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Research Article

Developing and Solving the Multi-Objective Flexible and Sustainable Job Shop Scheduling Problem With Reverse Flow and Job Rotation Considerations in Uncertain Situations

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Abstract

Flexible job shop scheduling problem (FJSP) has received a lot of attention in recent years, but the important point is that this field of study can be subject to many assumptions and lots of innovations can be considered. One of these can be reverse flow, which has been overlooked in many studies, while its effect on the cost and time of construction is undeniable. Other areas such as job rotation as well as issues related to sustainability can be of particular importance in this area and have not been reviewed in previous researches. Therefore, the present study seeks to provide a model to optimize the multi-objective flexible job shop scheduling problem concerning the issues of sustainability with reverse flow and job rotation considerations. For this purpose, a multi-objective mathematical scheduling model is developed, the first goal of which is to minimize the construction time and the second goal is to minimize the issues related to sustainability. To solve the model, two methods were used: Sensitivity analysis and meta-heuristic. The whale optimization algorithm (WOA) was employed in the meta-heuristic method. The results of the implementation of WOA indicate the efficiency of the proposed algorithm, while the findings of the sensitivity analysis also point to the effect of research innovations on the objective functions of the problem.

Keywords: Flexible job shop; Job rotation; Scheduling; The whale optimization algorithm (WOA); Uncertain

1. Introduction

Today, many businesses increase their product range by developing new products according to customer demands to survive competitively. For this reason, businesses expanding their product range prefer job shop manufacturing among manufacturing processes. In job shop-type manufacturing, a job may require more than one operation. The operations of a job are performed on certain machines. Flexible job shop manufacturing is a type of job shop manufacturing in which more than one machine can do the same job, and thus the job routes become flexible. It provides essential services to businesses, such as shortening the completion times of jobs. However, the scheduling of jobs is much more complicated than in traditional job shop manufacturing. (Tutumlu & Tugba Saraç 2023)

The flexible job shop scheduling problem (FJSP) is an NP-hard combinatorial optimization problem, which has wide applications in the real world. The complexity and relevance of the FJSP have led to numerous research works on its modeling and resolution. (Dauzère-Pérès et al., 2023)

Job shop scheduling can be described as 'the workload distribution between the machines and the determination of the process sequences of these jobs on their machines' (Stevenson, 1996). In a job shop scheduling problem where there are n jobs and m machines, there are (n!)m schedules. As the number of jobs and machines increases, schedules increase exponentially, making it difficult to obtain the optimum schedule. Job shop scheduling problems are NP-hard problems. (Tutumlu & Tugba Saraç 2023)

The FJSP is based on the classical job shop scheduling problem (JSP) considering machine flexibility. In the FJSP, each operation can be processed by a machine selected from the corresponding processing machine set, which makes it more in line with the actual production situation than the JSP. Due to the NP-hard characteristic of the JSP, the FJSP considering machine flexibility also belongs to NP-hard problem. In the recent decades, an increasing number of extended versions of FJSP have been proposed by researchers to make the scheduling models more in line with the real production environment. (Gong et al., 2024)

Flexible job shop scheduling is one of the most common systems in the production of various parts and is a subset of the job shop problem. This manufacturing environment includes a number of tasks that must be programmed to be processed on a set of machines. Each task contains a sequence of operations (path), which must be performed to complete a task. Each of these operations related to each task must be executed on a machine from a set of available machines. The set of machines available for each task is a subset of all the machines that exist in the manufacturing environment. In this case, n tasks are scheduled and planned on m machines. Parallel and identical machines are used for this type of problem that are capable of performing all operations. Each task consists of several operations and has a specific performing process different from that of other tasks. This developed issue is called classic job shop scheduling problem (Saeedi and Fattahi, 2007).

In the case of flexible job shop, the machines used for the operation are not predetermined. This means that each

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machine can perform the operations of all n different tasks and there is no limit to the number of operations that can be performed by machines. Therefore, this feature removes the constraint of machine selection and greatly extends the solution space of job shop scheduling problem (Kacem et al., 2002). FJSP is one of the NP-Hard problems, which means that by adding one dimension to the problem, the time of problem solving increases exponentially (Kacem et al., 2002).

The essential issue for modeling flexible job shop production scheduling problem is to take into consideration, the reverse flows in this system. In production scheduling, reverse flows are introduced through the production units of montage/separation/duplication. This research presents a multi-objective model for flexible and sustainable job shop scheduling problem with the reverse flow of tasks for the purpose of duplication, in which considerations related to workforce allocation and job rotation based on learning factors are also examined. As mentioned, the model will be multi-objective with the objectives of minimizing the maximum completion time, minimizing the environmental impact, and minimizing the cost of workforce allocation.

It should be noted that according to the investigations, research has been done on the issue of job shop scheduling with reverse flows, but due to the breadth and the importance of the subject, other studies can be conducted in this field. Regarding the issue of flow shop production scheduling with limited resources, we can refer to Yazdani and Naderi (2016) as one of the latest relevant researches. In these types of issues, in addition to scheduling tasks, limited and available resources are allocated to tasks. The important innovation of this type of research is the attention directed towards the issue of limited resources allocation. Fattahi et al. (2018) have proposed the flexible job shop production by considering one montage step as well as the preparation time depending on the sequence. In respect of modeling and scheduling montage flow shop problems, we can point out Bashi Warsho Saz et al. (2018) who explored montage in two stages with non-identical montage machines. But it is apparent that the researches conducted abroad are greater in number and variety. For example, Shen et al. (2018) have analyzed task scheduling with regard to the sequence-dependent start-up time which has been evaluated as an important variable in significant research studies.

Recent research includes that of Budala et al. (2019) which suggests learning-based optimization based on an integrated approach with the aim of minimizing maximum completion time. Huang and Yang (2019) have also attempted to provide an integrated version of the genetic algorithm to solve the scheduling problem considering the transfer time. Lonardi et al. (2019) have discussed assigning operations to machines and determining the order of sequence by observing the preliminaries. Among the latest research, Kress et al. (2019) have looked into the issue of flexible job shop scheduling with sequence-dependent start-up time, taking into account the heterogeneity of machines and operator-dependent processing time.

But in the meantime, it is evident that many issues have not been addressed in the flexible flow shop problem such as sustainability, reverse flow, etc. This research aims to provide a model for the problem by considering processing constraints (focusing on reverse flow, job rotation, learning effect, and sustainability) with respect to reverse flow and its solution using meta-heuristic algorithms. There is a dynamic offline problem in which the factors that cause uncertainty, such as the time required for duplication, have been examined indefinitely.

In this study, a number of products of each workstation that need to be duplicated after quality control so that the model and the assumed actual production system correspond, are referred back to the station and the desired process is performed on them again.

One of the cases considered in this research is the dimensions of sustainability (economic, social, and environmental); However, the economic and environmental dimensions are just considered in the present study. The economic dimension includes minimizing production costs and allocating workforce; The environmental dimension includes minimizing the emission of pollutants and waste of production.

In this research, the allocation of skilled workforce and job rotation is also surveyed and in this regard, the learning factor is considered. In order to allocate workforce, the learning factor of the workforce is taken into consideration; In this way, the effect of human factors in scheduling is noted. In the present research model, the learning coefficient is defined, which is calculated as k^a each time a workforce is allocated to a machine (a is the learning coefficient, k is the number of times the workforce is allocated).

In the present study, the duplication time parameter is considered fuzzy. A multi-objective mathematical model with fuzzy parameters is presented and then implemented in the environment of GAMS to check the validity of the model. Finally, meta-heuristic algorithms such as whale optimization will be used to solve the model after validating.

2. Literature Review

The flexible job shop scheduling problem (FJSP) has been studied extensively over the past decades, mainly because of its practical significance for managers to make production decisions in manufacturing environments. (Gong et al., 2024).

The flexible job shop scheduling problems (FJSP) has been developed many different mathematical models and solution approaches have been developed. (Guo et al, 2022) This section provides an overview of articles conducted in the field of flexible job shop. Articles from 2000 to 2024 have been reviewed over a period of 20 years and their main concepts will be presented.

Jonsen et al. (2000) proposed a polynomial algorithm to solve FJSP once by considering and then, not considering the interruption of the operations.

Kacem et al. (2002) developed a genetic algorithm controlled by the localization method and applied it to a flexible job shop problem with single and multiple objective functions.

Chan et al. (2004) used genetic algorithms to solve the problem. First, the genetic algorithm was proposed to solve the allocation problem, and then the second genetic algorithm was implemented to determine the sequence of tasks on the machines.

Torabi et al. (2005) suggested a way to solve the problem of economic scheduling of multi-product groups with a general cycle in a flexible job shop. In this paper, flexible job shop has been studied as a combination of two job shop issues and parallel machines.

Low et al. (2005) solved the flexible processing problem with multi-objective function with a precise optimization algorithm.

Ong et al. (2005) offered an immune system-based algorithm to solve FJSP with job re-rotation. The objective function of this problem was the maximum completion time.

Axia and Wu (2005) proposed an efficient hierarchical practical method for solving the flexible job shop problem with multi-objective functions. This method used particle swarm optimization to assign operations to machines and simulated annealing to schedule operations on each machine.

Rossi and Dini (2007) used the ant colony optimization (AOC) algorithm to minimize the maximum completion time of tasks in flexible job shop environments. In the case under study, sequence-dependent preparation time, as well as job rotation time in the job shop, have been considered.

Tao (2007) solved the problem of flexible job shop by combining distribution rules and genetic algorithms.

Gao et al. (2008) solved the job shop problem, using genetic algorithms along with variable neighborhood search, with the aim of minimizing the maximum completion time and workload of machines and their total workload by weighting objective functions.

Pezzella et al. (2008) proposed a genetic algorithm for the flexible job shop production scheduling problem and used different rules for population growth, selection of individuals, and combination operator.

De-Ming et al. (2008) discussed job shop scheduling with random processing time in natural dispersion. The developed Giffler-Thompson randomization method was first presented and some operations related to random processing time were defined. Then a new permutation-based method was suggested in which the sub-string corresponded to each permutation machine.

In a study by Gen et al. (2009), a multi-step genetic algorithm with a change in bottlenecks was developed for the flexible job shop problem. The genetic algorithm used two vectors to represent each candidate for flexible job shop problem solving. Crossover operators and phenotypic mutations have been proposed to adapt to the chromosome-specific structures and characteristics of this problem.

The following are articles presented in the field of job shop in the 2010s.

Akhshabi et al. (2014) have introduced a mixed linear programming model for a specific open-shop scheduling problem, taking into account the separation and configuration time. This algorithm provides good results in problems with sufficient discontinuity but lacks the necessary efficiency in complex problems.

Mirabi et al. (2014) have introduced two genetic algorithms for the permutation flow shop scheduling problem. The authors have used the modified NEH method to find the initial solution to the flow shop problem.

Mascia et al. (2015) also provided two simple local search methods based on the interactive greedy algorithm. This algorithm has two steps: the demolition step in which some tasks are removed from the initial response and the construction step in which the deleted tasks are assigned to the initial response using the NEH innovative method.

Gao et al. (2016) utilized the harmony search algorithm to solve the multi-objective flexible job shop problem with the goals of minimizing the maximum completion time and the average earliness and tardiness. They first designed an innovative method for assigning a task to the machine and then used the algorithm to determine the sequence of tasks on the machine and the sequence of machines for each task, which is combined with an improvement procedure (involving several neighborhood search operators).

Zandieh et al. (2016) proposed a new multi-objective tabu search algorithm for the open-shop scheduling problem with two objectives based on the fuzzy multi-objective decision-making approach.

Zhou et al. (2017) addressed the flexible job shop scheduling problem as the selected research objective and the mathematical model with the aim of minimizing the maximum reconstruction time. Using a company's reverse gear production line, for example, a genetic algorithm was applied to a flexible job shop scheduling problem to obtain specific optimal results with MATLAB.

Hamm (2017) has studied FJSP with a parallel batch processing machine. First, a mixed-integer programming (MIP) formulation is proposed.

Wang et al. (2017) investigated FJSP with the aim of minimizing the maximum completion time and proposed an improved ant colony optimization algorithm to solve it. They first provided a mathematical model for this problem and then, due to the two weaknesses of the ant algorithm, namely, low computational efficiency and local optimum, improved the algorithm and used it to solve the mathematical model.

Eddineh Nouri et al. (2018) solved FJSP using integrated meta-heuristic algorithms based on a multifactorial system. They presented how to solve FJSP using integrated metaheuristic algorithms and clustered holonic multifactorial models.

Shen et al. (2018) have solved FJSP by considering the assumption of sequence-dependent preparation times. They first presented a mathematical model with the aim of minimizing the maximum completion time.

Lou et al. (2018) used an improved genetic algorithm to solve the flexible flow shop scheduling problem. They offered a mathematical model with the aim of minimizing the maximum completion time and solved 3 problems in the literature using their proposed algorithm.

Shen et al. (2018) dealt with FJSP with sequence-dependent start-up time, where the goal was to minimize the time interval.

Rooeinfar et al. (2019) have proposed a new mathematical model for a flexible flow shop problems in uncertainty.

Raissi et al. (2019) proposes a new more realistic mathematical model which considers both the PM and holding cost of jobs inside the buffers in the stochastic flexible flow shop scheduling problem. The holding cost is controlled in the model via the budget constraint.

Sajjadi et al. (2019) also presented a model for FJSP and used a genetic algorithm to solve it. In this paper, robust optimization of FJSP has been discussed, considering the failure of random machines.

Buddalla et al. (2019) have solved FJSP. In their paper, teaching-learning-based optimization (TLBO) was proposed to solve FJSP based on an integrated approach with the aim of minimizing the maximum completion time.

Huang and Yang (2019) prepared an integrated version of the genetic algorithm to solve FJSP by considering the transfer time. They proposed a multi-objective mathematical model for scheduling, which also considered transfer time. They also used a multi-objective genetic algorithm combined with a simulated annealing algorithm to solve the model.

Kress et al. (2019) investigated FJSP with sequence-dependent start-up time, taking into account the processing time and operator-dependent heterogeneous machines. They developed a bi-objective model, dividing the scheduling problem into a vehicle routing problem with priority constraints and a job-machine assignment problem, using precise methods to solve the model.

Beheshtinia and Ghazi Vakili (2019) have studied and solved multi-objective FJSP. In this paper, three objectives including the minimization of maximum completion time (Cmax), maximum working load of the machine (Wmax), and total work (WT) were investigated and after presenting the mathematical model for the problem, a genetic algorithm named the reference group genetic algorithm (RGGA) was used to solve the issue.

Basan et al. (2019) proposed a decomposition algorithm based on mixed-integer programming for FJSP. They assumed that the manufacturing environment is multi-stage and includes heterogeneous machines. They suggested a dual-stage model for this problem, in which the maximum completion time was optimized in the first stage and then the resource consumption was minimized in the second stage while maintaining the Cmax optimization.

Jun et al. (2019) have provided learning rules, using the Random Forest method to solve FJSP. In this paper, they addressed FJSP to minimize penalty interest using learning rules.

Cao et al. (2019) have solved FJSP with the aim of minimizing the maximum completion time. In their paper, an adaptive scheduling algorithm was proposed to minimize the makespan in dynamic FJSP. Instead of a linear order, a directed cycle graph was used to show the complex prerequisite constraints between operations of tasks.

Renna (2019) has discussed FJSP with the effect of learning rate and based on multifactorial models. This research presented a scheduling method to support a production system influenced by learning/forgetting.

Luo et al.(2020) have provided An improved genetic algorithm is proposed to overcome the shortcomings of traditional genetic algorithm, such as weak searching ability and long running time when solving FJSP. The simulation results proved that the improved algorithm has better performance than some other algorithms.

Ding & Gu (2020) have provided a hybrid HLO-PSO algorithm, which utilizes various combinations of the proposed improved PSO and proposed scheduling strategies to solve FJSP under the algorithm architecture of HLO. by using it to solve several groups of FJSP instances, the result comparisons with other related algorithms reveal that HLO-PSO can efficiently solve most of single-objective FJSP.

Apornak et al. (2021) used data envelopment analysis (DEA) as a multi-criteria decision making techniques to seek more appropriate assignment.

Fan et al. (2021) have provided a hybrid Jaya algorithm integrated with Tabu search is proposed to solve FJSP for makespan minimization. The proposed algorithm is compared with several state-of-the-art algorithms on three well-known FJSP benchmark sets. Extensive experimental results suggest its superiority in both optimality and stability.

Lou et al. (2022) have provided a multi-objective FJSP considering human factors (MO-FJSPHF) to simultaneously minimize makespan, maximum machine workload, and total machine workload. Experimental results show that proposed algorithm outperforms four state-of-the-art algorithms on forty-three test instances and three real-world cases from a casting workshop.

Lei et al (2022) have provided an end-to-end deep reinforcement framework to automatically learn a policy for solving a flexible Job-shop scheduling problem (FJSP) using a graph neural network.

Prause da Silva & et al (2024) the DM Fuzzy TOPSIS algorithm proves to be a valuable tool for supporting decision-making in production systems, assisting in the selection of the best production schedule among the optimal or near-optimal solutions obtained from the Pareto set. By integrating multi-objective optimization and decision-making techniques, this research contributes to more efficient and informed production scheduling practices, ultimately enhancing overall system performance.

Tutumlu & Sara (2023) have provided addresses the Flexible Job-Shop Scheduling Problem (FJSP) with job-splitting, determining how many sub-lots each job should be split into and the size of each sub-lot. A MIP model is proposed for the considered problem. In the model, the size and number of sub-lots of a job are not predefined or bounded. The objective function of the model is to minimize the makespan.

We can see that a variety of extended versions of FJSP have been studied by many researchers to help the production managers to implement more reasonable scheduling strategies. However, to the best of our knowledge, there is no research attention to machine flexibility and operation flexibility simultaneously, which have a great impact on production efficiency. In order to take the machine flexibility and operation flexibility full into consideration, a lot of work is needed in this area to fill the gap. (Gong et al., 2024).

3. Research Methods

The present research is theoretical and analytical. In this study, a mathematical model of fuzzy multi-objective mixed-integer programming will be presented. Pareto archived multi-objective meta-heuristic algorithms will be proposed to solve the model. The reason for using the Pareto archived solution method is that the objectives of the research are not in the same direction and are in conflict with each other. This method is the one that solves multi-objective problems and tries to achieve near-optimal solutions by improving all the objectives at the same time. In this research, the aforementioned method will be utilized to find the responses close to the Pareto principle. First, the model is solved in a GAMS environment in small segments in order to measure the validity of the model, and then the results of GAMS are compared with the results of the same problem solved by the proposed meta-heuristic algorithm, namely, Pareto archived multi-objective whale optimization algorithm. Then, the validity of the model and algorithm is checked. It is worth noting that, the multi-objective model is transformed into a single-objective model, using the LP-metric method, in order to solve the model in the GAMS environment. After validating the model and algorithm, the multi-objective model is solved using the proposed Pareto-archived algorithm suitable for various sample problems that are designed based on previous research.

Besides, the results of the mentioned algorithm will be compared to the known multi-objective genetic algorithm to prove the efficiency of the proposed algorithm. Comparative characteristics of quality, dispersion, and uniformity designed for multi-objective problems will be used to compare the results of the proposed algorithm and the multi-objective whale optimization algorithm. The research steps are presented in figure 1.

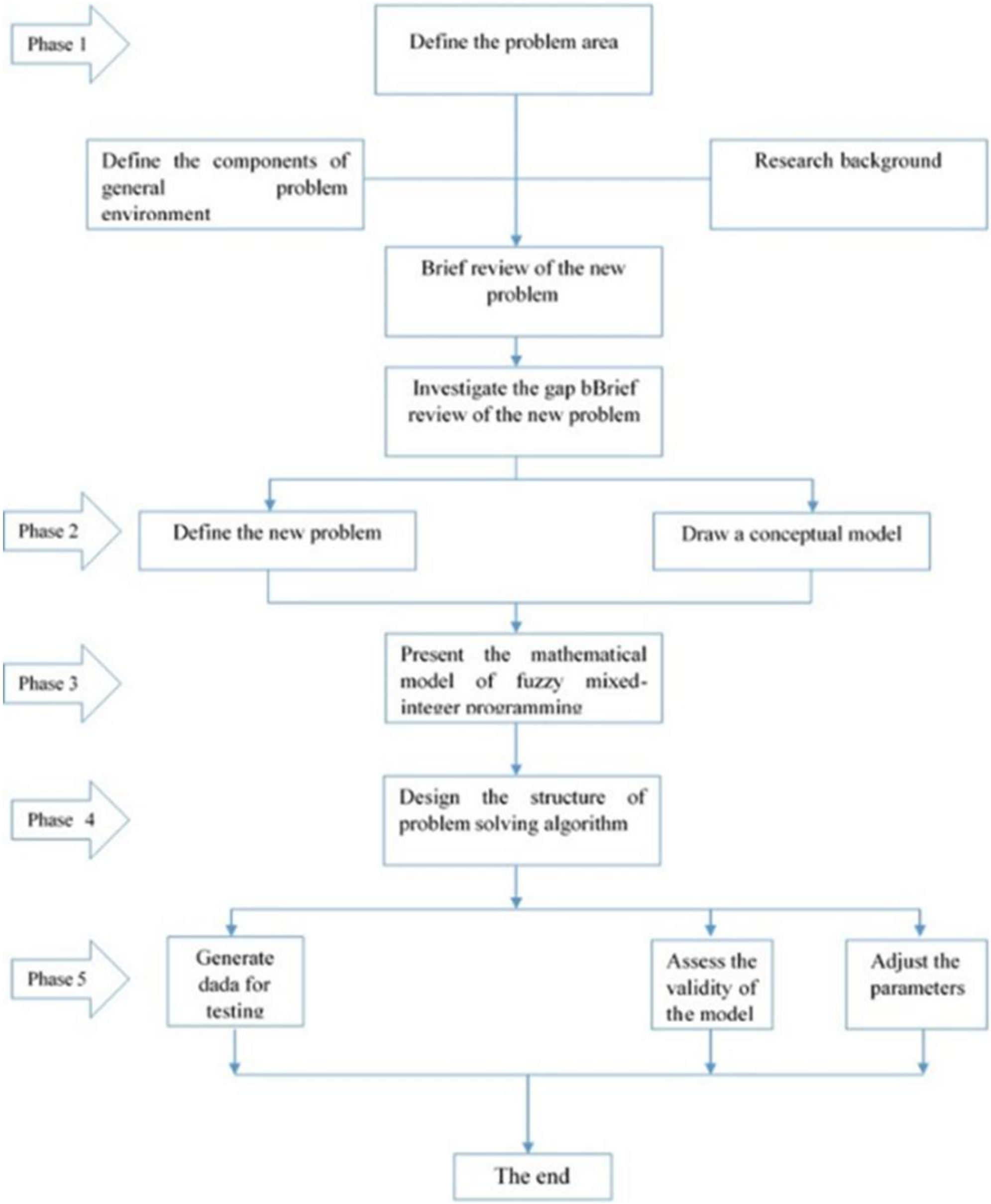

Fig. 1. Research steps

3.1. Mathematical model

The problem is based on the assumption that a number of tasks (jobs) or activities must be processed by a number of machines. The operation has a certain duration, which is called processing time. In this case, the machines are always available, but the tasks may not always be available, which means that the part is ready to be processed at this time. The objective of this problem is to find the sequence of operations on the machines in such a way that, first, they are compatible with technological constraints, that is, a possible schedule is created, and second, it is optimized according to a number of performance criteria.

In addition to the question of flexible job shop scheduling, three other important issues are addressed in the present paper. One of these issues is paying attention to sustainability or environmental concerns, which is considered as one of the objective functions of the problem, seeking to manage energy and reduce fuel consumption. The second issue studied in the present study is the discussion of reverse flow in scheduling, which has been less investigated in the literature. Of course, it should be noted that many subjects have been examined under the title of reverse logistics in supply chain research, but in studies related to operation scheduling, either flow shop or job shop has been less observed.

Another discussion in the present model which has rarely been remarked in scheduling research is the discussion of job rotation. The purpose of the present model is to maximize and optimize job rotation and in fact, to pay serious attention to this point due to its significance in the field of manpower and organizational behavior

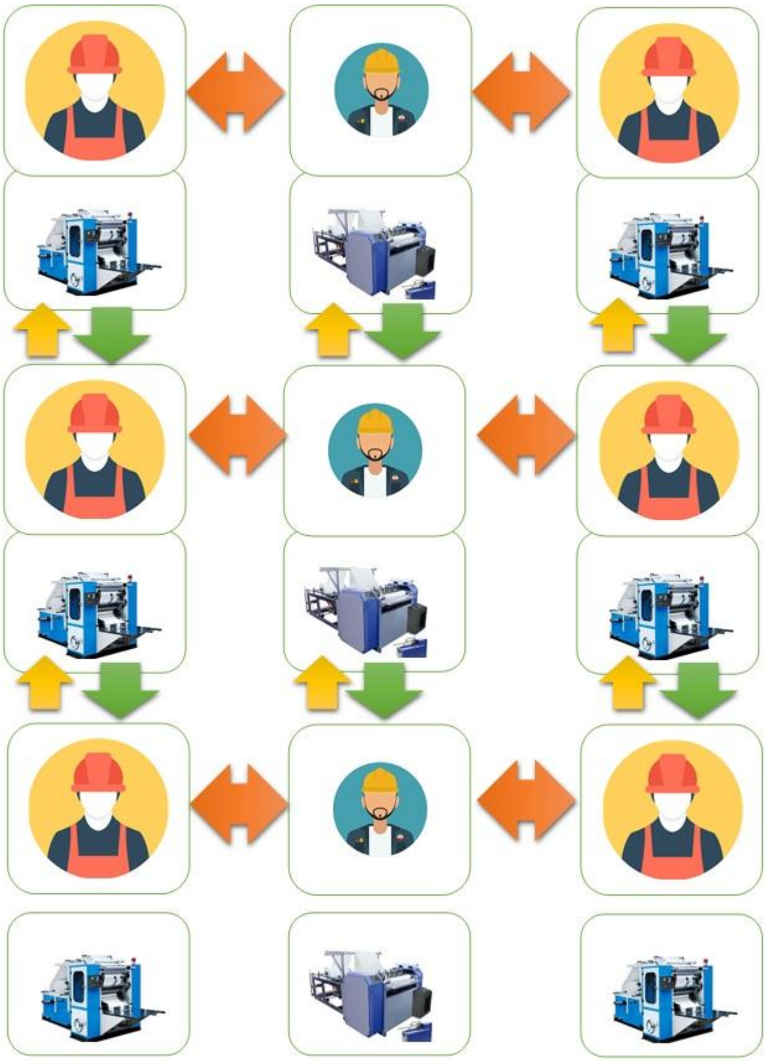

Fig. 2. General schema of the model

Figure 2 indicate how the machine are assigned based on assumptions of research. Meanwhile the reverse flow is indicate with orange arrows that the possibility for reversing is considered here.

Assumptions:

1. It is impossible to process two operations of a task at the same time, i.e., each task can only be processed by one machine at a time.
2. Each task has a distinct operation that is performed on one machine.
3. Interruption of operations is not allowed, that is, when an operation is started on a machine, it must continue

without interruption until completion, and the start of another operation on that machine is not allowed before the completion of this operation.
4. Canceling the task is not allowed. That is, each task must be processed until it is completed.
5. Each machine can only perform one operation at a time.
6. There exists only one machine of each type.
7. Machines may become idle.
8. Machines never break down and are available throughout the scheduling period.
9. Preparation times are independent of the sequence of operations.
10. The time of transferring tasks between machines can be ignored.
11. Task saving is allowed during construction. That is, tasks can wait for their next stage of operation.
12. Technological constraints are clear and unchangeable from the beginning.
13. Some parameters are uncertain.
14. Environmental considerations are discussed in the present model.
15. Improvement or maximization of job rotation considerations are noted.
16. Reverse flow is considered in the production scheduling in the present model.

Indexes
Task: i
Machine: k
Number of machines: m
Position of the machine: t
Number of activities: n
Workdays: d
Operator: l

Parameters
The number of operations for task i: s_i
Operation j of task i: $o_{i,j}$
Number of replacement machines for operation $o_{i,j}$: $m_{i,j}$
The maximum number of positions for machine k: P_k
Uncertain time of performing operation o on machine k: $\widetilde{Pt}_{i,j,k}$
Uncertain power consumption to perform operation o on machine k: $\widetilde{P}_{i,j,k}$
Maximum on-off strategy time for machine k: N_k
The energy required to perform operation o on machine k:$P_{power_{i,j,k}}$
Power consumed by machine k to turn on and off machine k: energy S_k
Energy consumed by machine k during idle time: P_k^{idle}
Payback period of machine k: TB_k
A large number: M

Decision variables
$X_{i,j,k}$: The binary decision variable takes the value 1 if operation o is performed on machine k and otherwise it is zero
$Y_{i,j,k,t}$: The binary decision variable takes the value 1 if operation o occupies position t of machine k and otherwise it is zero
$Z_{k,t}$: The binary decision variable takes the value 1 if the on-off strategy is implemented between position t and t+1 on machine k and otherwise it is zero
$B_{i,j}$: Continuous decision variable for operation o start-up time
$E_{i,j}$: Continuous decision variable for operation o completion time
$S_{k,t}$: Continuous decision variable for position t start-up time on machine k
$F_{k,t}$: Continuous decision variable for position t completion time on machine k
R_l : The number of different machines to which operator l is assigned
$RD_{l,d}$: If operator l is assigned on day d, the value is 1 and otherwise it is zero
$XR_{l,d,k}$: If operator l is assigned to machine k on day d, the value is 1 and otherwise it is zero

$$\min z1 = Cmax \tag{1}$$

$$\min z2 = \sum_{k=1}\sum_{t=1} energy_{k,t} + \sum_{i}\sum_{j}\sum_{k} Ppower_{i,j,k} . \tilde{P}_{i,j,k}.X_{i,j,k} \tag{2}$$

$$energy_{k,t} \geq energy \quad S_k Z_{k,t} \tag{3}$$

$$energy_{k,t} \geq \left(S_{k,t+1} - F_{k,t}\right)P_k^{idle} - MZ_{K,T} \tag{4}$$

$$\sum_{k=1} X_{i,j,k} = 1 \tag{5}$$

$$\sum_{k=1} Y_{i,j,k,t} = X_{i,j,k} \tag{6}$$

$$\sum_{i}\sum_{j} Y_{i,j,k,t} \leq 1 \tag{7}$$

$$\sum_{i}\sum_{j} Y_{i,j,k,t} \geq \sum_{i}\sum_{j} Y_{ii,jj,k,t+1} \tag{8}$$

$$E_{i,j} = B_{i,j} + \sum_{k} (\widetilde{pt}_{i,j,k}.X_{i,j,k}) \tag{9}$$

$$F_{k,t} = S_{k,t} + \sum_{i}\sum_{j} \left(\widetilde{pt}_{i,j,k}.Y_{i,j,k,t}\right) \tag{10}$$

$$S_{k,t} \leq B_{i,j} + M(1 - Y_{i,j,k,t}) \tag{11}$$

$$S_{k,t} + M(1 - Y_{i,j,k,t}) \geq B_{i,j} \tag{12}$$

$$S_{k,t+1} + F_{k,t} \geq TB_k - M(1 - Z_{k,t}) \tag{13}$$

$$S_{k,t+1} + F_{k,t} \leq TB_k - MZ_{k,t} \tag{14}$$

$$F_{k,t} \leq S_{k,t+1} \tag{15}$$

$$E_{i,j} \leq B_{i,j+1} \tag{16}$$

$$Cmax \geq E_{i,s} \tag{17}$$

$$\sum_t Z_{k,t} \leq N_k \quad (18)$$
$$B_{i,1} \leq M(1 - X_{i,j,1}) \quad (19)$$
$$B_{i,k} \leq M(1 - X_{i,j,1}) \quad (20)$$
$$B_{k,j} + \widetilde{pt}_{i,j,k} - B_{(k+1),j} \leq 0 \quad (21)$$
$$B_{k,j} + \widetilde{pt}_{i,j,k} - B_{(k-1),j} \leq 0 \quad (22)$$
$$R_l \leq \sum_d RD_{ld} \quad (23)$$
$$R_l \leq \sum_d XR_{l,d,l} \quad (24)$$
$$X_{i,j,k} \in \{0,1\} \quad (25)$$
$$Y_{i,j,k,t} \in \{0,1\} \quad (26)$$
$$Z_{k,t} \in \{0,1\} \quad (27)$$
$$B_{i,j} \geq 0 \quad (28)$$
$$E_{i,j} \geq 0 \quad (29)$$
$$S_{k,t} \geq 0 \quad (30)$$
$$F_{k,t} \geq 0 \quad (31)$$
$$R_l \geq 0 \quad (32)$$
$$RD_{ld} \in \{0,1\} \quad (33)$$
$$XR_{l,d,k} \in \{0,1\} \quad (34)$$

- The first objective function or equation 1 seeks to minimize the total production time.
- The second objective function or equation 2 seeks to minimize energy consumption and in fact issues related to sustainability.
- Equations 3 and 4 ensure that the energy consumed by the machine during idle time exceeds the energy consumed when the machine is switched on and off.
- Equation 5 ensures that each operation is assigned to exactly one of the machines.
- Equation 6 shows that if operation o is assigned to machine k, it is assigned to exactly one position of machine k, otherwise, it cannot be assigned to any position of machine k.
- Constraint 7 states that each position of a machine can be assigned to a maximum of one operation.
- Constraint 8 ensures that the positions of each machine must be assigned consecutive operations.
- Constraint 9 links the completion time of an operation to its start-up time.
- Constraint 10 ensures that the completion time of a position is the sum of the start-up time and the processing time of the operation assigned to it.
- Constraints 11 and 12 together ensure that the start-up time of the operation is equal to the start-up time of the position if the operation is assigned to position t of machine k.
- Constraints 13 and 14 limit the machine on-off strategy.
- Constraint 15 states that for two adjacent operations on one machine, the next operation can start if only the previous operation is perfectly completed.
- Constraint 16 is the precedence and latency constraint and states that any operation of any task can only begin after the previous operation has been completed.
- Constraint 17 determines the construction time.
- Constraint 18 limits the maximum time of on-off strategy for all machines
- Constraints 19 and 20 ensure that the first and the last machines start processing tasks at time zero.
- Constraints 21 and 22 reflect the constraints of precedence and latency between operations of a task. For a direct operation, the processed operation on machine k must precede the processed operation on machine k+1.
- For reverse tasks mentioned by constraint 23, the opposite is true.
- Constraints 24 and 25 determine the number and variety of job rotations for operators.
- Constraints 26 to 34 indicate the range of binary variables and the integer of the present study.

Due to the fuzzy nature of the problem, equations with fuzzy parameters must be de-fuzzified. Therefore, equations 2,9,10,21, and 22 which have a fuzzy parameter, are de-fuzzified in the form of equations 35 to 39.

$$\min z2 = \sum_{k=1}\sum_{t=1} \boldsymbol{energy}_{k,t} + \sum_i\sum_j\sum_k \boldsymbol{Ppower}_{i,j,k} . \frac{\boldsymbol{p1}_{i,j,k} + 2\boldsymbol{p2}_{i,j,k} + \boldsymbol{p3}_{i,}}{4} \quad (35)$$

$$\boldsymbol{E}_{i,j} = \boldsymbol{B}_{i,j} + \sum_k \frac{\boldsymbol{p1}_{i,j,k} + 2\boldsymbol{p2}_{i,j,k} + \boldsymbol{p3}_{i,j,k}}{4} \ .\boldsymbol{X}_{i,j,k}) \quad (36)$$

$$\boldsymbol{F}_{k,t} = \boldsymbol{S}_{k,t} + \sum_i\sum_j \left(\frac{\boldsymbol{p1}_{i,j,k} + 2\boldsymbol{p2}_{i,j,k} + \boldsymbol{p3}_{i,j,k}}{4} .\boldsymbol{Y}_{i,j,k,t}\right) \quad (37)$$

$$\boldsymbol{B}_{k,j} + \frac{\boldsymbol{p1}_{i,j,k} + 2\boldsymbol{p2}_{i,j,k} + \boldsymbol{p3}_{i,j,k}}{4} - \boldsymbol{B}_{(k+1),j} \leq \boldsymbol{0} \quad (38)$$

$$\boldsymbol{B}_{k,j} + \frac{\boldsymbol{p1}_{i,j,k} + 2\boldsymbol{p2}_{i,j,k} + \boldsymbol{p3}_{i,j,k}}{4} - \boldsymbol{B}_{(k-1),j} \leq \boldsymbol{0} \quad (39)$$

$\min z2$ =Equation 35 is the de-fuzzified form of equation 2 and the second objective function. Equations 36 to 39 include defuzzification constraints.

3.2. Problem solving method

Classical and meta-heuristic methods are used to solve the above problem. First, the problem is solved in small segments and the whale optimization algorithm is utilized for larger segments. The algorithm begins with a set of random solutions and in each repetition, the search parameters update their position randomly or based on the best solution obtained according to each of the search parameters. Parameter a has been reduced from 2 to zero, in order to provide exploration and exploitation. Two conditions are considered to update the position of search parameters. If $|A| > 1$, then the random search parameter is selected, and if $|A| < 1$, then the best solution is selected. Depending on the value of p, the whale algorithm is able to switch between spiral and circular movements. Finally, the whale algorithm terminates after reaching the set satisfaction criterion. The figure 3 shows the pseudocode of the algorithm.

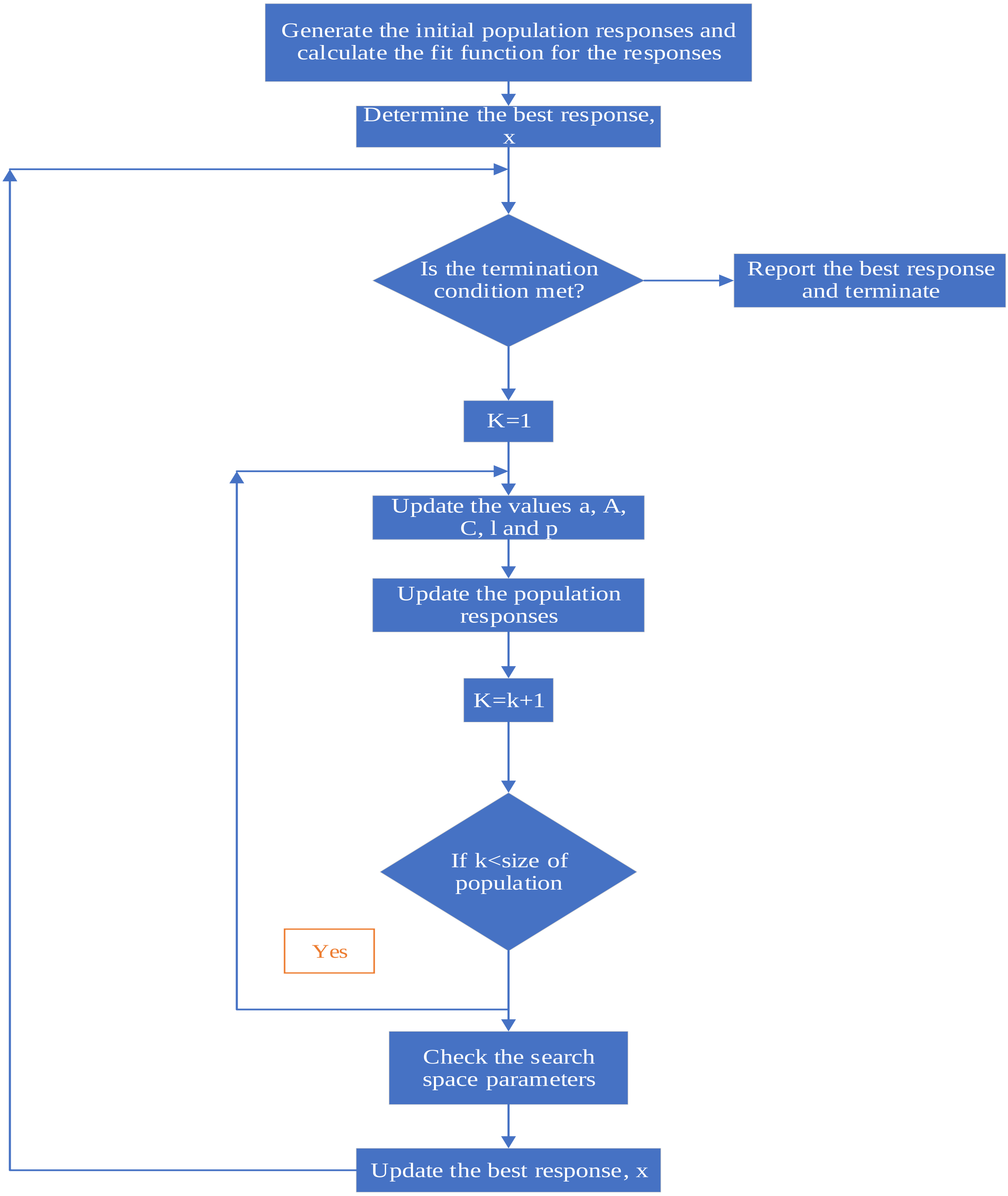

Fig. 3. General structure of the whale algorithm

Figure 3 shows the general structure of the whale algorithm. In the present study, the design of the algorithm is Pareto archived which is updated at the end of each iteration of the algorithm. Also, in each iteration, an improvement procedure is used, therefore the structure of the algorithm of the present study will be shown as figure 4.

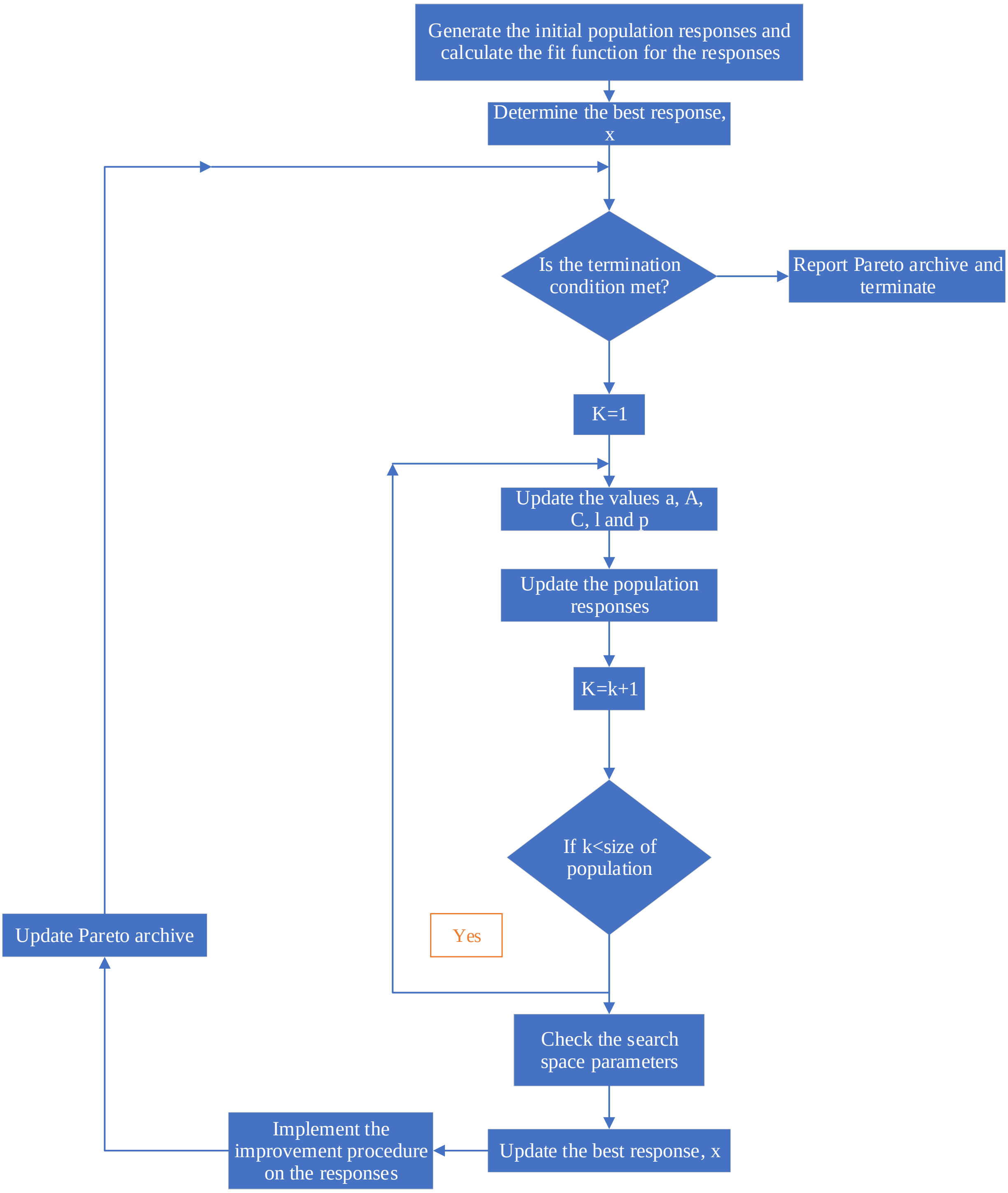

Fig. 4. THE structure of the algorithm of the present study

4. Numerical Examples

In this section, the problem is analyzed in small segments and then the results are explained separately based on the innovations presented in the research. First, each of the objectives is considered as a separate problem and then solved as a bi-objective problem such as the model presented in the third chapter. The results are proposed below using GAMS.

4.1. Calculation of Cmax without considering energy consumption

In this section, the variable Cmax is solved, which is, in fact, the total construction time, without considering the second objective function or minimizing energy consumption and the results are set forth once with and then without reverse flow condition

Table 1
Problem solving regardless of reverse flow

	machine		
activitie	1	2	3
1	0	238	50
2	238	191	293
3	293	125	94
4	50	147	359

Table 2
Problem Solving, regarding reverse flow

	machine		
activite	1	2	3
1	0	331	79
2	331	284	386
3	386	178	123
4	79	200	452

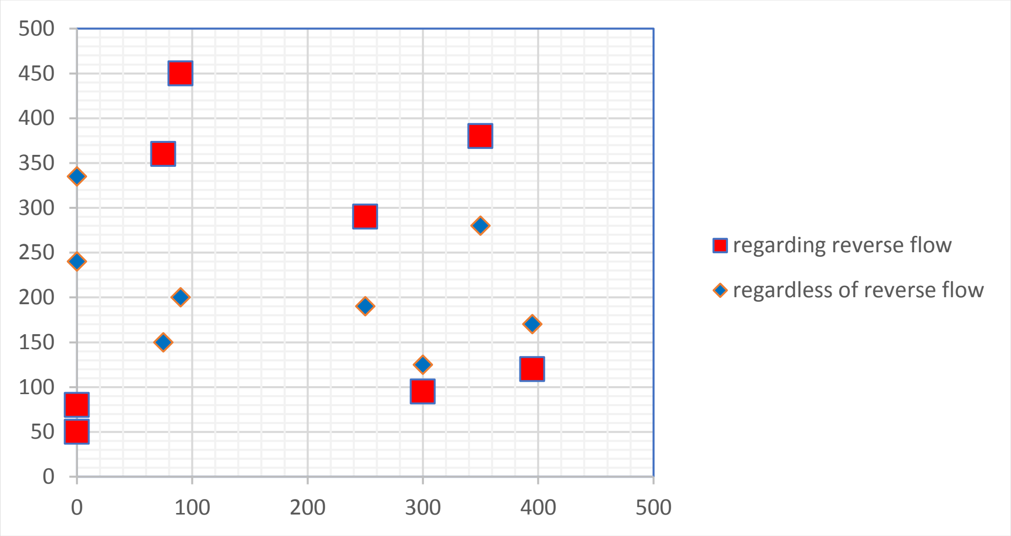

Fig. 5. Comparison of problem solving with and without reverse flow condition

As reflected by the Figure and tables above, the problem of minimizing Cmax is solved once regarding the reverse flow and then regardless of reverse flow and the results signify a significant difference in both the value of the objective function and the decision variables of start-up time and completion time. In general, solving the problem in the reverse flow condition has led to a worse response and an increase in Cmax. The value of the objective function is as table 3.

Table 3
Cmax value with and without reverse flow

Cmax	**with reverse flow**	**without reverse flow**
value	400	493

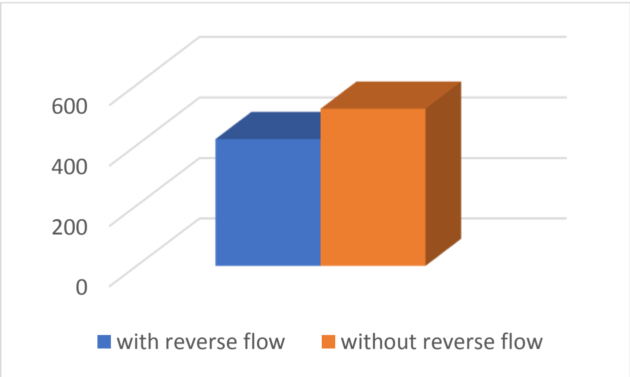

Fig. 6. Cmax value with and without reverse flow

As highlighted, the consideration of reverse flow has led to an increase in production time by approximately 25% and 93 units of time, which indicates the relatively high impact of implementing reverse flow on production. Job rotation of the employees is considered separately for each operator of each machine.

Table 4
Consideration of job rotation

	Machine		
Operator	1	2	3
1	2	2	3
2	3	2	2
3	2	3	3
4	3	3	2

The table 4 shows the machine assignment arrangement to each of the operators within 30 workdays. As is evident, the best scenarios for assigning the number of workdays to each operator are shown in table 4

4.1. Minimizing energy consumption regardless of Cmax

In this section, the second function of the problem, i.e., minimizing energy regardless of Cmax is considered. In table 5, the results related to the optimal amount of energy consumption by each machine are provided separately for each task.

Table 5
The optimal amount of energy consumption by each machine separately for each task

	Machine		
Activity	1	2	3
1	140	100	320
2	150	240	120
3	200	225	250
4	500	450	320

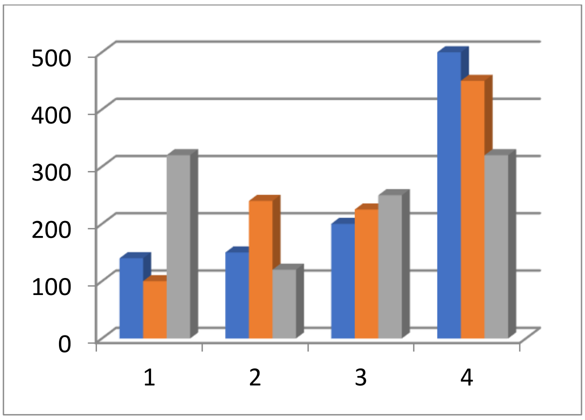

Fig. 7. Presents the optimal amount of energy consumption by each machine, separately for each task and the value of the objective function, i.e., total energy consumption

Table 6
Energy consumption regardless of construction time

Energy consumption	Objective function
1285	value

4.2. Bi-objective situation

In this section, two objective functions are examined simultaneously, while all the desired innovations, namely reverse flow, sustainability as well as job rotation are considered. To solve the problem, first, the Pareto distribution is presented and then the values of the objective function are described.

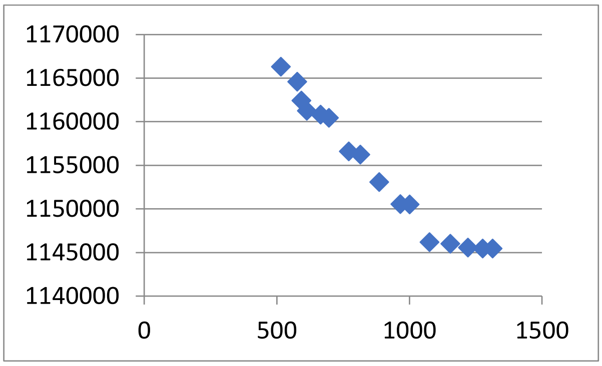

Fig. 8. Pareto distribution curve of bi-objective problem

By looking at the Fig. 8, we can understand the conflict between the two objective functions; In other words, we see an increase in cost by reducing energy consumption and vice versa. Each point in the Pareto distribution curve indicates the intersection of the optimal solutions for those two problems.

Table 7
Values of objective functions

Second objective functions	**First objective functions**	**objective functions**
1145455	515	Value

The table 7 presents the values of the objective functions for two problems of minimizing energy consumption and also minimizing the production time. The table 8 is provided to clarify the difference between bi-objective and single-objective situations.

Table 8
Comparison of single-objective and bi-objective situations for the first and second objective functions

Second objective functions	**First objective functions**	**Result**
1285	400	Single objective situations
1145455	515	bi-objective situations

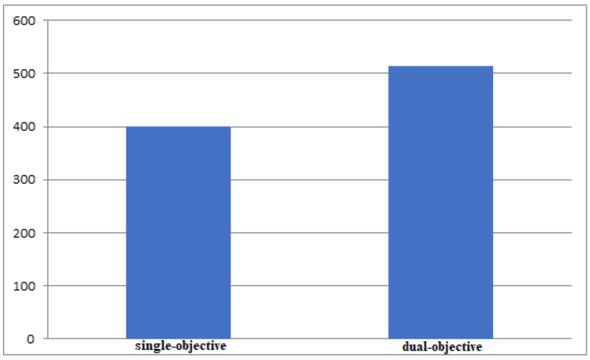

Fig. 9. Comparison of the first objective function in single-objective and bi-objective situations

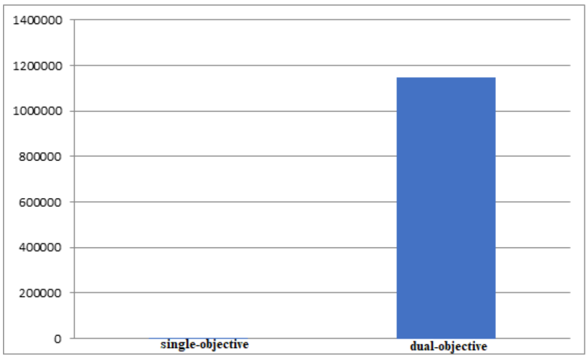

Fig. 10. Comparison of the second objective function in single-objective and bi-objective situations

As evidenced, there is a relatively significant difference between the values of the objective function in single-objective and bi-objective situations. This difference in the second objective function i.e., minimizing energy consumption, is much greater than that of the first objective function. As a result, one might argue that due to the worsening of the problem in the bi-objective situation compared to the single-objective situation, the designed model has acceptable validity and its solution is possible in large segments, using meta-heuristic algorithms.

4.3. Solving the model by whale algorithm

As mentioned in the previous section, it was possible to solve the problem in small segments using GAMS. In this section, large and medium segments were solved using the whale algorithm. Here, the efficiency of the algorithm is examined. First, the various segments in which we intend to solve the problem are introduced, and then the model solution is presented using GAMS as well as the whale meta-heuristic algorithm. The table 9 presents the segments of problem solving

Table 9
Large and medium segments of problem solving

Problem	activity	Machine	Machine position	Day time	oprator
1	17	7	4	20	5
2	18	7	4	21	6
3	19	8	5	22	7
4	20	8	5	23	8
5	20	9	6	24	9
6	20	10	6	25	10
7	11	11	7	26	11
8	21	11	7	27	12
9	22	12	8	28	13
10	22	12	8	29	14
11	23	13	9	30	15
12	24	14	3	31	7
13	23	14	3	32	6
14	23	15	4	33	6
15	24	15	4	34	6

Table 10
Problem solving results using GAMS and multi-objective whale algorithms

Problem	GAMS results				Whale algorithm results			
	The number of Pareto curve points	Dispersion index	MID	Problem solving time in seconds	The number of Pareto curve points	Dispersion index	MID	Problem solving time in seconds
1	4	31.5	0.76	133	4	32.6	0.86	145
2	3	219	0.76	147	4	37	0.85	144
3	5	83.7	0.75	137	6	45	0.89	159
4	6	5.61	0.74	151	5	6.65	0.83	161
5	8	101.534	0.77	149	7	65	0.85	151
6					6	86	0.86	167
7					7	8.9	0.85	191
8					8	51	0.84	201
9					6	31	0.91	211
10					5	34	0.92	232
11					6	6.5	0.88	255
12					5	8.6	0.89	251
13					4	19.5	0.88	254
14					5	13.5	0.89	256
15					8	12.5	0.87	254

In table 10 the results of solving problems by Gams and whale alghorithm are compared. It should be noted that the GAMS part is implemented by epsilon consntrats method.
In the Fig. 10, the problem solving result is presented using the multi-objective whale algorithm.

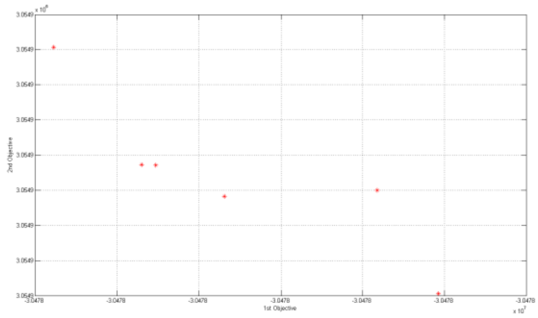

Fig.11. Pareto graph of multi-objective whale algorithm

As demonstrated, the optimal results of the multi-objective algorithm are presented in the above Pareto graph for a problem, which shows that this algorithm has been able to solve the problem. The results of table 3 also indicate the comparison of the performance of GAMS and multi-objective whale algorithm in problem solving, the results of which are presented as a Fig.12.

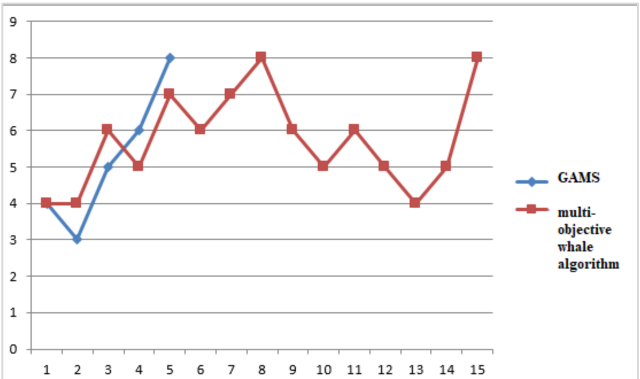

Fig. 12. Comparison of the performance of GAMS and multi-objective whale algorithm in production of Pareto points.

Apparently, the multi-objective whale algorithm produced more Pareto points for second and third problems, whereas GAMS generated more Pareto points for fourth and fifth problems. This indicates the ability of both precise and imprecise methods to solve the problem.

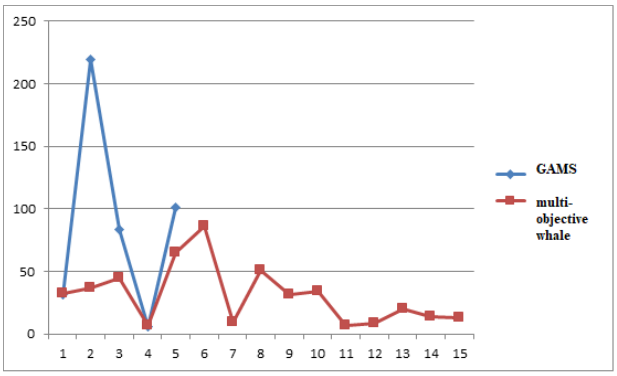

Fig.13. Comparison of the results of two methods in terms of dispersion index

As per Fig. 13, there is not much difference in the initial problems in terms of dispersion index. However, it should be noted that GAMS was able to solve the problems 1 to 5, therefore, the results of the multi-objective whale algorithm are examined in terms of dispersion index for the problem 5 onwards, which signifies a better solution for larger problems.

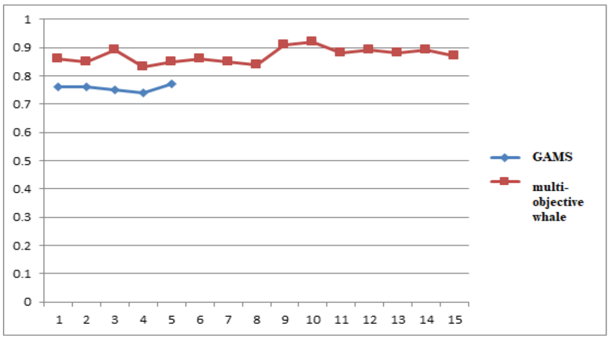

Fig. 14. Comparison of the results of two methods in terms of distance to the ideal point

It is observed in the first 5 problems that the results of the precise method are better than those of the meta-heuristic method and the distance to the ideal point is less, while the following shows that the meta-heuristic algorithm has obtained relatively sustainable results in terms of distance to the ideal point.

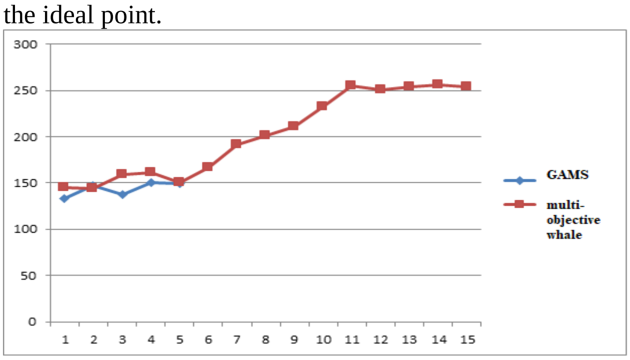

Fig. 15. Comparison of the results of two methods in terms of calculation time

It is evident that the calculation time of the meta-heuristic method was less than that of GAMS and GAMS was able to solve the problem up to the 5th. The calculation time increased as the problem segments extended in size, but reached a maximum of 250 seconds, of which it can be inferred that acceptable results have been obtained.

5. Conclusion

The present study was conducted with the aim of developing and solving a multi-objective, flexible and sustainable job shop scheduling problem with reverse flow, considering the job rotation. Sustainability, reverse flow as well as job rotation were considered as innovations of the present study, and their effect on flexible job shop scheduling was investigated. The results of the reverse flow effect indicate a worse response and an increase in total production time. In addition, the best responses of assigning the number of workdays to each operator were obtained using the precise method. Regarding sustainability issues, it is observed that there is an inverse relationship between the first objective function and the second objective function, which is related to energy issues, therefore the cost should be increased to a certain extent in order to improve energy consumption.

Besides, the results of problem solving using the whale algorithm show the high efficiency of this algorithm in problem solving, because the presentation of the Pareto curve indicates the initial efficiency of the algorithm, while GAMS is only capable of solving the problem in small segments. The precise method was unable to solve the problem 6 onwards, whereas the whale algorithm achieved desired results by providing suitable Pareto points as well as reducing the dispersion and sustaining the distance to the ideal point.

References

Apornak. A, Raissi, Mohammad Reza Pourhassan, (2021) Solving flexible flow-shop problem using hybrid multi-criteria Taguchi-based computer simulation model and DEA approach, *Journal of Industrial and Systems Engineering*, Vol. 13, No. 2, pp. 264-276, Spring (April) 2021.

Bashi Varshosaz. M., Nadri. B., & Mohammadi. M. (2018) “Modeling and Scheduling Two-stage Assembly Flow Shop Problems with Non-Identical Assembly Machines,” *Computational Methods in Engineering*, 37(1), 65 – 81.

Beheshtinia. M, & Hassani Bidgoli. M. (2017) “Scheduling in Flexible Jobshop Environment Considering Assembling and Sequence Dependent Processing Times,” *Journal of Operational Research and Its Applications*, 13 (4), 21-37.

Błażewicz, J. Pesch, E, & Sterna, M. (2000) “The disjunctive graph machine representation of the job shop scheduling problem,” European Journal of Operational Research, 127 (2), 317-331.

Dauzère-Pérès, S., Ding, J., L.& Karim Tamssaouet, K. (2023).The flexible job shop scheduling problem: A review, *European Journal of Operational Research*, In press.

Degardi, H. & Rahimi Qashqaei, M. (2010) “Flexible job shop scheduling problem considering maintenance and repair constraints,” *Journal of industrial engineering and management of Sharif Univesity*; 26(1), 101-115.

Ding, H. & Gu, X.(2020).Hybrid of human learning optimization algorithm and particle swarm optimization algorithm with scheduling strategies for the flexible job-shop scheduling problem, *Neurocomputing*, Vol. 414, 313-332. https://doi.org/10.1016/j.neucom.2020.07.004.

Fan, J., Shen, W., Gao, L., Zhang, C. & Zhang, Z.(2021). A hybrid Jaya algorithm for solving flexible job shop scheduling problem considering multiple critical paths, *Journal of Manufacturing Systems*,Vol.60,298-311. https://doi.org/10.1016/j.jmsy.2021.05.018.

Fattahi, P. & Daneshamooz, F. (2016) Providing a model for workshop production line scheduling by considering the cargo flow along with a parallel assembly step,” *Operation research and its applications*, 13 (3), 53-73.

Fattahi. P, Bagheri Rad. N., & Daneshamooz, F. (2018) “Solving flexible job shop scheduling problem with an assembly stage and sequence-dependent setup time,” *Sharif Journal of Industrial Engineering and Management*, 34.1 (2.2), 119-131.

Gao, J., Sun, L., & Gen, M. (2008) "A hybrid genetic and variable neighborhood descent algorithm for flexible job shop scheduling problems", *Computers & Operations Research*, 35(9), 2892 – 2907.

Gen, M., Gao, J., & Lin, L. (2009) “Multistage-Based Genetic Algorithm for Flexible Job-Shop Scheduling Problem”, *Intelligent and Evolutionary Systems*, 187, 183-196. 5] Graham, R. (1966) "Bounds for certain multiprocessing anomalies", *Bell System Technical Journal*, 45 (9), 1563–1581.

Gong, G., Tang, J., Huang, D., Luo, Q., Zhu, K. & Peng, N. (2024). Energy-efficient flexible job shop scheduling problem considering discrete operation sequence flexibility, *Swarm and Evolutionary Computation*, Vol. 84, February 2024, 101421. https://doi.org/10.1016/j.swevo.2023.101421.

Haji. R, & Taghizadeh. H. (2005) “Algorithm of determining Product's Batch Size and Sequence of the Operations of n Job on m Machine,” *Journal of future studies management*, 65, 41-53.

Ham, A. (2017) “Flexible job shop scheduling problem for parallel batch processing machine with compatible job families”, *Applied Mathematical. Modeling*, 45, 551-562.

Ho, N.B., Tay, J.C., & Lai, E.M. (2006) "An effective architecture for learning and evolving flexible job-shop schedules", *European Journal of Operational Research*, 179, 316-333.

Hurink, E., Jurisch, B., & Thole, M. (1994) "Tabu search for the job shop scheduling problem with multi-purpose machines", *Operations Research Spectrum*, 15, 205-215

Hurtubise, S. Olivier, C. & Gharbi. A, (2004). “Planning Tools for Managing the Supply Chain*”. Computers & Industrial Engineering*, 46, 763–779.

Kim, Y.K., Park, K., & ko, J. (2003) "A symbiotic evolutionary algorithm for the integration of process planning and job shop scheduling," Computers & Operations Research, 30, 1151-1171.

Kreipl, S. & Pinedo, M. (2009) “Planning and Scheduling in Supply Chains: An Overview of Issues in Practice,” Production and Operations Management 13(1), 77–92.

Lei, K., Guo, P., Zhao, W.,Wang, Y., Qian, L., Meng, X. Liansheng Tang, L.(2022). A multi-action deep reinforcement learning framework for flexible Job-shop scheduling problem, *Expert Systems with Applications*, Vol. 205, 117796, https://doi.org/10.1016/j.eswa.2022.117796.

Li, X. Guo, X.,Tang, H. Wu, R., Wang, L., Pang, S.,Liu, Z., Xu, W. & Li, X. (2022). Survey of integrated flexible job shop scheduling problems, *Computers & Industrial Engineering*, Vol. 174, 108786, https://doi.org/10.1016/j.cie.2022.108786.

Lou, H.,Wang, X., Zhiming Dong, Z. & Yang,Y.(2022). Memetic algorithm based on learning and decomposition for multiobjective flexible job shop scheduling considering human factors, *Swarm and*

Evolutionary Computation,Vol.75,101204, https://doi.org/10.1016/j.swevo.2022.101204.

Luo, X., Qian, Q. & Fa Fu,Y.(2020).Improved Genetic Algorithm for Solving Flexible Job Shop Scheduling Problem, *Procedia Computer Science*,Vol.166,480-485.https://doi.org/10.1016/j.procs.2020.02.061.

Asadzadeh, L. & Zamanifar, K. (2010) "An agent-based parallel approach for the job shop scheduling problem with genetic algorithms," Math. Comput. Model, 52, 11–12.

Manne. A. S. (1960) "On the job shop scheduling problem," Operational Research, 8(2), 219-223.

Mastrolilli, M. & Gambardella, L.M. (2002) "Effective neighborhood functions for the flexible job shop problem," Journal of Scheduling, 3, 3-20.

Nahavandi, N. & Abbasian, M. (2011) "Multi-Objective Flexible Dynamic Job-Shop Scheduling Problem Considering Maintenance Constraint," *International Journal of industrial engineering and production management*, 22 (1), 13-26.

Namazi, A. & Golmakani, H. (2013) "Multiple Route Job Shop Scheduling with Maintenance Activity Constraints," *International Journal of industrial engineering and production management*, 23 (4), 459-470.

Pezzella, F., Morganti, G., & Ciaschetti, G. (2008) "A genetic algorithm for the flexible Job-shop scheduling problem," Computers & Operations Research, 35(10) 3202-3212.

Prause da Silva, D.,Santos Inoue, R.& Rodrigues Kato,E.(2024). Scheduling choice method for flexible job shop problems using a fuzzy decision maker, *Intelligent Systems with Applications*,Vol.21, 200302, https://doi.org/10.1016/j.iswa.2023.200302.

Raissi. S, Rooeinfar. R, Ghezavati. VR, (2019) Three Hybrid Metaheuristic Algorithms for Stochastic Flexible Flow Shop Scheduling Problem with Preventive Maintenance and Budget Constraint, Journal of Optimization in Industrial Engineering, Vol. 12, Issue 2, 131-147.

Reiter, S. (1966) "A system for managing job-shop production," Journal of Business, 39 (3), 371– 393.

Rooeinfar. R, Raissi. S, Ghezavati. VR, (2019), Stochastic flexible flow shop scheduling problem with limited buffers and fixed interval preventive maintenance: a hybrid approach of simulation and metaheuristic algorithms, Journal of Simulation, Vol.95, Issue 6, 509-528.

Rossi, A. & Dini, G. (2007) "Flexible job-shop scheduling with routing flexibility and separable setup times using ant colony optimization method," Robotics and Computer-Integrated Manufacturing, 23(5), 503-516

Saidi-Mehrabad, M. & Fattahi, P. (2006) "Flexible job shop scheduling with tabu search algorithms," The International Journal of Advanced Manufacturing Technology, 32, 563-570.

Shen, L., Dauzère-Pérès, S. & Neufeld, J. S. (2018) "Solving the flexible job shop scheduling problem with sequence-dependent setup times", European Journal of Operational Research, 265(2), 503-516.

Tutumlu, B. & Tugba Saraç, T. (2023).A MIP model and a hybrid genetic algorithm for flexible job-shop scheduling problem with job-splitting, *Computers & Operations Research*, Vol.155,106222, https://doi.org/10.1016/j.cor.2023.106222.

Wagner, H. (1959) "An integer linear-programming model for machine scheduling," Naval Research logistics Quarterly, 6 (2), 131-140.

Yazdani, M. & Naderi, B. (1998) "Modelling Multi-Mode Resource-Constrained Flow Shop Scheduling Problem", *Industrial management studies*, 47, 151-167.