Journal of Optimization in Industrial Engineering
Vol.14, Issue 1, Winter & Spring 2021, 105-119
DOI:10.22094/JOIE.2021.578550.1599

Research Article

A Benders-Decomposition and Meta-Heuristic Algorithm for a Bi-Objective Stochastic Reliable Capacitated Facility Location Problem Not Dealing with Benders Feasibility-Cut Stage

AmirHossein Zahedi-Anaraki [a,c,*], Gholamreza Esmaeilian [b]

[a] *Department of Industrial Engineering, University of PayamNoor, Tehran, Iran*
b Department of Industrial Engineering, University of PayamNoor, Tehran, Iran
[c] *Department of Engineering, University of Applied science and Technology, Tehran, Iran*

Received 05 November 2018; Revised 13 October 2020; Accepted 10 January 2021

Abstract

This paper addresses a bi-objective two-stage stochastic mixed-integer linear programming model for a stochastic reliable capacitated facility location in which the optimum numbers, locations and as well as shipment quantity of the product between the network nodes for all scenarios should be determined. Unlike most of previous relevant works, multiple levels of capacities available to the manufacturers in different scenarios are permitted in this study. The proposed objectives of the model include: the minimization of expected sum of installation, production, transportation under uncertainty of parameters, such as transportation and production and disruption of facilities, as well as minimizing expected standard deviation of network costs for whole scenarios. Since one of the most important reasons for researchers' reluctance to apply Benders-decomposition algorithm in facility-location concept is the time-consuming nature of its feasibility-cut stage, one of the most outstanding innovation in this paper is to add a strengthening redundant constraint to the proposed model in order to eliminate the mechanism related to feasibility cuts in master problem. to the best of our knowledge, it is the first time that this technique, not being involved in keeping master-problem feasibility, is used to solve a reliable capacitated facility location problem. In this approach, in terms of time-consuming the Benders algorithm is able to powerfully compete with metaheuristic algorithms, but with an exact solution. To prove advantage of this algorithm satisfying both ultimate solution optimality and appropriate running time compared to metaheuristic algorithms at the same time, one metaheuristic algorithm, namely Imperialist Competitive Algorithm (ICA), is presented. Usefulness and practicality of the proposed model and solution method demonstrated through a case example in different class with variable size.

***Keywords**:* Capacitated facility location; Reliability; Stochastic Programming; Benders Decomposition

1. Introduction

To design an efficient supply chain network the most challenging and crucial decision is related to facility location issues. Once the decision on facility location has been made, changing the locations incurs a huge cost to the whole network and thus reverting especially in a short-term period is impossible. As an example, the location of wheat distribution centers cannot be changed as a result of fluctuations in customer demands, government taxes, and natural disasters. Companies have to bear financial troubles in the case of inefficient facility locations even though inventory management, production planning and information systems are enough flexible in response to inconsistent situations.

In classical facility location problem (FLP), it is assumed that the facilities are always available to serve customers; however, unexpected natural or man-made events such as earthquake and terrorist attacks bring about serious issues for companies since these miserable incidents make a full blockage in the flow of their supply chain network for a considerable amount of time. As stated by Shari et al. (2003), Motorola suffered significant supply chain delay after SARS outbreak. This example indicates that keeping supply chain networks away from disruptions is of great importance.

Usually, in traditional distribution network design primarily once a facility is incapable to serve customers, two approaches can be considered. The first is finding an alternative source of supply while the other imposes a penalty for unsatisfied demands. One should note these approaches may result in customer losing and big costs.

Triggered by these driving forces, this paper addresses the possibility of random facility disruption in the traditional

*Corresponding author Email address: zahedi.anaraki@gmail.com

capacitated facility location problem (CFLP). The remainder of this paper is organized as follow. The next section presents a brief overview of the key related research. Section 3 proposes a scenario-based formulation for the CFLP with random facility failure. Section 4 elaborates a Benders decomposition-based solution approach to solve the proposed model. The computational performance of the solution approach us analyzed in Section 5. Section 6 discusses concluding remarks and possible future guideline.

2. Literature Review

Based on considering capacity constraints, a wide variety of studies in the CFLP models have been addressed in literature. A Lagrangean relaxation method is proposed in the work of (Geoffrion and Bride, 1978) for CFLP. (Wentges, 1996) proposed a modified Benders decomposition algorithm with a fast convergence behavior to solve the CFLP. (Chen and Ting, 2008) applied Lagrangean heuristics to solve the single source capacitated facility location problem and evaluated the performance of their proposed solution method through two sets of benchmark problems. Moreover, a comprehensive and recent review on facility location models is presented by (Melo, Nickel and Saldanha-Da-Gama, 2009) to address a variety of future research directions. In past decades, the studies has conducted researches on developed version of CFLP such as competitive CFLP(Harks and von Falkenhausen, 2014; Rodrigues and Xavier, 2017; Beresnev and Melnikov, 2018) and multi-period CFLP (Correia and Melo, 2016).

(Ball and Lin, 1993) proposed an integer programming optimization model with valid inequalities for a reliable emergency service vehicle location problem and applied a branch and bound procedure to solve the problem. (Lim *et al.*, 2010) studied a reliable facility location problem in the presence of random facility disruptions incorporating two kinds of reliable and unreliable facilities. As a detailed study, (Snyder and Daskin, 2007) classified literature reviewing the unreliability of facilities in making location choices using a wide range of risk criteria to construct reliable facility location systems. The sample average approximation method is applied by (Gade and Pohl, 2009) to solve a capacitated facility location model with unreliable facilities. (Li and Ouyang, 2010) proposed a continuum approximation approach for the incapacitated facility location problem considering spatially correlated disruptions. For a review of designing supply chain models under facility disruption the reader can consult (Snyder, 2006) in which they presented an exhaustive review and categorize these models by the existing network status.

The review of stochastic facility location models can be found in works (Laporte, Louveaux and van Hamme, 1994; Snyder, 2006; Revelle, Eiselt and Daskin, 2008). (Razmi, Zahedi-Anaraki and Zakerinia, 2013) do a research on a reliable redesign model under uncertainty of parameters of demand and variable costs. Demand-side uncertainty has been discussed in the most of aforementioned papers, while this paper considers supply-side uncertainty and transportation cost uncertainty. Notably, it is addressed an additional objective function, minimization of the standard deviation of operational costs, in addition to the traditional sum of first stage investment cost and the expected second-stage transportation and non-utilize capacity penalty costs.

Research works involving multi-objective optimization under uncertainty are considerably smaller in number in comparison with those under deterministic situations. In (Cardona-Valdés, Álvarez and Ozdemir, 2011), lead time as a responsiveness measure as well as traditional total costs was addressed as a bi-objective two-stage stochastic programming formulation to cope with uncertainty in customer demands. A solution approach based on an extension of Benders decomposition method was developed to solve the proposed model. (Azaron *et al.*, 2008) addressed a robust multi-objective stochastic programming model for logistics network problem and applied goal attainment procedure to handle the conflicting objective functions. (Snyder and Daskin, 2005) proposed an integer programming model in which they studied a classical location problem taking into account failure of facilities with a known probability. The model had two objectives, the first one was classical objective that overlooks disruptions and the second was expected costs after realizing disruptions. Also, they minimized these objectives using a weighted sum method. The main drawbacks of traditional stochastic CFLPs come in the following manners:

- Taking account into reliability, stochastic and multi-objective issue simultaneously have been rarely addressed. Motivated by above discussion, this paper addresses these disadvantages by developing a bi-objective reliable capacitated facility location problem for designing a distribution network under uncertainty.
- In the approach, not only demands but also transportation, non-utilize capacity costs and facility capacities are all addressed as the uncertain parameters. To represent facility failure, potential scenarios are utilized, in order that these scenarios have been formerly identified by managers.

- Since the CFL has been known as strategic problem, achieving an optimal solution play an essential key in decreasing network costs while in the past decades, the most studies have focused on approximate method.
- To the best our knowledge, it is the first time that a Benders Decomposition algorithm, not being involved in keeping master-problem feasibility, is used to solve a scholastic reliable capacitated facility problem.
- Until now, most of researched Benders decomposition algorithm has to add feasibility cut to master problem iteratively, thereby not preventing to involving infeasibility space, which significantly boosts elapsed time. Therefore, in this paper, by adding sets of strengthening innovative redundant constraints to master problem, this algorithm is capable not to deal with these cuts. Additionally, efficiency of such constraints is confirmed via proving a mathematical theorem.

3. Problem description

Typically, in classical stochastic programming, there are two sets of decision variables available to decision makers the expected value of which constitutes objective function. The first-stage variables include decisions that are unaffected by uncertainty and required to made here-and-now. On the other hand, the second-stage variables are so-called wait-and-see ones and indicate decisions that should be made after realization of the random events. The second-stage decisions are confined by limitations that imposed by the second-stage problem.

Without loss of generality, suppose a system with a set of potential facilities exposed to failure, and fixed customer zones. There is a set of scenarios with given probability of occurrence with respect to demands, capacity, transportation and non-utilized penalty costs, and facilities failure. Moreover, customers demand is satisfied by the facilities that are not disrupted. Under these situations, the problem is to determine the location of each facility in the first stage and then the allocation of customers demand to be served by an open available facility after the realization of one scenario. Particularly, a penalty cost for a given scenario is also considered to avoid non-utilization of the capacity of each facility. The proposed model considers two objectives encompassing minimization of the expected total cost as well as standard deviation of operational costs under the following assumptions:

- The demand occurs for a single commodity and it can be served from multiple facilities
- Transportation and non-utilized capacity costs are linear
- There is no shortage at the whole system
- Multiple facilities can disrupt independently at the same time
- When facility failure occurs, it provides no service
- The capacity of each facility may change in every scenario

Moreover, the notations used to formulate the problem mathematically are as follows:

Sets

I: set of potential facility points

J: fixed locations of customer zones

S: set of potential scenarios

Parameters

d_{js}: demand of customer j in scenario s

b_i: maximum throughput production capacity at facility i

w_{is}: maximum expected warehousing capacity of facility i in scenario s

f_i: fixed cost of opening facility i

c_{ijs}: unit shipping cost from facility i to customer j in scenario s

π_{is}: unit penalty cost of non-utilize capacity at facility i in scenario s

p_s: occurrence probability of scenario s

a_{is}: 1 if facility i is failed in scenario s and 0 otherwise

N: upper limit on number of facilities which can be opened

ρ: Weight coefficient in objective function

Decision Variables

x_{ijs}: quantity of demand shipment from facility i to customer j in scenario s

z_{is}: non-utilized capacity at facility i in scenario s

y_i: 1 if facility i is opened; otherwise 0

Accordingly, the concerned objectives are as follows:

$$g_1 = \sum_{i\in I} f_i\, y_i + \sum_{s\in S}\sum_{i\in I}\sum_{j\in J} p_s\, c_{ijs}\; x_{ijs} + \sum_{s\in S}\sum_{i\in I}\sum_{j\in J} p_s\, \pi_{is}\; z_{is} \tag{1}$$

$$g_2 = \sum_{s\in S} p_s \left| \sum_{i\in I}\sum_{j\in J} c_{ijs}\, x_{ijs} + \sum_{i\in I}\sum_{j\in J} p_s\, \pi_{is}\; z_{is} - \Big(\sum_{s\in S}\sum_{i\in I}\sum_{j\in J} p_s\, c_{ijs}\; x_{ijs} + \sum_{s\in S}\sum_{i\in I}\sum_{j\in J} p_s\, \pi_{is}\; z_{is}\Big) \right| \tag{2}$$

Objective g_1 computes the summation of fixed opening costs and expected transportation and penalty costs. Objective g_2 considers the standard deviation of total operational costs. The model minimizes a weighted sum of the two objectives $g_1 + (1-\rho)\, g_2$, where $0 \le \rho \le 1$. The reliable capacitated facility location problem is formulated as follows:

$$Min \;\; \rho\, g_1 + (1-\rho)\, g_2 \qquad (3)$$

$$\sum_{i \in I} x_{ijs} \ge d_{js} \qquad \forall j \in J, \forall s \in S \qquad (4)$$

$$\sum_{j \in J} x_{ijs} + z_{is} = (1 - a_{is})\, b_i\, w_{is}\, y_i \qquad \forall i \in I, \forall s \in S \qquad (5)$$

$$\sum_{i \in I} y_i \le N \qquad (6)$$

$$x_{ijs}, z_{is} \ge 0,\; y_i \in \{0,1\} \qquad (7)$$

Constraint (4) ensures that total quantity of delivered shipment meets demand in each scenario. Constraint (5) is related to facilities reliability and prevents a customer from being allocated to a facility which is failed in a given scenario. Constraint (6) limits the number of facilities that can be opened. Finally, constraint (7) enforces non-negativity and binary restrictions on the corresponding decision variables.

3.3. Linearization of the proposed model

The proposed mathematical model (3)-(7) is a mixed-integer nonlinear programming model (MINLP) because of the nonlinear term, the second objective function, in equation (3). Transformation into a linear equivalent formulation requires manipulation and substitution which will result in an accurate transformation of the problem. To do so, the new non-negative variables v_s^+ and v_s^- are introduced and thus the second objective function in equation (3) is rewritten as follows:

$$(1-\rho) \sum_{s \in S} p_s\, v_s^+ \qquad (8)$$

With respect to such rewritten terms, the following constraint must be added to the original formulation:

$$v_s^+ - v_s^- = \sum_{i \in I} \sum_{j \in J} c_{ijs}\, x_{ijs} + \sum_{i \in I} \sum_{j \in J} p_s\, \pi_{is}\, z_{is} - \left(\sum_{s \in S} \sum_{i \in I} \sum_{j \in J} p_s\, c_{ijs}\, x_{ijs} + \sum_{s \in S} \sum_{i \in I} \sum_{j \in J} p_s\, \pi_{is}\, z_{is}\right) \qquad \forall s \in S \qquad (9)$$

In this manner, the optimal solution obtained from the equivalent mixed-integer linear model, consisting of constraints (3)-(7) and constraint (9) with the linearized form of the second term in the weighted objective function, i.e. equation (8), is equivalent to the optimal solution of the original mixed-integer nonlinear model (3)-(7).

4. Benders-Decomposition Algorithm

Benders decomposition algorithm involves decomposing the overall formulation into a master problem and a dual sub-problem, and then solving them iteratively by utilizing the solution of one in the other (Benders, 1962). The master problem involves integer variables and the dual sub-problem, on the other hand, is a linear program incorporating the integer variables as parameters whose values are determined solving the master problem. One should note that the solutions of the dual sub-problem generate feasibility or optimality cut for the corresponding integer solution that might be added to the master problem. In an iteration of the overall solution procedure, the current master problem is solved to determine a lower bound for the overall problem along with the corresponding values of the integer variables. Then the dual sub-problem is solved and an upper bound obtained utilizing its objective value along with the cost components implied by the master problem solution (Üster and Agrahari, 2011) . According to relevant literature, Benders decomposition algorithm usually is successful for problem with complicated variables.

Benders decomposition technique has been applied for solving location problems in the works of (Wentges, 1996) for the capacitated facility location problem,(Balas, 1965) for the incapacitated facility location problem, in (Magnanti and Wong, 1981) for P-Median problems and (Sodagari and Sadeghi, 2015) for integrated logistics

Network designing problem with multiple capacities. In order to find a holistic review of Benders decomposition application on location problems interested readers is referred to (Costa, 2005).
One should note that the underlying problem belongs to NP-Hard class of problems as is the combination of large number of potential scenarios with capacitated version of incapacitated facility location problem, a well-known NP-Hard problem (Cornuejols et al. 1990). Thus, the proposed reliable capacitated facility location problem with large numbers of scenarios and such a structure with complicated variable, shown in Figure 1, seems a suitable candidate for Benders decomposition approach as it inhabits an exploitable primal configuration.

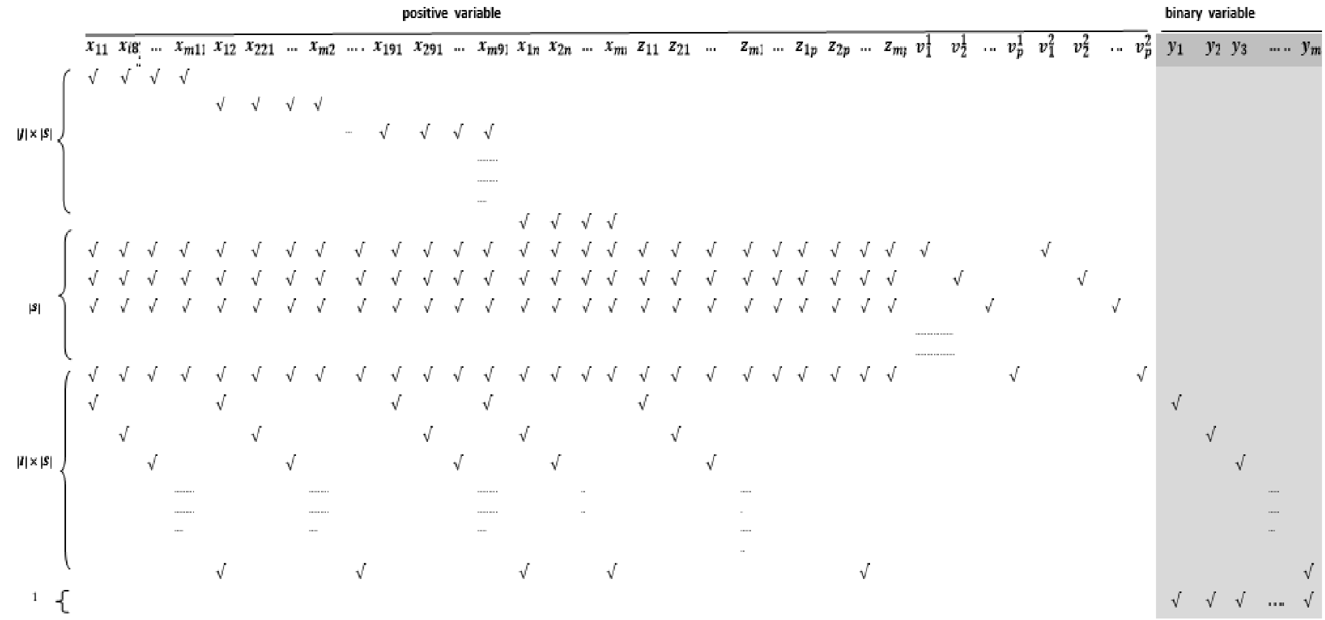

Fig. 1. Model variable structure

4.1. Master problem and the dual sub-problem

By fixing the binary location variable $y=\overline{y_i}$ to yield a feasible solution for the linearized mixed-integer programming model, the following problem called sub-problem will be obtained:

$$Min\ \rho\ (\sum_{i\in I} f_i\,\bar{y}_i+\sum_{s\in S}\sum_{i\in I}\sum_{j\in J}p_s\,c_{ijs}\ x_{ijs}+\sum_{s\in S}\sum_{i\in I}\sum_{j\in J}p_s\,\pi_{is}\ z_{is}+(1-\rho)\ \sum_{s\in S}p_s\,v_s^+ \tag{10}$$

$$v_s^+-v_s^-=\sum_{i\in I}\sum_{j\in J}c_{ijs}\,x_{ijs}+\sum_{i\in I}\sum_{j\in J}p_s\,\pi_{is}\ z_{is}-(\sum_{s\in S}\sum_{i\in I}\sum_{j\in J}p_s\,c_{ijs}\ x_{ijs}+\sum_{s\in S}\sum_{i\in I}\sum_{j\in J}p_s\,\pi_{is}\,z_{is}\qquad \forall s\in S \tag{11}$$

$$\sum_{i\in I}x_{ijs}\geq d_{js}\qquad \forall j\in J,\forall s\in S \tag{12}$$

$$\sum_{j\in J}x_{ijs}+z_{is}=(1-a_{is})\,b_i\,w_{is}\ \bar{y}_i\qquad \forall i\in I,\forall s\in S \tag{13}$$

$$x_{ijs},z_{is}\geq 0 \tag{14}$$

The dual sub-problem by associating the dual variablesα_s,β_{js} and δ_{is} to the constraint (11) - (13), respectively can be written as follow:

$$Max\ \sum_{j\in J}\sum_{s\in S}d_{js}\ \ \beta_{js}+\sum_{j\in J}\sum_{s\in S}(1-a_{is})\,b_i\,w_{is}\,\bar{y}_i\ \ \delta_{is} \tag{15}$$

$$\alpha_s\leq p_s\,(1-\rho)\qquad \forall s\in S \tag{16}$$

$$-\alpha_s\leq 0\qquad \forall s\in S \tag{17}$$

$$-c_{ijs}\,\alpha_s + \sum_{s\in S} p_s c_{ijs}\,\alpha_s + \beta_{js} + \delta_{is} \le \rho\, p_s\, c_{ijs} \qquad \forall i \in I, \forall j \in J, \forall s \in S \qquad (18)$$

$$-\pi_{is}\,\alpha_s + \sum_{s\in S} p_s \pi_{is}\;\alpha_s + \delta_{is} \le \rho\, p_s\, \pi_{is} \qquad \forall i \in I, \forall s \in S \qquad (19)$$

$$\beta_{js} \ge 0 \; and \;\; \delta_{is}, \alpha_s{:}\, free\ in\ sign \qquad (20)$$

By adding $\sum_{j\in J}\sum_{s\in S}(1-a_{is})\, b_i\, w_{is} y_i \ \ge \sum_{j\in J} d_{js}$ that it is the redundant constraint for proposed model, we obtain the master problem as follows:

$$Max \;\; \varphi + \sum_{i\in I} f_i\, y_i \qquad (21)$$

$$\varphi \ge \sum_{j\in J}\sum_{s\in S} d_{js}\;\overline{\beta_{Js}} + \sum_{j\in J}\sum_{s\in S}(1-a_{is})\, b_i\, w_{is}\;\overline{\sigma_{\iota s}}\; y_i \qquad (22)$$

$$\sum_{i\in I} y_i \le N \qquad (23)$$

$$\sum_{j\in J}\sum_{s\in S}(1-a_{is})\, b_i\, w_{is} y_i \ \ge \sum_{j\in J} d_{js} \qquad (24)$$

$$\varphi{:}\, free\ in\ sign\ and\ y_i \in \{0,1\} \qquad (25)$$

Up till now, most of researched Benders decomposition algorithm has to add feasibility cut to master problem iteratively, thereby not preventing to involving infeasibility space, which significantly boosts elapsed time. Therefore, in this paper, by adding sets of strengthening innovative redundant constraints to master problem, this algorithm is capable not to deal with these cuts. Additionally, efficiency of such constraints is confirmed via proving the below mathematical theorem.

Theorem. The DBSP (15)-(20) is always bounded.

Proof. It is a well-known property in operation research that if a dual problem is unbounded, then the corresponding primal problem is infeasible. As an equivalent predicate if one primal problem is feasible, then its dual problem will be bounded.

Let Δ be the set of all BMP feasible solutions in one iteration, i.e. $\{\mathbf{y}\} \in \Delta$, after introducing the redundant constraint (I) to the BMP and taking the fact into account that adding such surrogate constraint, although not needed for the correct formulation of the original mixed-integer formulation, ensures customer satisfaction in all scenarios, thus the BSP is always feasible in Δ and consequently, the DBSP is bounded.

Corollary. A Benders feasibility cut is no longer required to add to the BMP.

4.2. Benders decomposition algorithm

The Benders decomposition approach is formally described below, in which *UB* and *LB* are the upper and lower bounds consecutively. Notably,

Step 1: Initialization

Set UB=+∞, LB=-∞, ε =0.001, $Iter$=0, and $Max.Itr$ = 500.

Solve the current master problem using the surrogate constraint (21) to obtain values for $\overline{y_i}$ and Z_{MNP}. Set LB= Z_{MNP}.

Step 2*:*

Iter = Iter+1

Solve the dual sub-problem and let the optimal value by Z_{DSP}.

If $ZDSP_{+}\sum_{i\in I} f_i\, \bar{y}_i < UB$ then $UB = Z_{DSP+}\sum_{i\in I} f_i\, \bar{y}_i$.Store($\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\delta}$).

Step 3: Add the optimality cut (22) into the current master problem.

Solve the master problem and let the optimal value by Z_{MNP}. Set LB= Z_{MNP} and store ***y***.

Step 4**:** If ((UB-LB)/$UB \le \varepsilon$ *or* $Iter \ge Max.Iter$), terminate and return the corresponding optimal solution of the original problem. Otherwise, go to Step 2.

A lower bound on the original problem objective function is the master problem solution. The lower bound can be improved at each iteration by adding a strong cut, like the Benders cut. Finally, the optimal solution of the original problem has been obtained by converging lower bound and upper bound.

5. Proposed Meta-heuristic Algorithm

In this section, the meta-heuristic algorithm of Imperialist Competitive (Atashpaz-Gargari and Lucas, 2007) is introduced and applied to this problem. The pseudo-codes of the proposed algorithm are shown in Figure 2. Given this fact that the purpose of this article is to emphasize the effective role of decomposition-Benders especially without infeasible cuts in comparison with other related method, either exact or approximate, in this paper metaheuristic algorithm generally is described and defining the detailed comment related to ICA is prevented

.

Begin ICA

Generating the first population randomly

Calculating the objective function for the first population in following step:

Decreasing the first population based on their objective function costs

Specifying the imperialist states

Form the empires by sharing the colonies to the imperialist states

**** While (the specified decades are not passed)*** *do*

***** While (all empires selected)*** *do*

Choose the empire

****** While*** *(all colonies selected) do*

Choosing the colony

Moving the colony toward its imperialist state (assimilation)

Changing the direction of movement for some colonies (revolution)

Calculating the objective function cost for the populations in new positions in following step:

Evaluating two new costs and selecting the best one

Substituting colony with new one

End * while;**

Descending all colonies of empire based on their cost functions

Checking cost of all colonies in each empire

If *there is a colony with a lower cost than it's imperialist*

Exchanging the position of the colony and the imperialist

End if

Updating the location of the empire

End ** while;

Computing total cost of empires

Searching for the weakest empire

Giving one of its colonies to the empire which is more powerful

Checking the number of colony in each empire

If *there is an empire with no colony*

Omitting the empire and possessing its imperialist for the best empire

End if

End * while;

End ICA

Fig. 2. Pseudo-code of imperialist competitive algorithm proposed

6. Computational results

The proposed solution method was implemented in GAMS 23.20 and executed on a machine with 2.7 GHz Core 2 Duo CPU and 3 GB RAM. Furthermore, the original mixed-integer model given in Section 3 was tested in CPLEX 12.0, well-known optimization software. The purpose of the experiments is to determine the efficiency of the proposed Benders method by comparing the results with those of CPLEX within a reasonable time limit. The test instances generations are described in the following section.

6.1. Parameters tuning

In an optimization problem, the performance of approximate algorithms utilized depends on their parameters significantly. Thus, tuning ones plays an important role in helping to gain efficient solutions.

In this paper, the parameters of the algorithm are tuned by the response surface technique which is a well-known method in this way. Using quantitative data extracted from appropriate experiments, this technique can be considered as a statistical technique which controls and solves multi-variant equations. After distinguishing the effective parameters, two surfaces are definite, namely the upper surface and the lower surface. Then 2^n examinations in which n is the numbers of critical parameter, should be calculated on various surfaces of the parameters and after that, reports are evaluated based on the objective function. To conclude, the parameter values being critical for the algorithm, are proposed to use. In this paper, tuned parameters is divided to three separate classes of instances. There are four critical parameters for the ICA, that is to say N_{imp} , β , $p_{\mathrm{Re}volution}$ and ξ . Table 1 displays the values of the parameters proposed by RSM. In addition, these parameters, another parameter, namely the numbers of maximum iterations, which is equal to 200, is defined for all the instances in order to control the elapsed time.

Table1

Value of parameters tuned

	instance dimensions											
	S	*M*	*L*	*S*	*M*	*L*	*S*	*M*	*L*	*S*	*M*	*L*
parameters		N_{imp}			β			$P_{\mathrm{Re}volution}$			ξ	
	80	90	100	2	2	2	1	1	1	0	1	0

6.2. Experimental results

A set of test instances with realistic size is developed with changing the number of potential facility locations | *I* | (10, 15, 20, and 25), the number of customers | *J* | (100, 200, 300, 400, and 500), and the number of scenarios | *S* | (30, 40, 50, and 60).

Demand data was randomly generated from *U* [50, 200], unit fixed cost *U* [5000, 10000], unit penalty cost *U* [5, 10], and maximum throughput capacity *U* [0.4, 1]. To generate unit shipping costs first Euclidian distances between each facility *i* and customer *j* over a uniform square, [0, 1] × [0, 1], is calculated, and then multiplied by a random number chosen from *U* [10, 20]. Capacity of each facility is generated according to *U* [10×*avgCap*, 25× *avgCap*] where$avgCap = \frac{\sum_j \sum_s d_{js}}{|I||S|}$. Number of facilities, *N*, was derived from *U* [0.3× | *I* |, 0.9× | *I* |] and rounded to the nearest integer number. Probability of each scenario was originated from *U* [0.01, 1] in order that$\sum p_s = 1$. Finally, to generate facilities failure, a_{is} , first the $P_a = \alpha$ (in this problem0.1) is defined which illustrates the probability of facility disruption in each scenario, then a random number was generated between U [0,1], if the generated number is smaller than α, a_{is} =1, otherwise a_{is} =0.

6.3. Algorithmic performance

Table 2 compares the performance of the proposed Benders decomposition method, MIP solver, CPLEX and ICA with each other in terms of CPU time and solution qualities. The first six columns illustrate test problem characteristics; the next columns indicate the optimal solution, Bender CPU time, CPLEX CPU time, and Benders iteration number, consecutively. The two next columns are related to CPLEX and Benders optimality gaps and finally last two column is the optimal solution, Bender CPU time related to ICA. Remarkably, the optimality gap could be calculated as follows.

$$\frac{100\times(BDsol - CPLEXsol)}{CPLEXsol} \qquad (26)$$

Where *BD.Sol* and *CPLEX.Sol* denote the optimal solutions obtained by Benders decomposition and CPLEX, consecutively.

Noteworthy, in cases that CPLEX is unable to reach the optimal solution in a reasonable time, i.e. 1h, the optimality gap is calculated as the tightness between the last lower and upper bounds.

$$\frac{100\times(UB-LB)}{UB}$$

Table 2
Obtained results for the considered instance

Facilities number	Customer number	scenario number	Binary variable	positive variable Number	Constraint Number	Bender Optimal solution	Bender CPU_Time	CPLEX CPU_Time	Benders iteration	CPLELX Gap%	Benders Optimality Gap%	Meta Optimal-solution	Meta CPU_Time
$\|I\|=10$	$\|J\|=100$	$\|S\|=30$	10	30360	3331	1172680	66.1	236.3	25	0	0	1172680	59.5
		$\|S\|=40$	10	40480	4441	1182164	36.7	136.5	14	0	0	1182164	35.6
		$\|S\|=50$	10	50600	5551	1185034	27.7	85.5	19	0	0	1185034	25.8
		$\|S\|=60$	10	60720	6661	1204846	58.8	158	33	0	0	1204846	56.4
	$\|J\|=200$	$\|S\|=30$	10	60360	6331	2349202	22.8	59	14	0	0	2349202	21
		$\|S\|=40$	10	80480	8441	2353540	17.4	96	7	0	0	2494752.4	17.4
		$\|S\|=50$	10	100600	10551	2316163	67.6	388.2	24	0	0	2478294.4	62.9
		$\|S\|=60$	10	120720	12661	2339203	37	338.1	11	0	0	2479555.2	35.5
	$\|J\|=300$	$\|S\|=30$	10	90360	9331	3443641	49	640.1	20	0	0	3684695.9	47.5
		$\|S\|=40$	10	120480	12441	3460190	236.9	2362.7	36	0	0	3529393.8	227.4
		$\|S\|=50$	10	150600	15551	3504673	356	2662.4	31	0	0	3750000.1	327.5
		$\|S\|=60$	10	180720	18661	3512632	286.7	3461.4	31	0	0	3582884.6	275.2
	$\|J\|=400$	$\|S\|=30$	10	120360	12331	4555983	132.7	1027.3	26	0	0	4783782.2	128.7
		$\|S\|=40$	10	160480	16441	4554948	121.3	477.9	17	0	0	4646047	121.3
		$\|S\|=50$	10	200600	20551	4534424	224.4	2474	25	0	0	4761145.2	217.7
		$\|S\|=60$	10	240720	24661	4572524	195.2	1379	21	0	0	4663974.5	183.5
	$\|J\|=500$	$\|S\|=30$	10	150360	15331	5740821	83.7	431.9	16	0	0	5855637.4	79.5
		$\|S\|=40$	10	200480	20441	5878053	746.6	>3600	53	0.033	0	6289516.7	701.8
		$\|S\|=50$	10	250600	25551	5810607	452.1	>3600	29	0.006	0	5926819.1	425
		$\|S\|=60$	10	300720	30661	5599170	402.6	>3600	17	0.003	0	5823136.8	382.5
$\|I\|=15$	$\|J\|=100$	$\|S\|=30$	15	45510	3481	1138965	61.3	119.9	51	0	0	1207302.9	58.2
		$\|S\|=40$	15	60680	4641	1138738	102.7	150.5	69	0	0	1184287.5	101.7
		$\|S\|=50$	15	75850	5801	1161567	173	413.5	86	0	0	1231261	166.1
		$\|S\|=60$	15	91020	6961	1174033	267.4	663	107	0	0	1232734.7	243.3
	$\|J\|=200$	$\|S\|=30$	15	90510	6481	2310301	235.7	398.1	102	0	0	2425816.1	216.8
		$\|S\|=40$	15	120680	8641	2315467	507.5	1393.8	149	0	0	2431240.4	507.5

		$\|S\|=50$	15	150850	10801	2419469	779.1	1998.1	171	0	0	2516247.8	701.2
		$\|S\|=60$	15	181020	12961	2296318	321.6	1210.5	68	0	0	2365207.5	299.1
	$\|J\|=300$	$\|S\|=30$	15	135510	9481	3420724	116.1	779.9	35	0	0	3625967.4	113.8
		$\|S\|=40$	15	180680	12641	3417486	259.6	1121.6	52	0	0	3451660.9	244
		$\|S\|=50$	15	225850	15801	3461231	549.7	>3600	87	0.015	0	3634292.6	544.2
		$\|S\|=60$	15	271020	18961	3453534	785.3	>3600	114	0.028	< 0.001	3626210.7	769.6
	$\|J\|=400$	$\|S\|=30$	15	180510	12481	4405029	207	1175.4	43	0	0	4669330.7	207
		$\|S\|=40$	15	240680	16641	4579156	404.1	3146.4	61	0	0	4853905.4	383.9
		$\|S\|=50$	15	300850	20801	4569953	578.2	>3600	82	0.015	< 0.001	4752751.1	560.9
		$\|S\|=60$	15	361020	24961	46121565	645.4	>3600	111	0.028	< 0.001	49811290.2	613.1
	$\|J\|=500$	$\|S\|=30$	15	225510	15481	5609283	534.3	3329.4	53	0	0	6058025.6	529
		$\|S\|=40$	15	300680	20641	5465110	372.9	>3600	35	0.011	0	5465110	358
		$\|S\|=50$	15	375850	25801	5598789	789.1	>3600	73	0.028	< 0.001	5766752.7	749.6
		$\|S\|=60$	15	451020	30961	5721458	867.8	>3600	69	0.041	< 0.001	6179174.6	815.7
$\|I\|=20$	$\|J\|=100$	$\|S\|=30$	20	60660	3631	1131332	130.7	347.8	86	0	0	1233151.9	126.8
		$\|S\|=40$	20	80880	4841	1162654	339.9	712.6	154	0	0	1255666.3	319.5
		$\|S\|=50$	20	101100	6051	1154110	491.4	1285.1	173	0	0	1292603.2	486.5
		$\|S\|=60$	20	121320	7261	1175741	798.1	2214.8	196	0	0	1281557.7	734.3
	$\|J\|=200$	$\|S\|=30$	20	120660	6631	2200882	611.9	1100.8	35	0	0	2376952.6	550.7
		$\|S\|=40$	20	160880	8841	2226211	347.5	2647.1	84	0	0	2292997.3	347.5
		$\|S\|=50$	20	201100	11051	2261965	860	3424	184	0	0	2533400.8	860
		$\|S\|=60$	20	241320	13261	2219610	429.2	3029.3	73	0	0	2286198.3	429.2
	$\|J\|=300$	$\|S\|=30$	20	180660	9631	3321573	408.6	>3600	101	0.018	< 0.001	3321573	380
		$\|S\|=40$	20	240880	12841	3314394	391.5	>3600	73	0.003	< 0.001	3579545.5	387.6
		$\|S\|=50$	20	301100	16051	3348629	707.5	>3600	101	0.068	< 0.001	3750464.5	658
		$\|S\|=60$	20	361320	19261	3390731	851.8	>3600	98	0.089	< 0.001	3797618.7	809.2
	$\|J\|=400$	$\|S\|=30$	20	240660	12631	4385055	439.1	>3600	79	0.022	< 0.001	4735859.4	408.4
		$\|S\|=40$	20	320880	16841	4506314	810.2	>3600	110	0.043	< 0.001	4596440.3	810.2
		$\|S\|=50$	20	401100	21051	4456887	621.5	>3600	65	0.031	< 0.001	4858006.8	578

		$\lvert S\rvert=60$	20	481320	25261	4565214	796.9	>3600	86	0.084	< 0.001	5067387.5	773
	$\lvert J\rvert=500$	$\lvert S\rvert=30$	20	300660	15631	5450242	690.8	>3600	93	0.042	< 0.001	5886261.4	649.4
		$\lvert S\rvert=40$	20	400880	20841	5507908	1151.9	>3600	123	0.082	< 0.001	5673145.2	1059.7
		$\lvert S\rvert=50$	20	501100	26051	5421456	1998.1	Out of memory	175	-	< 0.001	5855172.5	1798.3
		$\lvert S\rvert=60$	20	601320	31261	5612458	2629.2	Out of memory	211	-	< 0.001	6005330.1	2418.9
$\lvert I\rvert=25$	$\lvert J\rvert=100$	$\lvert S\rvert=30$	25	75810	3781	11083743	67.1	213	37	0	0	12192117.3	61.7
		$\lvert S\rvert=40$	25	101080	5041	1142387	213	1201.6	84	0	0	1279473.4	193.8
		$\lvert S\rvert=50$	25	126350	6301	1153695	530.3	>3600	168	0.002	< 0.001	1257527.6	525
		$\lvert S\rvert=60$	25	151620	7561	1153483	937.5	>3600	225	0.008	< 0.001	1199622.3	862.5
	$\lvert J\rvert=200$	$\lvert S\rvert=30$	25	150810	6781	2198970	456.2	2348.6	70	0	0	2462846.4	428.8
		$\lvert S\rvert=40$	25	201080	9041	2198548	1096.0	>3600	226	0.004	< 0.001	2308475.4	1019.3
		$\lvert S\rvert=50$	25	251350	11301	2238548	708.5	>3600	117	0.003	< 0.001	2440017.3	687.2
		$\lvert S\rvert=60$	25	30620	13561	2298758	892.1	>3600	136	0.002	< 0.001	2459671.1	838.6
	$\lvert J\rvert=300$	$\lvert S\rvert=30$	25	225810	9781	3399151	750	>3600	145	0.019	< 0.001	3433142.5	682.5
		$\lvert S\rvert=40$	25	301080	13041	3317075	799.7	>3600	115	0.039	< 0.001	3715124	743.7
		$\lvert S\rvert=50$	25	376350	16301	3409764	1019.1	>3600	129	0.079	< 0.001	3648447.5	988.5
		$\lvert S\rvert=60$	25	451621	19561	3421458	1498.6	Out of memory	178	-	< 0.001	3695174.6	1468.6
	$\lvert J\rvert=400$	$\lvert S\rvert=30$	25	300810	12781	4418898	1397.3	Out of memory	165	-	< 0.001	4639842.9	1383.3
		$\lvert S\rvert=40$	25	401080	17041	4514569	1984.6	Out of memory	211	-	< 0.001	4920880.2	1984.6
		$\lvert S\rvert=50$	25	501350	21301	4498985	2011.1	Out of memory	159	-	< 0.001	5173832.8	1830.1
		$\lvert S\rvert=60$	25	601620	25561	4598547	2145.5	Out of memory	182	-	< 0.001	4966430.8	1952.4
	$\lvert J\rvert=500$	$\lvert S\rvert=30$	25	375810	15781	5579433	2065.6	Out of memory	147	-	< 0.001	6472142.3	1859
		$\lvert S\rvert=40$	25	501080	21041	5785698	2454.5	Out of memory	169	-	< 0.001	6132839.9	2233.6
		$\lvert S\rvert=50$	25	626350	26301	5798786	2758.3	Out of memory	211	-	< 0.001	6436652.5	2537.6
		$\lvert S\rvert=60$	25	751620	31561	5632154	3121.2	Out of memory	256	-	< 0.001	5632154	2996.4

By comparing the results, it is evident that Benders decomposition algorithm is much more efficient than the commercial optimization software in both terms of CPU time and solution qualities. As table 1 reveals, Benders decomposition approach is able to find the optimal solution of each test problem in a far shorter time than those of CPLEX. Furthermore, there is no significant difference between the obtained solutions by both methods (the optimality gap is zero). the Benders algorithm is able to reach to the solution in a reasonable time while CPLEX faces with the lack of memory, and is not even able to obtain an optimal solution in some cases, consequently. As obvious in Table 1, Benders's CP-time is acceptably more than ICA's, in interval 0% to 13%, while Benders's optimal solution is significantly less than ICA's, in interval 0% to 19%.

6.4. Tradeoff Curves

The tradeoff curve is constructed for the proposed CFLP using the class of 10-200-30 where they imply the number of facilities, number of customers, and number of scenarios, respectively. According Figure 3, the vertical axis plots the standard deviation of the operational costs, and the horizontal axis shows the total costs. Each point on the curves represent a different solution; the optimal solution of the problem considering only the first objective (α=1) is the right-most point on the curve, while the left-most point illustrates the optimal solution of the second objective function (α=0). All six different points (solutions) are obtained along varying α in 0, 0.2, 0.4, 0.6, 0.8, and 1. The conflict behavior of such objective functions demonstrates that the represented results are clearly Pareto ones.

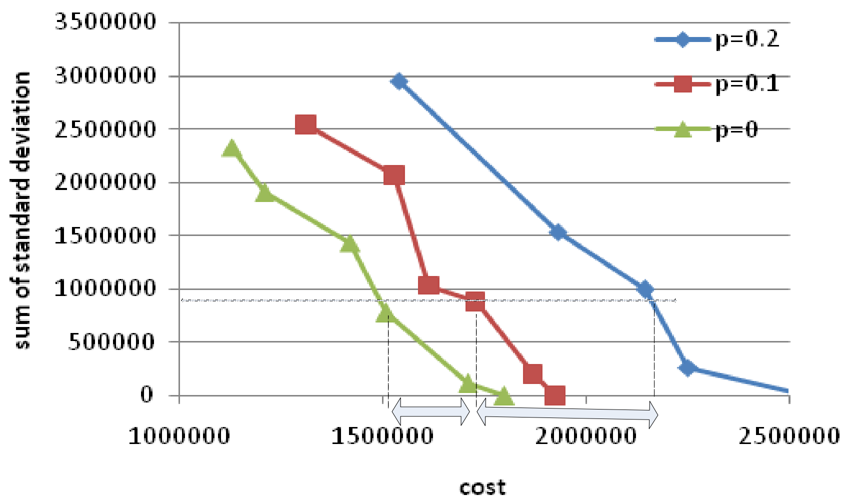

Fig. 3. Comparing pareto solution

Furthermore, three scenarios for occurrence of each scenario are considered along varying probability of these (p) in 0, 01, and 0.2, which 0 is the lack of any disruption. Three curves come from each relevant probability illustrated in green, red, and blue color, consecutively. As obvious, with gradually increasing probability, a significantly upward curve-shifting reaction is evident, which means that with boosting possibility of disruption, there are an exponential increase in both terms of bi-objectives, expected network cost and expected standard deviation.

7. Conclusion and Future Works

Facility location decisions are critical to the efficient and effective supply chain network. The classical facility does not take the facility into account; however, such events result in disruption or complete blockage in the supply chain. This paper presents a scenario-based stochastic programming model for incorporating reliability of facilities under an uncertain environment into classical CFLPs based on reliable supply chain network designing models. The objective is to determine a good set of locations while minimizing expected strategic and operational costs, as well as standard operating cost deviation in a bi-objective scheme. Taking into account features such as accessibility of facilities, penalty for nonoperational capacities, variable scenario-based capacity, and parameter uncertainty are properties of the proposed model. Therefore, this approach seems to be a good way of capturing the high complexity of the problem. The proposed formulation is solved using

Benders decomposition, with promising results. Adding a supporting innovative redundant constraint to the master's infeasibility in a master problem plays a significant and promising role in reducing the time-consuming process of the Benders algorithm. In this approach, in terms of time-consuming the Benders algorithm is able powerfully to compete with metaheuristic algorithms, but with an exact solution. To demonstrate the effectiveness of the Benders algorithm, a metaheuristic algorithm, namely ICA, is presented. The computational results of the samples illustrate that Benders algorithm run time is much less than the run time of the CPLEX algorithm in GAMS software, while in a large-sized instance, GAMS is not able to obtain optimal solutions also Benders algorithm can obtain a solution exact in contrast to ICA obtaining an approximate solution spending not much more time. The conflict behavior of such objective functions is demonstrated. With the boosting possibility of disruption, results indicate that there is an exponential increase in both terms of two goals, expected network cost and expected standard deviation. As a possible future research direction, one should note that the proposed model can be extended to multi-period case addressing inventory management decisions at different time intervals.

References

Atashpaz-Gargari, E. and Lucas, C. (2007) ‘Imperialist competitive algorithm: an algorithm for optimization inspired by imperialistic competition’, in *Evolutionary computation, 2007. CEC 2007. IEEE Congress on*. IEEE, pp. 4661–4667.

Azaron, A. *et al.* (2008) ‘A multi-objective stochastic programming approach for supply chain design considering risk’, *International Journal of Production Economics*, 116(1), pp. 129–138.

Balas, E. (1965) ‘An additive algorithm for solving linear programs with zero-one variables’, *Operations Research*, 13(4), pp. 517–546.

Ball, M. O. and Lin, F. L. (1993) ‘A reliability model applied to emergency service vehicle location’, *Operations research*, 41(1), pp. 18–36.

Benders, J. F. (1962) ‘Partitioning procedures for solving mixed-variables programming problems’, *Numerische mathematik*, 4(1), pp. 238–252.

Beresnev, V. and Melnikov, A. (2018) ‘Exact method for the capacitated competitive facility location problem’, *Computers & Operations Research*, 95, pp. 73–82.

Cardona-Valdés, Y., Álvarez, A. and Ozdemir, D. (2011) ‘A bi-objective supply chain design problem with uncertainty’, *Transportation Research Part C: Emerging Technologies*, 19(5), pp. 821–832.

Chen, C.-H. and Ting, C.-J. (2008) ‘Combining lagrangian heuristic and ant colony system to solve the single source capacitated facility location problem’, *Transportation research part E: logistics and transportation review*, 44(6), pp. 1099–1122.

Correia, I. and Melo, T. (2016) ‘Multi-period capacitated facility location under delayed demand satisfaction’, *European Journal of Operational Research*, 255(3), pp. 729–746.

Costa, A. M. (2005) ‘A survey on benders decomposition applied to fixed-charge network design problems’, *Computers & operations research*, 32(6), pp. 1429–1450.

Gade, D. and Pohl, E. A. (2009) ‘Sample average approximation applied to the capacitated-facilities location problem with unreliable facilities’, *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, 223(4), pp. 259–269.

Geoffrion, A. and Bride, R. M. (1978) ‘Lagrangean relaxation applied to capacitated facility location problems’, *AIIE transactions*, 10(1), pp. 40–47.

Harks, T. and von Falkenhausen, P. (2014) ‘Optimal cost sharing for capacitated facility location games’, *European Journal of Operational Research*, 239(1), pp. 187–198.

Laporte, G., Louveaux, F. V. and van Hamme, L. (1994) ‘Exact solution to a location problem with stochastic demands’, *Transportation Science*, 28(2), pp. 95–103.

Li, X. and Ouyang, Y. (2010) ‘A continuum approximation approach to reliable facility location design under correlated probabilistic disruptions’, *Transportation research part B: methodological*, 44(4), pp. 535–548.

Lim, M. *et al.* (2010) ‘A facility reliability problem: Formulation, properties, and algorithm’, *Naval Research Logistics (NRL)*, 57(1), pp. 58–70.

Magnanti, T. L. and Wong, R. T. (1981) ‘Accelerating Benders decomposition: Algorithmic enhancement and model selection criteria’, *Operations research*, 29(3), pp. 464–484.

Melo, M. T., Nickel, S. and Saldanha-Da-Gama, F. (2009) ‘Facility location and supply chain management–A review’, *European journal of operational research*, 196(2), pp. 401–412.

Razmi, J., Zahedi-Anaraki, A. and Zakerinia, M. (2013) ‘A bi-objective stochastic optimization model for reliable warehouse network redesign’, *Mathematical and Computer Modelling*, 58(11–12), pp. 1804–1813.

Revelle, C. S., Eiselt, H. A. and Daskin, M. S. (2008) ‘A bibliography for some fundamental problem categories in discrete location science’, *European Journal of Operational Research*, 184(3), pp. 817–848.

Rodrigues, F. C. and Xavier, E. C. (2017) ‘Non-cooperative capacitated facility location games’, *Information Processing Letters*, 117, pp. 45–53.

Snyder, L. V. (2006) ‘Facility location under uncertainty: a review’, *IIE transactions*, 38(7), pp. 547–564.

Snyder, L. V. and Daskin, M. S. (2005) 'Reliability models for facility location: the expected failure cost case', *Transportation Science*, 39(3), pp. 400–416.

Snyder, L. V. and Daskin, M. S. (2007) 'Models for reliable supply chain network design', in *Critical Infrastructure*. Springer, pp. 257–289.

Sodagari, N. and Sadeghi, A. (2015) 'A Benders Decomposition Method to Solve an Integrated Logistics Network Designing Problem with Multiple Capacities', *Journal of Optimization in Industrial Engineering*, 8(18), pp. 79–93.

Üster, H. and Agrahari, H. (2011) 'A Benders decomposition approach for a distribution network design problem with consolidation and capacity considerations', *Operations Research Letters*, 39(2), pp. 138–143.

Wentges, P. (1996) 'Accelerating Benders' decomposition for the capacitated facility location problem', *Mathematical Methods of Operations Research*, 44(2), pp. 267–290.

ZahediAnaraki, A., Esmaeilian, G. (2021). A Benders-Decomposition and Meta-Heuristic Algorithm for a Bi- Objective Stochastic Reliable Capacitated Facility Location Problem Not Dealing with Benders Feasibility-Cut Stage.
Journal of Optimization in Industrial Engineering, 14(1), 105-119.

http://www.qjie.ir/article_678991.html
DOI: 10.22094/JOIE.2021.578550.1599