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Research Article

The Design of Inverse Network DEA Model for Measuring the Bullwhip Effect in Supply Chain with Uncertain Demands

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Abstract

Two different bullwhip effects with equal scores may have different sensitivities and production patterns. As a result, the difference between these two seemingly equal scores has been ignored in previous methods (such as frequency response and moving average). So, the present study constructs a model of Inverse Network Data Envelopment Analysis, to introduce the relative and interval scores of the bullwhip effect magnitude, when a series of uncertain demands are made in a specific time interval. In the first stage of the proposed network, the uncertain demands and the forecasted uncertain data are regarded respectively as the model's inputs and outputs. These output data constitute the intermediate variables and consequently the inputs of the second stage of the study model. In the second stage, after considering the ordering policies, the uncertain orders are sent. Due to utilizing both the optimistic and pessimistic perspectives, the study methodology includes an interval value for measuring the bullwhip effect with relative attitude. In the optimistic perspective, the analyzed decision making unit has the optimal status in comparison with other decision making units. In the pessimistic perspective, the analyzed decision making unit has the worst status in comparison with other decision making units. The results show that time is an unfair factor in the size of the bullwhip effect. The impact of uncertainties on the bullwhip effect in the demand forecasting stage is greater than the ordering stage. According to the research findings, cross-sectional planning is possible at different times according to different conditions. Therefore, using the results of the research, a fair score of the bullwhip effect can be obtained by considering all perspectives.

***Keywords*:** Relative Bullwhip Effect; Dynamic Supply Chain; Inverse Network DEA; Uncertain Demands.

1. Introduction

The concept of supply chain management (SCM) is becoming more popular today due to increased global competition in the global marketplace (Asif et al., 2012). SCM may be defined as a set of relationships between suppliers, manufacturers, distributors, and retailers, which transform raw materials into end products (Li et al., 2014). Modeling and controlling such systems includes taking all commercial components into account in such complexity (Disney et al., 2003). SCM has recently attracted great attention among engineers in the processing system (Pin et al., 2004). The tendency of orders to increase in diversity as a supply chain move is commonly known as the bullwhip effect (BWE). Dejonckheere et al. (2003) initiated the analysis of this variance amplification phenomenon. Their work inspired many authors to develop business games to demonstrate the bullwhip effect.

Amplification of the demand variance concept in industrial systems was introduced by Forrester (1958). He was a pioneer who studied the bullwhip effect and defined it as demand amplification. He believed that this was the system dynamics problem that was controllable through reducing time delay. Forrester (1958) enumerated four critical sources of demand amplification: demand signal processing, rationing game, order batching and price variations. Demand amplification is not a new concept, and many researchers have been interested in working on it. In this regard, uncertain demands are one of the most important factors which lead to variance amplification. Other factors such as replenishment policy (Gaffari et al 2014), stochastic demand, stochastic noises (Sajjad Aslani et al 2019), and the forecasting method are also vital in affecting the efficiency of the supply chain system.

Over the years, a significant number of researches and techniques such as the control theory including simulation, mathematical, and statistical techniques have been utilized to measure the bullwhip effect in series or parallel structures of supply chains (Dejonckheere et al., 2003; Disney & Towill, 2003; Dejonckheere et al., 2004; Kim et al., 2006; Duc et al., 2008a; Duc et al., 2008b; Fu et al., 2015; Sadeghi et al., 2014). There is little research in the literature that has dealt with the data envelopment analysis (DEA) approach on measuring the BWE. Although DEA is capable of achieving relative efficiency scores of DMU, it is not able to consider internal processes in decision making units (DMUs). There are several studies on DEA, with the intention of resolving such problems using other perspectives. Kao and Hwang (2008) considered the series relationship of two sub-processes and proved that the overall efficiency was the product of the efficiencies of the two sub-processes. They modified the conventional DEA model by taking into account the series relationship of the two sub-processes within the whole process. In other words, the relational model developed in their paper is more reliable in measuring the efficiencies and consequently is capable of identifying the causes of inefficiency more accurately.

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The results show that proposed models can be extended to systems of multiple stages connected in series, and the efficiency of the whole process is the product of the efficiencies of individual sub-processes. Li et al. (2012) investigated Liang et al.'s (2008) methods. Li et al. (2012) considered a two-stage DEA model, in which the outputs of the first stage and additional input to the second stage were assumed as the inputs of the second stage. Moreno and Lozano (2014) applied a network DEA model for measuring the efficiency of NBA teams.

In another application of network DEA research, Khalili et al. (2015) developed uncertain network DEA with undesirable outputs to evaluate the efficiency of electricity power production and distribution processes. They introduced network DEA to evaluate the efficiency of electric power production and distribution processes. In the production phase, power plants consume fuels such as oil and gas to generate the electricity. In the distribution phase, regional electricity companies transmit and distribute the electricity to the customers in houses, industries, and agriculture. They evaluated final ranking of DMUs and sub - DMUs using a multi-attribute decision making (MADM) method.

According to SCM literature (Costantino et al., 2015, Disney and Lambrecht, 2008, Fu et al., 2015), BWE can be used to evaluate supply chain performance. BWE leads to a number of inefficiencies such as uncertain production planning, stock-outs which lead to high costs, low service level for all nods in the supply chain, direct impact on setting up and shutting down machines, excess inventory upstream levels, difficulty in forecasting policy and scheduling programs, and poor supplier/customer relationships (Cannella et al., 2013, Disney et al., 2007, Wang and Disney, 2016). Indeed, the lower the BWE, the more efficiency in supply chain management (SCM). Therefore, BWE can be considered as an undesirable criterion for supply chain performance. Amir Rezaie et al (2015) evaluated the performance of seven supply chains including the supplier, producer, distributors and customers by using DEA methods. By paying attention to the obtained results it can be seen that for evaluating industries, especially tile industries, supply chain criteria have better results that other factors. In a similar research Amirteimoori et al (2011) develops a DEA model for measuring the performance of suppliers and manufacturers in supply chain operations. They are proposed additive efficiency decomposition for suppliers and manufacturers in supply chain operations.

Since bullwhip effect is undesirable, worst-practice frontier (WPF) approach is considered by Goodarzi et al. Accordingly, they developed a new network worst practice model with undesirable outputs to calculate BWE of non-serial SCNs. In Goodarzi et al.'s research, for measuring the bullwhip effect, each separated supply chain was assumed as a decision making unit. As a result, in order to measure the bullwhip effect using this method, there should be gathered data from several decision-making units (measuring the relative bullwhip effect in several supply chains cannot be useful and realistic). However, it is difficult to access the information of supply chains in real-world issues, which increases the volume of calculations. To solve these problems, we propose to measure the relative magnitude of the bullwhip effect in several time intervals (Time intervals assumed as decision making units (DMUs)).

The bullwhip effect does not occur by itself. However, specific factors cause this phenomenon. In Goodarzi et al's research, there is no mention of the factors that cause the bullwhip effect, (time delay, stochastic noise, demand forecasting, uncertain demands, aggressive orders, etc.) and their impact effect on the scores. So, in this research, by presenting a network model of inverse DEA and introducing the effect of uncertain demands in a supply chain, the relative score and the effect of uncertainty on supply chain oscillation were discussed. The inherent nature of the BWE requires that its size increase over time (paying attention to how it propagates in the chain). In fact, the passage of time adds to the actual size of the BWE and creates unrealistic scores. So, it can be said that the time variable is an unfair factor for the size of the BWE. Selecting time intervals as DMUs is an important feature which can help to solve the abovementioned problem.

The rest of this paper is arranged as follows: The dynamic supply chain system with uncertain demand is formulated with the network IDEA in Section 2. Further, we propose a method in this section to measure the bullwhip effect. Classification of interval score of relative bullwhip effect is presented in Section 3. Section 4 provides an illustrative example of the supply chain system with uncertain demands to verify the advantage of the proposed strategy. Our conclusions are presented in Section 5.

2. Relative Approach and Design Inverse Network DEA for Measuring the BWE

Basically, excessive increase in the output (ordering) due to changes in the input (demand), in each node at different times, is called the bullwhip effect. In the methods of measuring this phenomenon, such as frequency response and moving average, the absolute size of this ratio is considered. For the following reasons, this absolute size in a real supply chain can be far from reality and lead to unpredictable losses on the chain.

Thus, identifying, ranking and measuring this phenomenon, according to their impacts on efficiency, are significant in this field. Assume that, at two different times, two bullwhip effects are equal in size, but they are produced with different variance of demand and orders.

$$\frac{\text{var (order=output)=0.4}}{\text{var(demand=input)=0.2}} = 2 = \text{bullwhip effect}$$
$$\frac{\text{var (order=output)=0.02}}{\text{var(demand=input)=0.01}} = 2 = \text{bullwhip effect} \quad (1)$$

On a superficial view and by using the previous methods of measuring the bullwhip effect (frequency response, etc.), no difference is observed between the two bullwhip effects. While in fact, the production and sensitivity of the two equal scores of these bullwhip effects are different. To overcome this problem and the distinction between these two values of the bullwhip effect at different times, we proposed the IDEA method. The efficient method for comparing and measuring any phenomenon is a relative measurement of this phenomenon. In fact, the score of any phenomenon must be measured in the society in which it occurs (DEA Approach).

We divided the checking time into equal intervals and considered each time interval as a decision making unit. In fact, a decision-making unit transforms into several decision-making units at different times. This partition describes a dynamic IDEA. All decision-making units have one type of input and one type of output. Due to the undesirable effect of the bullwhip phenomenon and the fact that the ideal of experts is to reduce the output in each unit, the inverse DEA was used.

As noted, uncertainty of the information is one of the factors that leads to the bullwhip phenomenon. Measuring the relative score of the bullwhip effect should be in the presence of uncertainty. To insert this inward factor, IDEA is not sufficient. Network IDEA models enable researchers to do this. Figure 1 is the proposed network IDEA model for measuring the bullwhip effect at a decision making unit in the presence of uncertainty.

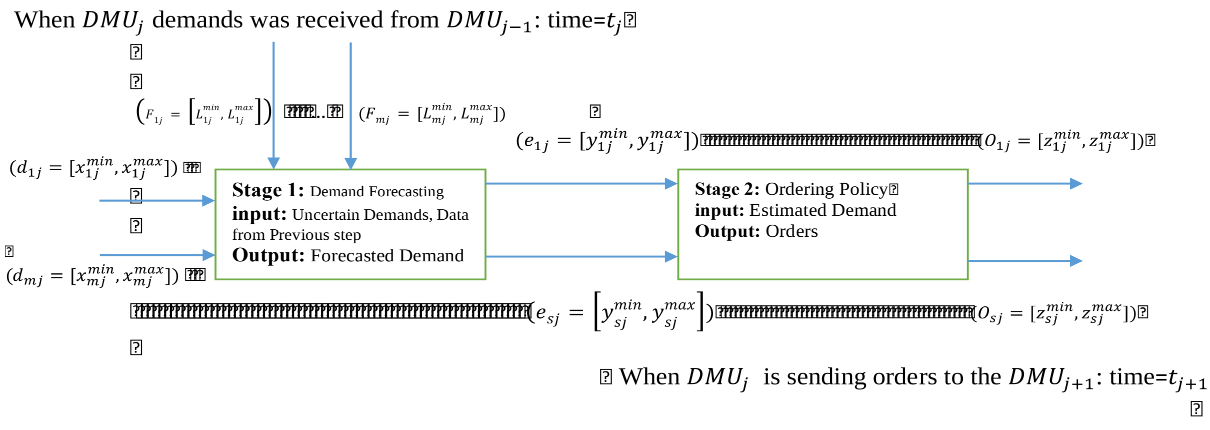

Fig. 1. The Network Supply Chain with Uncertain demands

Demand as the input of a node in a supply chain can experience uncertainty in various intervals. In time, this uncertainty can spread to other sectors of a manufacturing system, and result in the bullwhip effect.

In the first stage, the uncertainty level of demand (indicated by $(d_{1j}, \dots, d_{mj})$) is inserted into the model in a specific interval (indicated by j), and as a result comprises the input of this stage. Through exponential smoothing method, this input alongside other forecasted data of the decision making unit with the numerical values of $(F_{1j}, \dots, F_{mj})$ are fed to the hypothetical node for forecasting the status of the demand. Therefore, this stage is designed to forecast the status of the demand. The output of the first stage includes a series of uncertain data which have emerged from the demand forecast (with numerical values of $(e_{1j}, \dots, e_{sj})$), and consequently the intermediate and input variables of the second

In the second stage, ordering operations are applied on the data, and a series of uncertain orders (with numerical values of $(O_{1j}, \dots, O_{sj})$) will be the outputs. In this stage, the ordering policies are applied upon forecasted demands. It needs to be mentioned that the uncertainty levels of all the data are analyzed at various intervals.

The present study proposes that two relatively-set numerical values in a numerical interval – with maximal and minimal limits – should be considered so that the bullwhip effect can be measured through the study's optimistic (where the lowest bullwhip effect occurs) and pessimistic (where the highest bullwhip effect occurs) perspectives. In the optimistic perspective, the decision making unit has its most optimal status. In this status, the model generates the lowest number of outputs after being fed with the highest number of inputs. In the pessimistic perspective, the described condition is inverted for the analyzed decision making unit, and as a result, the highest number of outputs are generated by the lowest number of inputs.

2.1 The uncertain inverse network DEA model for measuring overall RBWE in dynamic supply chain

We first investigated a network IDEA model based on the theoretical aspects of IDEA models proposed by Khalili et al. (2015). In the proposed model, an interval relative bullwhip effect score $[\mathrm{RBWE}_o^L, \mathrm{RBWE}_o^U]$ is proposed on optimistic and pessimistic view points where, with an optimistic view the score is RBWE_o^L and with a pessimistic the score is RBWE_o^U. Suppose that $[\mathrm{RBWE}_o^L, \mathrm{RBWE}_o^U]$, $[\mathrm{RBWE}_o^{1L}, \mathrm{RBWE}_o^{1U}]$ and $[\mathrm{RBWE}_o^{2L}, \mathrm{RBWE}_o^{2U}]$ are the relative interval scores of the overall bullwhip effect, the relative interval score of the first stage and the relative interval score of the bullwhip effect of the second stage, respectively. By assuming a constant return to the scale and input-axis model, we propose model (2) based on optimistic view points in order to measure the lower bound of minimum achievable relative bullwhip effect of DMU_o:

$$RBWE_o^L = Min \frac{\sum_{r=1}^{s} u_r z_{ro}^{\min}}{\sum_{i=1}^{m} v_i x_{io}^{\max} + \sum_{i=1}^{m} v_i^{'} l_{io}^{\max}}$$

$$s.t \begin{cases} \dfrac{\sum_{r=1}^{s} u_r^{'} y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}^{\min} + \sum_{i=1}^{m} v_i^{'} l_{ij}^{\min}} \geq 1, & o \neq j = 1,\dots,n \\ \dfrac{\sum_{r=1}^{s} u_r^{'} y_{ro}}{\sum_{i=1}^{m} v_i x_{io}^{\max} + \sum_{i=1}^{m} v_i^{'} l_{io}^{\max}} \geq 1 & \\ \dfrac{\sum_{r=1}^{s} u_r z_{rj}^{\max}}{\sum_{r=1}^{s} u_r^{'} y_{rj}} \geq 1, & o \neq j = 1,\dots,n \\ \dfrac{\sum_{r=1}^{s} u_r z_{ro}^{\min}}{\sum_{r=1}^{s} u_r^{'} y_{ro}} \geq 1 & \end{cases} \tag{2}$$

$$u_r, u_r^{'}, v_i, v_i^{'} \geq 0, i = 1,\dots,m, r = 1,\dots,s$$

Model (2) is the nonlinear model for the relative interval score of the bullwhip effect, which can be converted to the linear model (4), using Equation (3).

$$\frac{1}{\sum_{i=1}^{m} v_i x_{io}^{\max} + \sum_{i=1}^{m} v_i^{'} l_{io}^{\max}} = t \tag{3}$$

We can rewrite:

$$RBWE_o^L = \min \sum_{r=1}^{s} u_r z_{ro}^{\min}$$

$$s.t \begin{cases} \sum_{i=1}^{m} v_i x_{io}^{\max} + \sum_{i=1}^{m} v_i^{'} l_{io}^{\max} = 1 & \\ \sum_{r=1}^{s} u_r^{'} y_{rj} - \sum_{i=1}^{m} v_i x_{ij}^{\min} - \sum_{i=1}^{m} v_i^{'} l_{ij}^{\min} \geq o, & o \neq j = 1,\dots,n \\ \sum_{r=1}^{s} u_r z_{rj}^{\max} - \sum_{r=1}^{s} u_r^{'} y_{rj} \geq 0, & o \neq j = 1,\dots,n \\ \sum_{r=1}^{s} u_r^{'} y_{ro} - \sum_{i=1}^{m} v_i x_{io}^{\max} - \sum_{i=1}^{m} v_i^{'} l_{io}^{\max} \geq 0 & \\ \sum_{r=1}^{s} u_r z_{ro}^{\min} - \sum_{r=1}^{s} u_r^{'} y_{ro} \geq 0 & \end{cases} \tag{4}$$

$$u_r, u_r^{'}, v_i, v_i^{'} \geq 0, i = 1,\dots,m, r = 1,\dots,s$$

The intermediate measures $(e_{1j}, \dots, e_{sj})$ in model (4) are outputs of the first stage and inputs to second stage. On the basis of optimistic view points, the output should be minimum and the input should be maximum, so intermediate measures appear in two sets. So, this attitude cannot be ideal for intermediate values. Kao and Liu (2011) introduced the two-level optimization method for specifying the optimum values of the intermediate measures in which the objective function is at its greatest possible value. Consequently, we proposed the two level optimization model (5) to evaluate these values:

$$Min(y_{rj}^{\min} \leq y_{rj} \leq y_{rj}^{\max})_{\forall j,r} \begin{cases} RBWE_o^L = \min \sum_{r=1}^{s} u_r z_{ro}^{\min} \\ \sum_{i=1}^{m} v_i x_{io}^{\max} + \sum_{i=1}^{m} v_i^{'} l_{io}^{\max} = 1 \\ \sum_{r=1}^{s} u_r^{'} y_{rj} - \sum_{i=1}^{m} v_i x_{ij}^{\min} - \sum_{i=1}^{m} v_i^{'} l_{ij}^{\min} \geq o, o \neq j = 1,\dots,n \\ \sum_{r=1}^{s} u_r z_{rj}^{\max} - \sum_{r=1}^{s} u_r^{'} y_{rj} \geq 0, \quad o \neq j = 1,\dots,n \\ \sum_{r=1}^{s} u_r^{'} y_{ro} - \sum_{i=1}^{m} v_i x_{io}^{\max} - \sum_{i=1}^{m} v_i^{'} l_{io}^{\max} \geq 0 \\ \sum_{r=1}^{s} u_r z_{ro}^{\min} - \sum_{r=1}^{s} u_r^{'} y_{ro} \geq 0 \\ u_r, u_r^{'}, v_i, v_i^{'} \geq 0, i = 1,\dots,m, r = 1,\dots,s \end{cases} \tag{5}$$

Where $y_{rj}, j = 1, \dots, m, r = 1, \dots, s$ are the decision variables for the outer optimization problem and are considered as the constant multipliers for the inner optimization problem. The aim is to convert the Model (5) into a single level optimization. So, Model (5) can be thought of as a single level optimization such as Model (6):

$$RBWE_o^L = \min \sum_{r=1}^{s} u_r z_{ro}^{\min}$$

$$s.t \begin{cases} \sum_{i=1}^{m} v_i x_{io}^{\max} + \sum_{i=1}^{m} v_i^{'} l_{io}^{\max} = 1 & \\ \sum_{r=1}^{s} u_r^{'} y_{rj} - \sum_{i=1}^{m} v_i x_{ij}^{\min} - \sum_{i=1}^{m} v_i^{'} l_{ij}^{\min} \geq 0, & o \neq j = 1,\dots,n \\ \sum_{r=1}^{s} u_r z_{rj}^{\max} - \sum_{r=1}^{s} u_r^{'} y_{rj} \geq 0, & o \neq j = 1,\dots,n \\ \sum_{r=1}^{s} u_r^{'} y_{ro} - \sum_{i=1}^{m} v_i x_{io}^{\max} - \sum_{i=1}^{m} v_i^{'} l_{io}^{\max} \geq 0 & \\ \sum_{r=1}^{s} u_r z_{ro}^{\min} - \sum_{r=1}^{s} u_r^{'} y_{ro} \geq 0 & \\ y_{rj}^{\min} \leq y_{rj}, & j = 1,\dots,n, r = 1,\dots,s \\ y_{rj} \leq y_{rj}^{\max}, & j = 1,\dots,n, r = 1,\dots,s \end{cases} \tag{6}$$

$$u_r, u_r^{'}, v_i, v_i^{'} \geq 0, i = 1,\dots,m, r = 1,\dots,s$$

Theorem 1: Model (6) is always feasible. Also, its optimal objective is larger than or equal to 1.

Proof: Suppose that $\theta, \lambda_j^1, \lambda_j^2, \lambda_o^1, \lambda_o^2, \alpha_{rj}, \beta_{rj}$ are dual variables, associated constraints 1,…,7 respectively. So, the dual form of Model (6) can be written as follows:

$$
\begin{aligned}
&Max\ \ \theta-\sum_{r=1}^{s}\sum_{j=1}^{n}\beta_{rj}y_{rj}^{\max}+\sum_{r=1}^{s}\sum_{j=1}^{n}\alpha_{rj}y_{rj}^{\min}\\
&s.t\begin{cases}
\sum_{o\neq j=1}^{n}\lambda_j^2 z_{rj}^{\max}+z_{ro}^{\min}\lambda_o^2\le z_{ro}^{\min}, r=1,\dots,s\\
\sum_{o\neq j=1}^{s}\lambda_j^1 y_{rj}-\sum_{o\neq j=1}^{m}\lambda_j^2 y_{rj}+y_{ro}\lambda_o^1-y_{ro}\lambda_o^2\le 0, r=1,\dots,s\\
\theta x_{io}^{\max}-x_{io}^{\max}\lambda_o^1-\sum_{o\neq j=1}^{n}x_{ij}^{\min}\lambda_j^1\le 0, i=1,\dots,m\\
\theta l_{io}^{\max}-l_{io}^{\max}\lambda_o^1-\sum_{o\neq j=1}^{n}l_{ij}^{\min}\lambda_j^1\le 0, i=1,\dots,m
\end{cases}\\
&\theta\ free,\alpha_{rj},\beta_{rj},\lambda_j^1,\lambda_j^2\ge 0, r=1,\dots,s, j=1,\dots,n
\end{aligned}
\tag{7}
$$

It is obvious that $\theta=1,\lambda_j^1=0,\lambda_j^2=0,\lambda_o^1=\lambda_o^2=1,\alpha_{rj}=\beta_{rj}=0$ is a feasible solution for Model (7). So, there is always a feasible solution for Model (6). We know that the optimal value of Model (7) will be less than or equal to objective function of any feasible solution of Model (6). So, optimal value of Model (6) will be bigger or equal to 1.

Definition 1: Optimal value of Model (6) is the lower bound of relative interval bullwhip effect score (RBWE) of total network for a dynamic supply chain in the presence of uncertain demand. If this score is 1, then the bullwhip effect in the optimistic view point does not occur.

Model (8) is the pessimistic view points in order to measure the upper bound of relative bullwhip effect score of supply chain (i.e. $RBWE_o^U$).

$$
\begin{aligned}
&RBWE_o^U=\min\sum_{r=1}^{s}u_r z_{ro}^{\max}\\
&s.t\begin{cases}
\sum_{i=1}^{m}v_i x_{io}^{\min}+\sum_{i=1}^{m}v_i' l_{io}^{\min}=1\\
\sum_{r=1}^{s}u_r' y_{rj}-\sum_{i=1}^{m}v_i x_{ij}^{\max}-\sum_{i=1}^{m}v_i' l_{ij}^{\max}\ge o, \qquad o\neq j=1,\dots,n\\
\sum_{r=1}^{s}u_r z_{rj}^{\min}-\sum_{r=1}^{s}u_r' y_{rj}\ge 0, \qquad o\neq j=1,\dots,n\\
\sum_{r=1}^{s}u_r' y_{ro}-\sum_{i=1}^{m}v_i x_{io}^{\min}-\sum_{i=1}^{m}v_i' l_{io}^{\min}\ge 0\\
\sum_{r=1}^{s}u_r z_{ro}^{\max}-\sum_{r=1}^{s}u_r' y_{ro}\ge 0
\end{cases}\\
&u_r,u_r',v_i,v_i'\ge 0, i=1,\dots,m, r=1,\dots,s
\end{aligned}
$$

Accordingly, the intermediate measures $(e_{1j},\dots,e_{sj})$ are outputs of the first stage and inputs to the second stage. On the basis of pessimistic view points, the output should be maximum and the inputs should be minimum, so intermediate measures appear in two sets. Therefore, this model cannot be appropriate for intermediate values. We have proposed the two level optimization model (8) to calculate these values:

$$
Max(y_{rj}^{\min}\le y_{rj}\le y_{rj}^{\max})_{\forall j,r}
\begin{cases}
RBWE_o^U=\min\sum_{r=1}^{s}u_r z_{ro}^{\max}\\
\sum_{i=1}^{m}v_i x_{io}^{\min}+\sum_{i=1}^{m}v_i' l_{io}^{\min}=1\\
\sum_{r=1}^{s}u_r' y_{rj}-\sum_{i=1}^{m}v_i x_{ij}^{\max}-\sum_{i=1}^{m}v_i' l_{ij}^{\max}\ge o, o\neq j=1,\dots,n\\
\sum_{r=1}^{s}u_r z_{rj}^{\min}-\sum_{r=1}^{s}u_r' y_{rj}\ge 0, o\neq j=1,\dots,n\\
\sum_{r=1}^{s}u_r' y_{ro}-\sum_{i=1}^{m}v_i x_{io}^{\min}-\sum_{i=1}^{m}v_i' l_{io}^{\min}\ge 0\\
\sum_{r=1}^{s}u_r z_{ro}^{\max}-\sum_{r=1}^{s}u_r' y_{ro}\ge 0\\
u_r,u_r',v_i,v_i'\ge 0, i=1,\dots,m, r=1,\dots,s
\end{cases}
\tag{8}
$$

Where $y_{rj}, j=1,\dots,m, r=1,\dots,s$ are the decision variable for the outer optimization problem and considered the constant multipliers for the inner optimization problem. The aim is to change the Model (8) into a single level optimization. So, for transforming two-level optimization Model (8) to a single-level optimization, dual form of inner optimization is presented in Model (9):

$$
\begin{aligned}
&Max\theta\\
&s.t\begin{cases}
\sum_{o\neq j=1}^{n}\lambda_j^2 z_{rj}^{\min}+z_{ro}^{\max}\lambda_o^2\le z_{ro}^{\max}, r=1,\dots,s\\
\sum_{o\neq j=1}^{s}\lambda_j^1 y_{rj}-\sum_{o\neq j=1}^{m}\lambda_j^2 y_{rj}+y_{ro}\lambda_o^1-y_{ro}\lambda_o^2\le 0, r=1,\dots,s\\
\theta x_{io}^{\min}-x_{io}^{\min}\lambda_o^1-\sum_{o\neq j=1}^{n}x_{ij}^{\max}\lambda_j^1\le 0, i=1,\dots,m\\
\theta l_{io}^{\min}-l_{io}^{\min}\lambda_o^1-\sum_{o\neq j=1}^{n}l_{ij}^{\max}\lambda_j^1\le 0, i=1,\dots,m
\end{cases}\\
&\theta\ free,\lambda_j^1,\lambda_j^2\ge 0, r=1,\dots,s, j=1,\dots,n
\end{aligned}
\tag{9}
$$

By implementation virtue of dual theorem, the optimal solution of Model (9) is equal to optimal solution of inner optimization Model (8). Model (10) is proposed as follows:

$$
Max(y_{rj}^{\min}\le y_{rj}\le y_{rj}^{\max})_{\forall j,r}
\begin{cases}
Max\theta\\
s.t.\sum_{o\neq j=1}^{n}\lambda_j^2 z_{rj}^{\min}+z_{ro}^{\max}\lambda_o^2\le z_{ro}^{\max}, r=1,\dots,s\\
\sum_{o\neq j=1}^{s}\lambda_j^1 y_{rj}-\sum_{o\neq j=1}^{m}\lambda_j^2 y_{rj}+y_{ro}\lambda_o^1-y_{ro}\lambda_o^2\le 0, r=1,\dots,s\\
\theta x_{io}^{\min}-x_{io}^{\min}\lambda_o^1-\sum_{o\neq j=1}^{n}x_{ij}^{\max}\lambda_j^1\le 0, i=1,\dots,m\\
\theta l_{io}^{\min}-l_{io}^{\min}\lambda_o^1-\sum_{o\neq j=1}^{n}l_{ij}^{\max}\lambda_j^1\le 0, i=1,\dots,m\\
\theta\ free,\lambda_j^1,\lambda_j^2\ge 0, r=1,\dots,s, j=1,\dots,n
\end{cases}
\tag{10}
$$

As the orientation of objective function in the inner and outer Model (10) are the same so, we can write Model (11) as follows:

$$
\begin{aligned}
& Max\,\theta \\
s.t\ & \begin{cases}
\sum_{o\neq j=1}^{n} \lambda_j^2 z_{rj}^{\min} + z_{ro}^{\max}\lambda_o^2 \le z_{ro}^{\max}, r=1,\dots,s \\
\sum_{o\neq j=1}^{s} \lambda_j^1 y_{rj} - \sum_{o\neq j=1}^{m} \lambda_j^2 y_{rj} + y_{ro}\lambda_o^1 - y_{ro}\lambda_o^2 \le 0, r=1,\dots,s \\
\theta x_{io}^{\min} - x_{io}^{\min}\lambda_o^1 - \sum_{o\neq j=1}^{n} x_{ij}^{\max}\lambda_j^1 \le 0, i=1,\dots,m \\
\theta l_{io}^{\min} - l_{io}^{\min}\lambda_o^1 - \sum_{o\neq j=1}^{n} l_{ij}^{\max}\lambda_j^1 \le 0, i=1,\dots,m \\
y_{rj}^{\min} \le y_{rj}, r=1,\dots,s;\ j=1,\dots,n \\
y_{rj} \le y_{rj}^{\max}, r=1,\dots,s;\ j=1,\dots,n
\end{cases} \\
& \theta\ free, \lambda_j^1, \lambda_j^2 \ge 0, j=1,\dots,n
\end{aligned} \tag{11}
$$

Theorem 2: Model (11) is always feasible. Also, its optimal objective is larger than or equal to 1.

Proof: It is obvious that $\theta = 1$, $\lambda_j^1 = 0$, $\lambda_j^2 = 0$, $\lambda_o^1 = \lambda_o^2 = 1$ is a feasible solution for Model (11). So, there is always a feasible solution for Model (11). We know that the optimal value of Model (11) will be bigger or equal to 1.

Definition 2: Optimal value of the Model (11) is the upper bound of the relative interval bullwhip effect score (RBWE) of total network for a dynamic supply chain in the presence of uncertain demand. If this score is 1, then the bullwhip effect in the pessimistic view point does not occur.

2.2 The uncertain inverse network DEA model for measuring RBWE in stage1 (Demand forecasting) of dynamic supply chain

By assuming a constant return to the scale and input-axis model, we propose model (12) based on optimistic view points in order to measure the lower bound of minimum achievable relative bullwhip effect of DMU_o in demand forecasting stage:

$$
\begin{aligned}
& RBWE_o^{1L} = \min \sum_{r=1}^{s} u'_r y_{ro} \\
s.t\ & \begin{cases}
\sum_{i=1}^{m} v_i x_{io}^{\max} + \sum_{i=1}^{m} v'_i l_{io}^{\max} = 1 \\
\sum_{r=1}^{s} u_r z_{ro}^{\min} = (RBWE_o^{L})^* \\
\sum_{r=1}^{s} u'_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij}^{\min} - \sum_{i=1}^{m} v'_i l_{ij}^{\min} \ge o, & o\neq j=1,\dots,n \\
\sum_{r=1}^{s} u_r z_{rj}^{\max} - \sum_{r=1}^{s} u'_r y_{rj} \ge 0, & o\neq j=1,\dots,n \\
\sum_{r=1}^{s} u'_r y_{ro} - \sum_{i=1}^{m} v_i x_{io}^{\max} - \sum_{i=1}^{m} v'_i l_{io}^{\max} \ge 0 \\
\sum_{r=1}^{s} u_r z_{ro}^{\min} - \sum_{r=1}^{s} u'_r y_{ro} \ge 0
\end{cases} \\
& u_r, u'_r, v_i, v'_i \ge 0, i=1,\dots,m, r=1,\dots,s
\end{aligned} \tag{12}
$$

Due to the effect of intermediate variables, the two-level optimization Model (13) is proposed as follows:

$$
Min(y_{rj}^{\min} \le y_{rj} \le y_{rj}^{\max}), \forall j,r
\begin{cases}
RBWE_o^{1L} = \min \sum_{r=1}^{s} u'_r y_{ro} \\
\sum_{i=1}^{m} v_i x_{io}^{\max} + \sum_{i=1}^{m} v'_i l_{io}^{\max} = 1 \\
\sum_{r=1}^{s} u_r z_{ro}^{\min} = (RBWE_o^{L})^* \\
\sum_{r=1}^{s} u'_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij}^{\min} - \sum_{i=1}^{m} v'_i l_{ij}^{\min} \ge o, & o\neq j=1,\dots,n \\
\sum_{r=1}^{s} u_r z_{rj}^{\max} - \sum_{r=1}^{s} u'_r y_{rj} \ge 0, & o\neq j=1,\dots,n \\
\sum_{r=1}^{s} u'_r y_{ro} - \sum_{i=1}^{m} v_i x_{io}^{\max} - \sum_{i=1}^{m} v'_i l_{io}^{\max} \ge 0 \\
\sum_{r=1}^{s} u_r z_{ro}^{\min} - \sum_{r=1}^{s} u'_r y_{ro} \ge 0 \\
u_r, u'_r, v_i, v'_i \ge 0, i=1,\dots,m, r=1,\dots,s
\end{cases} \tag{13}
$$

Model (13) can be transferred to Model (14):

$$
\begin{aligned}
& RBWE_o^{1L} = \min \sum_{r=1}^{s} u'_r y_{ro} \\
s.t\ & \begin{cases}
\sum_{i=1}^{m} v_i x_{io}^{\max} + \sum_{i=1}^{m} v'_i l_{io}^{\max} = 1 \\
\sum_{r=1}^{s} u_r z_{ro}^{\min} = (RBWE_o^{L})^* \\
\sum_{r=1}^{s} u'_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij}^{\min} - \sum_{i=1}^{m} v'_i l_{ij}^{\min} \ge o, & o\neq j=1,\dots,n \\
\sum_{r=1}^{s} u_r z_{rj}^{\max} - \sum_{r=1}^{s} u'_r y_{rj} \ge 0, & o\neq j=1,\dots,n \\
\sum_{r=1}^{s} u'_r y_{ro} - \sum_{i=1}^{m} v_i x_{io}^{\max} - \sum_{i=1}^{m} v'_i l_{io}^{\max} \ge 0 \\
\sum_{r=1}^{s} u_r z_{ro}^{\min} - \sum_{r=1}^{s} u'_r y_{ro} \ge 0 \\
y_{rj}^{\min} \le y_{rj} & r=1,\dots,s;\ j=1,\dots,n \\
y_{rj} \le y_{rj}^{\max} & r=1,\dots,s;\ j=1,\dots,n
\end{cases} \\
& u_r, u'_r, v_i, v'_i \ge 0, i=1,\dots,m, r=1,\dots,s
\end{aligned} \tag{14}
$$

Theorem 3:

Model (14) is always feasible and its optimal value is larger than or equal to 1.

Proof: Suppose that $\theta^1, \theta^2, \lambda_j^1, \lambda_j^2, \lambda_o^1, \lambda_o^2, \alpha_{rj}, \beta_{rj}$ are dual variables associated with constraints 1,…,8 respectively. So, dual form of Model (14) can be written as follows:

$$
\begin{aligned}
& Max\ \ \theta^1 + (RBWE_o^{L})^*\theta^2 - \sum_{r=1}^{s}\sum_{j=1}^{n} \beta_{rj} y_{rj}^{\max} + \sum_{r=1}^{s}\sum_{j=1}^{n} \alpha_{rj} y_{rj}^{\min} \\
s.t\ & \begin{cases}
\theta^2 z_{ro}^{\min} + \sum_{o\neq j=1}^{n} \lambda_j^2 z_{rj}^{\max} + z_{ro}^{\min}\lambda_o^2 \le 0, r=1,\dots,s \\
\sum_{o\neq j=1}^{s} \lambda_j^1 y_{rj} - \sum_{o\neq j=1}^{m} \lambda_j^2 y_{rj} + y_{ro}\lambda_o^1 - y_{ro}\lambda_o^2 \le y_{ro}, r=1,\dots,s \\
\theta^1 x_{io}^{\max} - x_{io}^{\max}\lambda_o^1 - \sum_{o\neq j=1}^{n} x_{ij}^{\min}\lambda_j^1 \le 0, i=1,\dots,m \\
\theta^1 l_{io}^{\max} - l_{io}^{\max}\lambda_o^1 - \sum_{o\neq j=1}^{n} l_{ij}^{\min}\lambda_j^1 \le 0, i=1,\dots,m
\end{cases} \\
& \theta^1, \theta^2\ free, \alpha_{rj}, \beta_{rj}, \lambda_j^1, \lambda_j^2 \ge 0, r=1,\dots,s, j=1,\dots,n
\end{aligned} \tag{15}
$$

It is obvious that $\theta^1 = 1 = \lambda_o^1$, $\lambda_j^1 = \lambda_j^2 = \theta^2 = \lambda_o^2 = 0, \alpha_{rj} = \beta_{rj} = 0$ is a feasible solution for Model (15). So, there is always a feasible solution for Model (14). We know that the optimal value of Model (15) will be less than or equal to objective function of any feasible solution of Model (14). So, optimal value of Model (14) will be bigger or equal to 1.

Definition 3: Optimal value of Model (14) is the lower bound of relative bullwhip effect score (RBWE) of stage1 for a dynamic supply chain in the presence of uncertain demands. If this score is 1, then the bullwhip effect in the optimistic view point does not occur.

Model (16) is the pessimistic view points in order to measure the upper bound of relative bullwhip effect score of supply chain in stage1(i.e. $RBWE_o^{1U}$).

$$
\begin{aligned}
& RBWE_o^{1U} = \min \sum_{r=1}^{s} u'_r y_{ro} \\
& s.t. \begin{cases}
\sum_{i=1}^{m} v_i x_{io}^{\min} + \sum_{i=1}^{m} v'_i l_{io}^{\min} = 1 \\
\sum_{r=1}^{s} u_r z_{ro}^{\max} = (RBWE_o^U)^* \\
\sum_{r=1}^{s} u'_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij}^{\max} - \sum_{i=1}^{m} v'_i l_{ij}^{\max} \ge o, & o \ne j = 1,\dots,n \\
\sum_{r=1}^{s} u_r z_{rj}^{\min} - \sum_{r=1}^{s} u'_r y_{rj} \ge 0, & o \ne j = 1,\dots,n \\
\sum_{r=1}^{s} u'_r y_{ro} - \sum_{i=1}^{m} v_i x_{io}^{\min} - \sum_{i=1}^{m} v'_i l_{io}^{\min} \ge 0 \\
\sum_{r=1}^{s} u_r z_{ro}^{\max} - \sum_{r=1}^{s} u'_r y_{ro} \ge 0
\end{cases} \\
& u_r, u'_r, v_i, v'_i \ge 0, i = 1,\dots,m, r = 1,\dots,s
\end{aligned}
\qquad (16)
$$

Due to the effect of intermediate variables, the two-level optimization Model (17) is proposed as follows:

$$
Max(y_{rj}^{\min} \le y_{rj} \le y_{rj}^{\max})\ \forall j,r
\begin{cases}
RBWE_o^{1U} = \min \sum_{r=1}^{s} u'_r y_{ro} \\
\sum_{i=1}^{m} v_i x_{io}^{\min} + \sum_{i=1}^{m} v'_i l_{io}^{\min} = 1 \\
\sum_{r=1}^{s} u_r z_{ro}^{\max} = (RBWE_o^U)^* \\
\sum_{r=1}^{s} u'_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij}^{\max} - \sum_{i=1}^{m} v'_i l_{ij}^{\max} \ge o, & o \ne j = 1,\dots,n \\
\sum_{r=1}^{s} u_r z_{rj}^{\min} - \sum_{r=1}^{s} u'_r y_{rj} \ge 0, & o \ne j = 1,\dots,n \\
\sum_{r=1}^{s} u'_r y_{ro} - \sum_{i=1}^{m} v_i x_{io}^{\min} - \sum_{i=1}^{m} v'_i l_{io}^{\min} \ge 0 \\
\sum_{r=1}^{s} u_r z_{ro}^{\max} - \sum_{r=1}^{s} u'_r y_{ro} \ge 0 \\
u_r, u'_r, v_i, v'_i \ge 0, i = 1,\dots,m, r = 1,\dots,s
\end{cases}
\qquad (17)
$$

Dual form of inner optimization is presented with Model (18):

$$
\begin{aligned}
& RBWE_o^{1U} = Max\ \theta^1 + (RBWE_o^U)^* \theta^2 \\
& s.t. \begin{cases}
\theta^2 z_{ro}^{\max} + \sum_{o \ne j=1}^{n} \lambda_j^2 z_{rj}^{\min} + z_{ro}^{\max} \lambda_o^2 \le 0, r = 1,\dots,s \\
\sum_{o \ne j=1}^{s} \lambda_j^1 y_{rj} - \sum_{o \ne j=1}^{m} \lambda_j^2 y_{rj} + y_{ro} \lambda_o^1 - y_{ro} \lambda_o^2 \le y_{ro}, r = 1,\dots,s \\
\theta^1 x_{io}^{\min} - x_{io}^{\min} \lambda_o^1 - \sum_{o \ne j=1}^{n} x_{ij}^{\max} \lambda_j^1 \le 0, i = 1,\dots,m \\
\theta^1 l_{io}^{\min} - l_{io}^{\min} \lambda_o^1 - \sum_{o \ne j=1}^{n} l_{ij}^{\max} \lambda_j^1 \le 0, i = 1,\dots,m
\end{cases} \\
& \theta^1, \theta^2\ free, \lambda_j^1, \lambda_j^2 \ge 0, r = 1,\dots,s, j = 1,\dots,n
\end{aligned}
\qquad (18)
$$

The optimal solution of inner optimization Model (17) is equal to optimal solution of optimization Model (18). So, Model (16) can be transferred to Model (19):

$$
Max(y_{rj}^{\min} \le y_{rj} \le y_{rj}^{\max})\ \forall\ ; j,r
\begin{cases}
RBWE_o^{1U} = Max\ \theta^1 + (RBWE_o^U)^* \theta^2 \\
\theta^2 z_{ro}^{\max} + \sum_{o \ne j=1}^{n} \lambda_j^2 z_{rj}^{\min} + z_{ro}^{\max} \lambda_o^2 \le 0, r = 1,\dots,s \\
\sum_{o \ne j=1}^{s} \lambda_j^1 y_{rj} - \sum_{o \ne j=1}^{m} \lambda_j^2 y_{rj} + y_{ro} \lambda_o^1 - y_{ro} \lambda_o^2 \le y_{ro}, r = 1,\dots,s \\
\theta^1 x_{io}^{\min} - x_{io}^{\min} \lambda_o^1 - \sum_{o \ne j=1}^{n} x_{ij}^{\max} \lambda_j^1 \le 0, i = 1,\dots,m \\
\theta^1 l_{io}^{\min} - l_{io}^{\min} \lambda_o^1 - \sum_{o \ne j=1}^{n} l_{ij}^{\max} \lambda_j^1 \le 0, i = 1,\dots,m \\
\theta^1, \theta^2\ free, \lambda_j^1, \lambda_j^2 \ge 0, r = 1,\dots,s, j = 1,\dots,n
\end{cases}
\qquad (19)
$$

Two-level optimization Model (19) can be transferred to single-level optimization Model (20):

$$
\begin{aligned}
& RBWE_o^{1U} = Max\ \theta^1 + (RBWE_o^U)^* \theta^2 \\
& s.t. \begin{cases}
\theta^2 z_{ro}^{\max} + \sum_{o \ne j=1}^{n} \lambda_j^2 z_{rj}^{\min} + z_{ro}^{\max} \lambda_o^2 \le 0, r = 1,\dots,s \\
\sum_{o \ne j=1}^{s} \lambda_j^1 y_{rj} - \sum_{o \ne j=1}^{m} \lambda_j^2 y_{rj} + y_{ro} \lambda_o^1 - y_{ro} \lambda_o^2 \le y_{ro}, r = 1,\dots,s \\
\theta^1 x_{io}^{\min} - x_{io}^{\min} \lambda_o^1 - \sum_{o \ne j=1}^{n} x_{ij}^{\max} \lambda_j^1 \le 0, i = 1,\dots,m \\
\theta^1 l_{io}^{\min} - l_{io}^{\min} \lambda_o^1 - \sum_{o \ne j=1}^{n} l_{ij}^{\max} \lambda_j^1 \le 0, i = 1,\dots,m \\
y_{rj}^{\min} \le y_{rj} \le y_{rj}^{\max}
\end{cases} \\
& \theta^1, \theta^2\ free, \lambda_j^1, \lambda_j^2 \ge 0, r = 1,\dots,s, j = 1,\dots,n
\end{aligned}
\qquad (20)
$$

Theorem 4: Model (20) is always feasible and its optimal value is larger than or equal to 1.

Proof: It is obvious that $\theta^1 = 1 = \lambda_o^1$, $\lambda_j^1 = 0, \lambda_j^2 = 0, \theta^2 = \lambda_o^2 = 0$ is a feasible solution for Model (20). So, there is always a feasible solution for Model (20). We know that the optimal value of Model (20) will be bigger or equal to 1.

Definition 4: Optimal value of Model (20) is the upper bound of relative bullwhip effect score (RBWE) of stage1 for a dynamic supply chain in the presence of uncertain demands. If this score is 1, then the bullwhip effect in the pessimistic view point does not occur.

2.3 *The uncertain inverse network DEA model for measuring RBWE in stage2 (Ordering strategy stage) of dynamic supply chain*

We propose Model (21) based on optimistic view points in order to measure the lower bound of minimum achievable relative bullwhip effect of DMU_o in ordering stage:

$$RBWE_o^{2L} = \min \sum_{r=1}^{s} u_r z_{ro}^{\min}$$
$$s.t \begin{cases} \sum_{i=1}^{m} u'_r y_{ro} = 1 \\ \sum_{r=1}^{s} u_r z_{ro}^{\min} - (RBWE_o^L)^* \sum_{i=1}^{m} v_i x_{io}^{\max} - (RBWE_o^L)^* \sum_{i=1}^{m} v'_i l_{io}^{\max} \geq 0 \\ \sum_{r=1}^{s} u'_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij}^{\min} - \sum_{i=1}^{m} v'_i l_{ij}^{\min} \geq o, \quad o \neq j = 1,...,n \\ \sum_{r=1}^{s} u_r z_{rj}^{\max} - \sum_{r=1}^{s} u'_r y_{rj} \geq 0, \quad o \neq j = 1,...,n \\ \sum_{r=1}^{s} u'_r y_{ro} - \sum_{i=1}^{m} v_i x_{io}^{\max} - \sum_{i=1}^{m} v'_i l_{io}^{\max} \geq 0 \\ \sum_{r=1}^{s} u_r z_{ro}^{\min} - \sum_{r=1}^{s} u'_r y_{ro} \geq 0 \end{cases} \quad (21)$$
$$u_r, u'_r, v_i, v'_i \geq 0, i = 1,...,m, r = 1,...,s$$

Due to the effect of intermediate variables, the two-level optimization Model (22) is proposed as follows:

$$Min(y_{rj}^{\min} \leq y_{rj} \leq y_{rj}^{\max})" ; j,r \begin{cases} RBWE_o^{2L} = \min \sum_{r=1}^{s} u_r z_{ro}^{\min} \\ \sum_{i=1}^{m} u'_r y_{ro} = 1 \\ \sum_{r=1}^{s} u_r z_{ro}^{\min} - (RBWE_o^L)^* \sum_{i=1}^{m} v_i x_{io}^{\max} - (RBWE_o^L)^* \sum_{i=1}^{m} v'_i l_{io}^{\max} \geq 0 \\ \sum_{r=1}^{s} u'_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij}^{\min} - \sum_{i=1}^{m} v'_i l_{ij}^{\min} \geq o, \quad o \neq j = 1,...,n \\ \sum_{r=1}^{s} u_r z_{rj}^{\max} - \sum_{r=1}^{s} u'_r y_{rj} \geq 0, \quad o \neq j = 1,...,n \\ \sum_{r=1}^{s} u'_r y_{ro} - \sum_{i=1}^{m} v_i x_{io}^{\max} - \sum_{i=1}^{m} v'_i l_{io}^{\max} \geq 0 \\ \sum_{r=1}^{s} u_r z_{ro}^{\min} - \sum_{r=1}^{s} u'_r y_{ro} \geq 0 \\ u_r, u'_r, v_i, v'_i \geq 0, \quad i = 1,...,m, r = 1,...,s \end{cases} \quad (22)$$

Model (22) can be transferred to Model (23):

$$RBWE_o^{2L} = \min \sum_{r=1}^{s} u_r z_{ro}^{\min}$$
$$s.t \begin{cases} \sum_{i=1}^{m} u'_r y_{ro} = 1 \\ \sum_{r=1}^{s} u_r z_{ro}^{\min} - (RBWE_o^L)^* \sum_{i=1}^{m} v_i x_{io}^{\max} - (RBWE_o^L)^* \sum_{i=1}^{m} v'_i l_{io}^{\max} \geq 0 \\ \sum_{r=1}^{s} u'_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij}^{\min} - \sum_{i=1}^{m} v'_i l_{ij}^{\min} \geq o, \quad o \neq j = 1,...,n \\ \sum_{r=1}^{s} u_r z_{rj}^{\max} - \sum_{r=1}^{s} u'_r y_{rj} \geq 0, \quad o \neq j = 1,...,n \\ \sum_{r=1}^{s} u'_r y_{ro} - \sum_{i=1}^{m} v_i x_{io}^{\max} - \sum_{i=1}^{m} v'_i l_{io}^{\max} \geq 0 \\ \sum_{r=1}^{s} u_r z_{ro}^{\min} - \sum_{r=1}^{s} u'_r y_{ro} \geq 0 \\ y_{rj}^{\min} \leq y_{rj} \quad r = 1,...,s; j = 1,...,n \\ y_{rj} \leq y_{rj}^{\max} \quad r = 1,...,s; j = 1,...,n \end{cases} \quad (23)$$
$$u_r, u'_r, v_i, v'_i \geq 0, i = 1,...,m, r = 1,...,s$$

Theorem 5:
Model (23) is always feasible and its optimal value is larger than or equal to 1.

Proof: Suppose that $\theta^1, \theta^2, \lambda_j^1, \lambda_j^2, \lambda_o^1, \lambda_o^2, \alpha_{rj}, \beta_{rj}$ are dual variables associated with constraints 1,…,8 respectively. So, dual form of Model (23) can be written as follows:

$$Max \;\; \theta^1 - \sum_{r=1}^{s} \sum_{j=1}^{n} \beta_{rj} y_{rj}^{\max} + \sum_{r=1}^{s} \sum_{j=1}^{n} \alpha_{rj} y_{rj}^{\min}$$
$$s.t \begin{cases} \theta^2 z_{ro}^{\min} + \sum_{o \neq j=1}^{n} \lambda_j^2 z_{rj}^{\max} + z_{ro}^{\min} \lambda_o^2 \leq z_{ro}^{\min}, r = 1,...,s \\ \theta^1 y_{ro} + \sum_{o \neq j=1}^{s} \lambda_j^1 y_{rj} - \sum_{o \neq j=1}^{m} \lambda_j^2 y_{rj} + y_{ro} \lambda_o^1 - y_{ro} \lambda_o^2 \leq 0, r = 1,...,s \\ -(RBWE_o^L)^* \theta^2 x_{io}^{\max} - x_{io}^{\max} \lambda_o^1 - \sum_{o \neq j=1}^{n} x_{ij}^{\min} \lambda_j^1 \leq 0, i = 1,...,m \\ -(RBWE_o^L)^* \theta^2 l_{io}^{\max} - l_{io}^{\max} \lambda_o^1 - \sum_{o \neq j=1}^{n} l_{ij}^{\min} \lambda_j^1 \leq 0, i = 1,...,m \end{cases} \quad (24)$$
$$\theta^1 \; free, \theta^2, \alpha_{rj}, \beta_{rj}, \lambda_j^1, \lambda_j^2 \geq 0, r = 1,...,s, j = 1,...,n$$

It is obvious that $\theta^1 = 1 = \lambda_o^2$, $\lambda_j^1 = \lambda_j^2 = \theta^2 = \lambda_o^1 = 0, \alpha_{rj} = \beta_{rj} = 0$ is a feasible solution for Model (24). So, there is always a feasible solution for Model (23). We know that the optimal value of Model (24) will be less than or equal to objective function of any feasible solution of Model (23). So, optimal value of Model (23) will be bigger or equal to 1.

Definition 5: Optimal value of Model (23) is the lower bound of relative bullwhip effect score (RBWE) of stage2 for a dynamic supply chain in the presence of uncertain demands. If this score is 1, then the bullwhip effect in the optimistic view point does not occur.

Model (25) is the pessimistic view points in order to measure the upper bound of relative bullwhip effect score of supply chain in stage1(i.e. $RBWE_o^{1U}$).

$$RBWE_o^{2U} = \min \sum_{r=1}^{s} u_r z_{ro}^{\max}$$

$$s.t \begin{cases} \sum_{i=1}^{m} u_r^{'} y_{ro} = 1 \\ \sum_{r=1}^{s} u_r z_{ro}^{\max} - (RBWE_o^U)^* \sum_{i=1}^{m} v_i x_{io}^{\min} - (RBWE_o^U)^* \sum_{i=1}^{m} v_i^{'} l_{io}^{\min} \geq 0 \\ \sum_{r=1}^{s} u_r^{'} y_{rj} - \sum_{i=1}^{m} v_i x_{ij}^{\max} - \sum_{i=1}^{m} v_i^{'} l_{ij}^{\max} \geq o, \qquad o \neq j = 1,\dots,n \\ \sum_{r=1}^{s} u_r z_{rj}^{\min} - \sum_{r=1}^{s} u_r^{'} y_{rj} \geq 0, \qquad o \neq j = 1,\dots,n \\ \sum_{r=1}^{s} u_r^{'} y_{ro} - \sum_{i=1}^{m} v_i x_{io}^{\min} - \sum_{i=1}^{m} v_i^{'} l_{io}^{\min} \geq 0 \\ \sum_{r=1}^{s} u_r z_{ro}^{\max} - \sum_{r=1}^{s} u_r^{'} y_{ro} \geq 0 \end{cases} \qquad (25)$$

$$u_r, u_r^{'}, v_i, v_i^{'} \geq 0, i = 1,\dots,m, r = 1,\dots,s$$

Due to the effect of intermediate variables, the two-level optimization Model (26) is proposed as follows:

$$Max_{(y_{rj}^{\min} \leq y_{rj} \leq y_{rj}^{\max}) \forall j,r} \begin{cases} RBWE_o^{2U} = \min \sum_{r=1}^{s} u_r z_{ro}^{\max} \\ \sum_{i=1}^{m} u_r^{'} y_{ro} = 1 \\ \sum_{r=1}^{s} u_r z_{ro}^{\max} - (RBWE_o^U)^* \sum_{i=1}^{m} v_i x_{io}^{\min} - (RBWE_o^U)^* \sum_{i=1}^{m} v_i^{'} l_{io}^{\min} \geq 0 \\ \sum_{r=1}^{s} u_r^{'} y_{rj} - \sum_{i=1}^{m} v_i x_{ij}^{\max} - \sum_{i=1}^{m} v_i^{'} l_{ij}^{\max} \geq o, \qquad o \neq j = 1,\dots,n \\ \sum_{r=1}^{s} u_r z_{rj}^{\min} - \sum_{r=1}^{s} u_r^{'} y_{rj} \geq 0, \qquad o \neq j = 1,\dots,n \\ \sum_{r=1}^{s} u_r^{'} y_{ro} - \sum_{i=1}^{m} v_i x_{io}^{\min} - \sum_{i=1}^{m} v_i^{'} l_{io}^{\min} \geq 0 \\ \sum_{r=1}^{s} u_r z_{ro}^{\max} - \sum_{r=1}^{s} u_r^{'} y_{ro} \geq 0 \\ u_r, u_r^{'}, v_i, v_i^{'} \geq 0, i = 1,\dots,m, r = 1,\dots,s \end{cases} \qquad (26)$$

Dual form of inner optimization is presented in Model (27):

$$Max \ \theta^1$$

$$s.t \begin{cases} \theta^2 z_{ro}^{\max} + \sum_{o \neq j=1}^{n} \lambda_j^2 z_{rj}^{\min} + z_{ro}^{\max} \lambda_o^2 \leq z_{ro}^{\max}, r = 1,\dots s \\ \theta^1 y_{ro} + \sum_{o \neq j=1}^{s} \lambda_j^1 y_{rj} - \sum_{o \neq j=1}^{m} \lambda_j^2 y_{rj} + y_{ro} \lambda_o^1 - y_{ro} \lambda_o^2 \leq 0, r = 1,\dots,s \\ -(RBWE_o^U)^* \theta^2 x_{io}^{\min} - x_{io}^{\min} \lambda_o^1 - \sum_{o \neq j=1}^{n} x_{ij}^{\max} \lambda_j^1 \leq 0, i = 1,\dots,m \\ -(RBWE_o^U)^* \theta^2 l_{io}^{\min} - l_{io}^{\min} \lambda_o^1 - \sum_{o \neq j=1}^{n} l_{ij}^{\max} \lambda_j^1 \leq 0, i = 1,\dots,m \end{cases} \qquad (27)$$

$$\theta^1 \ free, \theta^2, \alpha_{rj}, \beta_{rj}, \lambda_j^1, \lambda_j^2 \geq 0, r = 1,\dots,s, j = 1,\dots,n$$

The optimal solution of inner optimization Model (27) is equal to optimal solution of optimization Model (27). So, Model (26) can be transferred to Model (28):

$$Max_{(y_{rj}^{\min} \leq y_{rj} \leq y_{rj}^{\max}) \forall j,r} \begin{cases} Max \theta^1 \\ \theta^2 z_{ro}^{\max} + \sum_{o \neq j=1}^{n} \lambda_j^2 z_{rj}^{\min} + z_{ro}^{\max} \lambda_o^2 \leq z_{ro}^{\max}, r = 1,\dots,s \\ \theta^1 y_{ro} + \sum_{o \neq j=1}^{s} \lambda_j^1 y_{rj} - \sum_{o \neq j=1}^{m} \lambda_j^2 y_{rj} + y_{ro} \lambda_o^1 - y_{ro} \lambda_o^2 \leq 0, r = 1,\dots,s \\ -(RBWE_o^U)^* \theta^2 x_{io}^{\min} - x_{io}^{\min} \lambda_o^1 - \sum_{o \neq j=1}^{n} x_{ij}^{\max} \lambda_j^1 \leq 0, i = 1,\dots,m \\ -(RBWE_o^U)^* \theta^2 l_{io}^{\min} - l_{io}^{\min} \lambda_o^1 - \sum_{o \neq j=1}^{n} l_{ij}^{\max} \lambda_j^1 \leq 0, i = 1,\dots,m \\ \theta^1 free, \theta^2, \alpha_{rj}, \beta_{rj}, \lambda_j^1, \lambda_j^2 \geq 0, r = 1,\dots,s, j = 1,\dots,n \end{cases} \qquad (28)$$

Two-level optimization Model (28) can be transferred to single-level optimization Model (29):

$$Max \ \theta^1$$

$$s.t \begin{cases} \theta^2 z_{ro}^{\max} + \sum_{o \neq j=1}^{n} \lambda_j^2 z_{rj}^{\min} + z_{ro}^{\max} \lambda_o^2 \leq z_{ro}^{\max}, r = 1,\dots,s \\ \theta^1 y_{ro} + \sum_{o \neq j=1}^{s} \lambda_j^1 y_{rj} - \sum_{o \neq j=1}^{m} \lambda_j^2 y_{rj} + y_{ro} \lambda_o^1 - y_{ro} \lambda_o^2 \leq 0, r = 1,\dots,s \\ -(RBWE_o^U)^* \theta^2 x_{io}^{\min} - x_{io}^{\min} \lambda_o^1 - \sum_{o \neq j=1}^{n} x_{ij}^{\max} \lambda_j^1 \leq 0, i = 1,\dots,m \\ -(RBWE_o^U)^* \theta^2 l_{io}^{\min} - l_{io}^{\min} \lambda_o^1 - \sum_{o \neq j=1}^{n} l_{ij}^{\max} \lambda_j^1 \leq 0, i = 1,\dots,m \\ y_{rj} \leq y_{rj}^{\max}, j = 1,\dots,n; r = 1,\dots,s \\ -y_{rj} \leq -y_{rj}^{\min}, j = 1,\dots,n; r = 1,\dots,s \end{cases} \qquad (29)$$

$$\theta^1 \ free, \theta^2, \alpha_{rj}, \beta_{rj}, \lambda_j^1, \lambda_j^2 \geq 0, r = 1,\dots,s, j = 1,\dots,n$$

Theorem 6:
Model (29) is always feasible and its optimal value is larger than or equal to 1.
Proof: It is obvious that $\theta^1 = 1 = \lambda_o^2, \lambda_j^1 = 0, \lambda_j^2 = 0, \theta^2 = \lambda_o^1 = 0$ is a feasible solution for Model (29). So, there is always a feasible solution for Model (29). We know that the optimal value of Model (29) will be bigger or equal to 1.
Definition 6: Optimal value of Model (29) is the upper bound of relative bullwhip effect score (RBWE) of stage2 for a dynamic supply chain in the presence of uncertain demands. If this score is 1, then the bullwhip effect in the pessimistic view point does not occur.

3. Classification of the Relative Bullwhip Effect in an Uncertain Supply Chain

By using interval relative bullwhip effect scores of the network, interval relative bullwhip effect scores of stage1 (Demand Forecasting Stage) and interval relative bullwhip effect scores of stage 2 (Ordering Policy Stage), they can be categorized in four subsets due to the lower and upper bound of maximum achievable bullwhip effect scores as (30).

$$\begin{cases} RBWE_o^L = RBWE_o^U = 1 \rightarrow NonBullwhip_*^* \\ RBWE_o^L > 1, RBWE_o^U = 1 \rightarrow NonBullwhip^* \\ RBWE_o^L = 1, RBWE_o^U > 1 \rightarrow NonBullwhip_* \\ RBWE_o^L > 1, RBWE_o^U > 1 \rightarrow Bullwhip \end{cases} \quad (30)$$

Where there are DMUs in the network, in the demand forecasting stage and in the ordering policy stage, in class NonBullwhip_*^* they are in strong non-bullwhip effect position. DMUs in the network, in the demand forecasting stage and in the ordering policy stage in class NonBullwhip^* are in pessimistic non-bullwhip effect position. DMUs in the network, in the demand forecasting stage and in the ordering policy stage in class NonBullwhip_* are in optimistic non-bullwhip effect position. DMUs in the network, in the demand forecasting stage and in the ordering policy stage in class Bullwhip are in bullwhip effect position with optimistic and pessimistic view points. Obviously, DMUs (an interval time in the node) of the supply chain in class NonBullwhip_* is better than class NonBullwhip^*.

4. Numerical Example

In this study, Meshkin's match factory was considered as a real dynamic supply chain. The objective of this section was to calculate the relative score of the bullwhip effect produced with the uncertain data in the factory node. The test run time was about 90 days. In order to use the proposed method, we divided the 90-day period into nine equal time intervals. Therefore, information obtained from the first day until the ninth day was assumed as the first decision-making unit, and decision-making units 2 to 9 were also considered, accordingly. Table 1 presents uncertain inputs and outputs of the supply chain network. As shown in the table, all the bounds were normalized.

Table 1
Uncertain inputs, outputs in 90 day

DMU_j	Time $[t_j, t_{j+1}]$	D1	D 2	Info step 1	Info step 2	D F 1	D F 2	O 1	O 2
1	[0-10]	[1-2]	[2-3]	[1-2]	[2-3]	[2-4]	[3-4]	[5-10]	[5-7]
2	[10-20]	[2-3]	[3-4]	[1-2]	[2-3]	[3-6]	[4-5]	[5-10]	[5-9]
3	[20-30]	[3-5]	[1-3]	[2-3]	[3-4]	[4-6]	[1-3]	[5-8]	[2-7]
4	[30-40]	[1-2]	[1-2]	[3-5]	[1-3]	[2-4]	[2-3]	[3-8]	[5-8]
5	[40-50]	[2-4]	[2-4]	[1-2]	[1-2]	[3-6]	[3-4]	[5-11]	[4-8]
6	[50-60]	[2-3]	[1-5]	[2-4]	[2-4]	[3-4]	[2-7]	[4-8]	[4-9]
7	[60-70]	[3-6]	[3-4]	[2-3]	[1-5]	[3-7]	[4-7]	[5-9]	[6-12]
8	[70-80]	[4-6]	[4-5]	[3-6]	[3-4]	[4-7]	[5-7]	[6-12]	[6-13]
9	[80-90]	[2-4]	[3-4]	[4-6]	[4-5]	[3-5]	[5-6]	[4-8]	[6-9]

Table 2
Interval Relative bullwhip effect score using the proposed uncertain inverse network DEA

DMU_j	Time $[t_j, t_{j+1}]$	RBWE in stage1 (Demand Forecasting)	RBWE in stage2 (Ordering Policy)	RBWE in Network	Classification for Network
1	[0-10]	[1-1.021]	[1-1.121]	[1-1.121]	NonBullwhip_*
2	[10-20]	[1-1.102]	[1-1.1222]	[1-1.23]	NonBullwhip_*
3	[20-30]	[1-1.3]	[1-343]	[1-1.677]	NonBullwhip_*
4	[30-40]	[1-1.66]	[1-1.4566]	[1-2.11]	NonBullwhip_*
5	[40-50]	[1-1.576]	[1-1.676]	[1-1.534]	NonBullwhip_*
6	[50-60]	[1-1.2022]	[1-1.222]	[1-1.345]	NonBullwhip_*
7	[60-70]	[1-1.0601]	[1-1.0001]	[1-1]	NonBullwhip_*^*
8	[70-80]	[1.1-1.011]	[1.101-1.11]	[1.121-1.211]	Bullwhip
9	[80-90]	[1-1.20033]	[1-1.233]	[1-1.112]	NonBullwhip_*

Using the proposed model results in the interval bullwhip effect scores of the overall process, forecasting stage (i.e., sub-process 1) and ordering stage (i.e., sub-process 2) for 9 interval times. The results are given in the Table 2. In Table 2, according to the explanations presented, all DMUs (1-6) are classified in NonBullwhip_* category (that is the bullwhip effect happens in pessimistic point of views), DMU_7 is in NonBullwhip_*^* category (that is the bullwhip effect does not happen in strong point of views), DMU_8 is in Bullwhip category (that is the bullwhip effect happens inevitably) and DMU_9 is in NonBullwhip$_*$ category in the overall process (that is the bullwhip effect happens in pessimistic points of view).

The results in Table 2 show that in the demand forecasting stage, the relative scores of the bullwhip effect were low. Uncertainties are one of the most important reasons for the effect of the bullwhip phenomenon. The proposed method showed that the bullwhip effect occurred at the second stage of the network with an uncertain demand. In the ordering stage the relative scores had large variances.

5. Conclusion

Demand uncertainties in the manufacturing systems caused the bullwhip effect which can decrease the performance level of the supply chain, and create numerous economic and managerial challenges. Identification and measuring of the bullwhip effect is the first step towards its efficient management. This is a summary of what was done in this study: the model of Inverse Network Data Envelopment Analysis was proposed for measuring the bullwhip effect. Considering demand uncertainties in various intervals, the study regarded each interval as a decision making unit. Each unit includes two stages. In the first stage, the decision making unit forecasts the demand for the next interval through exponential smoothing. A series of forecasted demands (with a considerable level of transferred uncertainty) are regarded as the outputs of this stage. In the second stage, the outputs are fed to the model, and the decision making unit places its order. The study verified the feasibility and the optimality of its proposed model, which had been customized and developed for the study. Finally, the theoretical results were analyzed through a numerical example from a real supply chain. These results and their numerical verification and analysis, attested the efficiency of the study's proposed methodology.

The magnitude of the bullwhip effect increases over time from downstream to upstream, and magnification occurs in the variance of demand and orders. In classical methods, no reference is made to the relative causes and internal factors of the chain components. Therefore, managers do not take into account the relative effects of time and internal processes in dealing with the effect of RBWE, which causes unintentional errors. In this study, we were able to provide a true and fair measure of the BW by introducing the relative effects of scores (relative to time). Relativity in scores, in turn, can provide important information for managers. We used an interval level for indeterminate demands and extended this view throughout the chain. This approach allows managers to identify the effect of the RBWE in both optimistic and pessimistic modes, and to plan carefully for each situation. Due to the turmoil in the market, favorable and unfavorable conditions are always possible. Planning for ideal and non-ideal conditions has been a concern for managers.

Network analysis allows managers to examine the relative effect of the BWE on internal supply chain network processes. Hidden weaknesses and strengths are revealed at this stage. As the results of the case study showed, the size of the RBWE in the first stage (demand forecasting process) is larger than the second stage (ordering process). Therefore, managers should focus more on the demand forecasting stage. It can be said that the demand fluctuations at the beginning of the chain are more sensitive. The RBWE measured across the entire network is significantly larger at each step. In fact, the simultaneous adoption of forecasting and ordering policies complicates the chain, and this has intensified RBWE. It is recommended that managers adopt policies in accordance with the results of this study.

One of the important features of the presented model was the division of time into consecutive intervals. In this way we were able to assume time intervals as decision making units. This ensured network dynamics and also introduced the relative effects of time into measurements. Managers can use the results of this attitude to make different decisions over time and tailored to time periods. Unlike classical sizes, the relative scores obtained for RBWE were larger at the beginning of the chain than at the end of the chain. This is because the decision-making units were time periods. The relative role of time was well demonstrated. This approach identifies gaps that were hidden in classical methods.

In general, it can be stated that the present study was able to create a suitable structure to identify the BWE with the use of factors such as dynamics (decision units are time intervals), uncertainty, optimistic and pessimistic views and networking.

Recommendations for future research directions include an envelopment form of the Uncertain Inverse Network DEA model proposed in this paper which can be used alternatively. In the present study, we used a constant return to scale/input-oriented model to evaluate the bullwhip effect. An interesting stream of research is additional assumptions for variable return to scale/output-oriented models. We modeled uncertain demands in the data using the inverse network DEA. Other factors such as stochastic demands, time delay and aggressive ordering can be investigated. We used the proposed approach to evaluate the bullwhip effect of a match factory, which can be used in other applications and similar systems. There are some methods in IDEA, such as ranking, return to scale speech, and stability analysis; these methods provide techniques for optimization. In future researches, one can analyze the sensitivity of other factors causing the bullwhip effect in network IDEA models.

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