\Delta C$ | $S_0^*$ | $S_1^*$ | $TC^S(S^*)$ | $\%\Delta C$ | $S_0^*$ | $S_1^*$ | $TC^S(S^*)$ | $\%\Delta C$ |
| 0.1 | 6 | 1 | 121.7 | -29.2% | 5 | 1 | 85.0 | -28.8% | 4 | 1 | 62.3 | -25.7% |
| 0.2 | 5 | 1 | 139.1 | -13.0% | 4 | 1 | 100.7 | -8.6% | 3 | 2 | 70.0 | -11.9% |
| 0.3 | 4 | 1 | 151.0 | -4.1% | 4 | 1 | 113.2 | **3.4%** | 4 | 2 | 72.2 | -8.4% |
| 0.4 | 4 | 1 | 158.7 | **0.9%** | 4 | 2 | 121.5 | **10.0%** | 4 | 2 | 75.0 | -4.4% |
| 0.5 | 3 | 1 | 164.8 | **4.6%** | 4 | 2 | 123.7 | **11.5%** | 4 | 2 | 78.8 | **0.6%** |
| 0.6 | 3 | 1 | 169.7 | **7.4%** | 4 | 2 | 126.8 | **13.7%** | 4 | 2 | 82.6 | **5.2%** |
| 0.7 | 3 | 1 | 173.6 | **9.5%** | 4 | 2 | 128.9 | **15.1%** | 4 | 2 | 86.4 | **9.3%** |
| 0.8 | 4 | 1 | 177.5 | **11.4%** | 3 | 2 | 132.8 | **17.6%** | 4 | 2 | 90.9 | **13.8%** |
| 0.9 | 3 | 1 | 178.3 | **11.9%** | 3 | 2 | 135.5 | **19.2%** | 4 | 2 | 95.3 | **17.8%** |
| 1 | 2 | 1 | 181.7 | **13.5%** | 3 | 2 | 138.2 | **20.8%** | 4 | 2 | 99.4 | **21.2%** |

Table 2
Numerical results for $m = 1$ and $\pi = 40$

| $P$ | 5 | | | | 10 | | | | 20 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T^*$ | 0.60 | | | | 0.68 | | | | 0.85 | | | |
| $TC^T(T^*)$ | 91.4 | | | | 109.4 | | | | 136.8 | | | |
| $\tau_1$ | $S_0^*$ | $S_1^*$ | $TC^S(S^*)$ | $\%\Delta C$ | $S_0^*$ | $S_1^*$ | $TC^S(S^*)$ | $\%\Delta C$ | $S_0^*$ | $S_1^*$ | $TC^S(S^*)$ | $\%\Delta C$ |
| 0.1 | 5 | 1 | 76.9 | -19.0% | 5 | 1 | 85.0 | -28.8% | 5 | 1 | 100.6 | -36.0% |
| 0.2 | 4 | 1 | 93.6 | **2.3%** | 4 | 1 | 100.7 | -8.6% | 4 | 1 | 114.8 | -19.1% |
| 0.3 | 5 | 2 | 98.8 | **7.5%** | 4 | 1 | 113.2 | **3.4%** | 4 | 1 | 125.5 | -9.0% |
| 0.4 | 5 | 2 | 102.1 | **10.4%** | 4 | 2 | 121.5 | **10.0%** | 4 | 1 | 134.4 | -1.8% |
| 0.5 | 5 | 2 | 105.2 | **13.1%** | 4 | 2 | 123.7 | **11.5%** | 4 | 1 | 140.5 | **2.7%** |
| 0.6 | 5 | 2 | 110.5 | **17.3%** | 4 | 2 | 126.8 | **13.7%** | 3 | 1 | 146.7 | **6.8%** |
| 0.7 | 4 | 2 | 114.4 | **20.1%** | 4 | 2 | 128.9 | **15.1%** | 3 | 1 | 150.9 | **9.3%** |
| 0.8 | 4 | 2 | 117.4 | **22.1%** | 3 | 2 | 132.8 | **17.6%** | 2 | 1 | 155.2 | **11.8%** |
| 0.9 | 4 | 2 | 122.5 | **25.4%** | 3 | 2 | 135.5 | **19.2%** | 2 | 1 | 158.3 | **13.6%** |
| 1 | 3 | 2 | 126.0 | **27.4%** | 3 | 2 | 138.2 | **20.8%** | 2 | 1 | 162.1 | **15.6%** |

Table 3
Numerical results for $m = 1$ and $p = 10$

| $\pi$ | 20 | | | | 40 | | | | 60 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T^*$ | 1.00 | | | | 0.68 | | | | 0.58 | | | |
| $TC^T(T^*)$ | 83.3 | | | | 109.4 | | | | 122.8 | | | |
| $\tau_1$ | $S_0^*$ | $S_1^*$ | $TC^S(S^*)$ | $\%\Delta C$ | $S_0^*$ | $S_1^*$ | $TC^S(S^*)$ | $\%\Delta C$ | $S_0^*$ | $S_1^*$ | $TC^S(S^*)$ | $\%\Delta C$ |
| 0.1 | 4 | 1 | 68.4 | -21.9% | 5 | 1 | 85.0 | -28.8% | 7 | 1 | 101.6 | -20.8% |
| 0.2 | 4 | 1 | 73.1 | -14.1% | 4 | 1 | 100.7 | -8.6% | 5 | 2 | 128.3 | **4.3%** |
| 0.3 | 3 | 1 | 76.6 | -8.8% | 4 | 1 | 113.2 | **3.4%** | 5 | 2 | 132.5 | **7.3%** |
| 0.4 | 3 | 1 | 79.5 | -4.8% | 4 | 2 | 121.5 | **10.0%** | 5 | 2 | 137.8 | **10.9%** |
| 0.5 | 2 | 1 | 82.2 | -1.4% | 4 | 2 | 123.7 | **11.5%** | 4 | 2 | 145.0 | **15.3%** |
| 0.6 | 2 | 1 | 84.0 | **0.8%** | 4 | 2 | 126.8 | **13.7%** | 5 | 2 | 151.6 | **19.0%** |
| 0.7 | 2 | 1 | 85.3 | **2.3%** | 4 | 2 | 128.9 | **15.1%** | 4 | 2 | 158.9 | **22.7%** |
| 0.8 | 2 | 1 | 86.8 | **4.0%** | 3 | 2 | 132.8 | **17.6%** | 4 | 2 | 165.3 | **25.8%** |
| 0.9 | 2 | 1 | 87.7 | **5.0%** | 3 | 2 | 135.5 | **19.2%** | 4 | 2 | 172.3 | **28.8%** |
| 1 | 1 | 1 | 88.6 | **5.9%** | 3 | 2 | 138.2 | **20.8%** | 4 | 2 | 178.5 | **31.2%** |

Finally, based on the observed results, when both shortage unit cost and transportation time from the central warehouse to the retailers are high and the perishing unit cost is low, the (1,*T*) policy is preferred. Moreover, for fixed values of system parameters, there is a fixed value of transportation time from the central warehouse to the retailers for which the (1,*T*) policy outperforms the (*S*–1, *S*) policy. Further, as the lead time increases, this superiority is more pronounced.

## 6. Conclusions and Future Research

In this paper, a two-echelon inventory system with a single perishable-on-the-shelf item was considered. The considered system consists of a central warehouse and a number of identical retailers. Two scenarios were investigated to deal with the inventory control of the system. In the first one (1, *T*) policy was employed, whereas in the second (*S*-1, *S*) policy was applied. When (1, *T*) policy is used for retailers, the central warehouse faces a deterministic demand. Therefore, the central warehouse can place its orders to the external supplier in such a way that these orders arrive at the central warehouse and are dispatched to the retailers at the same time. Consequently, the holding cost at the central warehouse is zero. This is an interesting advantage of employing (1, *T*) policy for multi-echelon inventory systems. Moreover, the system total cost rate of (1, *T*) policy is independent of the transportation time from the central warehouse to the retailers.

Furthermore, a numerical study was carried out to compare two considered scenarios. Accordingly, for fixed values of system parameters, there is a fixed value of transportation time from the central warehouse to the retailers for which (1, *T*) policy is better than (*S*–1, *S*)

policy. Further, as the lead time increases this superiority is more pronounced.

An attractive direction for future research is to develop (1, *T*) policy to deal with a two-echelon inventory system with non-identical retailers. Furthermore, considering a model from which the items start to age at the central warehouse would be another challenging direction. Such a model is more complex than ours, since the retailers concern the items with stochastic shelflife. Considering the ordering cost is also important and would be a challenge for future research.

**Appendix A.** Proof of Proposition 1

We should obtain the total cost rate for $m \le T$. To do this, we can interpret the inventory problem at the retailers as a *D*/*M*/1 queue with impatient customers, a single channel queueing system in which the inter-arrival times are constant, equal to *T*, the service times have exponential distribution with mean $1/\mu_1$, and a customer (items in inventory system) leaves the queue system whenever his sojourn time is larger than *m*. It is clear that the number of customers (units) in this queueing system is equal to the inventory on hand in our inventory system. Let $W_q$ denote the queue waiting time of an arriving customer (item) in the queueing system in steady state, $W_s$ denote a customer (item) average waiting time in the queueing system in steady state, $S$ denote the random variable of the required service time of a customer (item) in steady state, and $\overline{S}$ represent the random variable of the occurred service time of a customer (item) in steady state. Consequently, using this queueing system, the total cost rate of the considered inventory system is obtained as follows.

It is clear that $W_q = 0$ for $m \le T$. Thus we have

$$\overline{S} = \begin{cases} S & if \quad S \le m \\ m & if \quad S > m \end{cases}, \tag{A.1}$$

From (A.1), it is clear that

$$W_s = E(\overline{S}) \tag{A.2}$$

Using Little's formula and (A.2), the long-run average number of units at a retailer is obtained as

$$I_1 = \frac{E(\overline{S})}{T}$$

Hence,

$$H_1 = h_1 I_1 = h_1 \frac{E(\overline{S})}{T} \tag{A.3}$$

Since $W_q = 0$, the probability that an item is perished at the retailers is obtained as follows.

$$\alpha = \Pr(S > m) = e^{-\mu_1 m}$$

Therefore, the proportion of products that is perished in a retailer is $\frac{\alpha}{T}$. Thus

$$OC = \frac{p\alpha}{T} = \frac{pe^{-\mu_1 m}}{T} \tag{A.4}$$

Furthermore, the proportion of time that the inventory system is out of stock, $P_0$, could be interpreted as the proportion of time for which the server in queueing system is idle. Hence, $P_0 = 1 - \lambda / \mu_{effective}$. Where in our case $\lambda = 1/T$ and $\mu_{effective} = 1/E(\overline{S})$. Therefore,

$$P_0 = 1 - \frac{E(\overline{S})}{T} \tag{A.5}$$

Since $P_0$ is the proportion of time that the system is out of stock, the proportion of demand that is lost is $\mu P_0$. Hence,

$$\Pi = \pi\mu_1 P_0 = \pi\mu_1 (1 - \frac{E(\overline{S})}{T}) \tag{A.6}$$

In addition, the amount of items purchased per time unit at the central warehouse is $N/T$, Thus

$$PC = \frac{Nc}{T} \tag{A.7}$$

Form (A.3), (A.4), (A.6), and (A.7), the total cost rate of the system for $m \le T$ can be written as:

$$TC^T = PC + N\left(OC + \Pi + H_1\right) =$$

$$N\left(\frac{c}{T} + h_1 \frac{E(\overline{S})}{T} + \frac{pe^{-\mu_1 m}}{T} + \pi\mu_1 (1 - \frac{E(\overline{S})}{T})\right) \Rightarrow$$

$$TC^T = N\left(\frac{c}{T} + (h_1 - \pi\mu_1)\frac{E(\overline{S})}{T} + \frac{pe^{-\mu_1 m}}{T} + \pi\mu_1\right) \tag{A.8}$$

From (A.8), if $\frac{c}{T}+(h_1-\pi\mu_1)\frac{E(\bar{S})}{T}+\frac{pe^{-\mu_1 m}}{T}\geq 0$, It is clear that $T^*=\infty$. Moreover, if $\frac{c}{T}+(h_1-\pi\mu_1)\frac{E(\bar{S})}{T}+\frac{pe^{-\mu_1 m}}{T}<0$, evidently $TC^T$ is increasing with respect to *T*, also it is assumed that $m\leq T$, thus $T^*=m$. Therefore, if $T^*\neq\infty$, $T^*$ will be in the interval of (0,*m*].

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