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Process Capability Analysis in the Presence of Autocorrelation

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Abstract

The classical method of process capability analysis necessarily assumes that collected data are independent; nonetheless, some processes such as biological and chemical processes are autocorrelated and violate the independency assumption. Many processes exhibit a certain degree of correlation and can be treated by autoregressive models, among which the autoregressive model of order one (AR (1)) is the most frequently used one. In this paper, we discuss the effect of autocorrelation on the process capability analysis when a set of observations are produced by an autoregressive model of order one. We employ a multivariate regression model to modify the process capability estimated from the classical method, where the AR (1) parameters are utilized as regression explanatory variables. Finally, the performance of the presented method is investigated using a Monte Carlo simulation.

Keywords: Process capability analysis; Statistical process control; Autocorrelation; AR (1).

1. Introduction

Process Capability Indices (PCIs) are introduced to give a proper indication of the capability of a manufacturing process by quantifying the relation between the desired engineering specifications and the actual performance of the process. In fact, PCIs are organized to determine whether a process is capable of visiting specification limits on the quality features of interest or not. The quantitative measure of PCI indicates the amount of customer requirements obtained from quality characteristics. Generally, a bigger amount of PCI illustrates a better process performance and a smaller amount of PCI shows a worse process performance. PCIs have been extensively applied in different production systems and can be regarded as an effective and superior means of determining product quality.

Basic assumptions in using PCIs are: 1) The observations collected are assumed to be identically distributed; 2) the observations are always assumed to be independent; 3) the observations are normally distributed with mean μ and variance σ^2. According to the above-mentioned definitions and assumptions, we can use the following well-known capability indices:

$$C_p = \frac{USL - LSL}{6\sigma} \tag{1}$$

$$C_{pl} = \frac{\mu - LSL}{3\sigma} \quad , \quad C_{pu} = \frac{USL - \mu}{3\sigma}$$

$$C_{pk} = \min\left\{C_{pl}, C_{pu}\right\} \tag{2}$$

Eq. (1) determines the capability of a central process. According to Eq. (2), we can calculate the capability of a process when its specification interval mean is not equal to its process mean. The *USL* and *LSL* indicate the upper specification limit and the lower specification limit, respectively. Furthermore, the μ is the process mean and the σ is the standard deviation. In general, products with multiple features could usually contain huge non-central specifications and central specifications. In fact, whenever all process capabilities of each characteristic satisfy preset specifications, consumers will not reject products. It is clear that a single PCI is not able to visit the consumer requirements stated above, and it seems that many crucial problems are concentrating on central quality characteristics.

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Despite the fact that PCIs are predominantly defined under the independency assumption, most of the processes in the real world produce autocorrelated data. For instance, data exhibit some degree of autocorrelation for chemical processes like the production of pig iron. In addition, some biological processes are autocorrelated and violate the independency assumption. For these processes it is proved that the autocorrelation dramatically affects the amount of PCIs defined under the independency assumption (Mingoti and Oliveira 2011). Nevertheless, no significant research has been conducted about the impacts of autocorrelation on the amount of PCIs. Therefore, this subject is addressed in this study, and a robust estimation method for process capability is proposed.

2. Literature Review

In the literature there are quite a few studies dealing with process capability analysis when one of the three basic assumptions is not met. For a comprehensive review of these studies, see the review paper with discussion by Kotz and Johnson (2002). A comprehensive literature review of classified studies on process capability indices in the time span of 2000–2009 can also be found in Yum and Kim (2010). Yet, the non-normality assumption is more investigated (for example, see Clement 1989; Somerville and Montgomery 1996; Tang and Than 1999; Hosseinifard et al. 2008). In this paper, we present a new way of calculating the PCI when data display an inner dependent behavior. More specifically, we explore process capability estimation in AR(1) processes.

There are a few studies dealing with PCI estimation for autocorrelated processes. Shore (1997) described some of the undesirable effects that autocorrelations may have on the sampling distribution of the estimates of mean and standard deviation, and, hence, on the PCIs calculated through the Monte Carlo simulation. He remarked that both C_p and C_{pk} are biased upward when autocorrelation is present, and therefore critical values and confidence intervals extracted under the assumption of independent data which should not be used as type I and type II errors may be high. Finally, he carefully studied differences between the two approaches suggested for use in the presence of autocorrelation, and demonstrated them numerically for an AR(1) model when autocorrelation existed. The first procedure was *model-free* and estimated the μ and σ, by $\bar{X}$ and MR/d_2, where the MR was the average range observed in the subsamples the size of n while the process was in control. This procedure appraised the instantaneous capability but the customer wished to be aware of the long-term capability of the process, which also contained changes in the mean while the process was engaged in producing. The second procedure was able to estimate long-term capability. It was *model-dependent* and estimated the μ and σ, by the $\bar{\bar{X}}$ and S^2 of all data. As a result, Shore (1997) believed that the *model-free* approach was better than the *model-dependent* approach when both performance and convenience with regard to the application are considered, and process capability analysis should be used along the lines of the *model-free* approach. It seemed that more research was needed yet. Thus, in some other studies including Noorossana (2002), Scagliarini (2002), Chen et al. (2003), and Vannman and Kulahci (2008) the effect of autocorrelation on the process capability was estimated. Noorossana (2002) showed through an example that autocorrelated data could lead to biased estimates of PCI, and eventually to wrong decisions about performance. He employed a two-step procedure based on multivariate regression and time series modeling to remove the autocorrelation that may exist in the data and also to estimate model parameters correctly. Scagliarini (2002) described properties of the estimator of C_p for autocorrelated observations in the presence of measurement errors. He derived the performance of the estimator of C_p in the case of measurement errors for an autoregressive model of order one (AR (1)) and compared it with the results achieved in the error free case. Vannman and Kulahci (2008) presented a new way to perform process capability analysis when observations are autocorrelated. It was called the "Iterative skipping" strategy, in which the data set was divided into subsamples by skipping a pre-determined number of observations. It is clear that for the obtained sets the independence assumption may be valid. Consequently, Equations (1) and (2) could be used to estimate the PCI for each subsample of data.

In another study, the Taguchi method was applied by Jing et al. (2009) in order to estimate the PCIs of autocorrelated observations. Using this method, the impacts of autocorrelation on the μ, σ, and probability density function (pdf) were evaluated for the models of order one. Moreover, Jing et al. (2010) developed a comparison method for five different estimation strategies of process capability when observations were not independent. Eventually, they succeeded in finding an estimation method for process capability analysis when observations are autocorrelated.

It is clear that the existing studies on process capability analysis do not predominantly check the autocorrelation of observations although it is something common in industries and should not be overlooked. As a result, this study presents a robust estimation method for process capability analysis when a set of observations are autocorrelated and produced by an autoregressive model of order one. In this regard, the rest of the paper is organized as follows: we explain our suggested model in the next section, and then Monte Carlo simulation studies are presented in Section 3. In Section 4, we investigate the performance of the proposed method by applying statistical analysis. Section 5 provides conclusions and final remarks.

3. Description of the Proposed Model

Hereinafter the estimated PCI for autocorrelated processes is called $C_{\text{p-au}}$. Since AR(1) parameters impact on the bias in $C_{\text{p-au}}$ in comparison to the PCI which is known for independent observations, we propose to use these parameters to diminish autocorrelation effects on the PCI estimation. To accomplish this purpose, a straightforward way is incorporating AR (1) parameters in the PCI formula. Here, we use a multivariate regression analysis and nominate the PCI as follows:

$$C_{p-au} = C_{pu} + bias \tag{3}$$

We apply a multivariate regression model as shown in Eq. (4) to calculate the $C_{\text{p-au}}$. It has been known that $C_{\text{p-au}}$ is a linear combination of C_{pu}, φ and ε, where the C_{pu} denotes the PCI in Eq. (2) based on the independence assumption, and the φ and ε give us the correlation coefficient and model parameter, respectively.

$$C_{p-au} = C_{pu} + \beta_1\varphi + \beta_2\varepsilon \tag{4}$$

The model coefficients β_1 and β_2 are estimated from the observations of the process by the multivariate regression. In fact, the effects that the φ and ε may have on the PCIs are the main motivation for using the proposed model in the presence of autocorrelation. We will present a two-step procedure in order to calculate β_1 and β_2. The first step involves generating sets of data which are autocorrelated and calculating the C_{pu} for each set of data. Then, the β_1 and β_2 are estimated by the multivariate regression in the second step. To check the performance of the model, a Monte Carlo simulation is also used. This simulation can be classified into two major steps briefly described below:

(1) Generating sets of autocorrelated data and estimating the C_{pu} for each set of data.
(2) Using the multivariate regression to estimate the β_1 and β_2 for each set of data.

First, we consider an AR(1) process in Eq. (5) to generate sets of autocorrelated data where the e_t is a random variable that represents the amount by which the t^{th} measurement will differ from the mean owing to the effect of common causes. Typically, e_1, e_2, e_3, …, e_n are regarded as a sequence of independent and identically distributed random variables with the mean being zero and the standard deviation being σ, and parameters μ and σ are estimated from the data of the process.

$$x_t = \varepsilon + \varphi x_{t-1} + e_t \tag{5}$$

Consider a situation where $C_{\text{p-au}}$=1 in this simulation, and then make simulation for certain φ, ε and a pre-determined sample size of 10000 (N=10000). Afterwards, this procedure must be repeated for twenty iterations with different φ and ε in each iteration. Eq. (5) can be used to obtain the upper specification limit of each set of 10000 observations.

$$C_{p-au} = \frac{\phi^{-1}\left[\int_{-\infty}^{usl} f(x)dx\right]}{3} = \frac{\phi^{-1}[\Psi_x(usl)]}{3} \tag{6}$$

Where the $f(x)$ and Ψ_x are the probability density function and the cumulative density function of x, respectively. It is clear that Eq. (6) can be rewritten as $\phi(3C_{\text{p-au}})=p$. Because we initially set $C_{\text{p-au}}$=1 in each iteration, the value of p can be easily determined. Hence, it is enough to organize the observations of each set in the increasing order, and then choose the $p\times10000^{\text{th}}$ observation as the USL. In fact, the USL is the $p\times10000^{\text{th}}$ observation since we have $\Psi_x(USL)=p$ according to Eq. (6). σ must be also calculated in order to estimate the $C_{\text{p-au}}$, but it has been known that calculating σ isn't correct when data are autocorrelated. Therefore, a new technique is created to transform the autocorrelated data into independent data. In this technique, x_t-φx_{t-1} is used instead of x_t $(\forall t \in 2,3,\ldots,10000)$, and it is proved that x_t-φx_{t-1} $(\forall t \in 2,3,\ldots,10000)$ is always independent. Firstly, Eq. (5) is rewritten as x_t -$\varphi x_{t-1}=\varepsilon+e_t$. Obviously, the independence assumption for $\varepsilon+e_t$ is valid because the ε is $N(\varepsilon,0)$ and the e_t is $N(0, \sigma^2)$. It is clear that the summation of these two independent parameters is independent. Since x_t-φx_{t-1} is equal to $\varepsilon+e_t$, it seems reasonable to conclude that x_t-φx_{t-1} is $N(\varepsilon, \sigma^2)$, t=2,3,…,10000, and it is subsequently an independent variable.

C_{pu} can be obtained for each set by using the sample standard deviation of x_t-x_{t-1}, t=2, 3, … , N instead of the σ in Eq. (2) and the $p\times10000^{\text{th}}$ data as the USL. At the end, a multivariate regression is used to estimate the β_1 and β_2 when the generation of 10000 observations is repeated for twenty times. It should be noted that we use different $C_{\text{p-au}}$=0.5,0.6,0.7,0.8,0.85,0.9,0.95,1,1.05,1.1,1.2,1.25,1.3, 1.35,1.4,1.45,1.5,1.6,1.7,1.8 for each time, so 400 sets of observations are created. For instance, the generated observations for $C_{\text{p-au}}$=0.9, 1 are shown in Table 1 in order to make it easy for the readers to understand how $\hat{C}_{\text{p-au}}$ can be estimated. The estimated $C_{\text{p-au}}$ is shown in the table, and it can be used for estimating β_1 and β_2 when $0<\varphi<0.2$.

Let us recall that $C_{\text{p-au}}=C_{\text{pu}}+\beta_1\varphi+\beta_1\varepsilon$. Before assuming β_1, β_2 as the model coefficients, it is vital to determine whether the $C_{\text{p-au}}$, φ and ε are related or not. In this regard, it is necessary to test the null hypothesis H_0: $\beta_1=0$ and H_1: $\beta_1\neq0$ for the former, which shows whether the $C_{\text{p-au}}$ and φ are related or unrelated. In testing this null hypothesis, the statistic $t=\frac{\hat{B}_1-B_1}{S\hat{B}_1}$ and the confidence interval $A=\left[-t_{\frac{\alpha}{2},n-3}, t_{\frac{\alpha}{2},n-3}\right]$ are used at the significance level of α, respectively. Using 400 sets of generated observations, the statistics are equal to -9.19, -11.81, -10.14 and -48.22 when $0<\varphi<0.2$, $0.2<\varphi<0.4$, $0.4<\varphi<0.6$ and $0.6<\varphi<0.8$, respectively.

Table 1
The estimated $C_{p\text{-}au}$ when $C_{p\text{-}au}$=0.9, 1 and 0<φ<0.2

Iter. No	$C_{p\text{-}au}$	$\hat{C}_{p\text{-}au}$	φ	ε	μ		Iter. No	$C_{p\text{-}au}$	$\hat{C}_{p\text{-}au}$	φ	ε	μ
1	0.9	0.76	0.03	3	3.1		1	1	0.98	0.12	10.39	11.83
2	0.9	0.97	0.16	11.27	13.4		2	1	0.88	0.004	7.63	7.67
3	0.9	0.99	0.08	17.9	19.6		3	1	1.26	0.004	21.8	21.92
4	0.9	0.78	0.18	22.3	27.1		4	1	0.71	0.038	0.38	0.37
5	0.9	0.88	0.14	6.7	8		5	1	1.11	0.11	19.2	21.77
6	0.9	0.82	0.13	6.37	7.4		6	1	0.9	0.012	24.3	24.55
7	0.9	0.87	0.06	21.65	23.2		7	1	1.08	0.07	24.7	26.7
8	0.9	0.77	0.034	5.8	6		8	1	1.38	0.12	19.7	22.5
9	0.9	0.89	0.03	20.13	20.7		9	1	0.95	0.14	10.9	12.8
10	0.9	1.11	0.04	22.7	26.3		10	1	1.06	0.13	12.4	14.4
11	0.9	0.53	0.08	5.8	6.35		11	1	1.13	0.017	5.4	5.4
12	0.9	0.79	0.17	6	7.2		12	1	0.87	0.09	16	17.7
13	0.9	0.69	0.1	1.25	1.38		13	1	1	0.08	8	8.7
14	0.9	0.46	0.16	1.9	2.31		14	1	1.06	0.07	24	25.8
15	0.9	0.89	0.09	16	17.73		15	1	0.86	0.03	18.2	18.8
16	0.9	0.77	0.092	4.8	5.27		16	1	0.83	0.13	10.3	11.9
17	0.9	0.91	0.09	21.1	23.2		17	1	0.87	0.14	18.7	21.7
18	0.9	0.67	0.08	4.35	4.7		18	1	0.94	0.145	6.7	7.9
19	0.9	0.57	0.18	4.27	5.2		19	1	1.02	0.096	11	12.2
20	0.9	0.78	0.002	24.86	24.9		20	1	1.11	0.11	23.34	26.2

Thus, it seems reasonable to reject the null hypothesis for 0<φ<0.2, 0.2 <φ<0.4, 0.4 <φ<0.6 and 0.6 <φ<0.8 in that the confidence interval is $A = [-1.96, 1.96]$ at the 5% significance level. Note that $\beta_1 \neq 0$ means the $C_{p\text{-}au}$ relates to the φ. Likewise, the null hypothesis H_0: β_2=0 and H_1: $\beta_2 \neq 0$ should be tested for the latter, which shows whether the $C_{p\text{-}au}$ relate to the ε or not. Here, the statistics are correspondingly equal to 0.0014, 0.0008, 0.0009 and -0.0001, so the null hypothesis H_0: β_2=0 is not rejected for 0 <φ<0.2, 0.2 <φ<0.4, 0.4 <φ<0.6 and 0.6 <φ<0.8. As a result, the importance of φ and ε are considered as 'more important' and 'important' in the present study, and we presume that both of them are the model coefficients to increase performance.

Since $C_{p\text{-}au}=C_{pu}+\beta_1\varphi+\beta_1\varepsilon$, we can assume $C_{p\text{-}au}$-C_{pu} as a response variable of the multivariate regression, and β_1, β_2 can be also assumed as the model coefficients that are estimated from the observations of the process by the multivariate regression. The regression equations of $C_{p\text{-}au}$-C_{pu} have been estimated for different intervals of φ and displayed in Figure 1 as follows:

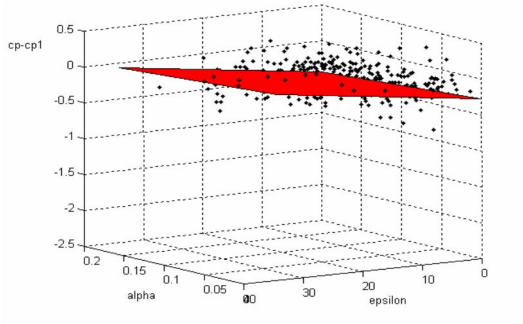

Fig. 1(a). Regression equations of $C_{p\text{-}au}$-C_{pu} when 0<φ<0.2

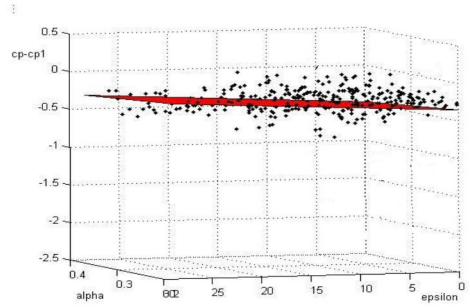

Fig. 1(b). Regression equations of $C_{p\text{-}au}$-C_{pu} when 0.2<φ<0.4

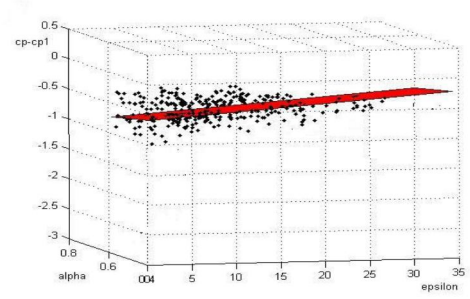

Fig. 1(c). Regression equations of $C_{p\text{-}au}$-C_{pu} when 0.4<φ<0.6

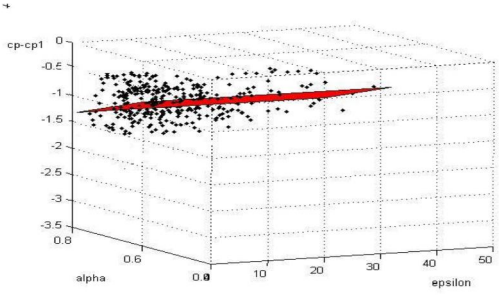

Fig. 1(d). Regression equations of $C_{p\text{-}au}$-C_{pu} when 0.6<φ<0.8

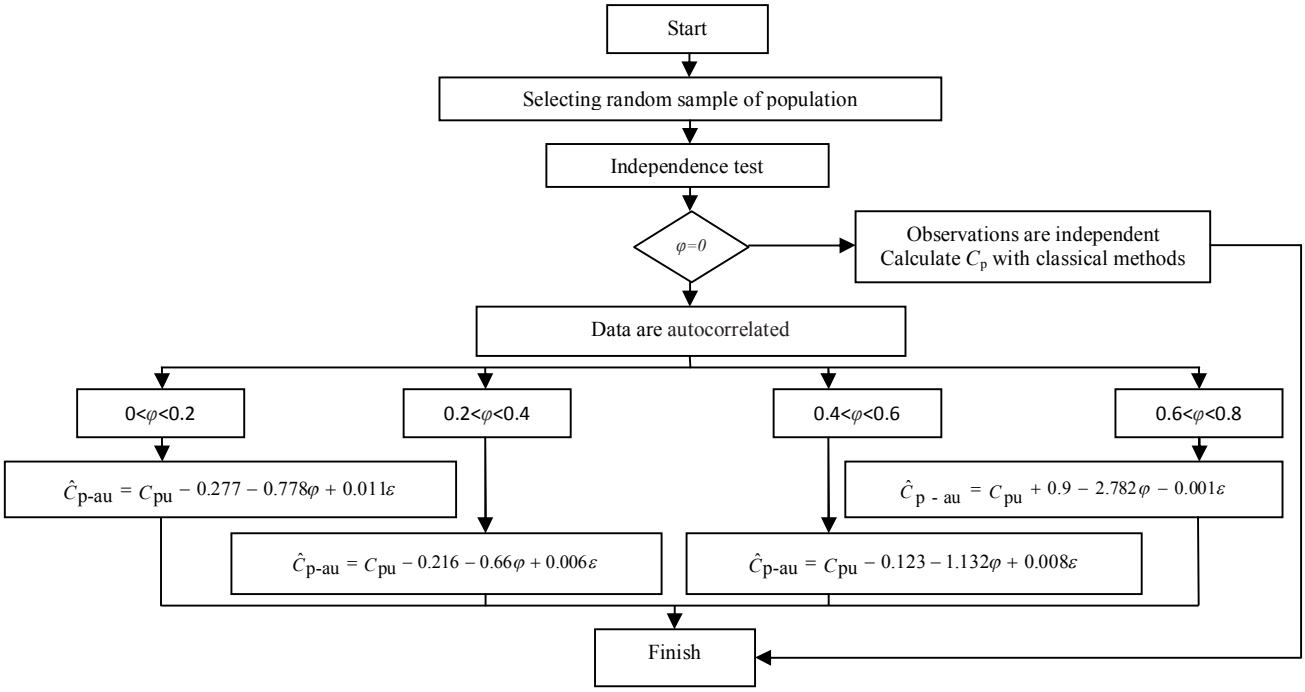

Fig. 2. The estimated $C_{p\text{-}au}$ when data are autocorrelated with 0<φ<0.2 , 0.2<φ<0.4 , 0.4<φ<0.6 , 0.6<φ<0.8

The equations customarily utilized to estimate $C_{p\text{-}au}$ are obtained by using the multivariate regression and displayed on the classification of Figure 2. The quality of the output from an autocorrelated process can be easily managed by using this classification to monitor the difference between customer requirements and actual performance of an autocorrelated process. On the other hand, this classification is able to create products that meet customer requirements, and then maintain the autocorrelated process in a 'capable' state.

4. Statistical Analysis and the Model's Validity

We should investigate whether there is a significant difference between the capability index and the estimated capability index at a given significance level. To obtain a suitable decision rule, the hypothesis can be formulated as:

$$H_0: C_{p\text{-}au} - \hat{C}_{p\text{-}au} = 0 \qquad H_1: C_{p\text{-}au} - \hat{C}_{p\text{-}au} \neq 0 \qquad (7)$$

We can use the hypothesis to obtain a suitable decision rule. To test the hypothesis, we use *t* statistic where the standard deviation of the pair differences is not known. The *t* statistic is defined as follows:

$$t = \frac{\bar{d}-0}{s_{\bar{d}}} \qquad (8)$$

And the confidence interval is defined as:

$$A = \left[-t_{\frac{\alpha}{2},n-1}, t_{\frac{\alpha}{2},n-1}\right] \qquad (9)$$

Note that d_i is obtained by subtracting the i^{th} member of $C_{p\text{-}au}$ from the i^{th} member of $\hat{C}_{p\text{-}au}$. Subsequently, the average of d_i can be determined as $\bar{d} = \frac{\sum_{i=1}^{n} d_i}{n}$ for a population the size of *n*. The null hypothesis will be rejected whenever $t > t_{\frac{\alpha}{2},n-1}$ or $< -t_{\frac{\alpha}{2},n-1}$, where the constant $t_{\frac{\alpha}{2},n-1}$ is determined so that the significance level of the test is α for a sample size of *n* observations. If the null hypothesis is not rejected, there is no significant difference between $C_{p\text{-}au}$ and $\hat{C}_{p\text{-}au}$. Table 2 shows $C_{p\text{-}au} - \hat{C}_{p\text{-}au}$ for the given 0<φ<0.2.

Table 2
Calculating $C_{p\text{-}au} - \hat{C}_{p\text{-}au}$ when 0<φ<0.2

Iter. No	$d = C_{p\text{-}au} - \hat{C}_{p\text{-}au}$	$\hat{C}_{p\text{-}au}$	$C_{p\text{-}au}$
1	-0.005	1.005	1
2	0.103	0.797	0.9
3	-0.013	0.936	0.95
4	-0.037	1.087	1.05
5	0.004	1.096	1.1
6	0.03	1.17	1.2
7	-0.01	1.26	1.25
8	-0.08	1.38	1.3
9	-0.13	1.48	1.35
10	-0.1	1.5	1.4
11	-0.08	1.53	1.45
12	0.04	1.46	1.5
13	0.14	1.46	1.6
14	0.21	1.49	1.7
15	0.2	1.6	1.8
16	0.07	0.78	0.85
17	0.03	0.77	0.8
18	0.13	0.57	0.7
19	0.1	0.5	0.6
20	0.03	0.47	0.5

According to Table 2, we can calculate the test statistic as follows:

$$t = \frac{\bar{d}-0}{s_d} = \frac{0.0316}{\sqrt{\frac{0.00905}{20}}} = 1.486093 \qquad (10)$$

The nominal significance level in this test is 0.05 for a sample of n=20 and 0<φ<0.2; therefore, the confidence interval can be defined as $A = [-t_{0.025,19}, t_{0.025,19}] = [-2.093, 2.093]$. It has been known that the null hypothesis will be accepted whenever $|P-\text{value}| \leq t_{0.025,19}$. Under these assumptions, the calculated P-value is equal to 0.15. This means that there are no significant differences between $C_{\text{p-au}}$ and $\hat{C}_{\text{p-au}}$ in this situation. According to this procedure, the statistics are also equal to 0.8, 0.151 and 0.1 when 0.2 <φ<0.4, 0.4 <φ<0.6 and 0.6 <φ<0.8, respectively, so it seems reasonable to conclude that there are no significant differences between $C_{\text{p-au}}$ and $\hat{C}_{\text{p-au}}$ for these values of φ .

5. Conclusion

It is common to come across biological and chemical processes which, because of their inborn nature, produce autocorrelated data. If these autocorrelated data are treated independently during capability analysis, the conclusions may lead to wrong decisions. To prevent such wrong decisions, we proposed a strategy to handle problems that occur during capability analysis when the observations are not independent. This strategy is based on subtracting consecutive observations from each other in order to obtain samples with independent observations, and then using regression analysis to calculate the PCI at different levels of autocorrelation. Using this strategy, we can find powerful decision rules to determine the capability of a process at a given significance level. In this study, a Monte Carlo simulation was then employed to evaluate the obtained results. For future research, we recommend the extension of this strategy to estimate process capability index for AR(P) or non-normal processes.

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