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Research Article

A Model for Determining Optimum Process Mean in the Presence of Inspection Errors by Considering the Cycle Time

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Abstract

Any production process should be adjusted based on a target value. The problem of process mean determination in a production system with two markets is investigated. An absorbing Markov chain is employed to formulate the flow of items. All items are inspected and if the value of the quality characteristic falls below a lower limit then the item is scrapped and when it falls above an upper limit then the item is reworked. Since some items are reworked thus the cycle time of production is computed in the presence of the inspection errors. Numerical studies are performed to analyze the results.

Keywords: Taguchi loss function; Cycle time; Markov chain; Optimum process mean

1. Introduction

In quality control charts, optimum process mean should be determined when the deviation of a quality characteristic in one direction is more costly than in the opposite direction. (Abbasi et al, 2006). Determination of the process parameters (mean and variance) is an important problem because selecting the optimal parameters can be effective on the quality of the product, costs, and the customer satisfaction (Al-Sultan & Pulak, 2000). At the start of production process, each quality characteristics should be adjusted at a predetermined value. The quality inspector should check whether the item complies with the specification. If the items fall within the specification limits then they are sold in the first or second market, otherwise they are being scrapped or reworked. Items which satisfy the first desired specification limit are sold in a primary market at a regular price and the items that fall within the second one are sold in a secondary market at a reduced price. When the item should be reworked then it is returned to the production process and a corrective action is conducted (Reworking loops).

The optimum adjustment of production process parameters leads to reduce the cost of production and improve the quality. Springer was the first to consider the problem of process adjustment in order to minimize the total cost (Springer, (1951)). Hunter and Kartha developed a model with one specification limit that the conforming items are sold in a primary market and non-conforming ones are sold in a secondary market. The model takes into account the regular and reduced selling prices, the give-away cost, and the process variability (Hunter & Kartha, (1977)). Bisgaard et al. presented an economic model for the problem of determining optimum process mean for an industrial process. The analysis was illustrated by considering the problem of choosing the optimal amount of overfill in a filling operation (Bisgaard et al., (1984)). Hong had considered problem of optimizing the process mean and screening limits for each market in situations where there are several markets with different price/cost values. It was assumed that that quality characteristic is normally distributed with an unknown mean and a known variance. A profit model was developed which involves selling price, production cost, penalty cost and inspection cost (Hong, (1999)). Also, Hong and Elsayed investigated the effect of measurement errors on the optimal mean value for the case of a two-class inspection process using Markov modeling of optimal process adjustment considering several potential markets and imperfect inspection (Hong & Elsayed, (1999)). Lee and Elsayed had considered the problem of determining the optimum process mean and screening limits of a surrogate variable associated with product quality under a two-stage screening procedure. The surrogate variable is inspected first to decide whether an item should be accepted, rejected or additional observations should be taken. The optimum process mean and screening limits were obtained by maximizing the expected profit which includes selling price, production, reprocessing, inspection and penalty costs (Lee & Elsayed, (2002)). Al-sultan and Pulak presented a model for two machines in series. The product is assumed to have two attributes which are related to the processing of the product, by machine 1 and machine 2. Each attribute has a lower specification limit (LSL) set for it, and if the measured attribute for a certain product is less than its LSL, the product is recycled at a specified cost (Al-sultan & Pulak (2000)). Zinlong and Enriuedel obtained the mean and variance of the process through minimizing the sum of costs including the costs of deviation from the target and the costs of fixed adjustments (Zinlong & Enriuedel, (2006)). Jinshyang et al applied the issues associated with production setup and raw material

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procurement into the process mean problem. A two-echelon model was formulated for a single-product production process, and an efficient algorithm was developed for finding the optimal solution (Jinshyang et al, (2000)). Wang et al, presented method of optimal process adjustment based on the approach of integrated control (Wang et al, (2004)). Duffuaa and Gaally developed a multi-objective optimization model which includes profit function and income. They had used Taguchi quadratic loss function (Duffuaa and Gaally, (2012)). Chen and Lai proposed a model with quadratic quality loss function of product within the specification limits. They assumed that the non-conforming items in the sample of accepted lot are replaced by conforming ones (Chen and Lai, (2007)). Shokri and Walid discussed the problem of determining the means of a set of processes in series. Depending on the value of the quality characteristic, an item can be reworked, scrapped or forwarded to the next process. An item is reworked at the same stage. They presented a recursive form of the profit function that yields a very efficient method for determining the means (Shokri and Walid, (2011)). Park et al obtained mean and inspection limits through maximizing the profit function using frequent method of Gauss-seidl (Park et al, (2011)). Chung and Hui considered the production cost, inspection cost, rework cost, scrap cost, and the use cost of customers in the model. The quadratic quality loss function was used in evaluating the use cost of customers (Chung & Hui, (2009)). Lee et al (2007) investigated different aspects of optimal process adjustment problem (Lee et al, (2007)).

In this research, the production process is formulated using an absorbing Markov chain. The item is inspected and if it does not conform to its specifications, it is either scrapped or reworked. The reworked item will be inspected again. Two potential markets are available. The inspected items are classified into four categories. Conforming items are sold in a primary market and nonconforming items are either sold in a secondary market or reworked or scrapped depending on the value of its quality characteristics. Markov models of production process have been presented in some studies including (Fallahnezhad & Niaki, (2010)) and (Fallahnezhad & Hosseininasab, (2012)) and (Bowling, et al, (2004)). We have extended their models by considering the cycle time of production in profit objective function. The cycle time is the time between productions of two successive items that is determined based on the time of bottle-neck station. The main contributions of presented model are as follows,

- Considering two markets for the sale of items.
- Considering the process cycle time in objective function. Due to the use of reworking loops in the production process, one item may be processed several times that leads to increase the cycle time of production.
- Considering inspection errors in the model along with analyzing their effects on the sales and the total profit of manufacturer.
- Considering loss functions in the model. The costs of quality are usually analyzed by reworking cost or scrapping cost. However, Taguchi considered cost to customers. Since after sale of the item, the consumer bears quality loss either in repairs or the purchase of a new item thus the manufacturer will bear the costs of quality loss due to the negative feedback from the customers. Therefore, any item manufactured away from the target value would lead to some losses to the manufacturer. Ignoring this cost can prevent the suppliers from operating efficiently according to market needs (Fallahnezhad & Ahmadi, (2014)).

The time that an item spends at a workstation, from arrival to its departure is known as the cycle time. Since the production process may rework the items thus the cycle time may increase for each item thus reworking time affects the number of produced conforming parts and the throughput of the manufacturing system.

The cycle time should be included in process adjustment because some items may be reworked hence the item may be reprocessed on the workstation several times. Thus reworking time reduces total available time of production. When the process mean is adjusted at a higher level then it may lead to increase the time of reworking process for each item and consequently it increases the cycle time for each item. Therefore if we do not consider the cycle time in process adjustment problem then unpractical results will be obtained.

The conforming items that are accepted and the nonconforming items that are rejected will be the suitable decisions for the inspection. The conforming item that is rejected is a loss for the producer, and the nonconforming item that are sent to the customers is a loss for the customer. Hence, we are dealing with two cases (1) conforming item would be rejected and (2) nonconforming item would be accepted. These cases are typically known as inspection errors. These errors can be effective on quality where the inspection may reject some conforming items in the presence of first type error or it may accept some nonconforming items due to the presence of second type error. The second type of error is more important because it leads to selling nonconforming items to the customers. Consequently if we do not consider these types of error in the optimization model then it may strongly distort the results from the optimal solution and wrong process adjustment leads to more costly and time consuming process.

Assuming two different markets for the sales of products is a common approach for dealing with the needs of different customers. We have assumed that some items are sold in a primary market while others are sold in a secondary market.

It is elaborated that all concepts applied in the proposed methodology are important factors of each production process. The discussed factors are separately considered in previous works but it is tried in this research to develop an optimization model that considers all of these factors.

The problem of optimal process adjustment is mostly encountered in filling or cutting problems (Abbasi et al, (2006)). When the value of quality characteristics in a cutting process was above an upper specification limit then the item can be reworked to a conforming one by applying the cutting process one more time but when the value of quality characteristics was below a lower

specification limit then the item is categorized as scrapped. For example, the item should be melted and then reprocessed in the production process as raw material.
The rest of this article is organized as follow. Section 2 presents the required notations used in the model development. The model development is the topic of section 3. The Numerical demonstration of the proposed methodology is given in section 4. Sensitivity analysis of key parameters is the topic of section 5. The inspection errors are investigated in section 6. Finally, the conclusion is given in section 7.

2. Notations

The parameters of the model are defined in this section.

U_2 : The upper specification limit of quality characteristic for items in primary market

U_1 : The upper (lower) specification limit of quality characteristic for items in the secondary (primary) market

L : The Lower specification limit of quality characteristic for items in the secondary market

P_{ij} : The probability of going from state *i* to state *j*

f_{ij} : The long run probability of going from a non-absorbing state (*i*) to an absorbing state (*j*)

a : The item price at the primary market

r : The item price at the secondary market

g : Give-away cost per unit of excess material in the primary market

g': Give-away cost per unit of excess material in the secondary market

R :Scrapping cost

K_1 : The Coefficient of quality loss function for the quality characteristics at the primary market

K_2 : The Coefficient of quality loss function for the quality characteristics at the secondary market

TP :The total profit

P: The transition probability matrix

Q: The transition probability matrix of going from a non-absorbing state to another non-absorbing state

R : A matrix containing all probabilities of going from a non-absorbing state to another absorbing state (i.e., accepted or rejected item)

I : The identity matrix

O : A matrix with zero elements

M : The fundamental matrix

F: The absorption probability matrix

C : The production cycle time

T : The time of producing or processing one item in each operation

H : Total production time in period of decision making

C:The coefficient of production cost per item

i : The inspection cost per item

$\phi(.)$: Normal cumulative distribution function

3. Model development

Duffuaa, and El-Gaaly employed the sampling plan for inspecting produced items and considered two markets for selling the produced items (Duffuaa, & El-Gaaly, (2013)). We extended their model and considered one serial production system in which all items are inspected. We assume that there is an inspection station after the production station and rework loops are applied for inspecting the items. It means that if an item needs reworking then it is returned to production process. The quality characteristics of an item are represented by the random variable x with an adjustable mean μ and a constant variance σ^2. An item is sold in one of two markets with different profit structures or it will be scrapped or reworked. A produced item is called conforming if its quality characteristic falls within U_1, U_2 $(U_1 < x < U_2)$ and it is sold in a primary market at a price $\$a$. The item is called nonconforming If x falls within L and U_1 $(L < x < U_1)$ then it will be sold in a secondary market at a reduced price $\$r$ $(a > r)$. When x falls above U_2 then it will be reworked and it is returned to the production process. The item is reworked, accepted and sold at the primary market, accepted and sold at the secondary market or scrapped. Raw materials come into the production system and finally the finished items are produced. A Markov chain is designed to model different states of the production system including raw materials, i.e., reworking, scrapping, accepting and selling at the primary markets or accepting and selling at the secondary markets. An absorbing Markov chain for production is presented in Pillai & Chandrasekharan, (2008).
Thus total profit can be obtained as follows,

$$TP = \frac{H}{C} E(RP) \tag{1}$$

Consider a production system with the following states:
State 1: An item is processed in the production process
State 2: An item is sold at the primary market
State 3: An item is sold at the secondary market
State 4: An item is scrapped
Transition probability matrix is as follows:

$$\mathbf{P} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \tag{2}$$

P_{11} is the probability of reprocessing an item, P_{12} is the probability of selling an item in the primary market, P_{13} is the probability and selling an item in the secondary market. P_{14} is the probability of scrapping an item in production process. Thus followings are obtained,

$$P_{11} = \int_{U_2}^{+\infty} f(x)dx = 1 - \phi(U_2)$$
$$P_{12} = \int_{U_1}^{U_2} f(x)dx = \phi(U_2) - \phi(U_1)$$
$$P_{13} = \int_{L}^{U_1} f(x)dx = \phi(U_1) - \phi(L)$$
$$P_{14} = \int_{-\infty}^{L} f(x)dx = \phi(L) \quad (3)$$

The analysis of absorbing Markov chains is presented in Bowling et al, (2004) and the results are as following,

$$M = (I - Q)^{-1} = m_{11} = \frac{1}{1 - p_{11}}$$
$$F = \begin{pmatrix} \frac{P_{12}}{(1-P_{11})} & \frac{P_{13}}{(1-P_{11})} & \frac{P_{14}}{(1-P_{11})} \end{pmatrix} = \begin{pmatrix} f_{12} & f_{13} & f_{14} \end{pmatrix} \quad (4)$$

Also the cycle time of production (C) is obtained as follows,

$$C = T \times m_{11} \quad (5)$$

Since the value of m_{11} is equal to the expected number of times that an item is processed before absorption occurs and T is the production time. Thus the expected total processing time of one item (cycle time) is as following,

$$C = T \times m_{11}$$

Thus following is obtained,

$$TP = \frac{H}{C}\begin{pmatrix} [(a - g(X' - U_1) - \\ K_1 \frac{\int_{U_1}^{U_2} (\frac{1}{X^2}) f(x)dx}{\int_{U_1}^{U_2} f(x)dx}) \times f_{12} + \\ (r - g'(X'' - L) - \\ K_2 \frac{\int_{L}^{U_1} (\frac{1}{X^2}) f(x)dx}{\int_{L}^{U_1} f(x)dx}) \times f_{13} - \\ m_{11}(c\mu + i) - (R \times f_{14})] \end{pmatrix} \quad (6)$$

The parameter f_{12} is the absorption probability of selling an item in the primary market and X' is defined as the conditional expectation of the quality characteristic X given that its value is between U_1 and U_2 . Also the parameter $X' - U_1$ is the amount of the excess material sold in the primary market. Thus following is obtained,

$$X' = E\left(X \middle| U_1 < X_1 < U_2\right) = \frac{\int_{U_1}^{U_2} x f(x)dx}{\int_{U_1}^{U_2} f(x)dx}. \quad (7)$$

The loss function for customer is as follows (Duffuaa, & El-Gaaly, (2013))

$$L(x) = k\,\frac{1}{x^2} \quad (8)$$

If the value of the quality characteristic increases then it would be better for customers thus the concept of loss function is used to quantify the loss for customers (Taguchi et al, (1989)). The function $L(x)$ in the Eq. (8) is the larger the better type. Since the ideal value of the quality characteristic for customer is infinity thus the loss for customer will be zero in this case. Also as the value of the quality characteristic increases then more production and give-away costs are incurred. Hence, the optimum mean value will be in a point of compromise between these costs and cost of nonconformity (Duffuaa, & El-Gaaly, (2013)).

The expected value of loss function per item in primary market is as follows:

$$E\left(loss \middle| U_1 < X_1 < U_2\right) = \frac{E(loss, U_1 < X_1 < U_2)}{P(U_1 < X_1 < U_2)} = K_1 \frac{\int_{U_1}^{U_2} (\frac{1}{x^2}) f(x)dx}{\int_{U_1}^{U_2} f(x)dx} \quad (9)$$

Thus the profit for the primary market is obtained as following,

$$\left(a - g(X' - U_1) - K_1 \frac{\int_{U_1}^{U_2} (\frac{1}{X^2}) f(x)dx}{\int_{U_1}^{U_2} f(x)dx} \right) \times f_{12} \quad (10)$$

The profit for the secondary market is obtained in the same procedure as following

$$\left(r - g'(X'' - L) - K_2 \frac{\int_{L}^{U_1} (\frac{1}{x^2}) f(x)dx}{\int_{L}^{U_1} f(x)dx} \right) \times f_{13} \quad (11)$$

Thus the expected scrapping cost is obtained as follows:

$$R \times f_{14} \quad (12)$$

Also, expected processing cost is determined as follows:

$$m_{11}(c\mu + i) \quad (13)$$

Since 100% inspection system is performed thus the inspection cost is included in the processing cost. Also, it is assumed that processing cost is a coefficient of the process mean.

4. Numerical Examples

Consider a production system with two markets for selling the items and the following parameters:

$$a = 80, R = 4, c = 6, i = 1, g = g' = 2, \sigma = 1,$$

$$K_1 = K_2 = 1, L = 8, U_1 = 11, U_2 = 13, r = 67.5$$

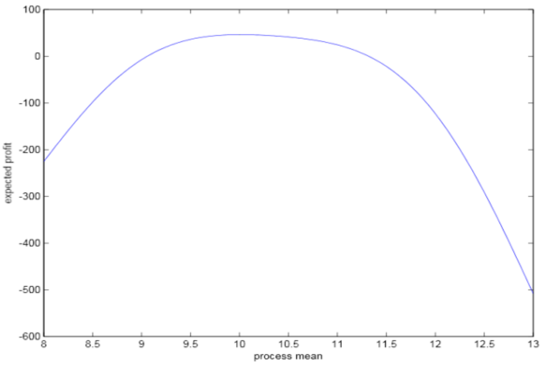

Fig. 1. The expected profit

The expected profit is maximized at the level of $\mu^* = 10.025$ with the expected profit of production that is equal to $TP^* = 45.9972$. The function TP is plotted versus the decision variable μ in Figure (1). Figure (1) shows the expected profit that is a concave function of the process mean.

5. Sensitivity Analysis

A sensitivity analysis of the proposed model is performed for illustrating the effects of parameters on the optimal process mean and optimal total profit. All parameters were varied in this production system and their effects have been analysed. Table 1 shows the behaviours of the optimal process mean and the optimal expected profit when the parameters of production system varies.
Following results are obtained,

- The optimal expected profit and optimal process mean significantly increase as the selling price of the items in primary market increases.
- the values of μ^* and TP^* slightly decrease by increasing the value of g. This result is reasonable because by increasing the give-away cost, we try to decrease the amount of excess material.
- The optimal expected profit decreases by increasing the value of g' and the optimal value of the μ^* is a convex function of g'.
- When the value of r increases then the value of μ^* decreases but the value of TP^* increases because, we try to sell more in secondary market by increasing the price in the secondary market.
- The value of μ^* increases and the value of TP^* decreases by increasing the value of R. As scrapping cost increases then we try to decrease the probability of scrapping therefore the optimal process mean increases.
- The optimal expected profit and optimal process mean significantly decrease by increasing the value of c.
- When the value of T increases then optimal process mean remains constant but the optimal expected profit decreases.
- The value of TP^* significantly decreases and the value of μ^* increases.by increasing value of σ. Increasing the value of σ leads to increase the probability of scrapping and reworking and their cost thus the optimal mean value increases in order to decrease the cost of scrapping.
- Shift in the value of i has no effect on the value of μ^* but the value of TP^* decreases by increasing the value of i.
- When the values of K_1 and K_2 increase then the value of TP^* slightly decreases but optimal process mean remains fixed.

6. Inspection Error

In this section, we assume that all items are inspected for analysing the value of quality characteristic in the presence of inspection errors. In general, there are two types of inspection errors in each inspection system. These inspection errors occur mostly because of not calibrating the inspection tools and the human errors.

- First type error (type I error, risk of producer) that means to reject an conforming item
- Second type error (Type II error, risk of consumer) that means to accept a nonconforming item.

The probability distribution of quality characteristic is assumed to be known and transition probabilities can be determined. After producing an item, it is possible to reject some conforming items or accept some nonconforming items because of the inspection errors that it can definitely influence on the profit. Thus, inspection error has impacts on the optimum value of process mean by effecting the total profit. Although, the probability distribution of quality characteristics is estimated based on the historical inspection data but the inspection error does not have any effect on the estimated probability distribution and it can be effective on the decisions about the quality of produced items.
As mentioned there are two types of inspection errors in the inspection namely type I and II errors. Type I error is classifying a conforming item as nonconforming. Type II error is classifying a nonconforming item as conforming. Therefore, the inspector rejects some conforming items and accepts other nonconforming ones due to the presence of the two types of error. Assume that α is the probability of Type I error, and β is the probability of Type II error. If P'_{ij} denotes the probability of transition from state *i* to state *j* then we have

$$P'_{12}=(1-\alpha)P(U_1\le X\le U_2)+ \beta(1-P(U_1\le X\le U_2))= (1-\alpha)P_{12}+\beta(1-P_{12}) \quad (14)$$

$$P'_{13}=(1-\alpha)P(L\le X\le U_1)+ \beta(1-P(L\le X\le U_1))= (1-\alpha)P_{13}+\beta(1-P_{13})$$

$$P'_{14}=(1-\beta)P(X<L)+ \alpha(1-P(X<L))= (1-\beta)P_{14}+\alpha(1-P_{14})$$

Also, the following is obtained:

$$P'_{11}=1-P'_{12}-P'_{13}-P'_{14}= 1-(1-\alpha)(P_{12}+P_{13})- \beta(2-(P_{12}+P_{13}))- P_{14}(1-\beta)-(1-P_{14})\alpha$$

Now, the objective function in Eq. (6) can be optimized in order to determine the optimum process mean in the presence of inspection errors. Table 2 shows the results of sensitivity analysis for different values of inspection errors.

Table 1
Behaviors of optimal mean and expected profit with the variation of the parameters

Sensitivity analysis for a production system with two sale markets				
Cost parameter	Case #	Value parameter	μ^*	TP^*
a	1	80	10.025	45.99
	2	100	11.20	146.56
	3	130	11.525	368.52
	4	150	11.625	527.41
	5	200	11.75	937.69
g	6	1	10.075	47.06
	7	2	10.025	45.99
	8	4	9.975	44.06
	9	7	9.90	41.50
	10	10	9.85	39.20
g'	11	1	10.05	64.13
	12	2	10.025	45.99
	13	4	9.975	9.80
	14	7	9.75	-43.60
	15	10	11.175	-84.70
r	16	67.5	10.025	45.99
	17	80	9.80	177.32
	18	125	9.650	661.26
	19	150	9.625	931.37
	20	168	9.60	1.0936e+003
R	21	2	10	46.56
	22	4	10.025	45.99
	23	8	10.075	44.95
	24	12	10.125	44.018
	25	16	10.175	43.16
c	26	4	10.65	303.91
	27	6	10.025	45.99
	28	8	9.70	-199.72
	29	10	9.50	-439.04
	30	15	9.125	-1.190e+003
T	31	50	10.025	73.59
	32	80	10.025	45.99
	33	100	10.025	36.79
	34	120	10.025	30.66
	35	150	10.025	24.53
σ	36	0.25	8.60	163.97

	37	0.5	9.025	110.66
	38	1	10.025	45.99
	39	1.25	10.20	24.23
	40	1.5	10.15	-13.80
i	41	1	10.025	45.99
	42	3	10.025	20.99
	43	5	10.025	-4.002
	44	7	10.025	-29.002
	45	10	10.025	-66.50
K_1	46	1	10.025	45.99
	47	5	10.025	45.93
	48	10	10.025	45.86
K_2	49	1	10.025	45.99
	50	5	10.05	45.56
	51	10	10.05	45.02

Table 2

Behaviours of optimal mean and expected profit with the variation of inspection errors

inspection errors	Case #	Value parameter	μ^*	TP^*
(α,β)	**1**	**(0,0)**	**10.025**	**45.99**
	2	**(0.05,0.05)**	9.85	55.04
	3	**(0.1,0.05)**	9.75	1.21
	4	**(0.75,0.05)**	8.025	-544.68
	5	**(0.1,0.2)**	9.47	199.03
	6	**(0.1,0.3)**	9.25	335.32
	7	**(0.3,0.1)**	9.25	-138.42

Table 3

Behaviours of optimal mean and expected profit with the variation of inspection error with considering penalty cost

inspection errors	Case #	Value parameter	μ^*	TP^*
(α,β)	**1**	**(0,0)**	**10.025**	**45.99**
	2	**(0.05,0.05)**	9.95	-9.24
	3	**(0.1,0.05)**	9.85	-63.72
	4	**(0.1,0.1)**	9.87	-64.06
	5	**(0.1,0.2)**	9.87	-64.70
	6	**(0.1,0.3)**	9.90	-65.31
	7	**(0.1,0.4)**	9.92	-65.89
	8	**(0.2,0.1)**	9.67	-171.42
	9	**(0.3,0.1)**	9.50	-276.15
	10	**(0.4,0.1)**	9.30	-377.42
	11	**(0.5,0.1)**	9.00	-473.41

- The optimum value of μ^* decreases and the optimum value of TP^* increases by increasing the value of β .
- The optimum value of μ^* and TP^* decreases by increasing the value of α .

Since nonconforming items may be sold in each market and it will lead to an unsatisfied customer that is the most important issue in marketing thus we should consider the state of selling nonconforming items in the market. These nonconforming items should be replaced with guarantee cost and other related cost. To consider the cost of this event, the state 5 is added to the model and a cost parameter is considered for absorbing to this sate that is a penalty cost PEC .

State 5: A nonconforming item is sold at the market

If P'_{ij} denotes the probability of going from state *i* to state *j* in this case, the transition probability matrix is as following:

$$\mathbf{P} = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \overset{\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array}}{\begin{bmatrix} P'_{11} & P'_{12} & P'_{13} & P'_{14} & P'_{15} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}} \qquad (15)$$

Where,

$$P'_{12} = (1-\alpha)P(U_1 \le X \le U_2) = (1-\alpha)P_{12}$$
$$P'_{13} = (1-\alpha)P(L \le X \le U_1) = (1-\alpha)P_{13}$$
$$P'_{14} = (1-\beta)P_{14} + \alpha(P_{12} + P_{13})$$
$$P'_{15} = \beta P_{14}$$
$$P'_{11} = 1 - P'_{12} - P'_{13} - P'_{14} - P'_{15} \qquad (16)$$

Now, the penalty cost is included in the objective function as following,

$$TP = \frac{H}{C}\left(\begin{array}{l} [(a - g(X' - U_1) - \\ K_1 \dfrac{\int_{U_1}^{U_2} (\frac{1}{X^2}) f(x)dx}{\int_{U_1}^{U_2} f(x)dx}) \times f_{12} + \\ (r - g'(X'' - L) - \\ K_2 \dfrac{\int_{L}^{U_1} (\frac{1}{X^2}) f(x)dx}{\int_{L}^{U_1} f(x)dx}) \times f_{13} - \\ m_{11}(c\mu + i) - (R \times f_{14}) - (PEC \times f_{15})] \end{array} \right)$$

(17)

All terms in the objective function have been determined before and the value of f_{15} is obtained as follows,

$$f_{15} = \frac{P'_{15}}{1 - P'_{11}}$$

Assume $PEC = 20$ and the numerical example is solved to determine the optimal process adjustment under the presence of inspection errors. Table 3 denotes the results of sensitivity analysis for different values of the inspection errors. The results are as following,

- The optimum values of μ^* and TP^* decreases when the values of α increases, because the Type I error leads to reject some conforming items.
- The optimum value of μ^* increases and the optimum value of total profit decreases by increasing the value of β because the Type II error leads to accept some nonconforming items that results in a penalty cost.
- The penalty cost has a considerable effect on the total profit and it can lead to bankruptcy because of negative profit.

7. Conclusion

In this paper, an absorbing Markov chain model for a production is introduced that products are sold in two markets and the objective is to maximize the expected profit per item. All items are inspected and they are classified as conforming to good quality that are sold in a primary market, conforming to medium quality that are sold in secondary market, scrapped, and reworked ones. The cycle time of the production has been formulated. Also, performance of the proposed methodology in the presence of inspection errors is investigated. The penalty cost of selling nonconforming items is considered in the model.

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