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Research Article

Optimizing Inventory Management Costs in Supply Chains by Determining Safety Stock Placement

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Abstract

For a production business, the measurement of safety inventory to be maintained during each step of a supply chain is a key concern and requires providing the clients an irregular state of management. The stock held should be small to reduce holding costs and capacity while maintaining the capacity to service customers in time to satisfy much, if not all, the demand. This paper discusses this issue by using a deterministic time structure and provides a measure of the security position of stock in supply chains for the overall network. First, prove that the overall problem is NP-hard. Then set up a couple of parameters that characterize an optimal overall network structure. To take care of these problems, a polynomial approximation is considered. An arrangement of computational tests to survey the execution of the general-network calculation and to decide how to set different parameters for the calculation is selected. In addition to the general network case, the two-layer network issues are considered. Also, a nonlinear model for determining the level of safety stock in different components of the supply chain to minimize the related safety stock costs is developed.

Keywords: Supply chains; Network problems; Safety stock; Optimization algorithms.

1. Introduction

Intense competition in global markets, the introduction of products with short life cycles, and the ever-changing expectations of customers have led companies to spend a lot of energy and cost on the supply chain and its management. In a supply chain, raw materials provided by suppliers and products are produced in one or more factory, and are transferred to warehouses and from there to retailers or end customers will be offered. The supply chain (SC) Consists of all parts that are directly or indirectly involved in the completion of customer orders.

Supply chain management (SCM) is a set of approaches that are used to integrate suppliers, manufacturers, warehouses, and shop effectively. These approaches help to needed products released to customers at the needed time and place and optimal performance. Inventory management has always been one of the pillars of the SCM (Sahebjamnia, Goodarzian, & Hajiaghaei-Keshteli, 2020). Inventory management in SC is usually faced with uncertainty. Uncertainty in demand, lead times, and production capacity are among the issues that are facing managers.

Inventory throughout the SC is kept in different forms and for different reasons. Ballou estimates that these inventories could have 20 to 40 percent of the costs annually (Ballou, 2007). Although the inventory to increase customer service and reduce the cost of distribution is essential, their scientific management to keep minimum inventory level will bring good economic outcomes and decrease (Ballou, 2007).

An inventory may take the form of raw materials, building work, or finished products at any point in the manufacturing process. The central warehouse, convenience store, or either of these locations can also be stocked. In all of these cases:

- Downstream bases cause demands for upstream bases;
- These uncertain demands in combination with the uncertain time of transfer or production will retain the balance of the base is high.

Lee and Belington noted that there are many opportunities in inventory management. According to them, inventory management in the supply chain, informs the decision-making between various stages; coordinates the resources in uncertain conditions, and helps establish a standard of good practice in the supply chain. What is certain is when the lowest level of inventory is reached if the entire supply chain has to be considered as a system (Lee & Billington, 1995). This coordination in decisions makes great results in Xerox and HP corporations and the inventory level has declined more than 25 percent (Stenross & Sweet, 1991).

Sometimes different components of the supply chain hold safety stock to reduce the effect of the losses caused by uncertainty and raise the level of customer satisfaction (Zhao, Lai, & Lee, 2001). But holding the safety stock in different parts of the supply chain imposes costs on these components and eventually to the entire supply chain. These costs will ultimately drive up the cost of goods. Therefore, holding the safety stock at the appropriate level will have a

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great impact on effectiveness. In this paper, the main research question is determining the level of safety stock held by different parts of the multilevel supply chain. This aim is realized about modeling problems in nonlinear programming format with some assumptions.

In the supply chain safety stock is held to raising the response rate for demands. The response rate is the proportion of the number of demands that have been met on time to demands from the customers (Chopra & Meindl, 2007). Generally, inventory costs in the supply chain are divided into four groups.

1. Holding costs
2. Ordering costs
3. Missed opportunity costs
4. Buying costs

Holding the safety stock is reducing the missed opportunity costs, but significantly increases the inventory costs (Maia & Qassim, 1999). So, managers should always try to keep safety stock at the appropriate level. Part of the common methods of determining the optimal level of safety stock is helping to create a balance between the cost of maintaining safety stock and the cost of the lost order.

In this paper, two main problems which need to be addressed in a manufacturing inventory network are considered. On the one side, stock around the chain must be reduced to have advantages as cheaply as possible with limited advantages. Again, customers expect an abnormal state of management that includes on-time transportations. Both questions depend upon the measurement of the stocks in each region of the supply chain of the company. The problem described in the document manages to determine the security stock in each step of a supply chain. The amount must be so high that a group company can service its customers in time to satisfy much of the order, if not all of them. However, the inventory calculation must be small to minimize the expense of carrying and of capacity.

A manufacturer can protect itself against unverifiable demand by resolving the problem and give its customers an irregular administrative status. In supply chains, various methods of showing security stocks are found that possibly deem most important in the paper's written audit. The paper is primarily aimed at breaking down the problem and providing an algorithm for positioning security stock in supply chains in the general network (Fakhrzad & Goodarzian, 2021).

2. Literature Review

2.1. Safety stock modeling

In the literature, a few examples of researchers' consideration of models and techniques in the field of inventory management and the optimum level of safety stock in the supply chain could be looking up. The model used in this study covers two ranges of topics: multi-echelon inventory system and Order splitting inventory system. One of the oldest models of multi-echelon inventory models was presented by Sherbrook. He considered a two-echelon inventory system with a central warehouse and some retail and to finding an optimal inventory policy invented a method called METRIC. Graves explained the METRIC method with the estimation of mean and variance parameters and fit a binomial distribution function for determining the optimum inventory policy. Recent research has also pointed to the work of Axater and Diks (Axater, 2000). On the other hand, in another important class of models, decision-making is based on local information. In this viewpoint, more control methods are decentralized. Even if the control variables are defined using local information, determining the optimal control policy is centralized. Multi-echelon inventory control models that have decentralized control policies are less than the amount in the literature, but such investigations are expanding.

The subject of buying systems using one supplier versus multiple suppliers as a result of Japanese companies' success is widely studied. Banerjee has studied Japanese companies in the use of one supplier. He concluded that using one supplier enhances the long-term relationship between buyer and suppliers and encourages providers to invest in new technology and improve the quality of the goods. Decision-making about whether a company uses one, two, or more suppliers, not only under the influence of qualitative factors that were presented by Banerjee but also a few quantitative factors involved in this decision.

Surveys of various modeling approaches can be found in Graves (Graves, 1985), Graves and Willems (Graves & Willems, 2003), Minner (Minner, 2003), Minner (Minner, 2012). Two kinds of inventory models as far as service models are concerned: assured benefits and stochastic service models. Specifically, the solution receives the inventory network device representation and accepts that any step should specify a service period it can meet. Stochastic serving models assume, of course, to be subject to the accessibility of the products at the stage in terms of the time of operation between phases. The main assumption of the models is that the request is constrained to make service time deterministic. This suspicion enables the phases to be able to guarantee a period of operation on a small inventory. The purpose of models of this kind is therefore to determine the internal service terms to minimize the aggregate costs of stocks in supply chains. Table 1 provides a rundown of the examination in the above areas.

2.2. Complexity of concave minimization

In this section, the multi-faceted quality of these topics will be examined. It is understood that the general problem of inner minimization is NP-hard. The verification of this reality can be found in Sahni or Vavasis (Sahni, 1974; Vavasis, 1990). For instance, in Vavasis, the creator demonstrates that quadratic programming issues are NP-hard by changing an occasion of SAT (Korte and Vygen (Korte & Vygen, 2005)) to an occurrence of the quadratic program (Vavasis, 1990):

Table 1
A summary review of diverse research in the area of safety stock modeling

Main features	Example Researches	Remarks and model description
Deterministic service time	Simpson, Graves and Willems, Inderfurth, Inderfurth and Minner, Minner	The papers adopt the framework of Kimball, whose paper was originally written in 1955. Kimball describes a one-stage inventory model that operates under a base-stock policy and requires that customer demand is satisfied within a specific service time (Kimball, 1988). Simpson extends Kimball's principles of inventory control to accommodate a serial production system (Simpson Jr, 1958). Graves states that a basic dynamic machine can be used for solving the serial method (Graves, 1987). Inderfurt applies the Simpson model to the general device model that diverges. It follows the same structure as the deterministic service times cited in the downstream nodes (Inderfurth, 1991). In Inderfurt and Minner (Inderfurth & Minner, 1998) general networks of the supply chain are considered. The authors mention the issue of finding the security stock in the supply chain in Graves and Willems's article (Graves & Willems, 2003).
	Humair and Willems	Provide an algorithm for a network that consisting of clusters of commonality, where a cluster of commonality is a two-layer general network (Humair & Willems, 2006).
	Lesnaia	describes the structure of the solution for the two-layer networks with the requirement that the nodes i that face demand quote the outbound service times to the end customers such that $S_i \leq T_i$ (Lesnaia, 2003).
	Magnanti et al.	In the paper, the authors approximate the objective function with piecewise linear functions and solve the problem by a branch-and-bound algorithm using CPLEX (Magnanti, Shen, Shu, Simchi-Levi, & Teo, 2006).
	Daniel and Rajendran	The paper deals with the problem of the determination of installation base-stock levels in a serial supply chain. The problem is treated first as a single-objective inventory-cost optimization problem, and subsequently as a multi-objective optimization problem by considering two cost components, namely, holding costs and shortage costs (Daniel & Rajendran, 2006).
	Chaharsooghi and Heydari	An incentive scheme based on credit options has been developed to encourage the buyer to participate in the coordination model. In this model, the downstream member has the option of using credit to purchase goods during the credit time, subject to its commitment to a jointly agreed order quantity and reorder point (Chaharsooghi & Heydari, 2010).
	Moncayo-Martínez, Zhang	The problem is to select one resource option to perform each stage, and based on the selected options to place an amount of inventory (in-progress and on-hand) at each stage, to offer a satisfactory customer service level with as low as possible total supply chain cost (Luis A. Moncayo-Martínez & Zhang, 2013).
	Liang et al.	The purpose of this paper is to provide a multi-objective label correcting algorithm (MLC) to solve supply chain modeling problems (Liang, Huang, Lin, Chang, & Shih, 2013).
	L.A. Moncayo-Martínez et al.	The paper aims to solve the problem of placing safety stock over a Logistic Network (LN) that is represented by a Generic Bill of Materials (GBOM) (L. A. Moncayo-Martínez, Reséndiz-Flores, Mercado, & Sánchez-Ramírez, 2014).
	Rezapour et al.	A mathematical model is developed for integrated flow planning in forward and after-sales supply chains of a company supplying repairable product-warranty packages to markets (Rezapour, Allen, & Mistree, 2016).
	Luangkesorn and Bidanda	This paper uses Bayesian methods to analyze operational data where data on parameters required in logistics models are unavailable, then models production and inventory management in systems that have the potential for major disruptions (Luangkesorn, Klein, & Bidanda, 2016).
	Hua and Willems	We prove it is preferable to synchronize the supply chain by employing the same type of option, either low-cost long lead-time or high-cost short lead-time, at both stages. We prove that the selection threshold for high-cost short lead-time options is lowest at just the downstream stage, highest for just the upstream stage, and between these extremes if such a candidate is selected for both stages (Hua & Willems, 2016).
Stochastic service time	Clark and Scarf	The biggest distinction from the assured service method about stochastic service is that the refill time is stochastic for any point of the supply chain network. The models cover three forms of uncertainty: demand (volume and mix), processes (performance, system downtime, the efficiency of transport), and distribution (part quality, delivery reliability) (Clark & Scarf, 1960).
	Sherbrooke	The METRIC model of Sherbrooke is a decentralized control model. Two levels with 1 depot node that have n repositories for the second echelon are included in the model (Sherbrooke, 1966).
	Lee and Billington	In their model, an overall network model is analyzed. The first phase model assumes that demand is normally spread and a target service or base inventory level is established (Hau L Lee & Billington, 1993)
	Ettl et al.	The Lee and Billington models have been expanded. In this model, the authors adopt an individual refilling strategy, usually disperse the demand and formulate a basic level dilemma. In the definition of the lead time the deviation from the previous model lies (Ettl, Feigin, Lin, & Yao, 2000).
	Liu et al.	Adopt the Ettl et al. inventory-service improvement paradigm, but concentrate on output capacity-induced queueing delays (Liu, Liu, & Yao, 2004).
	Zipkin and Graves and Willems	A more systematic overview of the stochastic service models (Graves & Willems, 2003; Zipkin, 2000).
	Brander and Forsberg	Considers the problem of scheduling the production of multiple items, each with random demand, on a single facility (Brander & Forsberg, 2006).
	Jha and Shanker	This paper considers a two-echelon supply chain inventory problem consisting of a single-vendor and a single-buyer. In the system under study, a vendor produces a product in a batch production environment and supplies it to a buyer facing a stochastic demand, which is assumed to be normally distributed (Jha & Shanker, 2009).

Main features	Example Researches	Remarks and model description
	Osman and Demirli	Two models are proposed; a decentralized safety stock placement model and a centralized consolidation model (Osman & Demirli, 2012).
	Kim et al.	This paper studies a two-stage supply chain where returnable transport items (RTIs) are used to ship finished products from the supplier to the buyer. Empty RTIs are collected by the buyer and returned to the supplier (Kim, Glock, & Kwon, 2014).
	Govindan	This paper seeks to find the supply chain that minimizes system cost by comparing performance between traditional and VMI systems (Govindan, 2015).
	Escalona et al.	This paper analyzes the design of a distribution network for fast-moving items able to provide differentiated service levels in terms of product availability for two demand classes (high and low priority) using a critical level policy (Escalona, Ordóñez, & Marianov, 2015).
	Ameknassi	This paper develops a programming model, which combines logistics outsourcing decisions with some strategic Supply Chains' planning issues, such as the Security of supplies, customer Segmentation, and the Extended Producer Responsibility (Ameknassi, Aït-Kadi, & Rezg, 2016).
	Cobb	Model inventory at each stage in a supply chain for returnable transport items. Optimal values for inspection run length and time between container purchases (Cobb, 2016).
	Clemons and Slotnick	We investigate the effect of supply-chain disruption on a firm's decisions to invest in quality, and on ordering decisions, when there is a variable rate of knowledge transfer and a choice between two suppliers (Clemons & Slotnick, 2016).
Multi-echelon inventory system	Sherbrook	He considered a two-echelon inventory system with a central warehouse and some retail and to finding an optimum inventory policy invented a method called METRIC(Sherbrooke, 1966).
	Graves	Explained the METRIC method with the estimation of mean and variance parameters and fit a binomial distribution function to determining the optimum inventory policy (Graves, 1985).
	Diks and Axater	The offered models are focused on control policies based on the information; so that decision-making in each database requires access to information in each subordinate database (Axater, 2000; Diks & De Kok, 1998).
	Lee	Offered a comparable procedure for a serial system based on Clark and Scarf model, so that databases in the system are considered as a cost center. This procedure is based on transfer payments between different databases (Hau L. Lee, 1987).
	Muckstadt	In both types of research, decentralized policies for multi-echelon inventory systems with low volume demand in the emergency supply model are considered. They compared using single-stage models with terms of service against multi-stage model performance for the same system (Muckstadt, 1973).
	Lee and Billington	Instead of designing a new structure, they organized the decentralized control by using conditions that provide a level of service to upstream bases (Hau L Lee & Billington, 1995).
	Anderson et al.	He investigated on decentralized control branch distribution system under access to information restrictions (Anderson et al., 1998).
	Forrester; Chen et al.	The bullwhip effect is a well-known phenomenon that is first raised by Forrester. It refers to increasing swings in inventory in response to shifts in customer demand as one moves further up the supply chain (Forrester, 1997). There are different methods for quantifying it. Chen et al. used to determine demand deviations (Chen, Drezner, Ryan, & Simchi-Levi, 2000).
	Alwan et al. ; Dejonckheeree et al.; Disney and Towill	Different suggested models for measuring deviations adjusted the Forrester model (Alwan, Liu, & Yao, 2003; Dejonckheere, Disney, Lambrecht, & Towill, 2004). Although, Disney and Towill reported that this model has not much predictability feature (Disney & Towill, 2003).
	Lee et al.; Gilbert	Lee et al. suggested offered a retailer's replenishment policy based on an ARMA (1,1) process that a function of autoregressive parameters of demand and lead time of replenishment which was confirmed later by some researchers (Gilbert, 2005; Hau L Lee, So, & Tang, 2000).
	Hosoda and Disney	Considered a three-level supply chain model and analyze the bullwhip effect and net inventory deviations in each level (Hosoda & Disney, 2006).
	Aghezza et al.	With a distribution center and a series of points of sale that had a specific demand, considered an inventory routing problem that the aim of the program is minimizing the total cost of distribution and holding without facing shortage (Aghezzaf, Raa, & Van Landeghem, 2006).
	Caggiano et al.	They describe and validate a practical method for computing channel fill rates in a multi-item, multi-echelon service parts distribution system. A simulation study is presented which shows that, in a three-echelon setting, our estimation errors are very small over a wide range of base stock level vectors (Caggiano, Jackson, Muckstadt, & Rappold, 2009).
	Sue-Ann et al.	This paper focuses on the operational issues of a Two-echelon Single-Vendor–Multiple-Buyers Supply chain (TSVMBSC) under vendor-managed inventory (VMI) mode of operation. The optimal sales quantity for each buyer in TSVMBC is determined using a mathematical model available in the literature (Sue-Ann, Ponnambalam, & Jawahar, 2012).
	Guo; Pal et al	They both investigate integrated supplier selection and inventory control problems in supply chain management by developing a mathematical model for a multi-echelon system (Guo & Li, 2014; Pal, Sana, & Chaudhuri, 2014).
	Patriarca et al.	Based on METRIC, this paper defines a system-approach model for determining the stock levels of repairable items in a complex network, by a genetic algorithm optimization process (Patriarca, Costantino, & Di Gravio, 2016).
	Eruguz et al.	Many real-world supply chains are multi-echelon systems consisting of several stages of procurement, manufacturing, and transportation. In such systems, it is not obvious how to allocate safety stocks to meet the target service levels at the lowest cost. The guaranteed-service model (GSM) is among the existing approaches that allow this problem to be addressed. In this paper, we conduct a comprehensive review of the GSM literature (Eruguz, Sahin, Jemai, & Dallery, 2016).
	Maghsoudlou et al.	A novel bi-objective three-echelon supply chain problem is formulated in this paper in which cross-

Main features	Example Researches	Remarks and model description
		dock facilities to transport the products are modeled as an M/M/m queuing system (Maghsoudlou, Kahag, Niaki, & Pourvaziri, 2016).
	Dondo et al.	The paper presents a column-generation based decomposition-approach for finding near-optimal solutions to the problem (Dondo & Méndez, 2016).
Order splitting inventory system	Shulli and Wu	They determine the mean and variance of demand during the delivery time and in an inventory system with two suppliers. They assumed that delivery time with normal distribution and reorder level at both suppliers are the same (Sculli & Wu, 1981).
	Kelle and Silver	They considered a system with n supplier so that the delivery time of each supplier is the same and have Weibull distribution. They took advantage of the system with n suppliers compared with one supplier (Kelle & Silver, 1990).
	Ramasesh et al.	They considered a system with two suppliers so that demand is deterministic and delivery times are equally having a uniform or exponential distribution and the order quantity is separated equally. They determined the order quantity and reorder point in such a way that the holding and shortage costs in the time unit are minimized (Ramasesh, Ord, Hayya, & Pan, 1991).
	Lau and Zhao	They considered a system with two suppliers and extend Ramasesh's model whereas lead time could have any distribution and the breakdown of order is a decision variable (Lau & Zhao, 1993).
	Benton and Chiang	They considered a system with two suppliers. In their model suppliers have the same delivery time with exponential distribution; supplier's price is the same and orders are equally separated between two suppliers, but demand is stochastic and has a normal distribution. They determined the reorder point and optimum order amount to minimize the total expected cost contains ordering cost, inventory holding cost, and shortage cost (Chiang & Benton, 1994).
	Kotabe and Helsen	Another study that has been done in the field of sourcing, is the global sourcing strategy. They studied in this field and said that this type of strategy needs closed coordination between various activities of the organization within borders (Kotabe & Helsen, 1998).
	Gurnani	He proposed a more comprehensive model for the study order separation strategy to understand the behavior of the system in systems with several suppliers with a more accurate assessment of the costs of the system. They considered the general inventory system with n suppliers in which demand and delivery times per time unit are stochastic variables. They proposed a model to determining a reorder point and the breakdown of optimal order quantity for general systems with n suppliers and with numerical study determine the optimum number of suppliers (Gurnani, 2001).
	Kotabe and Murray	They proposed a model for using global sourcing for competitive advantage (Kotabe & Murray, 2004).
	Haksever and Moussourakis	They present a mixed-integer programming model for ordering items in multi-product multi-constraint inventory systems from suppliers who offer incremental quantity discounts (Haksever & Moussourakis, 2008).
	Mendoza and Ventura	This model determines an optimal inventory policy that coordinates the transfer of items between consecutive stages of the system while properly allocating orders to selected suppliers in stage 1 (Mendoza & Ventura, 2010).
	Abginehchi et al.	They develop a mathematical model which considers multiple-supplier single-item inventory systems. The lead times of the suppliers and demand arrival rate are random variables. All shortages are back-ordered. Continuous review (s, Q) policy has been assumed (Abginehchi, Farahani, & Rezapour, 2013).
	Zhu et al.	In this paper, we consider a single-item periodic-review stochastic inventory system with both minimum order quantity (MOQ) and batch ordering requirements. In each period, the firm can order either none or at least as much as the MOQ (Zhu, Liu, & Chen, 2015).
	Konur et al.	This study analyzes an integrated inventory control and delivery scheduling problem in a stochastic demand environment with economic and environmental considerations. In particular, we examine a bi-objective continuous review inventory control model with order splitting among multiple suppliers, where both expected costs and carbon emissions per unit time are minimized (Konur, Campbell, & Monfared).
	Roni et al.	This paper proposes a hybrid inventory policy with split delivery under regular and surging demand. The combination of regular and surge demand can be observed in many areas, such as healthcare inventory and humanitarian supply chain management. The arrival rate of regular demand is typically higher than the arrival rate of surge demand, whereas the volume of regular demand is typically lower than the volume of surge demand (Roni, Eksioglu, Jin, & Mamun, 2016).
	Pazhani et al.	They propose a mixed-integer nonlinear programming model to determine the optimal inventory policy for the stages in the supply chain and allocation of orders among the suppliers at the initial stage (Pazhani, Ventura, & Mendoza, 2016).

$$\begin{aligned} \min \quad & \sum y_i(1-y_i) \\ s.t. \quad & Ay \le b \\ & 0 \le y_i \le 1 \end{aligned} \tag{1}$$

The issue is an uncommon instance of the curved minimization issue. In this manner, curved minimization is NP-hard.

2.3. Methods

Extensive reviews of most basic methods produced for continuous concave minimization issues are given in Benson (Benson, 1995), Benson (Benson, 1996), Horst (Horst, 1984), Pardalos and Rosen (Pardalos & Rosen, 1986), which use as references for the area. There are two classes of ways to deal with inward minimization: deterministic and stochastic. Here, the deterministic methodologies are just examined. The most prevalent deterministic methodologies are enumeration, cutting plane, progressive guess, and branch and bound.

This division of the algorithms is just hypothetical. In all actuality, most of the algorithms are mixes of the methodologies. Also, a few procedures were recommended for the issues with particular target capacities, for example, quadratic, detachable, factorable, and so forth, or when the attainable set has some particular geometry.

2.4. Complexity and algorithms for the safety stock problem

The general problems of concave minimization are NP-hard. However, given that characterized by a wellness stock problem on a particular polyhedron, in any case, we can create a polynomial-time approximation at times, such that the problem is NP-hard. The decision on the algorithms in this paper will be justified. In particular, it is rational to incorporate branch and linked algorithms on the basis that no polynomial algorithm can be found from now on.

In applying mathematical programming; decision-makers realize that many real-life problems could be considered in the form of multi-objective decision making. Some authors consider the nonlinear multi-objective problem in which all parameters are defined fuzzily. In this formulation, uncertainties are normally presented as membership functions. Inventory models are developed in a fuzzy environment expressing the goals and parameters by fuzzy functions and/or parameters and thus inventory problems are reduced to fuzzy decision-making problems, which are solved by different fuzzy programming methods. Till now, only a few fuzzy inventory models have been available in the literature (Sadeghi, Sadeghi, & Saidi Mehrabad, 2011). In this paper, a new fuzzy multi-objective inventory model is developed. The resulted model has been formulated in the fuzzy environment and has been transformed into some crisp form where an ordinary nonlinear programming tool, e.g. sequential quadratic programming, can be implemented.

2.5. Conclusions from literature review and expression of research challenges

Most of the models presented to determine the contingency reserve do so independently in the units involved in the supply of a product to customers and do not pay attention to the effect of supply chain component response parameters on the amount of safety stock. Some researchers have made limited attempts to determine the safety stock by considering the response rate of supply chain components, such as Maya and Qassim (Maia & Qassim, 1999), Persona et al.(Persona, Battini, Manzini, & Pareschi, 2007), Karim et al. (Karim, Samaranayake, Smith, & Halgamuge, 2010), Amirjabbari and Bhuiyan (Amirjabbari & Bhuiyan, 2014), Ngubia (Ngubia, 2018), Gonçalves et al. (Gonçalves, Carvalho, & Cortez, 2020), etc. The supply chain considered in Maia and Qassim's model is a two-level supply chain that includes one supplier level (consisting of several suppliers) and one manufacturer (Maia & Qassim, 1999). In this paper, the proposed model is tried to be extended to multi-supplier and multi-manufacturer. Also, an innovative algorithm has been used to solve the model.

In most of the mentioned methods, the basis of control is to maintain the existing with the help of available values of variables and with the classic offline method, but in the used method, optimizing the flow values between the supply chain nodes and making them compact in a decentralized structure along with estimating the variables is the most important challenge. The effect of looking forward to local and final forecasts is also examined. The issue of advanced control of the supply chain system, especially in its decentralized and constrained form, is new. In inventory control topics in the literature section, the greatest focus is on classical control, which is offline and has difficulty coping with changes in demand and the effect of the Bullwhip effect. Basically, in advanced work, a comprehensive model is used in these fields, and due to the application of restrictions and proper adjustment of horizons and weights, its advanced predictive and online control is by no means simple.

3. Assumptions and formulation

3.1. Assumptions

Considering the supply chain illustrated in Figure 1.

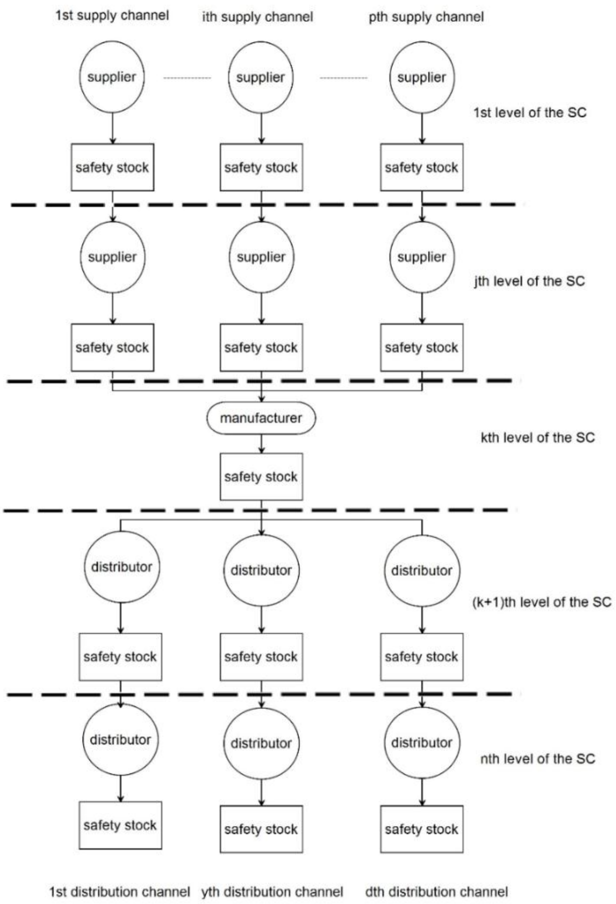

Fig 1. schematic of a supply chain

As you see in Figure 1, this chain is composed of *n* steps. With the help of these n steps, the final product is produced and shipped to end customers. Production and final product assembly take place in the k^{th} stage ($1<k<n$). Products offered by this supply chain composed of m types of semi-

finished products or raw materials each of which is supplied by a separate supply channel. After production, the commodity is reached to the customer by d distribution channel. The main assumptions of the model are as follows

1. Each of the components of the supply chain before the final assembly stage produced semi-finished parts and delivered them to the next stage. Because the response rate of each of the components to the assembly stage, not 100%, each of the components is held some safety stock to appropriately respond to the next level. Also in the production stage, held some final product as well as safety stock.
2. Each component of the supply chain, demands from the later stages can respond in two ways. A) By-products or components and materials that were received from existing components at the previous stage at that moment and carried out production processes. B) By safety stock from their products that are kept in stock.
3. There is no relationship between distribution and supply channels to receive products and services and each of them is works independently. Thus, there is no horizontal communication between components and connections is just vertically, as illustrated in Figure 1.

In the considered supply chain, the demand is entered from the customers to the end of distribution channels. This demand causes the components at the end of the distribution channels of the chain, send these demands to their predecessors and this demand moves in the direction of distribution channels to the manufacturer. The producer needed to provide parts or raw materials from suppliers to meet demand. In other words, the considered supply chain is a stretching one.

In model definition, the aim is to determine the optimal amount of safety stock levels by minimizing the inventory costs. In this model, the basis for calculating the costs is the operation of the supply chain components. The purpose of the operation of the supply chain components is the response rate of each component to demands referred to it from the next stages of the supply chain.

3.2. Formulation

Suppose that β_j is the base stock and *I* is the stock level. Organize the $I_j(t)$ submission for *j* watch at time *t* and start to renew the request. It submits an application to the top nodes for the information materials and fills out the application at the moment $t+\tau_j+T_j$.

The basic stock level to the measurement of the stock is related. The occurrences of occasions from 0 to *t* are shown in Figure 2. We are currently detailing the *P* problem of the discovery of ideal guaranteed outbound service times τ_1 and inbound service times τ_2 with a certain end objective to reduce safety costs in the chain.

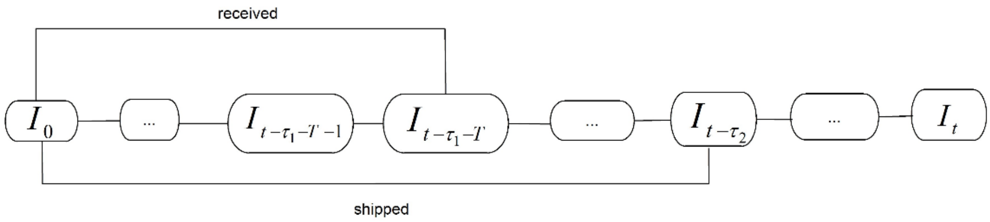

Fig. 2. Base-stock mechanics

$$P \qquad \min \quad \sum_{j=1}^{N} h_j\{D_j(\tau_1+T_j-\tau_2)-(\tau_1+T_j-\tau_2)\mu_j\}$$
$$\tau_1+T_j-\tau_2 \geq 0 \qquad j=1,\dots,N$$
$$\tau_2 \leq \tau_1 \tag{2}$$
$$\tau_2 \leq d_j \qquad j \in D$$
$$\tau_1,\tau_2 \geq 0$$

Where h_j shows the expense of keeping per unit of inventory at stage *j*. It is a question of reducing the inner capacity of a polyhedron. The goal capacity is the total cost of maintaining a security stock in the production network, consisting of individual expenditures in each hub. The primary condition safeguards the non-negative charging period and the goal power. The second criterion makes incoming operation times no more remarkable than their swift providers' outgoing administration time. This is because a hub cannot start to be created before all data sources are accessible. These limitations reflect the need not more remarkable than the assured gain times quoted to end-customers for all outbound service times τ_2 of request nodes established by the chain. Finally, all administrative hours must be numbered, to express the administration times in the essential items of the general schedule, which may be a day or a week.

4. Problem Characteristics and Modeling

I k_{ij} f is the response rate of the i^{th} component in the j^{th} level before production (*i=1,2,…,m; j=1,2,…,r-1*), then this new rate in the supply chain is defined as follows:

$$k_{ij}=\frac{q_{ij}^{*}+x_{ij}}{q_{ij}}=p_{ij}+\frac{x_{ij}}{q_{ij}} \tag{3}$$

Where in

q_{ij} is the total demand reached to *the* i^{th} component in the j^{th} level from its successor component

q^*_{ij} is the total demand that i^{th} component in the j^{th} level responded by getting goods or materials from its predecessor in the supply chain.

x_{ij} is the safety stock that i^{th} component in the j^{th} level holding for responsiveness for its successors.

p_{ij} is the proportion of demand that i^{th} component in the j^{th} level responded by commodity from the previous components.

If holding cost for the product in stock for the i^{th} component in the j^{th} level is equal to $\tilde{c}_{ij}$, then the holding cost of safety stock for this component of the supply chain is calculated using the following equation

$$\tilde{C}_{ij} = \tilde{c}_{ij} x_{ij} \tag{4}$$

The symbol "~" represents the fuzziness of the parameter.
By combining this equation with the previous, the total holding cost of safety stock can be obtained as follows:

$$\tilde{C}_J = \sum_{i=1}^{m} (\tilde{c}_{ij} q_{ij} (k_{ij} - p_{ij})) \tag{5}$$

According to the above equation, the total holding cost of the safety stock before production (CB) is obtained from the below equation

$$\tilde{C}\tilde{B} = \sum_{j=1}^{r-1} \sum_{i=1}^{m} (\tilde{c}_{ij} q_{ij} (k_{ij} - p_{ij})) \tag{6}$$

If assume that x_r is the safety stock of the final product that is kept in stock by the manufacturer and p_m is the response rate of the production line, then the manufacturer's response rate is calculated by this relation.

$$k_p = \frac{x_r}{\sum_{v=1}^{s} q_{yr}} + p_m \prod_{i=1}^{m} \prod_{j=1}^{r-1} k_{ij} \tag{7}$$

In this relation, q_{yr} is demand has arrived from y^{th} distribution channel that received from r^{th} or production level. If the holding cost of each unit in the manufacturer's stock is c_r, then the holding cost of the safety stock is calculating by relation (8)

$$\tilde{C}_r = \tilde{c}_r x_r \tag{8}$$

According to previous relations

$$\tilde{C}_r = \tilde{c}_r \sum_{y=1}^{s} (q_{yr} (k_p - p_m \prod_{i=1}^{m} \prod_{j=1}^{r-1} k_{ij})) \tag{9}$$

As mentioned earlier, this supply chain is composed of s distribution channel (*y=1,…,s*). Components of each of the distribution channels are in one step of *j=r,r+1,…,n*. If assume that q_{yj} is a demand that entered from successor component to y^{th} component, then k_{yj} that is the operation of j^{th} component in the y^{th} distribution channel is calculated as below

$$k_{yj} = p_{yj} + \frac{x_{yj}}{q_{yj}} \tag{10}$$

If assume that $\tilde{c}_{yj}$ is the holding cost of yj^{th} component, then the holding cost of the safety stock is calculated by relation (11)

$$C_{yj} = c_{yj} x_{yj} \tag{11}$$

By calculating x_{yj} from relation (10) and put it on the relation (11), the holding cost in component yj^{th} can be calculated by this relation

$$\tilde{C}_{yj} = \tilde{c}_{yj} q_{yj} (k_{yj} - p_{yj}) \tag{12}$$

So, the total holding cost in the j^{th} stage after production is calculated by relation (13)

$$\tilde{C}_j = \sum_{y=1}^{s} (\tilde{c}_{yj} q_{yj} (k_{yj} - p_{yj})) \tag{13}$$

Eventually, total holding cost after production (CA) is calculated by relation (14)

$$\tilde{C}\tilde{A} = \sum_{j=r+1}^{n} \sum_{y=1}^{s} (\tilde{c}_{yj} q_{yj} (k_{yj} - p_{yj})) \tag{14}$$

Now suppose the cost of lost order for each product in each distribution channel is $\tilde{c}\tilde{o}_y$, so the total lost order cost in the y^{th} channel is calculated by relation (15)

$$\tilde{c}\tilde{o}_y = \tilde{c}\tilde{o}_y q_y (1 - k_y) \tag{15}$$

In the above equation q_y is the demand comes to the y^{th} distribution channel and k_y is the y^{th} distribution channel's response rate that is calculated by relation (16)

$$k_y = \prod_{j=r+1}^{n} k_{yj} \tag{16}$$

So, the total cost of the lost order is calculated by relation (17)

$$\tilde{c}_y = \sum_{y=1}^{n} (q_y \tilde{c}\tilde{o}_y (1 - \prod_{j=r+1}^{s} k_{yj})) \tag{17}$$

So we can help relations 6, 9, 14, and 17 consider the following nonlinear programming model for determining the optimum amount of k_{ij} and p_{ij} after solving the model, achieve the optimum amount of x_{ij}, x_{yj} and x_r values. Also, the proposed model assumed that the parameter $\tilde{F}_i$ is the space required by each unit of product i, M_i is the total demand of product i during some given time interval, and L the maximum number of orders placed during the given period.

$$\begin{aligned}
\min \quad & Z_1 = \sum_{j=1}^{r-1}\sum_{i=1}^{m}(\tilde{c}_{ij} q_{ij}(k_{ij} - p_{ij})) + \tilde{c}_r \sum_{y=1}^{s} q_{kr}(k_p - p_m \prod_{i=1}^{m}\prod_{j=1}^{r-1} k_{ij}) \\
& + \sum_{j=r+1}^{n}\sum_{y=1}^{s}(\tilde{c}_{yj} q_{yj}(k_{yj} - p_{yj})) + \sum_{y=1}^{s}(q_y \tilde{c}\tilde{o}_y (1 - \prod_{j=r+1}^{s} k_{yj})) \\
\min \quad & Z_2 = \sum_{i=1}^{n} \tilde{F}_i q_i \\
s.t. \quad & \\
& \sum_{i=1}^{n}(\frac{M_i}{q_i}) \le \tilde{L} \\
& p_m \prod_{i=1}^{m}\prod_{j=1}^{r-1} k_{ij} \le k_p \le 1 \\
& p_{ij} \le k_{ij} \le 1 \qquad (i = 1,2,...,n;\ j = 1,2,...,r-1) \\
& p_{yj} \le k_{yj} \le 1 \qquad (y = 1,2,...,s;\ j = r+1, r+2,...,s)
\end{aligned} \tag{18}$$

4.1. Algorithm

The algorithm for general networks of the supply chain has been developed by formally stating the algorithm in the branch and bound algorithm.

<u>Branch and bound algorithm</u>

function computeGenMinCost
1 tree $\leftarrow$ createTree
2 lowerBound $\leftarrow$ computeTreeMinCost(tree, 0, 0, 0, 0)
3 upperBound $\leftarrow$ fixTree(tree)
4 if lowerBound = upperBound then return lowerBound
5 end if
6 branch(1)
7 return upperBound
end function

procedure branch(k)
8 $\forall$ valid paths P_{kj} do
9 $\forall$ i $\in P_{kj}$ compute S_i and SI_i
10 if all S_i and SI_i valid then set all S_i and SI_i
11 bound()
12 unset all S_i and SI_i
13 end if
14 end for
end procedure

procedure bound()
15 if all nodes have SI set then
16 branchCost $\leftarrow$ compute graph cost
17 if branchCost < upperBound then upperBound $\leftarrow$ branchCost
18 end if
19 else tree $\leftarrow$ createTree
20 branchLowerBound $\leftarrow$ computeTreeMinCost $(tree, SI, S, \underline{SI}, \overline{S})$
21 branchUpperBound $\leftarrow$ fixTree(tree)
22 if branchUpperBound < upperBound then upperBound $\leftarrow$ branchUpperBound
23 end if
24 if branchLowerBound < upperBound then $k \leftarrow$ next node without SI set
25 branch(k)
26 end if
27 end if
end procedure

The notation in the algorithm above is elaborated here. The initial limits on the optimal solution in lines 1-5 of the algorithm is calculated. First a spanning tree with *createTree* procedure (line 1) is created. Then the optimum cost of the tree network problem is calculated. As there are no service times or limits fixed at this point, passing 0 to *computeTreeMinCost* as service times and limits. The *branch(k)* procedure determines *k* node branches with paths. In line 8, the P_{kj} route is a valid route, where the conditions on the path are valid. Then placed S_i and SI_i on the road in line 9. Line 10 applies to all the service times determined on the route that follow the limits of the valid S_i and SI_i:

$$\underline{SI}_i \le SI_i \le M_i - T_i;$$
$$0 \le S_i \le \min\{M_i, \overline{S}_i\}$$

When the branch is connected, line 12 is set to SI_i and S_i. We write about the method bound(). Lines 16 through 18 refer to the situation in which the inbound service times for all network nodes are allocated. On line 16 the cost of the branch is calculated. *branchCost* in lines 17 to 18 is linked to the new *upperBound* solution. If there is no allocation of inbound network operating hours, lines 19 through 26 are executed.

After calculating the lower bound for the branch, correct the solution of the clamping tree in line 18. Then find the optimal answer from the top of the branch and the current best. Line 20 unassigns the time in the coming operation to the next node if the lower end of the branch is lower than the optimum current solution. Because some of the reception times of the path of nodes $i > k$ are set, the first thing you need to do is find the next node not receptive. Then log in through the input service times to this node (line 21).

The algorithm finishes with the perfect solution to the overall security problem of the method. The structure does not rely on the arrangement in which prepares the system nodes. All have, however, it is shown that the perfect cost speedier than to use others by applying a few requests from the hubs.

4.2. Solving fuzzy model

In this section, to solve the above fuzzy multi-objective, multi-item inventory model, the fuzzy non-linear programming (FNLP) method based on Zimmermann is used as follows (Zimmerman, 1983):

$$
\begin{aligned}
&\max \quad \alpha \\
&s.t. \\
&\sum_{j=1}^{r-1}\sum_{i=1}^{m}((c_{ij}-(1-\alpha)F_{c_{ij}})q_{ij}(k_{ij}-p_{ij}))+(c_r-(1-\alpha)F_{c_r})\sum_{y=1}^{s}q_{kr}(k_p-p_m\prod_{i=1}^{m}\prod_{j=1}^{r-1}k_{ij}) \\
&+\sum_{j=r+1}^{n}\sum_{y=1}^{s}((c_{yj}-(1-\alpha)F_{c_{yj}})q_{yj}(k_{yj}-p_{yj}))+\sum_{y=1}^{s}(q_y(co_y-(1-\alpha)F_{co})(1-\prod_{j=r+1}^{s}k_{yj}))\le b_1+(1-\alpha)P_1 \\
&\sum_{i=1}^{n}(F_i-(1-\alpha)F_{F_i})q_i\le b_2+(1-\alpha)P_2 \\
&\sum_{i=1}^{n}(\frac{M_i}{q_i})\le L+(1-\alpha)P_L \\
&p_m\prod_{i=1}^{m}\prod_{j=1}^{r-1}k_{ij}\le k_p\le 1 \\
&\lfloor q_1,q_2,\dots,q_n\rfloor T\ge 0 \\
&\quad p_{ij}\le k_{ij}\le 1 \qquad (i=1,2,\dots,n;\ j=1,2,\dots,r-1) \\
&\quad p_{yj}\le k_{yj}\le 1 \qquad (y=1,2,\dots,s;\ j=r+1,r+2,\dots,s)
\end{aligned}
\tag{19}
$$

Where

P_1: the maximally accepTable violations of the aspiration levels of b_1

P_2: the maximally accepTable violations of the aspiration levels of b_2

P_L: the maximally accepTable violations of the aspiration levels of L

F_i : the tolerance of i^{th} parameter

5. Computations

In this section, the computational consequences of actualizing the algorithm from the last segment are introduced. Our objectives here first to exhibit the execution of the algorithms and second outlining a numerical case to show the pertinence of our proposed algorithm. By doing a progression of investigations, pick the parameters of the algorithm to get the best execution is demonstrated.

The algorithms have been implemented on Pentium IV 3.4 GHz with RAM 16 GB desktop running Windows 7.The CPU time in milliseconds spent on explaining the occurrences. The season of creating occasions is excluded in the outcomes. Presently, the safety stock issue cases on a system with N nodes and M arcs are depicted.

A progression of examinations to locate the best settings of the algorithm is composed. Additionally, a few parameters of the algorithms must be investigated computationally. The parameters are outlined here.

1. Node request.
2. Tree sort.
3. Several initial bounds.
4. Several bounds per branching point (BBP).
5. Several global bounds per branching point (GBBP).
6. Tolerance limit.

For every test, various arbitrary cases are produced and determine the setting of the trial. The settings in a Table like Table 2 are outlined.

Table 2

Summary of the experimental settings

nodes	N
arcs	M
node order	cost, layer, random
tree type	same, smart, random
initial bounds	
bounds per branching point	
global bounds per branching point	
tolerance limit	
number of instances	

Experiment 1. Spanning tree computations

The execution of the Graves and Willems algorithm for a safety stock problem for a tree distribution inventory network is improved (Graves & Willems, 2003). The algorithm seen on the paper is a measure of pseudo-polynomial time. A polynomial is the latest algorithm. In this article, the importance to have a tree systems polynomial-time algorithm is shown. The tree structures for the general problem to achieve the lower limits are used. With the following example, we demonstrate the difference between the polynomial and pseudo-polynomial-time algorithms. By branch and bound algorithm, an overall framework problem was addressed.

Use the pseudo-polynomial algorithm first then subsequently use the polynomial algorithm. With 20 knots and 25 arcs, random schemes are generated. An odd variable to circulate constantly in between *[0, T_{max}]* to see the distinction in each hub was allowed.

T_{max} varies between 10 and 360. For each T_{max} estimate, 100 cases of secure storage and resolve the problems with polynomially recorded limits by branch and bound algorithms were generated. Related examples at this stage of a similar algorithm but with pseudo-polynomial restrictions will understand. The parameters of the trials are seen in Table 3 and the algorithm results in Table 4. Figure 3 shows the standard time to understand a general machine safety stock problem with the limits of time recorded by a tree algorithm by a polynomial.

Table 3

Settings for Experiment 1

nodes	20
arcs	25
node order	random
tree type	random
initial bounds	1
bounds per branching point	1
global bounds per branching point	0
error	0
number of instances	100

Table 4
Average time per instance in milliseconds for Experiment 1

Max lead time	Polynomial	Pseudo-polynomial
10	88	100
60	82	670
110	83	1938
160	83	3734
210	65	6483
260	81	9475
310	81	13309
360	62	17917

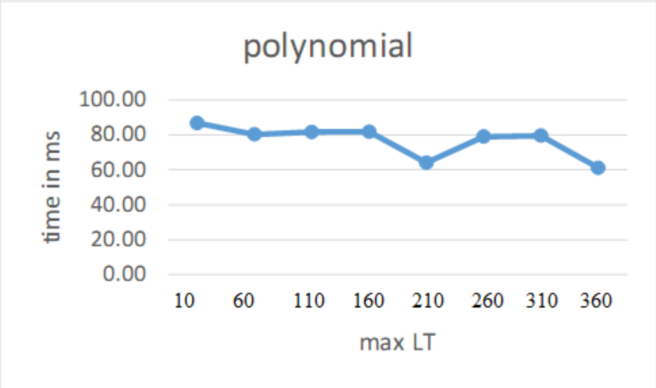

Fig. 3. Average time of solving a general network problem with 20 nodes and 25 arcs using polynomial bounds

Standard time as an aspect of T_{max} is added. Figure 4 shows the usual time to understand the general machine security problem with the pseudo-polynomial time limits which spread through the algorithm of the tree. The effect on a related figure is determined by the aftereffects of the general algorithm with polynomial limits.

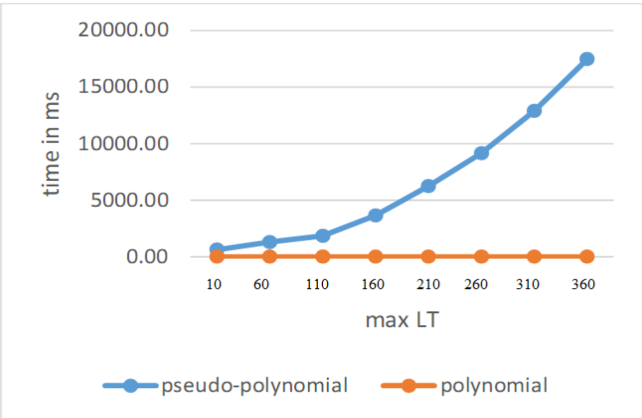

Fig. 4. Average time of solving a general network problem with 20 nodes and 25 arcs using polynomial and pseudo-polynomial bounds.

From the computer studies, for T_{max} different uses remain the standard time of the running of the general algorithm with polynomial limits. Meanwhile, as T_{max} increases, the ordinary time for a general time algorithm with the pseudo-polynomial limit increases.

Experiment 2. Register the initial limit against the global limit at any point of distribution

The best running conditions of the two strategies in this test are studied. The main technique initially records all the global limits. The approach is called the technology of the IB. For any stretching point, the second strategy works out one global connection. The GBBP approach is the methodology. Table 5 shows that for most of the tried and tested problems the IB technique is most normal. Table 6 includes the number of situations overlooked by the tactics in only ten minutes. We can see that almost 2% of such events are among the tried and tested problems and that both solutions have comparative data. Cases can also be explained by algorithms. Extra algorithms, for example, show that in 30 minutes, the GBBP Technique explains the event declared in Table 6 with 50 hubs and 59 curves.

Acquiring the best running time for the IB technique requires improving the number of initial bounds notwithstanding advancing the number of limits per branching point. To enhance BBP, picking 15 functions admirably for the issues considered here. However, upgrading the number of initial bounds for the IB strategy is a more troublesome assignment. Looking at the normal circumstances of taking care of the safety stock issue utilizing the calculation depicted in the paper to the circumstances that appeared in Magnanti et al. (Magnanti, Shen, Shu, Simchi-Levi, & Teo, 2006), would be considered. At last, a case of the time dispersion for the GBBP technique is demonstrated. 100 systems with 50 hubs and 62 circular segments are considered. As indicated in Table 5, the algorithm takes care of an issue of this size in 27 seconds by and large for the occasions explained under 10 minutes. For this set, 99 out of 100 cases were settled in *n* under 10 minutes.

Table 5
Best average time per instance in milliseconds for the two methods in Experiment 2.

Nodes	Arcs	GBBP best time	IB best time	Best method
20	20	7	3	IB
20	30	240	170	IB
20	40	2789	2512	IB
20	45	63200	5726	IB
25	25	25	12	IB
25	40	2830	2463	IB
25	45	7580	6628	IB
25	50	20112	18112	IB
30	30	16	3	IB
30	40	897	580	IB
30	45	22120	1960	IB
30	55	14250	13257	IB
35	35	26	6	IB
35	40	485	496	GBBP
35	50	7360	7214	IB
35	55	18025	22976	GBBP
40	40	46	28	IB
40	50	4860	1635	IB
40	55	17680	18630	GBBP
40	60	21790	21241	IB
50	50	150	60	IB
50	55	1090	146	IB
50	60	2026	1860	IB
50	65	27168	19112	IB

Table 6
The number of instances not solved in 10 minutes by IB and GBBP methods in Experiment 2.

nodes	arcs	GBBP	IB
30	50	0	1
35	55	1	1
40	53	0	1
45	57	2	2
50	60	1	1
55	65	1	1

Experiment 3. Two-layer algorithm vs. general algorithm
The efficiency of the general and two-layer network algorithms applied to two-layer network problems is compared in this experiment was created. The experiment settings are seen in Table 7. With 10 components and 10 demand nodes, 200 instances of a two-layer network problem. The arc count varies from 8 to 39. Each case is solved with the algorithm of two layers. Then, using the general network algorithm the same instances were solved. Table 8 and Figure 5 are the typical periods at which the issue is resolved. The two-layer algorithm is easier for the two-layer network problems with a few nodes than the general algorithm.

Table 7
Settings for Experiment 3

components	10
demand nodes	10
arcs	8 to 39
node order	layer
tree type	random
initial bounds	1
bounds per branching point	20
global bounds per branching point	1
tolerance limit	0
number of instances	200

Table 8
Average time in milliseconds for solving a two-layer problem with 10 components and 10 demand nodes using the two-layer and general network algorithms in Experiment 3.

arc	general	two-layer
20	10	4
21	11	7
22	17	8
23	25	10
24	28	17
25	38	15
26	52	18
27	74	29
28	101	38
29	178	29
30	279	45
31	409	52
32	503	41
33	780	49
34	1242	53
35	1939	94
36	2841	64
37	4312	66
38	6271	81
39	8635	83

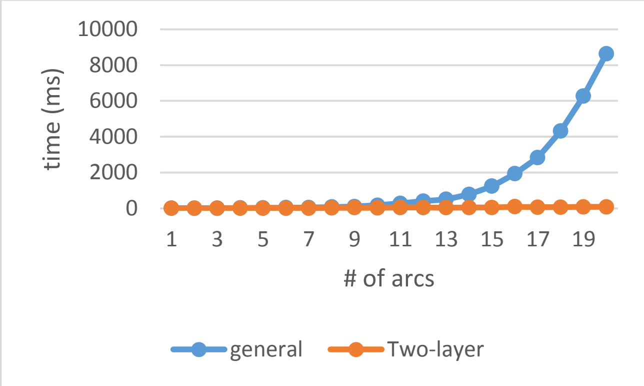

Fig. 5. Average time in milliseconds for solving a two-layer problem with 10 components and 10 demand nodes using the two-layer and general network algorithms in Experiment 3.

The discrepancy between the two algorithms will become negligible as the numbers of nodes in sparse networks rise. Networks of 40 elements and 40 requirements of 80 to 90 arcs were considered. For the two algorithms as seen in Table 9 the estimated time for resolving the problem is comparable. Furthermore, the number of cases that the two algorithms cannot resolve within 10 minutes is comparable as seen in Table 10.

Table 9
Average time in milliseconds for solving a two-layer problem with 40 components and 40 demand nodes using the two-layer and general network algorithms in Experiment 3.

arcs	general	two-layer
80	77	67
81	29	27
82	4606	4410
83	7756	7111
84	4110	3808
85	3507	3367
86	6689	6471
87	16939	15971
88	34348	32347
89	53239	50999
90	50331	48170

Table 10
Number of instances that general and two-layer algorithm failed to solve in 10 minutes for problems with 50 components and 50 demand nodes in experiment 3

arcs	general	two-layer
80	0	0
81	0	0
82	0	0
83	0	0
84	0	1
85	1	2
86	2	3
87	3	2
88	2	2
89	2	10
90	10	3

As can be seen, in the 10-component, the 10-node trend for graphs and the general algorithm is slower as the arc number increases. For networks with a greater number of arcs, the two-layer algorithm is also optimal. For graphs of up to 80 nodes, it is best to use the two-layer algorithm. Using the general algorithm for sparse graphs with more nodes would be considered.

In this section, to demonstrate the applicability of the model, a numerical example and calculation of several safety stocks in each of the components of the supply chain are considered. In our example, the supply chain has five levels and the manufacturer is in the 3rd level. Each of the components of this supply chain to respond to uncertainty caused by the response rate of their predecessor components hold some safety stock.

In this section, the optimum level of safety stock by using the proposed model in the previous section is determined. In this supply chain in is assumed that the holding cost in different components of the supply chain before production is two units and the holding cost of each product in the production stage and distribution channels is one unit. Also, in the above model, the amount of lost profiles in each of the distribution channels is 10 units. The amount of demand in each of the distribution channels is expressed in Table 11. The total demand (in three distribution channels) for the final product is 4000 units. Also, according to the product tree, it is assumed that the demand received by each of the supply channels is illustrated in Table 12.

Table 11
Demand in each of the distribution channels

Distribution channel	Demand
1	1000
2	2000
3	1000

Table 12
Received demand to each supply channel

Supply channel	Demand
1	4000
2	8000
3	12000

The variables in the model to find the optimal value p_{ij}, k_{ij} is. The nonlinear programming model for the above supply chain has 26 variables and 26 constraints. Given the number of variables and constraints, for solving the model Lindo software was used. Results are illustrated in Table 9. In this Table, after determining the number of p_{ij}, k_{ij} variables, the needed safety stock for each of the components of the supply chain also be calculated. As shown in Table 13, the amount k_{ij} before production is significantly less than the response rate of the components in the distribution channels that the main reason for this issue is considering the lost order cost for each component of the distribution channels. On the other hand, the amount of calculated safety stock for each of the distribution or supply channels is proportional to the received demand to that channel because of the same condition of each distribution channel. These conditions are the holding cost of the safety stock and also the cost of lost opportunity.

Table 13
Model solving results and calculating several safety stocks

variable	Variable val	demand	Safety stock val	variable	Variable val	demand	Safety stock val
k_{11}	0.87	4000	360	k_{14}	0.690	1000	80
p_{11}	0.78			p_{14}	0.880		
k_{12}	0.89	4000	360	k_{15}	0.690	1000	80
p_{12}	0.8			p_{15}	0.880		
k_{21}	0.88	8000	720	k_{24}	0.680	2000	160
p_{21}	0.79			p_{24}	0.870		
k_{22}	0.83	8000	720	k_{25}	0.890	2000	160
p_{22}	0.74			p_{25}	0.900		
k_{31}	0.86	12000	1080	k_{34}	0.390	1000	80
p_{31}	0.77			p_{34}	0.850		
k_{32}	0.87	12000	1080	k_{35}	0.490	1000	80
p_{32}	0.78			p_{35}	0.860		
kp	0.9	4000	400				

Determining the amount of safety stock in different levels of the supply chain has been always one of the challenges facing managers in the supply chains. Safety stock existence imposes holding costs to the supply chain and on the other hand, prevents the costs such as the cost of lost opportunity or the cost of stopping the production line. According to the above, managers are always trying to find ways to determine the optimal amount of safety stock.

In this paper, the goal is to present a nonlinear programming model for calculating safety stock in a multi-level supply chain with several supply and distribution channels. This model is designed by minimizing related safety stock holding costs. The proposed nonlinear programming model in this paper is solved by common operations research software. Results show that the amount of safety stock in each component of the supply chain, according to the conditions of that sector of the chain. Conditions such as the cost of lost opportunity and the cost of inventory holding in the supply chain.

5.1. Sensitivity Analysis

The efficiency of the algorithm for various tolerance limits can be seen in this section. Table 14 summarizes the configurations of the experiment. With 100 nodes and 100 to 125 arcs, the general problem was solved. 100 cases of each kind are produced and solve for the 1%, 5%, and 10% tolerance limits. Table 15 and Figure 6 show the findings. Table 16 shows the number of cases in which the algorithm was not resolved within 10 minutes. It is assumed that in this experiment the algorithm will easily obtain an approximation of the optimum solution. The 10 percent tolerance limit for the graphs in question is averaged in less than 20 seconds.

Table 14
Settings for sensitivity analysis

nodes	100
arcs	100 to 123
node order	layer
tree type	random
initial bounds	10
bounds per branching point	15
global bounds per branching point	1
tolerance limit	2%, 5%, 10%
number of instances	100

Table 15
Average time in milliseconds for solving a problem with 100 nodes and 100 to 123 arcs for 1%, 5%, 10% tolerance limits in the sensitivity analysis

Arc	Tolerance limit (%)		
	1	5	10
100	53	21	8
101	896	21	8
102	140	46	8
103	437	64	10
104	9321	106	21
105	827	99	24
106	3438	127	43
107	11280	136	63
108	7339	203	78
109	13818	508	81
110	13874	566	95
111	12138	578	109
112	19776	1984	192
113	43681	3158	158
114	70725	5282	197
115	64245	11043	232
116	127704	15175	377
117	149944	30255	672
118	181794	57837	1424
119	217293	65241	3661
120	233800	110338	4319
121	266000	165249	7738
122	287000	207327	15092
123	329000	230434	31819

Table 16
Number of instances that the algorithm 2%, 5%, 10% tolerance limits in the sensitivity analysis

Arc	Tolerance limit (%)		
	1	5	10
100	1	0	0
101	0	0	0
102	0	0	0
103	0	0	0
104	1	0	0
105	0	0	0
106	0	0	0
107	0	0	0
108	0	0	0
109	3	0	0
110	2	0	0
111	1	0	0
112	4	1	0
113	7	1	0
114	6	1	0
115	9	0	0
116	5	2	0
117	17	2	0
118	21	1	0
119	25	2	0
120	28	4	0
121	33	9	0
122	37	20	0
123	42	23	0

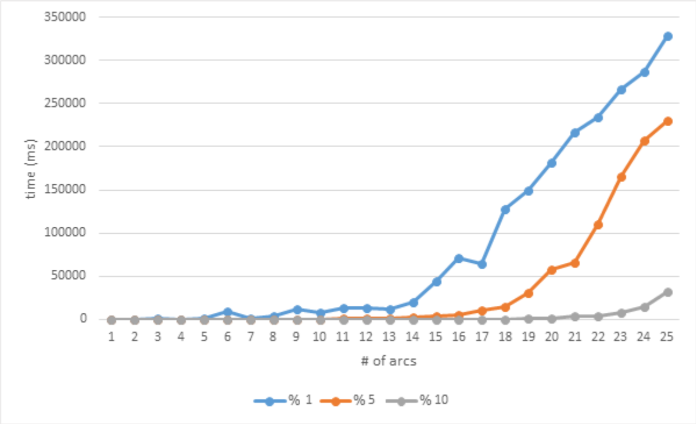

Fig. 6. Average time in milliseconds for solving a problem with 100 nodes and 100 to 123 arcs for 1%, 5%, 10% tolerance limits in the sensitivity analysis

6. Conclusion

In this paper, the question of determining the safety stock situation in the supply chain of the general network is considered. To handle the problem, the deterministic service times and suggested algorithm is assumed. In particular, the supply chain as a network device node is described that represents the production company's processing functions as well as security positions. The safety stock strategy applies to each level of the supply chain. Stages in the service quote for the supply chain to neighboring downstream and final customers. The model's main assumption is that consumer requirements are restricted. The model's main assumption is that consumer requirements are restricted. Each step ensures that 100% of consumer demand is met to the assumed limit.

It shows that the general safety issue is NP-hard and includes an algorithm to deal with the problem. The scheme is breaking down and discovered that any possible contenders can rely on for the perfect administration times for the hub by constructing each way of a hub to another hub in the system. Also, methods for calculating a growing tree are used. The boundaries are lowered for the spreading branches of the tree by addressing the problem of a crossing tree. A polynomial-time algorithm is built to comprehend the spreading over the tree safety stock problem.

Also, computer experiments to check the execution of the generated algorithms in the computational form are arranged. How to set the algorithm parameters to perform the best numerical performance is examined. The measurement divisions of hubs start from hubs and complete with segments is proposed. A random output is the best solution to creating crossing trees. The recording of 15 lower and top limits per division provides excellent results for the tried and tested insufficient diagrams. Using the polynomial-time spread around the tree algorithm for processing different global bounds in an ideal way is considered.

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