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Research Article

Failure Mode and Effect Analysis using Robust Data Envelopment Analysis (Case Study: Automobile Oil Filter)

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Abstract

Risk management improves and increases the speed of development and optimal implementation of the company's strategy to achieve a competitive advantage. Risk identification and assessment are known as one of the main tools of safety management, which helps the safety manager better select risk reduction measures and standardization of automobile oil filter by creating a suitable information platform. In this regard, evaluating and analyzing failure modes and their effects is an appropriate tool for risk management and improving product quality. Due to the weaknesses of the traditional method the complexities of the fuzzy method, a new type of risk non-priority is presented by assigning different weights to each of the risk factors under uncertainty and the proposed method is compared with the traditional and fuzzy methods simultaneously. The purpose of this study is to analyze the failure mode and risks in operation and various stages of manufacturing automobile oil filter, then to prioritize and compare risks by applying the fuzzy theory method and robust data envelopment analysis. Oil filter is an essential part of the automobile that its standardization reduces fuel consumption, improves engine performance and consequently decreases environmental pollution. This research has used the combination of Failure Mode and Effect Analysis(FMEA) method for analyzing the reliability of the oil filter and fuzzy theory has been used to record experts' opinions on failure modes and calculate the risk priority of each subsystem under uncertainty. In order to eliminate the existing defects, a new method is introduced for calculating the risk priority number in the failure mode and effect analysis based on the data envelopment analysis method. In this research, the robust optimization method covers the results of the data envelopment analysis (DEA) and is less complex than the fuzzy method has been used. The results of the case study indicate that the proposed model is more effective and reliable than the traditional and fuzzy Risk Priority Number (RPN) and also the proposed method has less has complexity than the fuzzy method. This method provides a complete ranking and convincing prioritization of failure modes. After calculating the RPNs, the operations related to the spiral tube, fiber folding, ring bending and cutting, and fiber folding are the highest number of RPNs, respectively, and their corrective actions were also determined.

***Keywords*:** FMEA technique; Fuzzy theory; Risk management; Reliability; Failure modes and effects analysis; Robust Data Envelopment; safety management

1. Introduction

Engine oil filter is one of the most important parts is a car lubrication system. This part helps to achieve longer engine life. While the air filter is responsible for preventing dust particles from entering the air, causing pollution, and wear and tear of the vehicle, the oil filter must filter the particles that want to enter or for some reason exist inside the engine. These contaminants may be consisted of worn metal, dust particles in the combustion air, soot, or corrosion products. The oil filter does not affect the chemical changes of the oil in the engine . It only performs the ability to separate the waste particles to a certain extent physicallyThe definite and certain conditions of the past give way to uncertain and ambiguous conditions. In this situation, decision-making as the most current issue in human life, has faced many challenges, so if there is a strong technique that can help humans make the right decision in a timely manner, it is quite noticeable. Today, quality plays a key role in conquering the product sales market. Therefore, improving the quality of products and services provided by firms is the first and foremost factor in overtaking competitors and gaining a major market share.

The Failure Mode and Effect Analysis technique (FMEA) were first used by the US military in 1949. This technique is an engineering method to identify and eliminate problems and potential errors in the system, production process and service delivery prior to the occurrence , as well as in various ways to examine both types of quality errors and safety and health hazards. It is an analytical method for risk assessment that seeks to identify and rank, as far as possible, the potential risks within the scope of the assessment and the causes and effects associated with it. In fact, this technique is a pre-event action and a Proactive rather than a Reactive technique.

In general, the main shortcomings in FEMA are categorized as follows:

1) Accurate determination of the probability of failure is difficult or impossible.

2) The effective parameters in RPN including severity, detection and occurrence are usually considered with

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equal weight. It means that there is no difference between the effectiveness of each parameter on the final RPN with another parameter. Therefore, a criterion is needed for prioritizing the failures in it.
3) There is no clear border between the scores in FMEA.
4) In some cases, identical RPNs are obtained with different S, O and D values, which confuse prioritization.
It is a step-by-step and systematic process for identifying potential failures before they occur, to , intending to eliminate or minimize the risk associated with the failures identified (Mhetre et.al, 2012, Ambekar and et.al,2013). Carl S. Carlson (2012) also articulated an advice that FMEA should guide the development of a complete set of actions that will reduce the risk associated with the system, subsystem, and component or manufacturing/assembly process to an acceptable level.
The second part of this research reviews the literature and research background in this field, then in the third part, the Fuzzy decision-making method is explained, and in the fourth part, the proposed model for analyzing car oil filter malfunctions RPN calculation steps are presented. The fifth part reviews the research methodology and in the sixth part, the data and results are analyzed and in the final part, the final conclusion is presented.

2. Literature Review

The FMEA method is widely used in risk management of industries such as manufacturing, automotive, and aerospace. Vinodh and Santhosh., (2011) applied FMEA method to an automotive leaf spring manufacturing organization in India. In failure analysis, the methods and principles of the system are discussed in sections on maintenance performance auditing, cost recording and tracing, reliability-centered maintenance planning and control, condition monitoring and online feedback control, and integrated maintenance planning and control. Baghery et al. (2016) and Yousefi et al. (2018) identified risks involved in production process of automotive spare parts by FMEA method. PY et al., (2001). Liu et al., (2011) proposed an FMEA using the fuzzy evidential reasoning (FER) approach and grey theory to solve the problems and enhance the effectiveness of the traditional FMEA. As tested by the numerical example, the presented FMEA can well capture FMEA team members' diversity of opinions and prioritize failure modes under different types of uncertainties. Wang et al. (2020) utilized a cloud model (CM) for improving the FMEA and reducing the uncertainty in the evaluation process and considered enabling the fuzziness and randomness. (IAHP) and (CRPN) improved the accuracy of the traditional RPN and applied in coal-to-methanol plant in Yinchuan, China. Liu et al. (2014) proposed the integrated Fuzzy AHP-VIKOR and FMEA method to deal with uncertainty from human' subjective perception. They implemented their method in the general anesthesia process and sensitivity analysis verified the advantage of the proposed method. Boral et al. (2020) investigated the industrial equipment failure mode from the sustainable point of view to establish sustainable manufacturing strategy. They utilized linguistic terms to evaluate the risk factors by cross-functional experts. They proposed the integrated Interval Type-2 Fuzzy Decision-Making Trial and Evaluation Laboratory (IT2F-DEMATEL) and Modified Fuzzy Multi-Attribute Ideal Real Comparative Analysis (Modified FMAIRCA).
Ru-xin et al. (2018) proposed a hybrid risk evaluation model using FMEA with utilizing multi granular linguistic distribution assessments. For determining subjective and objective combined weights Best-worst and maximizing derivation methods were used. Their proposed model was applied in supercritical water gasification (SCWG) and verified the reliability of the model. Yazdi (2018) for dealing with uncertainty proposed an interactive approach using fuzzy theory. He used the AHP process and entropy technique to handle the subjective and objective uncertainty weight and applied his proposed approach in construction period of a refinery. Yang et al. (2017) proposed integrating case-based reasoning (CBR) and BN-based diagnosis method. They implemented a prototype of hybrid expert system for the diagnosis of embedded software by integrating CBR with Bayesian network (BN) through F-CBR by the corresponding failure spectra as the bridge. Wan et al. (2019) developed novel model that incorporated a fuzzy belief rule approach with Bayesian networks to evaluate the risk factors of maritime supply chain under uncertainty and identified transportation of dangerous goods, fierce competition and fluctuation of fuel price as the most significant risk factors. Nazeri and Naserikia (2017) utilized fuzzy hybrid approach including FMEA, DEMATEL and ANP method. They applied the proposed method in railway of Iran to select a proper maintenance strategy to have available and reliable tamping equipment. Mutlu and Altuntas (2019) presented a new approach based on FMEA that integrates fault tree analysis (FTA) method and proposed the belief in fuzzy probability estimations of time (BIFPET) algorithm to enhance the performance of the FMEA method and employed in fabric dyeing department of a textile company. Chang and Paul Sun (2009) have used DEA method to improve the ability to the assessment of FMEA method. The SOD factors instead of RPN were used in proposed method. And applying the proposed method in the case study indicates that DEA could be the complement to the traditional FMEA method. Chang and Chung (2012) utilized FMEA and DEA method to reduce critical failures and maintain long-term profit service of companies. They assessed the risk prioritization of Which can be represented as triangular fuzzy numbers (a^L, a^m, a^r). Figure 1 shows this membership function.

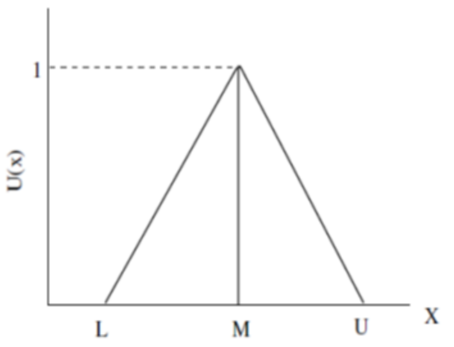

Fig. 1. Fuzzy triangular numbers

The critical potential service failure modes in third-party complaints about Taiwanese outbound group package tours. Dedimas & Gebeyehu. (2019) applied FMEA for efficient and cost-effective manufacturing. They have showed the economic gain from the reduced high downtime in the Bair Dar Textile Share Company by taking the advantages of FMEA. The findings of their research indicated that the firm can mitigate the total downtime from its 178 loom machines by 299.04hrs/day. Shaghaghia & Rezaie, (2012) have proposed a FMEA which uses generalized mixture operators to determine and aggregate the risk priorities of failure modes. They applied their proposed model in LGS gas type circuit breaker product in Zanjan Switch Industries in Iran. Chin et al. (2009) presented an integrated FMEA- DEA method that measures minimum and maximum risk for each failure mode. The geometric mean of both risks is calculated to measure the overall risk of failure modes. And Risk prioritization in terms of general risks instead of maximum and minimum risks is calculated, as well. Wang et al. (2019) considered team member's psychological behavior, interaction relationships among risk factors and uncertainty by developing a hybrid FMEA framework Integrating Interactive and Multi-criteria Decision Making) approach and Choquet integral method TO cover the conventional FMEA drawback. This research by adopting generalized trapezoidal fuzzy numbers to describe the uncertainty in risk assessment. Afterward, for FMEA team member an improved GTrFN-WAA (weighted arithmetic utilized to aggregate risk assessment information. Wei Lo et al. (2020) have considered environmental protection and anticipated costs factors in the FMEA model. They firstly used the DEMATEL method to establish an influential network relationship map of risk factors. Eventually identified failure modes prioritized by TOPSIS method.

Qin et al. (2020) to cope with some deficiency of the traditional l FMEA approach and deal with uncertainty proposed the integrated interval type-2 fuzzy sets (IT2FSs) with evidential reasoning (ER) method and applied the proposed method in the steam valve system to demonstrate the effectiveness of the method. Li et al (2021) considered failure modes, failure causes and critical system and deal with uncertainty and minimized the catastrophic failures on floating offshore wind turbines. They investigated fifteen failure scenarios and suggested corrective actions to reduce failure impacts. Ribas et al. (2021) utilized linguistic variables a two-stage evaluation. Firstly computed RCI to computed the RCI through the combined effect of Occurrence and Severity. They computed the FIS-RPN in the second stage by combining the RCI with detection. They applied their model in dam to identity the failure modes that have catastrophic impacts. André Filz et al. (2021) presented a data-driven Failure Mode and Effect Analysis (FMEA) using dep learning to support the maintenance planning for industrial investment goods by improving transparency and enhancing fault prediction significantly.

Given that the analysis of failure modes and their effects play an important role in quality management and cost reduction, the main purpose of this study is to address the disadvantages of the traditional and fuzzy RPN score using DEA. So far, little research has been done on the use of data envelopment analysis in FMEA, It can also be done by using DEA one of the main disadvantages of traditional FMEA is the allocation of the same weights to each of the risk priority factors. Since uncertainty in reality is inevitable and the mentioned models are also realistic models, the development of these models in an atmosphere of uncertainty is also necessary. Therefore, the use of the data envelopment analysis model in a robust optimization space, which is one of the new approaches to deal with uncertainty, is another selected model of this research.

Considering the investigation of preceding studies, it's found that they are classified into two groups. The majority of these researches, notwithstanding the defects of traditional and fuzzy RPN scores , prioritize risks in their effort to utilize this score. Other categories of paper, aiming to cover some of RPN score's defects, provide a new approach for assessment and prioritization of system's risks. This paper attempt to cover the disadvantages of conventional and fuzzy RPN score mentioned in the introduction while using the FMEA method for oil filter Failure Mode and the evaluation of its effects, . So that the score obtained from the proposed approach are more authentic than the traditional and fuzzy RPN scores, since moreover to consider other indicators except for SOD factors, uncertainty in extra indicators are considered in risks evaluations.

3. Research Approach

All research pursues three different object. Sometimes the purpose of research is to test theories, explain the relationships between phenomena, and add to the body of knowledge available in a specific field. Such research is called fundamental research. In the present study, based on previous research, a new model for the FMEA has been presented, so the research is fundamental. The use of mathematical models to determine the amount of efficiency is also the reason for the experimental-mathematical nature of this paper. Broadly speaking, the steps of this research are as follows:

1. Study related paper to FMEA, Data envelopment analysis (DEA), Robust optimization and combination of FMEA and DEA

2. Design and selection of model FMEA- DEA suitable for use to determine the number of risk priority
3. Design and run the proposed model by GAMZ software
4. Design and selection of RODEA Based on FMEA to robust the data in the space of uncertainty.
5. Solve numerical examples (automobile oil filter) in the above-mentioned models to test and justify the models
6. Conclusion and analysis of tables and information obtained from it
7. Comparison of the obtained results from RODEA, Fuzzy and conventional FMEA method

The current study's proposed approach is based on FMEA and RODEA, and it takes into account a variety of evaluation factors and output uncertainty. In the RODEA technique, decision-making unit (DMU) options (in this case, automobile oil filter) are evaluated for each risk based on the system's input and output values compared to other risks. In this procedure, factors of the decision-maker seeks to increment are considered as outputs, and those to be decrease are referred as inputs. As a result, As a result, SOD variables are seen as inputs, as management strives to lower the degree of risk, the likelihood of occurrence, and the failure to detect.

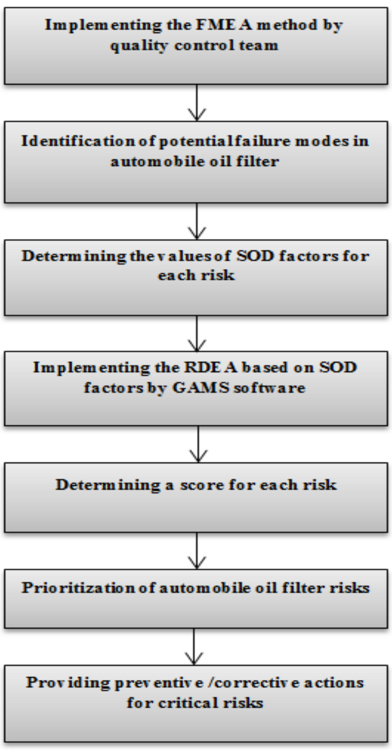

Fig. 2. Decision-making procedure used in this paper

Prior to investigating the RODEA method we examined the Fuzzy method.

4. Fuzzy Numbers and Fuzzy Sets

The theory of fuzzy sets was first proposed by Lotfizadeh (1965). This theory is used in situations of ambiguity and uncertainty and is able to express many inaccurate concepts and expressions in mathematical language and provides the basis for reasoning, inference, control, and decision-making in conditions of uncertainty. According to this theory, a fuzzy number is a special fuzzy set of form $\tilde{A} = x \in \frac{R}{\mu\tilde{A}}$. Where x accepts the real values of the member of the set R and its membership function is in the form $\mu\tilde{A}(x)$. Triangular and trapezoidal fuzzy numbers are the most common types of fuzzy numbers that are used in both theory and practice. Triangular fuzzy numbers are more commonly used due to simpler calculations. A triangular fuzzy number A is defined by relational linear membership function as Eq (1):

$$\mu\tilde{A}(x) = \begin{cases} \frac{(x - a^t)}{(a^m - a^l)}, a^L \le x < a^m \\ \frac{(a^4 - x)}{(a^r - a^m)}, a^m < x < a^r \end{cases} \quad (1)$$

Two fuzzy numbers are triangular and their algebraic operations are shown in the following of

$\tilde{B} = (b^L, b^m, b^n)$, $\tilde{A} = (b^L, b^m, b^u)$.

$$A + B = [a^L + b^L, a^m + b^m, a^u + b^u\} \quad (2)$$

$$\tilde{A} - \tilde{B} = [a^l - b^l, a^m - b^m, a^u - b^u\} \quad (3)$$

$$\tilde{A} \otimes \tilde{B} = [a^l b^l, a^m b^m, a^u b^u\} \quad (4)$$

5.The Proposed Model for Assessing the Risks of Making Automobile Oil Filters

This research has presented a model for assessing the risks of making an automobile oil filters based on failure modes and analyzing the effects and fuzzy theory. The proposed steps for assessing the risks of making a automobile oil filter are shown in Figure 2.

Step 1: Building an assessment team

Occurrence, severity and detection values are based on the group's expertise. Therefore, the present group should include experienced and specialized experts. Since each expert has a different impact on the results, the weight of each expert must be determined. Expert weight is calculated according to the method provided by Cebi and Kahraman, Eq (5);

$$W_{ei} = \frac{P_{ei}}{\sum_{i=1}^{z} P_{ei}} i = 1,2,3,\dots,z \quad (5)$$

Where ei and P represent the i-th expert and his score, respectively.

Step 2: Analysis and determination of potential failure modes

At this stage, the FMEA table is formed and completed, in which the potential failure modes, the effects of the potential failure modes, and the potential causes of the failure are determined. Reliability analysis in the use of oil filters can cause damage could be prevented. At the beginning, the probability of occurrence, detection and severity should be determined by experts using fuzzy linguistic variables.

Step 3: Determination of the probability of Occurrence, Severity and Detection Score

O, S and D are evaluated by the 10-point scale described in Tables (1) - (3). Based on the rating scale, experts can determine their views on the value of OSD for each failure mode. Since it is diffifcult to determine the exact values of the three risk factors (D, S, O) and people often use verbal variables to express their judgment. In this research, experts have been asked to determine these values in Tables (1) - (4) by fuzzy verbal variables.

Table 1
Occurrence Ranking Scale

Potential failure rate	Description	Rank
Failure occurs at least once a day or Failure occurs at least at any time	Certain probability of failure	10
Predictable failure occurs or occurs every 3 or 4 days	Failure is almost inevitable	9
Failure occurs frequently or occurs almost once a week	Very high probability of failure	8.7
Failure occurs almost once a month	Relatively high probability of failure	6.5
Failure occurs occasionally or occurs every three months	Moderate probability of failure	4.3
Failure is rare and occurs almost once a year	Low probability of failure	2
Failure never happens	Rare probability of failure	1

Table 2
Severity Ranking Scale

Potential failure rate	Description	Rank
Failure can cause the death of a customer or the failure of the entire system without any prior warning.	Extremely dangerous	10
Failure can cause a major, permanent, or serious disruption to the system and disrupt service (or prior warning)	Very dangerous	9
Failure causes minor to moderate damage with a high degree of customer dissatisfaction or a major system problem that requires major overhaul or rework.	Dangerous	7
Failure causes minor damage with relative customer dissatisfaction and major system problems.	Medium risk	6.5
Failure can cause minor or no damage, but customers are annoyed and cause minor problems that can be fixed with minor changes to the system or workflow.	Medium risk	4.3
Failure does not cause any damage and the customer does not notice the problem at all, although there is damage to the system.	Low risk	2
Failure did not cause any problems and did not affect the system's performance .	Safe	1

Table 3
Severity Detection Scale

Potential failure rate	Description	Rank
There is no known mechanism for detecting failures	No probability for detection	10
Damage can only be detected by careful inspection. Therefore, the detection is not practical or not easy	Probability of detection is improbable	9.8
The failure can be detected by manual inspection, but there is no procedure.	The probability of detection is unlikely	7
There is a process for re-inspection or inspection, but it is not automated and applies to only one sample	Medium probability of detection	6.5
There is a process for inspection or 100% inspection but it is not automatic	High probability of detection	4.3
There is an automated process for inspection or 100% inspection	The probability of detection is very high	2
There are automatic restrictions that prevent failures	The probability of detection is almost certain	1

to evaluate the importance and weight of each failure mode, the 5-level scale of fuzzy verbal variables shown in Table (5) has been used.

Table 4
Linguistic variables for the weight of criteria

Very poor	(0.0. 0.0, 0.2)
poor	(0.0. 0.2, 0.4)
moderate	(0.3. 0.5, 0.7)
good	(0.6, 0.8, 1.0)
Very moderate	(0.8. 1.0, 1.0)

Suppose that $D_{jk}^m, S_{jk}^m, O_{jk}^m$ are the values of occurrence, severity, and detection, where S, O represent the occurrence of failure and the severity of failure, respectively, D is the ability to detect failure such as product and customer arrival, these values are evaluated by expert m for the relationship j and the failure mode k. w_{jk}^m represents the weight of the importance of expert m's solution for the relationship between the two the failure modes of k. $D_{jk}^{m}, S_{jk}^{m}, O_{jk}^{m}$ are in the form of triangular fuzzy numbers which are shown as Eq (9) - (6).

$$\tilde{S}_{jk}^{m} = (LS_{jk}^{m}, MS_{jk}^{m}, US_{jk}^{m}), S_{jk}^{m} \in T, \quad \circ \leq LS_{jk}^{m} \leq MS_{jk}^{m} \leq US_{jk}^{m} \leq 10 \quad (7)$$

$$\tilde{D}_{jk}^{m} = (LD_{jk}^{m}, MD_{jk}^{m}, UD_{jk}^{m}), D_{jk}^{m} \in T, \quad \circ \leq LD_{jk}^{m} \leq MD_{jk}^{m} \leq UD_{jk}^{m} \leq 10 \quad (8)$$

$$\tilde{W}_{jk}^{m} = (LW_{jk}^{m}, MW_{jk}^{m}, UW_{jk}^{m}), W_{jk}^{m} \in S, \quad \circ \leq LW_{jk}^{m} \leq MW_{jk}^{m} \leq UW_{jk}^{m} \leq 10 \quad (9)$$

If D_{jk}, S_{jk}, O_{jk} are the values of occurrence, severity and detection of the i-th relationship and the failure mode k in the opinion of experts, it will be calculated as equations (10-12).

$$\tilde{O}_{jk} = \tilde{O}_{jk}^{1} * \tilde{W}_{e1} + \tilde{O}_{jk}^{2} * \tilde{W}_{e2} + \ldots. + \tilde{O}_{jk} * W \quad (10)$$

$$\tilde{S}_{jk} = \tilde{S}_{jk}^{1} * \tilde{W}_{e1} + \tilde{S}_{jk}^{2} * \tilde{W}_{e2} + \ldots. + \tilde{S}_{jk}^{m} * \tilde{W}_{em} \quad (11)$$

$$\tilde{D}_{jk} = \tilde{D}_{jk}^{1} * \tilde{W}_{e1} + \tilde{D}_{jk}^{2} * \tilde{W}_{e2} + \ldots. + \tilde{D}_{jk}^{m} * W_{em} \quad (12)$$

W_{jk} denote the importance assessed by the experts for the i-th relationship and the failure mode and W_{je} is the weight of the i-th expert, while m is the number of experts.

$$\tilde{W}_{jk} = \tilde{W}_{jk}^{1} * \tilde{W}_{e1} + \tilde{W}_{jk}^{2} * \tilde{W}_{e2} + \ldots. + \tilde{W}_{jk} * \tilde{W}_{em} \quad (13)$$

Step 4: Calculation of RPN score

Eq (14) shows the calculation of the risk priority number (RPN_T), which is the product of fuzzy values of occurrence, intensity and detection.

$$RPN_T = \tilde{O}_{jk} \otimes \tilde{S}_{jk} \otimes \tilde{D}_{jk} \otimes \tilde{W}_{jk} \quad (14)$$

Step 5: Calculation of the total RPN score for each of the subsystems

At this stage, the total RPN score is calculated by Eq (15) for the failure modes of each subsystem compared to the risk of the target subsystem.

$$RPN_T = \tilde{O}_{jk} \otimes \tilde{S}_{jk} \otimes \tilde{D}_{jk} \otimes \tilde{W}_{jk} \quad (15)$$

RPN_T is the total RPN score of the km subsystem and RPN_{kj} represents the RPN score of the j-th failure of the k-th subsystem. j denotes the number of failure modes in the subsystem.

Step 6: Obtaining the results

After step 5, the subsystem with the highest RPN score can be obtained. Failure modes with a higher RPN score are more important and will be prioritized for correction.

6. Research Methodology

The present study is applied in terms of objective type and the analysis of failure modes and its effects has been done in an oil filter engine of Peugeot, Pride and Toyota automobiles. In order to implement the proposed model, coding has been done in MATLAB and Lingo software.

6.1. Data analysis

The steps to perform a failure modes and effects analysis are as follows:

Step 1) at this stage, an evaluation team group was formed to evaluate the oil filter in the presence of the production manager, quality control manager and supervisor of each production line, which are 2 people. The weight of each expert is based on his/her personal characteristics and experience in this research, which has been calculated by Eq (1).

Table 5
Weight of experts W_{ei}

DM2	DM1	Experts
0.33	0.66	**weights**

Step 2) in this step, all operations, failure modes, effects and causes are formed in a table along with their weight of
importance, which are determined by fuzzy verbal variables.

6.2. DEA models for measuring the efficiency of decision units

A decision-making unit includes the units that convert data into outputs. Decision-making units are units that perform the same tasks and have common goals. Data envelopment analysis is a multi-criteria decision-makingtechnique to measure the relative efficiency scores of a set of decision-making units (DMUs) with homogenous inputs and outputs which was first introduced by Charnes et al. (1978). This method applies a linear programming optimization to separate DMUs into two efficient and inefficient categories each decision-making unit is scored using standard theory definitions to calculate performance which this score is calculated by special scales to maximize the efficiency score of the unit.

6.2.1. Input and output in data envelopment analysis

Input is a factor whose increase causes the efficiency increment while maintaining all other factors and decreases efficiency while maintaining all other factors. Output is also a factor that its increase causes the decrease in efficiency while maintaining all other factors and decreases efficiency while maintaining all other factors. Measuring the efficiency of decision-making units is possible by comparing their inputs and outputs. The data envelopment analysis model maximize the services of a unit. to achieve this goal, the data envelopment analysis method divides the linear composition of unit outputs by the linear composition of inputs of the same unit (provided that similar units provide different but similar services) to obtain the results of a scalar number without no unit between zero and one. The following model (Eq (16)) is the main model used in this research.

$$\begin{aligned} & Max \sum_{r=1}^{s} u_r \, y_{r0} \\ & s.t: \\ & \sum_{r=1}^{s} u_r \, y_{rj} - \sum_{j=1}^{m} v_i \, x_{ij} \leq 0 \qquad j = 1,2, \ldots, n \\ & u_r \geq \varepsilon \quad r = 1,2, \ldots, s \qquad (16) \end{aligned}$$

$$v_i \geq \varepsilon \qquad r = 1,2,\ldots,m$$

After solution, (v_i^*, u_r^*) are the optimal solutions of the model, then DMU_i is evaluated with these optimal weights DMU as equation (17):

$$E_{ij} = \frac{u_r^* y_{rj}}{v_i^* x_{ij}} \tag{17}$$

6.3. Optimistic efficiency

If we choose the highest value among the E_{ij} solutions, it is called the optimistic efficiency function.

$$Max\theta_0 = \sum_{r=1}^{s} u_r\, y_{ri}$$
$$s.t:$$
$$\sum_{r=1}^{s} u_r\, y_{rj} - \sum_{j=1}^{m} v_i\, x_{ij} \leq 0 \qquad j = 1,2,\ldots,n$$
$$u_r \geq \varepsilon \qquad r = 1,2,\ldots,s$$
$$v_i \geq \varepsilon \qquad r = 1,2,\ldots,m \tag{18}$$

6.4. Pessimistic efficiency

If we choose the most negligible value among the solutions, it is called the optimistic efficiency function.

$$Min\psi_0 = \sum_{r=1}^{s} u_r\, y_{r0} \qquad j = 1,2,\ldots,n$$
$$s.t:$$
$$\sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} \geq 0$$
$$= 1,2,\ldots,n$$
$$\sum_{i=1}^{m} v_i\, x_{io} = 1,$$
$$u_r \geq \varepsilon \qquad r = 1,2,\ldots,s$$
$$v_i \geq \varepsilon \qquad r = 1,2,\ldots,s \tag{19}$$

The DEA method has been selected as the main research model for evaluating efficiency and ranking. Because this method has a more realistic evaluation than other methods.

6.5. Average efficiency

Average efficiency includes not only the optimistic efficiency of the decision unit but also the pessimistic efficiency of the DMU. In fact, average efficiency measures overall efficiency by considering both modes (optimistic and pessimistic). The integration of both optimistic and pessimistic efficiencies is undoubtedly much more meaningful and comprehensive than efficiencies individually. For this purpose, Wang et al. (2009) in their research proposed the average geometric efficiency as follows:

$$\phi_J = \sqrt{\psi_J^*\ \theta_J^*}\ ,\ j = 1,\ \ldots\ ,n \tag{20}$$

Table 6
Application DEA in FMEA

Number of potential failure modes	i = 1,2,…, m
Number of risk factors of the parameters	j = 1,2,…, n
Ranking of potential failure modes on risk factors	r_{ij}
A set of risk factors	RF_j
Set of potential failure modes	FM_i
The set of the weight of risk factors	W_j
Risk set of potential failure modes	R_i

Consider N failure modes represented by FM_i = (i, … , n) that should be prioritized over risk factors. r_{ij} is the rank of FM_i on RF_j and, W_j represents the weight of the risk factors. Since RPN as a product of three risk factors S, O and D in terms of mathematical formula due to the same behavior and weights of the three risk factors have been widely criticized, the risk of failure modes is calculated by the following Eq:

$$R_i = \sum_{j=1}^{m} w_j\, r_{ij}\ ,\ i = 1,\ \ldots\ ,n \tag{21}$$

The Eq (21) shows the risk of each failure condition as the weighted sum of m risk factors. In this study, the weights of risk factors are determined by the DEA. Conventional DEA often assigns too many zeros to the input and output weights, which leads to very high optimistic efficiency or too low pessimistic efficiency. Therefore, to prevent the occurrence of this case in the FMEA, an imposing limit has been applied on the ratio of maximum weights to half-weights.

According to the Analytical Hierarchy Process (AHP) 4, the maximum value is considered as a ratio of the relative importance of one index against another, 3. Therefore, in this research, the ratio of maximum to minimum weights is in the form of Eq (22):

$$1 \leq \frac{max\{w_1,\ldots,w_m\}}{min\{w_1,\ldots,w_m\}} \leq 9 \tag{22}$$

Considering the number 3 as the maximum is for the following reasons:

- Mathematical pairwise comparison matrix in AHP to estimate the relative importance of the weights of decision options or indicators in which the maximum ratio between the relative importance of two options or indicators is usually not more than 3.
- All risk factors O, S and D have been rated from 1 to 10, while the number 1 indicates no risk, and since the absence of risk has no effect, it is better to rate from 1 to 9 instead of 1 to 10. As a result, the maximum ratio between the importance of the two risk factors is less than or equal to 4, and the equation is rewritten as follows:

$$w_j - w_k \leq 0 j, k = 1,\ \ldots\ ,m; k \neq j \tag{23}$$

Which eventually changes to the form of Equation (24):

$max\left\{\frac{w_j}{w_k}|j,k=1,\ \dots\ ,m;j\neq k\right\}\leq 9$ (24)

Now, according to the above sections, FMEA models are rewritten in the form of Eqs (25) - (26) to obtain the maximum and minimum risk:

R_0^{max}maximize$_0$

$$s.t:\begin{cases}R_i\leq 1\ ,i=1,\ \dots\ ,n\\ w_j-9w_k\leq 0, \quad j,k=1,\ \dots\ ,m;k\neq j\end{cases} \quad (25)$$

R_0^{min}minimize$_0$

$$s.t:\begin{cases}R_i\geq 1\ ,i=1,\ \dots\ ,n\\ w_j-9w_k\leq 0, \quad j,k=1,\ \dots\ ,m;k\neq j\end{cases} \quad (26)$$

The total risk as the geometric mean of the maximum risk and the minimum risk is as follows:

$$\bar{R}_i=\sqrt{R_i^{\max}.R_i^{\min}},\ \ i=1,\dots,n \quad (26)$$

Therefore, failure modes are prioritized using the geometric mean of the relevant risk, and the higher risk number leads to the higher priority. In the field of failure mode and effect analysis, determining the risk numbers related to the potential failure mode, including O, S and D, is the responsibility of experts and in fact the multifunctional team. In other words, these numbers are chosen by the decision-maker and also these numbers can include different values according to the decision of the decision-maker , so they do not have definite values. To analyze the failure modes in a stable space, we can refer to the robust optimization method which in case of uncertainty provides the best possible solution.

6.6. Robust optimization

The method of technical robust optimization is to stabilize the solution in conditions of uncertainty and obtain the most justified solution. Hence, the robust optimization methods were developed to model uncertainty. The First step was taken by Soyster (1973). He presented a column-wise uncertainty. Under this consideration model was too conservatism and release optimal solution to remain feasible. Determining the numbers O, S and D by a team of experts leads to the fact that these numbers do not have certain values and include different values according to the opinion of different decision-makers . In Equation (27), Γ represents the level of protection that can be calculated using the following equation:

$$\begin{aligned}&max\ z_0\\&s.t:\\&\sum_{i=1}^{m}v_ix_{io}=1\\&\sum_{r=1}^{s}u_ry_{ro}-z-\Gamma_o p_o-\sum_{j\in Ji}q_{ro}\geq 0 \quad (27)\\&\sum_{i=1}^{m}v_ix_{ij}-\sum_{r=1}^{s}u_ry_{rj}-\Gamma_j p_j-\sum_{j\in Ji}q_{ro}\geq 0\ , \quad j=1,\ \dots\ ,n\end{aligned}$$

$$\Gamma=1+\varphi^{-1}(1-e_i)\sqrt{n} \quad (28)$$

In Eq (27), φ represents the cumulative distribution function of the normal (Gaussian) random variable and n is the number of non-deterministic parameters of the problem and, of course for constraints whose number of non-deterministic constraints is less than 1. It is recommended to consider n, or in other words, the problem will be very conservative if we consider the complete uncertainty in the data. Determining the percentage of turbulence is a very difficult and complex matter that experts often consider a number between 5% and 10% as the percentage of turbulence. In this research, the model is formed and solved based on three numbers: e = 0.05, e = 0.01, e = 0.1.

6.7. Maximum risk mode

At this stage, if the new FMEA formula based on the DEA equation is placed in a robust optimization model, a new model is obtained for the maximum mode.

$$\begin{aligned}&max\ z\\&s.t:\\&\sum_{r=1}^{s}w_jr_{ij}-z-\Gamma_o p_o-\sum_{j\in Ji}q_{ro}\geq 0\\&-\sum_{j=1}^{m}w_ir_{ij}-\Gamma_i p_i-\sum_{j\in Ji}q_{ji}\leq -1\ , \quad i=1,\ \dots\ ,n\\&p_i+q_{ji}\geq er_{ij}z_j \quad ,\forall i,j\\&-z_j\leq w_j\leq z_j \quad ,\forall j\\&p_i,q_{ji}\geq 0\\&w_j-9w_k\leq 0 \quad ,j,k=1,\dots,m;k\neq j \quad (29)\end{aligned}$$

6.8. Minimum risk mode

Also, if the new FMEA model based on Eq (22) of DEA is included in the robust optimization model, we will have a new model for maximum risk mode:

$$\begin{aligned}&min\ z\\&s.t:\\&\sum_{r=1}^{s}w_jr_{ij}-z-\Gamma_o p_o-\sum_{j\in Ji}q_{ro}\geq 0\\&-\sum_{j=1}^{m}w_ir_{ij}-\Gamma_i p_i-\sum_{j\in Ji}q_{ji}\leq 1\ , \quad i=1,\ \dots\ ,n\\&p_i+q_{ji}\geq er_{ij}z_j \quad ,\forall i,j\\&-z_j\leq w_j\leq z_j \quad ,\forall j\\&p_i,q_{ji}\geq 0\\&w_j-9w_k\leq 0 \quad ,j,k=1,\dots,m;k\neq j \quad (30)\end{aligned}$$

7.Case Study

This section uses a numerical example of a fishing vessel to calculate RPN using fuzzy methods and robust data envelopment analysis. FMEA form is related to the design and manufacture of oil filter of one of the automobiles of Iran Khodro Automotive Company. In this study, 96 potential failure modes have been identified, which table (7) shows the risks, potential failure modes and potential causes of failure.

Table 7
Failure modes and effects, causes and weights of each risk

DM2	DM1	Operation	Potential failure modes	Potential failure effect	Potential cause of failure	Risk
I	VI	Cutting the shell sheet	Decrease of the width of the strip	Decrease and damage of the edge of the shell in the next step, loss of usable surface of the sheet, lack of proper waltz and sealing, failure of the shell	Failure to adjust the device and improper welding by the supplier	R1
I	I		Deformation and tears, folding and tingling		Failure to attach the sheet to the guillotine STOP and not to sharpen the guillotine blades, operator error Lack of mold adjustment, improper cutting	R2
I	VI	Stretching 1 shell	Tears, deformity, scratches, edge loss of the workpiece, tears of the edges	Improper appearance, part loss, failure to assemble and seal the filter, rupture, leakage	Improper operation of the operator in placing the part on the mold and the high press load of the machine, Lack of	R3
I	VI	Stretching 2 shells P15	Decrease in inner diameter	Lack of placement of the thick door and lack of waltzing on the shell	Umbilical cord amortization	R4
I	I		Decrease the height of the shell	Improper assembly in the next step	Heavy machine load, mold unbalance (mud)	R5
I	MI		High shell height	Improper assembly in the next step	Low device load	R6
I	VI		wrinkles, scratches, deformities and tears	Improper appearance and loss of the part, which causes it not to be transferred to the next station, do not seal during the assembly stage.	Excessive load of the press machine, improper performance of the operator, the presence of additional chips on the mold and workpiece and improper placement of the workpiece on the mold	R7
I	VI	edge loss of P20	Low edge diameter, low edge height and edge loss	Lack of filter sealing	Lack of adjustment of the press load, improper position of the mold and improper operation of the operator	R8
I	I		Large edge diameter and high edge height		Inadequate stretching	R9
I	MI		Existence of pleats on the edge and lack of proper edge slope	Lack of filter sealing and improper appearance	Blunt mold	R10
I	I	Degreasing P25	The presence of fat, waste and soot on the shell	Paint instability on the shell	Uncontrolled temperature and time, stopping and moving the part in the furnace	R11
I	I	P30 shell painting	Dye fading	Improper appearance	Lack of adjustment of the paint spraying due to non-adjustment of the pressure of the wind and spray nozzle and improper combination of paint and solvent	R12
I	I		Lack of full-color coverage	Improper appearance and improper performance against moisture	Non-adjustment of air pressure, lack of adjustment of the movement course, the presence of oil droplets in the windpipesand not clean surface of the workpiece, lack of adjustment of the dilution of the paint	R13

MI	I		Scratches	Improper appearance and improper performance against moisture	Improper operator performance in cleaning the surface of the shell before painting, and not placing cardboard on pallets before painting	R14
I	MI		hooked	Improper appearance	hook contact with the freshly painted part while removing part due to improper operator performance	R15
I	I		Rust and dirt on the surface of the shell	Improper performance against moisture, poor appearance and lack of full paint coverage	Improper performance of the operator in cleaning the surface of the shell before painting, flossing on the paint oven due to vibration on the surface of the workpiece	R16
U	MI		Paleness due to the connection of 2 pieces	Improper appearance and causes corrosion, rust and perforation of the filter at the site of paleness in a humid environment	Contact of the two painted pieces while placing on the conveyor by the operator	R17
VI	VI	Dye boiling of shell P35	Raw color, paint burn and flaking, blurring and deterioration of appearance	Improper appearance and improper performance against moisture	Lack of adjustment of the boiling temperature and time and failure to adjust the movement period of the conveyor	R18
I	VI	Folding P40 fiber	Existence of pleats, tears and crumbs	Lack of proper filtration and assembly of head and bottom doors in the next stage and serious damage to the engine	Lack of adjustment of left and right blades and perforation, being blunt	R19
I	I		Decreasing the thickness of the wrinkle	Reduction of the filtration area and assembly area in the next stage and improper oil purification	No adjustment of the distance between the left and right blades	R20
I	I		Increasing the thickness of the wrinkle	Increased filtration area and lack of proper oil circulation in the filter		R21
I	MI		Decreased fiber width	Improper assembly in the next step, lack of complete filtration due to reduced fiber filtration surface	Tilt of the fiber under the perforated blades of the machine	R22
I	VI		Increased fiber width	Improper assembly in the next step, excessive increase of the filtration surface which leads to non-rotation of the oil and its complete filtration	Increased size of the perforation blade distance	R23
I	VI		Inappropriate wrinkle (short/long)	Improper filtration surface/lack of proper assembly in the next step	Non-adjustment of the left and right blades of the folding machine	R24
I	VI		Fiber burn	Decreased filter quality, improper appearance, rupture and perforation	Non-adjustment of the temperature of the ironing elements and its temperature	R25
I	I		Not crushing the fiber (not separating the fiber properly)	Rupture and perforation and creation of pleats and lack of proper assembly in the next stage	Non-adjustment and bluntness of the blades of the perforation machine	R26

I	VI	Folded fiber cut P45	Improper cutting of wrinkle	Inadequate filtration surface and lack of proper assembly in the next step due to not being on the head and bottom and on the fiber, improper filtration and loss of fiber surface (usable) fiber	Due to the bluntness of the cutter	R27
I	I		Increasing number of wrinkle	Filling the filtration surface and not properly removing oil from the treatment site	The Exhaust of counter ink, improper performance of the operator in considering the location of the cutting mark	R28
I	I		The Declining number of wrinkle	Decreased filtration surface and lack of proper oil purification and improper assembly	Exhaust of the counter ink and non-adjustment of the counter of the machine Improper performance of the operator according to the location of the cutting mark	R29
I	I	Spiral pipe P 55	No drilling on a pipe	Lack of proper oil filtration	Non-adjustment of the forming roller and shortening the edges and teeth of the punching roller	R30
I	VI		Lack of proper waltz on the edges of the strip	Loss of the piece	Failure to adjust the waltz roller	R31
I	VI		Increasing diameter	No assembly in the next step	Non-adjustment of the waltzing roller and poor material of the sheet	R32
I	VI		Decrease in height	Loosening the spiral tube in the next step	Blunt cutting blades and non-adjustment of blades and electronic eyes	R33
I	VI		Increasing the height	Improper assembly of the head on bottom and complete non-adhesion of fiber to the series and bottom and lack of complete oil purification	Non-adjustment of electronic eye	R34
I	I		Lack of proper cut and pleated and protruding edges and loss of part of the spiral tube	Lack of proper assembly on the head and bottom	Failure to adjust the cutting edge distance of the cutting rollers and tubular shaft and failure to adjust the electronic eye	R35
MI	MI	Rolling the folded fiber and sealing with P50 glue	Improper tears and roll of fiber, improper direction of paper	Improper filtration and lack of proper oil purification	Improper operator performance in separating, rolling and connecting the beginning and end of the fiber to each other	R36
I	I		Lack of proper sealing	Improper filtration and lack of proper purification of	Improper performance of the operator in gluing the fiber	R37
I	I		Opening of the fiber adhesive	Lack of assembly in the next stage and improper filtration and lack of proper oil purification	Improper combination of glue and throwing fiber while throwing into the basket, which causes the fiber to open from the joint	R38

I	MI	Assembly of fiber and pipe P60	Irregular arrangement during the gathering the fiber around the spiral tube and the improper placement of the spiral tube in the fiber	Fiber opening or lack of complete placement of the fiber in the head and lack of proper assembly of the part in the next step, improper filtration by the fiber and lack of sealing of internal elements	Improper operator performance in spiral pipe assembly in fiber and disordered fiber arrangement in winding it around spiral pipe	R39
I	I	operation 1 of the head of element P65	Rupture and excessive wrinkling and deformity and edge loss	Rupture and loss of the part and lack of assembly in the next step	Excessive machine press load	R40
I	I	operation 2 P70 element head-operation 2 P70 element head	Shortening the edge	Lack of proper gluing on the piece (glue shergy) and lack of complete strength of fiber and pipe inside the head	Improper placement of the sheet under the mold	R41
I	VI		Increase and decrease in the diameter of the middle hole	Lack of proper assembly of thick doors in the next steps	Increasing and decreasing the load of the machine press	R42
I	I		Shortening the edge	Lack of proper gluing on the piece (shergy glue) and lack of complete strength of fiber and pipe inside the head	Improper placement of the sheet under the mold	R43
I	I		Low height of the neck	Not sticking the fiber on the head and leakage of the glue out of the head	Low machine press load	R44
I	MI	Gluing inside the P75 head	Non-uniformity of adhesive on the series	Lack of complete adhesion of fiber in all different areas on the element head and its lack of sealing and improper appearance	Lack of adjustment of injection pressure and time, improper performance of the operator gluing in the head, improper performance of the glue injection nozzle, lack of proper combination of composite materials for the preparation of adhesives	R45
I	MI		Low amount of adhesive	Complete non-adhesion of fiber on the element head and lack of sealing of internal elements	Low air pressure of the machine, improper performance of the operator in gluing inside the head and improper performance of the injection nozzle and head	R46
I	I		Low furnace temperature	Unbaked fiber	Lack of proper adjustment of the furnace temperature	R47
MI	VI		Too much adhesive	Improper appearance and sticking of several folds of fiber on the surface of the roll to each other	High air pressure of the device, improper operation of the glue injection nozzle in the head and improper performance of the operator	R49
I	VI	Putting P80 pipe fiber on the head	Improper placement on the head and deformation of the fiber	Lack of ball sealing	Improper operator performance in proper placement of tubular fiber on the head	R51
VI	VI	Boiling step 1 of P85 element	Fiber baking (rupture)	Rupture and perforation of the fiber due to burns and lack of proper oil filtration	Increased fiber baking time in the machine	R52

I	VI	head	Improper appearance and discoloration of the fiber	Increasing the fiber baking temperature in the machine		R53
I	VI		Raw adhesive	Lack of proper sealing of internal elements	Low temperature of the device	R54
I	VI	Inserting washers and springs and bending the forks of the P115 valve plug	Lack of inserting the gasket	Improper safety valve malfunction	Improper performance of the operator in not placing the gasket	R55
I	I		Improper bending of the tentacles on the spring	Improper safety valve malfunction	The device and tilting the two bottom valves on the device mold	R56
I	VI	Cutting of P135 ring sheet	Decrease the width of the strip	Improper waltz due to the low edge of the ring and the production of waste in the later stages of assembly and loss of the usable surface of the sheet	The sharpness of the guillotine blades	R57
VI	VI	Bending and cutting operation 1 of P140 ring	The low or high outer diameter of the	Improper waltz and lack of sealing	Increasing and decreasing the load of the machine press	R58
VI	VI		High diameter central hole	Enlargement of the hole in the ring operation 2 stage and improper assembly of the thick door and improper appearance	Increasing the press load of the machine and creating tension on the surface of the workpiece	R60
VI	VI		Deformity, tears, wrinkling and edge chipping of the piece	Improper waltz and lack of sealing	Improper performance of the operator in placing the part under the mold, the presence of chips and wastes on the mold and the surface of the workpiece, smooth and polished and not sharp edge of the mold	R61
VI	VI	Bending and cutting operation 2ring 141	Tears, wrinkling, deformity, scratches, and edge chipping	Lack of filter sealing, improper assembly of improper waltz gasket and oil leakage	Improper performance of the operator in placing the workpiece on the mold, high press load of the machine and the presence of pleats on the mold	R62
VI	VI		Reducing the width of the gasket hole	Lack of filter sealing and oil leakage	Improper assembly of surrounding gasket	R63
VI	VI		Increasing the width of the gasket hole		Mold depreciation	R64
VI	I		Reducing the depth of the gasket hole		Low machine press load	R65
I	I		Increasing the depth of the gasket hole		High machine press load	R66
I	MI		The large diameter of the hole in the middle of the ring	Enlargement of the hole and improper appearance and lack of sealing	High machine press load	R67

I	VI	Strip cutting of thick door sheet P145	Decreased width of the strip	Loss of usable surface of the sheet and lack of proper performance in later stages	Lack of proper adjustment of the device, non-compliance with the size specified on the desired product	R68
I	VI	Cutting of operation 1 thick door P150	Edge chipped of the piece	Lack of proper assembly at the boiling point stage	Tilt the piece under the mold	R69
I	VI		Increased and decreased diameter	Lack of proper placement and proper waltzing in later stages	Operator fault in improper placement of the sheet on the mold	R70
I	MI		Increase and decrease in the length of the square	Improper tip placement, improper welding in the next step	Improper placement of the sheet on the mold by the operator	R71
I	I		Existence of pleats on the cut edge	Lack of proper tapping in the tap stage and improper appearance	Blunting of the cutting edges of the mold	R72
I	VI	Thick door hole drilling P155	Lack of complete and proper drilling at this stage	Loss of the piece at this stage	Low machine press load	R73
I	VI		Lack of eccentricity of hole in the middle and around the piece	Inability to knock and damage the piece	Improper operator performance in tilting the workpiece on the machine mold	R74
I	VI	Forming P160	Rupture and deformity	Lack of assembly on the ring at the boiling point stage and finally oil leakage from the filter	Tilting the workpiece on the mold by the operator, low and high press load of the machine	R75
I	VI		Decreased part of the umbilical cord height	Lack of proper tapping in the next step	Excessive device press load and unsuitable condition of the device mold, mandrel depreciation and matrix	R76
I	VI		Increased diameter of the middle hole	Lack of proper tapping in the next step, having pleat, hardness and bluntness of the piece gears	Excessive load non-adjustment and improper performance of the operator in placing the workpiece on the mold of the machine	R77
U	VI		Inadequate neck	Lack of proper welding in the next process, pleated, hardness and slowness of the ribs of the piece	Non-adjustment of the load and improper performance of the operator in placing the workpiece on the mold of the machine	R78
I	VI	Tapping the thick door P 165	Swing and cramp during tapping, tilting and the presence of pleats	Creating eccentricity in the piece and its tilting and loosening of the tap or its tightening	Sticking the pleat in the tapping machine, tilting the tap to the shaft and the material of the thick door and the tap saucer in the shaft	R79
I	VI		Lack of complete parts of the piece	Loss of the part and not entering the gauge, not going and not mounting the part on the motor base	Lack of proper binocular adjustment and no adjustment of round trip speed	R80
VI	VI	Boiling point P170	Lack of weld strength	Detachment of thick door from ring and lack of sealing and improper waltz and oil leakage from filter	Improper operator performance	R81
I	I		Perforation of the weld	Lack of sealing and oil leakage from the holes	Power fluctuations	R82

I	VI	Boiling point P175	Rupture and deformity	Loss of the piece at this stage	Tilt under the mold, high press and machine load	R83
I	VI		Breaking the weld	Oil leakage	High machine press load	R84
I	I		No complete press	Eccentricity and oil leakage from the filter	Low machine press load	R85
VI	MI	Waltzing the shell and thick door P185- Waltzing shell and thick door P185	Scratches, rupture of the waltz	Leak at the edge of the filter waltz	Failure to adjust the machine pulleys and improper performance of the operator in placing the part on the waltz machine and large diameter of the thick door	R86
I	I		Scratches, rupture of the waltz	Improper appearance	Lack of adjustment of the device spools and large diameter of the thick door	R87
I	I		High and low waltz diameter	Loosening of the gasket and the filter and oil leak from the gasket	Non-adjustment of the pulleys of the device, high jack pressure	R88
VI	I	Insertion of sealing gasket	Lack of proper assembly	Leakage in the next step	Improper operator performance	R89
I	U	Dryer 245	Clean and dry filter surface, no oil	Lack of proper printing of brand and serial and improper and dirty appearance	Improper operator performance in cleaning and drying the surface of the part and damage to the blower fan	R90
VI	U	Brand printing P195	Lack of complete printing of the mark and illegibility of the mark	Lack of complete and legible printing and finally the impossibility of identifying and tracking the filter by the customer	Drying of paint on the stencil or nozzle (machine spatula), improper performance of the operator	R91
VI	U	Serial printing P200	Lack of full printing of the serial and illegibility of the serial	Lack of complete printing and the possibility of tracking and identifying customer dissatisfaction	No nozzle adjustment and speed of conveyor movement	R92
I	I	Shearing P205	Improper shearing	Improper appearance	Improper operator performance and misplacement	R93
I	U	Inserting into box P210	Tearing the box	Improper appearance	Improper operator performance by placing the filter inside the box	R94
MI	U	Packing P 215	Incomplete and improper placement inside the box	Improper appearance	Improper operator performance	R95
I	U	Vacuum P220	Improper vacuum	Improper appearance, and the possibility of damage to the box and the filters inside	Lack of adjustment of temperature and time of the device	R96

Step 3) The RPND values are calculated for all failure modes according to the relationships, after calculating the average opinions of experts for the weight of the importance of each failure mode.
Step 4) finally, the total RPNK score was calculated for each potential failure condition, and the total RPNK value can be obtained for each operation according to the relationship.
Step 5) Fuzzy and normal RPN values are converted to deterministic numbers using the center region finiteness method. After calculations, 15 potential failure modes were identified and ranked, as shown in the results table.

7.1.esults by FMEA on oil filter

As the results in the Table 8 show, the traditional RPN method cannot take into account the relative importance of the following three factors. Thus, it is not possible to separate the 15, 10, 27, and 38, 66 and 41, 43, 16 and 48, 73 and 83,53 failure modes from each other.
Table (8) clearly shows that several failure modes such as failure modes R_{15}, R_{10}, R_{27} and failure modes, R_{66}, R_{38} are not separated from each other. As a results, fuzzy RPN and traditional RPN cannot distinguish failure modes well. Consequently, it cannot separate failure modes R_{27}, R_{38} and failure modes R_{41}, R_{43} and R_{16} from each other.

Table 8
Average expert opinions for ODS of each failure mode and traditional RPN values for each failure mode

RPN	**S**	**D**	**O**	**Failure Model**	**Rank**
288	8	6	6	R_{76}	1
160	4	8	5	R_{33}	2
135	4	5	7	R_{15}	3
135	5	4	7	R_{10}	3
135	4	7	5	R_{27}	3
108	9	4	3	R_{38}	6
108	9	3	4	R_{66}	6
100	5	4	5	R_{41}	8
100	5	5	4	R_{43}	8
100	4	5	5	R_{16}	8
96	3	4	8	R_{48}	11
96	4	3	8	R_{73}	11
84	4	3	7	R_{67}	13
72	3	4	6	R_{83}	14
72	4	3	6	R_{53}	15

Table 9
Average expert opinions for ODS and weight of importance of each failure mode and fuzzy and deterministic RPN values for each failure mode

Rank	**Failure mode**	O	D	S	W	RPN_K	RPN_D
1	$\mathbf{R_{76}}$	(4.29,5,77,7.26)	(5.11,6.6,8.08)	(5.11,6.60,8.08)	(0.72,0.87,0.95)	(80.65,218.60,450.28)	249.66
2	$\mathbf{R_{33}}$	(3.46,4.95,6.43)	(6.76,8.25,9.24)	(2.64,4.12,5.16)	(0.72,0.87,0.95)	(44.45,146.37,316.64)	169.15
3	$\mathbf{R_{15}}$	(5.11,6.60,8.08)	(3.46,3.95,6.43)	(2.67,4.12,6.43)	(0.72,0.88,0.95)	(33.98,118.44,254.68)	135.73
4	$\mathbf{R_{10}}$	(4.29,5.77,7.26)	(2.66,4.14,5.58)	(3.42,4.93,6.48)	(0.6,0.75,0.9)	(28.21,102.37,258.74)	126.42
5	$\mathbf{R_{27}}$	2.64,4.12,5.61)(	(6.76,8.25,9.24)	(1.81,3.36,4.78)	(0.6,0.75,0.9)	(23.25,97.58,235,38)	118.73
5	$\mathbf{R_{38}}$	(6.76,8.25,9.24)	(2.64,4.12,5.61)	(1.81,3.36,4.78)	(0.6,0.75,0.9)	(23.25,97.58,235,38)	118.73
7	$\mathbf{R_{66}}$	(2.64,4.12,5.61)	(4.18,5.82,7.31)	(2.64,4.12,5.61)	(0.85,1,1)	(25.41,97.94,228.41)	117.25
8	$\mathbf{R_{41}}$	(4.29,5.77,7.26)	(2.64,4.12,5.61)	(3.46,4.95,6.43)	(0.6,0.75,0.9)	(23.51,88.25,235,69)	115.81
8	$\mathbf{R_{43}}$	(3.46,4.95,6.43)	(4.29,5.77,7.26)	(2.64,4.12,5.61)	(0.6,0.75,0.9)	(23.51,88.25,235,69)	115.81
8	$\mathbf{R_{16}}$	(3.49,4.92,6.43)	(4.29,5.77,7.26)	(2.64,4.12,5.61)	(0.6,0.75,0.9)	(23.51,88.25,235,69)	115.81
11	$\mathbf{R_{48}}$	(4.29,5.77,7,26)	(0.99,2.39,3.93)	(5.11,6.55,7.92)	(0.85,1,1)	(19.44,90.32,225.09)	111.28
12	$\mathbf{R_{73}}$	(6.76,8.25,9.24)	(2.64,4.12,5.61)	(1.81,3.32,4.78)	(0.6,0.75,0.825)	(19.43,84.22,223)	108.88
13	$\mathbf{R_{67}}$	(5.11,6.60,8.08)	(2.64,4.12,5.61)	(1.81,3.30,4.78)	(0.72,0.87,0.95)	(17.58,78.06,205.83)	100.49
14	$\mathbf{R_{83}}$	(5.95,7.32,8.86)	(2.64,4.12,5.61)	(1.81,3.30.4.78)	(0.6,0.75,0.825)	(17.10,74.64,196.01)	95.91
15	$\mathbf{R_{53}}$	(4.29,5.77,7.26)	(2.64,4.12,5.61)	(2.64,4.12,5.61)	(0.6,0.75,0.9)	(17.93,73.63,205.63)	93.93

Also, the ranking of operations and production processes of oil filter are shown in Table (9), where the values of 4 operations with the highest RPNT have been shown. Table (10) shows the results obtained from solving FMEA models by data envelopment analysis using GAMZ software.

Table 10
FMEA using DEA

Potential Failure Mode	Maximum Risk	Minimum Risk	Average Risk	Rank
R_{76}	1	3.16	1.78	1
R_{33}	0.93	2.32	1.47	2
R_{15}	0.94	2.16	1.43	3
R_{10}	0.90	1.96	1.33	4
R_{38}	0.93	1.85	1.31	5
R_{27}	0.8	2.05	1.28	6
R_{66}	0.90	1.76	1.26	7
R_{41}	0.83	1.74	1.20	8
R_{43}	0.87	1.60	1.18	9
R_{16}	0.78	1.60	1.12	10
R_{48}	0.94	1.23	1.08	11
R_{73}	0.84	1.12	0.97	12
R_{67}	0.80	1.08	0.93	13
R_{83}	0.75	1.03	0.88	14
R_{53}	0.71	1.01	0.83	15

The results obtained from Table (11) are as follows:
Failure modes R_{38} and R_{27} have been successfully distinguished from each other and the Failure modes R_{41}, R_{43} and R_{16} have also been successfully distinguished.
the relative importance of the three risk factors and the difference between the average geometric risk with their RPNs shows the difference between the proposed FMEA model and the fuzzy FMEA. Accurate differentiation and ranking between R_{38} and R_{27} failure modes and R_{41}, R_{43} and R_{16} show the new FMEA capabilities and its advantages over the traditional FMEA. The ranking is the most pessimistic mode after solving the model in robust optimization space. The obtained solutions are the most justified solutions is the uncertainty mode. Assuming that the opinions of the expert team are never deterministic, the best solutions for this mode are obtained by the mentioned method. According to the results obtained, all failure modes had identical risk priority number despite different values for severity, occurrence and detection and, consequently it had been possible to rank them and completely and, they have been separated through the RODEA technique and could have a separate ranking. For example, failure modes R_{41}, R_{43}, and R_{16} had the same rank and can be distinguished using the new method. As observed, the risk increases by considering the disturbances in the risk of failure modes. In FMEA, we calculate the risk that the higher risk causes higher damage and therefore faster investigation is needed. As observed, the fuzzy method performs better than the traditional method and also the RODEA method provides more accurate results than the fuzzy method.

Table 11
FMEA prioritization by using RODEA

i	Maximum Risk		Minimum Risk		Average Risk	Rank	Average Risk	Rank
	0.05e	0.1e	0.05e	0.1e	0.05e	0.05e	0.1e	0.1e
R_{76}	1	1	1	1.05	1	1	1	1
R_{33}	0.85	0.93	0.9	0.97	0.91	2	0.94	2
R_{15}	0.89	0.91	0.92	0.92	0.87	3	0.85	3
R_{10}	0.92	0.92	0.53	0.80	0.82	4	0.83	4
R_{38}	0.9	0.9	0.82	0.84	0.82	4	0.81	5
R_{27}	0.90	0.94	0.75	0.73	0.8	6	0.77	6
R_{66}	0.91	0.91	0.52	0.78	0.79	7	0.65	7
R_{41}	0.83	0.86	0.74	0.75	0.74	8	0.62	8
R_{43}	0.79	0.80	0.76	0.78	0.70	9	0.58	9
R_{16}	0.81	0.84	0.69	0.69	0.68	10	0.55	10
R_{48}	0.75	0.80	0.67	0.65	0.66	11	0.52	11
R_{73}	0.77	0.80	0.72	0.70	0.63	12	0.49	12
R_{67}	0.75	0.83	0.67	0.65	0.61	13	0.47	13
R_{83}	0.70	0.70	0.57	0.60	0.57	14	0.44	14
R_{53}	0.66	0,.67	0.51	0.55	0.52	15	0.41	15

After calculating the RPNs, respectively, the operations related to the spiral pipe (no hole drilling on the pipe, no proper waltz on the edges of the strip, increase in diameter, decrease in height, increase in height, lack of proper cut and pleat and edge protrusion and loss of part of the spiral pipe), fiber folding (presence of pleats, rupture and crushing, increasing and decreasing the thickness of the fold, increasing and decreasing the width of the fiber, improper folding, burning and not crushing the fiber), bending and cutting operation 2 of the ring (Rupture, wrinkling, deformity, scratches, edge loss, decreased and increased gasket driver diameter, decreased and increased gasket driver depth and high diameter of the middle hole of the ring) and painting the shell (Shergy and lack of complete coverage of paint, scratches, hook, stains and dirt on the surface of the shell and paleness due to the connection of 2 pieces are the most important operations and failure modes of their subset that corrective measures should be taken to eliminate them. Corrective measures include placing a roll on the fiber in the perforator to prevent the fiber width from increasing or decreasing, placing the temperature control loop and placing the temperature controller on the machine to prevent the fiber from burning, and placing the retaining screw beside the device eye in order not to prevent the movement on the moving rail and change the height of the spiral tube, determine the multiplication to sharpen the spiral tube blades, provide pressure gauge to prevent the depth of the gasket from increasing and decreasing, determining determine the mold multiplication for maintenance and repairs to correct the mandrel and scorification of the mold to prevent the increase of the width of the gasket is one of the measures that can be effective in reducing the RPN of high risk operations. Sloping while tapping, tilting and the presence of pleats, which is a subset of thick door tapping operations, has the highest RPN among failure modes. Because the pleat stays in the tapping device, the tap are tilted to the shaft, the raw material of the door is thick and the tap saucer is created in the shaft, which can be reduced by purchasing a flare to detect the defect in the RPN station. Decreasing the height of the spiral pipe, which causes the spiral pipe to loosen in the next step, ranked second in the RPN, which is due to the blunt cutting blades and the misalignment of the blades and the electronic eye in the spiral pipe. The presence of pleats on edge and its lack of proper slope, improper cutting of the fold and opening of the fiber adhesive are other failure modes with the highest RPN.

8. Conclusion

Oil filter along with engine oil plays an essential role in automobile engine health and the use of standard oils and oil filters reduces depreciation and increases engine efficiency. Clean and healthy air and oil filters reduce fuel consumption, improve engine performance and consequently reduce environmental pollution. Non-standard filters also cause premature erosion and engine failure. In this paper, a combined model of FMEA and DEA is presented that does not have problems in calculating the traditional RPN and its fuzzy model. By defining potential failure modes as the weighted total or weighted product of risk factors, DEA models are defined to measure the maximum and minimum risk of potential failure modes. Their geometric mean calculates the total risk of each failure mode and is therefore used to prioritize failure conditions. Considering that FMEA can include estimating ambiguous and incomplete information, robust optimization models have been used to eliminate this ambiguity. Also in this research, RODEA models have been developed for FMEA. The proposed FMEA model is proved by a numerical example and proves to be more practical and effective than the traditional RPN. Compared to its traditional RPN of its progress, the proposed FMEA has the following features:

The relative importance of risk factor weights is defined by DEA models by adding a weight range to the maximum weight to minimum weight ratio to avoid the relative importance of each higher or lower hand risk factor.

Risk factors are condensed in some cases, which is different from RPN. Risk factors are summed by a simple product and lead to a significant critique.

Potential failure modes are easily identified and fully rated and ranked. If there is vague and incomplete estimation of information in the problem, they are easily identified and examined.

In all industries, there are always risks that threaten the interests of the organization and its stakeholders. Because the automotive industry is one of the significant industries of the country, the risks of this supply chain are given special attention. The analysis of automobile oil filter failure modes has been done using fuzzy theory to prioritize them and take corrective measures to fix it. Risk assessment in FMEA is often influenced by uncertainty in real cases, in such a situation, fuzzy set theory and RODEA model has been proposed to deal with this type of problem. There are many quality factors in assessing the reliability of automotive equipment for this reason, fuzzy expert judgments are a more effective method. In this study, by forming a subset of the quality section of 96 different failure modes, its effects and causes were determined and explained. This classification was done in two forms of ranking based on failure modes and ranking based on oil filter manufacturing operations. In the end, it was found that the sloping of the tapping, the tilt and the presence of the pleat were identified as the most important state of failure and potential risk and spiral pipe and fiber folding operations have the highest RPN that, the corrective measures are needed to be at priority.

References

Ambekar, S.B. Edlabadkar A. & Shrouty, V. (2013), A Review: Implementation of Failure Mode and Effect Analysis, *International Journal of Engineering and Innovative Technology (IJEIT,)* 2 (8), February, 38

Arash Geramian, Ajith Abraham & Mojtaba Ahmadi Nozari (2018). Fuzzy logic-based FMEA robust design: a quantitative approach for robustness against groupthink in group/team decision-making, *International Journal of Production Research,*

Baghery, M., Yousefi, S., Rezaee, M.J., (2016). Risk measurement and prioritization of auto parts manufacturing processes based on process failure analysis, interval data envelopment analysis and grey relational analysis. *J. Intell. Manuf.*

Boral, S., Howard, I., Chaturvedi, S., McKee, K., Naikan, V.,(2020). A novel hybrid multi-criteria group decision making approach for failure mode and effect analysis: An essential requirement for sustainable manufacturing., Sustainable Production and Consumption., 21, 14–32

Chang, D.S., Chung, J.H., (2012). Risk evaluation of group package tour service failures that result in third-Party complaints. *J. Travel Tourism Mark*. 29 (8), 817–834.

Chang, D.S., Paul Sun, K.L., (2009). Applying DEA to enhance assessment capability of FMEA. *Int. J. Qual. Reliab. Manage*. 26 (6), 629–643.

Charnes, A., Cooper, W.W., Rhodes, E., (1978). Measuring the efficiency of decision making units. *Eur. J. Oper. Res. 2* (6), 429–444.

Chin, K.S., Wang, Y.M., Poon, G.K.K., Yang, J.B. (2009). Failure mode and effects analysis by data envelopment analysis. *Decis. Support Syst*. 48 (1), 246–256.

Dedimas, T., Gebeyehu, S., (2019), Application of Failure Mode Effect Analysis (FMEA) for Efficient and Cost-effective Manufacturing: A Case Study at Bahir Dar Textile Share Company, Ethiopia, Journal of Optimization in Industrial Engineering. 12, 23-29.

Ho, C-H. & C-J. Liao (2011). "The use of failure mode and effects analysis to construct an effective disposal and prevention mechanism for infectious hospital waste". *Waste Management*, 31(12), 2631-2637.

Interval type-2 fuzzy evidential reasoning method, *Applied Soft Computing Journal*. 89, 106-134.

jahangoshai Rezaee, M., Yousefi, S., Eshkevari, M. et al. (2020). Risk analysis of health, safety and environment in chemical industry integrating linguistic FMEA, fuzzy inference system and fuzzy DEA. *Stoch Environ Res Risk Assess* 34, 201–218

Li, H., Diaz, H., Soares, C., (2021). A failure analysis of floating offshore wind turbines using AHP-FMEA methodology, *Ocean Engineering*. 234, 109-126

Liu, H., Xin You, J., Yue You., Shan, M. (2014). A novel approach for failure mode and effects analysis using combination weighting and fuzzy VIKOR method ,

Liu, H-C.; Liu, L. Bian, Q-H. Lin, Q-L. Dong, N. & P-H. Xu (2011). "Failure mode and effects analysis using fuzzy evidential reasoning approach and grey theory". *Expert Systems with Applications*, 38(4), 4403-4415.

Lo, HW., Shiue, W., Liou, J.J.H. et al.(2020). A hybrid MCDM-based FMEA model for identification of critical failure modes in manufacturing. *Soft Comput* 24, 15733–15745.

Mhetre R.S., Dhake, R.J. (2012). Using failure mode effect analysis in precision sheet metal parts manufacturing company, *International Journal of Applied Sciences and Engineering Research*, 1(2), 302-311.

Nazeri, A., Naderikia, R.(2017) A new fuzzy approach to identify the critical risk factors in maintenance management. *Int J Adv Manuf Technol* 92, 3749–3783.

Qin, J., Xi, Y., Pedrycz, W., (2020). Failure mode and effects analysis (FMEA) for risk assessment based on

Safari, H., Faraji, Z. & Majidian, S.(2016). Identifying and evaluating enterprise architecture risks using FMEA and fuzzy VIKOR. J Intell Manuf 27, 475–486 .

Shaghaghia, M., Rezaie, K., (2012). Failure Mode and Effects Analysis Using Generalized Mixture Operators, *Journal of Optimization in Industrial Engineering*. 5, 1-10.

Soyster, A.L.,(1973). Technical note—convex programming with set-inclusive constraints and applications to inexact linear programming. *Operat. Res*. 21 (5), 1154–1157.

Tu PY, Yam R, Tse P, Sun AO (2001). An integrated maintenance management system for an advanced manufacturing company. *Int J Adv Manuf Technol* 17(9):692–703.

Vinodh, S., Santhosh, D. (2011). Application of FMEA to an automotive leaf spring manufacturing organization, *TQM Journal*, Vol. 24, No. 3, 260-274

Wan, C., Yan, X., Zhang, D., Qu, Z., Yang, Z., (2019). An advanced fuzzy Bayesian-based FMEA approach for assessing maritime supply chain risks., *Transportation Research Part E*., 125, 222-240.

Wang L, Yan F, Wang F, Li Z, (2020). FMEA-CM based quantitative risk assessment for process industries—A case study of coal-to-methanol plant in China, *Process Safety and Environmental Protection* .

Wang, W., Liu, X., Chen, X., Qin, Y., (2019). Risk assessment based on hybrid FMEA framework by considering decision-maker r's psychological behavior character, *Computers & Industrial Engineering*. 136, 516-527

Yang, S., Bian, C., Li, X. et al.(2018). Optimized fault diagnosis based on FMEA-style CBR and BN for embedded software system. *Int J Adv Manuf Technol*, 94, 3441–3453.

Yazdi, M.(2019). Improving failure mode and effect analysis (FMEA) with consideration of uncertainty handling as an interactive approach. *Int J Interact Des Manuf* 13, 441–458.

Yousefi, S., Alizadeh, A., Hayati, J., Baghery, M., (2018). HSE risk prioritization using robust DEA-FMEA approach with undesirable outputs: A study of automotive parts industry in Iran. *Safety Science*. 102, 144-158.

Zadeh Lotfi A. (1965). "Fuzzy sets", *Information and* control, 8 (3), pp. 338-353.

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