

# A Non-dominated Sorting Ant Colony Optimization Algorithm Approach to the Bi-objective Multi-vehicle Allocation of Customers to Distribution Centers

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## Abstract

This paper proposes a mathematical model as the bi-objective capacitated multi-vehicle allocation of customers to distribution centers. An evolutionary algorithm named non-dominated sorting ant colony optimization (NSACO) is used as the optimization tool for solving this problem. The proposed methodology is based on a new variant of ant colony optimization (ACO) specialized in multi-objective optimization problem. To help the decision maker to choose the best compromise solution from the Pareto front, the fuzzy-based mechanism is employed. For ensuring the robustness of the proposed method and giving a practical sense of this study, the computational results are compared with those obtained by NSGA-II. Results show that both NSACO and NSGA-II algorithms can yield an acceptable number of non-dominated solutions. In addition, the results show that while the distribution of solutions in the trade-off surface of both NSACO and NSGA-II algorithms do not differ significantly, NSACO algorithm is more efficient than NSGA-II with regard to optimality, convergence and the CPU time. Also, the results in some small cases are compared with those obtained by LP-metric method. The error percentages of objective functions in comparison to the LP-metric method are less than 2%. Furthermore, it can be seen that with increasing size of the problems, while the time of problem solving increases exponentially by using the LP-metric method, the running time of NSACO and NSGA-II are more stable.

**Keywords:** Bi-objective optimization, Capacitated allocation, Multi-vehicle, Distribution centers, Non-dominated sorting ant colony optimization, NSGA-II, LP- metric method.

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## 1. Introduction

In today's business environment, the competitiveness of a firm heavily depends on its ability to handle the challenges of reducing cost, increasing customer service, and improving product quality. In this competitive market, customer satisfaction is the most important factor for the success of the firm. In this regard, the supply chain network among different business entities like manufacturers, suppliers and distribution centers (DCs) needs to be effective enough to handle the changing demand patterns. Nowadays, efforts have been made to design and develop a more conducive and profitable supply chain network. Efficient allocation of customers to DCs always plays an important role in developing a flawless and reliable supply network.

One of the most active topics in manufacturing research over the last 10 years has been supply chain management (SCM). SCM is the management of material and information flows both in and between facilities, such as vendors, manufacturing, assembly plants and distribution

centers (Thomas & Griffin, 1996). Transportation network design is one of the most important fields of SCM. It offers great potential to reduce costs. In the several decades, there have been many researchers reported new models or methods to determine the transportation or the logistics activities that can lead to the least cost (Gen & Cheng, 1997). One of the important factors which influences on logistic system is to decide regarding the number of distribution centers. Geoffrion and Graves (1974) were the first researchers studied on two-stage distribution problem. Pirkul and Jayaraman (1998) presented a new mathematical formulation called PLANWAR to locate a number of production plants and warehouses and to design distribution network, so that the total operating cost can be minimized. They developed an approach based on Lagrangian relaxation to solve the problem. Hindi et al. (1998) stated a two-stage distribution planning problem. They supposed that each customer must be served from a single distribution center. The authors gave mathematical model for the problem and developed a branch and bound algorithm to solve the problem. Zhou et

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al. (2003) proposed a mathematical model and an efficient solution procedure for the bi-criteria allocation problem involving multiple warehouses with different capacities. Hajiaghahi-Keshteli (2011) considered two stages of supply chain network including distribution centers (DCs) and customers. His proposed model selects some potential places as distribution centers in order to supply demands of all customers and in order to solve the given problem, two algorithms, genetic algorithm and artificial immune algorithm, were developed. Chan and Kumar (2009) discussed a multiple ant colony optimization (MACO) approach in an effort to design a balanced and efficient supply chain network that maintains the best balance of transit time and customers service. The focus of their paper is on the effective allocation of the customers to the DCs with the two-fold objective of minimization of the transit time and degree of imbalance of the DCs.

## 2. Preliminaries

### 2.1. Multi-objective optimization problems

Many optimization problems in the real world involve the optimization of several objectives at the same time. To obtain the optimal solution, there will be a set of optimal trade-offs between the conflicting objectives, where the set of optimal solution is known as Pareto front (Abido & Bakhshwain, 2005). A multi-objective optimization problem is defined as the maximization or the minimization of many objectives subject to equality and inequality constraints. The multi-objective optimization problem can be formulated as follows:

$$\text{Max. /Min. } f_i(x), i=1, \dots, N_{obj} \quad (1)$$

Subject to constraints:

$$g_j(x) = 0, j=1, \dots, M$$

$$h_k(x) \leq 0, k=1, \dots, K \quad (2)$$

where  $f_i$  is the  $i$ th objective function,  $x$  is the decision vector,  $N_{obj}$  is the number of objectives,  $g_j$  is the  $j$ th equality constraint, and  $h_k$  is the  $k$ th inequality constraint.

There are techniques such as weighting method and  $\epsilon$ -constraint method which transfer multi-objective problems to a single-objective one, using different combinations of a weighting vector and constraints. Thus, each optimal solution can be assigned to a specific combination of weighting vector and constraint. Hence, in each run of the algorithm, a single solution can be achieved. However, multi-objective evolutionary algorithms are capable of finding almost all candidate solutions (Pareto) in a single run.

Figure 1 shows dominated and non-dominated relations between objective values in a bi-objective problem in which both objectives are optimized. In this figure, solutions labeled by 1 or 2 have non-dominating

conditions individually. Note that the set labeled 1 dominates the set labeled 2. In the optimization procedure, the best set of non-dominating solutions is called Pareto front. Thus, there are two Pareto in the Fig 1, in which the one labeled 1 is the Pareto front.

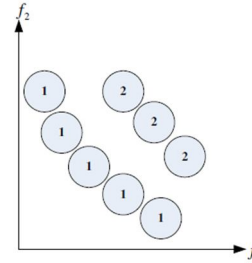


Fig1. Schematic of dominated and non-dominated conditions of solutions in a biobjective problem (Fallah-Mehdipour et al. 2012)

### 2.2. Multi-objective evolutionary algorithms

Evolutionary algorithms are based on evolutionary computations which can perform optimal/near-optimal solutions in all types of problems (linear/nonlinear, discrete/continuous, convex/ non-convex) using validated experimental theories of biological evolution and natural processes, particularly through activities of different species of animals. A set of solutions without using any techniques, are directly related to the decision-makers' opinions, is the most important advantage of evolutionary algorithms in the field of multi-objective optimization. Thus, these algorithms are used as optimization tools in the multi-objective optimization problems (Deb, 2001). In these algorithms, random decision variables are used as input data for a simulation model. Output data from the simulation model are then used as input data for an optimization model. In such a process, newly-generated decision variables, based on previously calculated ones, have been improved. This process continues up to the maximum number of iterations for determining the best solution. In traditional optimization methods, techniques such as the weighting approach are used in linear and non-linear programming (LP and NLP) to produce a single optimal solution. However, evolutionary algorithms can yield a set of non-dominated solutions, Pareto, as the optimal solutions.

Chen and Ting (2006) applied a multiple ant colony system and also developed a hybrid ant colony and lagrangian heuristic for the single source capacitated location problem (SSCFLP). Also, for discrete location problems in graphs, ant-based algorithms have been successfully applied, (e.g. Venables & Moscardini, 2006). Non-dominated Sorting Genetic Algorithm (NSGA), multi-objective ACO (MOACO), and multi-objective PSO (MOPSO) are few examples of multi-objective evolutionary optimization algorithms of this type (Xing & Qu, 2013). Kalhor et al. (2011) proposed a non-dominated archiving ant colony approach to solve the stochastic time-cost tradeoff optimization problem. Mostafavi and

Afshar (2011) used a powerful ant colony algorithm known as non-dominated archiving multi-colony ant algorithm (NAACO) to solve the optimal Waste Load Allocation as a multi-objective optimization problem.

In this paper, two-stage supply chain networks including the distribution centers and the customers, are considered. There are potential places which are candidate to be as distribution centers, called potential DCs, and customers with particular demands. Each of the potential DCs can ship to any of the customers. The two optimization objectives are to minimize transit time and total cost involving opening cost, assumed for opening a potential DC and shipping cost per unit from DC to the customers. The proposed model selects some potential places as distribution centers in order to supply demands of all customers, i.e. the model selects some potential DCs in such a way that the customer's demand can be satisfied at minimum DCs' opening cost and minimum shipping cost with minimum transit time. It is assumed that distribution centers have unequal capacities and each customer must be served from a single distribution center. Also in this paper, considering different types of vehicles caused more conflicting in these two objectives. We proposed an evolutionary algorithm, Non-dominated Sorting Ant Colony Optimization (NSACO) to tackle with the problem in this paper. In contrast to the traditional multiple objective programming techniques such as goal programming that require the decision maker to arbitrarily determine weighting coefficients and/or preferences on multiple criteria and consequently produce a dominated solution, the proposed algorithms was designed to generate a wide range of non-dominated solutions without the arbitrary determination of weights.

### 3. Description of Model

In this paper the allocation-based model on Zhou et al. (2003) is considered as a basic model. In their paper, optimization objectives are to minimize total transit time and total shipping cost. Since the shipping cost depends on distance and the value of the goods, both of these objective functions (minimizing time and cost) lead to allocation of customers to the nearest distribution centers to them. It means that both functions are aimed in one direction. While adding a decision criterion increases precision of the model, it increases the complexity of the problem, too. Therefore, there should be a balance between resolution and efficiency. In other words, in the multi-objective optimization, the aims are (1) to find Pareto optimal solutions and (2) to analyze the trade-off between conflicting objectives.

In this paper, different types of vehicles are considered to transport demand. In fact, considering heterogeneous vehicles lead to a more realistic model and cause more conflicting in the two objectives of the proposed model,

since a fast vehicle (because of high technology or having low capacity) has more cost and a vehicle with low cost can lead to higher transit time. Moreover, the choice of location of potential sites for the DCs also has been considered in the model. Then, in this paper, a location-allocation model for multi-vehicle single product in two-stage supply chain network is developed. This model includes distribution centers and customers with respect to two conflicting objectives consist of minimizing total transit time and total cost. The total cost here, involves opening cost, assumed for opening potential DCs and shipping cost from DCs to the customers. It is assumed that distribution centers have unequal capacities and each customer must be served from a single distribution center. It's possible that among potential DCs, all or some of them will be opened and deployed. Also, in this model the customers' demand is assumed to be deterministic.

Let us denote  $I$  as a set of nodes representing  $m$  customers,  $J$  as a set of nodes representing  $p$  potential distribution centers,  $V$  as a set of types of vehicles for transferring process so that the number of vehicles is assumed to be unlimited, and  $E$  as a set of edges representing a connection between customers and DCs.  $d_i$  denotes the demand of customer  $i$ ,  $f_j$  the fixed cost for opening a potential DC at site  $j$ ,  $s_v$  the capacity of type of vehicle  $v$ ,  $v \in V$ , and the associated capacity  $s_j$  for such DC;  $d_{ij}$  the distance between DC  $j$  and customer  $i$ ;  $c_{ij}^v$  is the cost of assigning customer  $i$  to DC located at site  $j$  with type of vehicle  $v$ ,  $t_{ij}^v$  is the transit time between customer  $i$  to DC located at site  $j$  with type of vehicle  $v$ . All parameters introduced above are assumed to be non-negative. The binary variable  $y_j$  is 1 if a DC is located at site  $j$  and 0 otherwise. Similarly, binary variable  $x_{ij}^v$  is equal to 1 if customer  $i$  is served by the DC located at site  $j$  with type of vehicle  $v \in V$  and 0 otherwise. In fact, here the dimension of vehicle type is added to the allocation variables and the allocation variables are considered as three-dimensional variables.

The bi-objective capacitated multi-vehicle allocation of customers to distribution centers problem can be formulated as the following binary integer programming:

$$\min z_1 = \sum_{v=1}^V \sum_{j=1}^p \sum_{i=1}^m d_i d_{ij} c_{ij}^v x_{ij}^v + \sum_{j=1}^p f_j y_j \quad (3)$$

$$\min z_2 = \sum_{v=1}^V \sum_{j=1}^p \sum_{i=1}^m t_{ij}^v x_{ij}^v \quad (4)$$

Subject to:

$$\sum_{v=1}^V \sum_{j=1}^p x_{ij}^v = 1 \quad \forall i=1, \dots, m \quad (5)$$

$$\sum_{v=1}^V \sum_{i=1}^m d_i x_{ij}^v \leq s_j y_j \quad \forall j=1, \dots, p \quad (6)$$

$$\sum_{v=1}^V \sum_{j=1}^p d_i x_{ij}^v \leq \sum_{v=1}^V \sum_{j=1}^p x_{ij}^v s_v \quad \forall i=1, \dots, m \quad (7)$$

$$x_{ij}^v, y_j \in \{0,1\}, \quad \forall i=1, \dots, m, \quad \forall j=1, \dots, p \quad \forall v=1, \dots, V \quad (8)$$

The first objective function (Eq.3) minimizes the total cost of opening distribution centers and assigning customers to such distribution centers, while the second objective function (Eq.4) minimizes total transit time between distribution centers and customers allocated to them. Constraints (Eq.5) guarantee that each customer is served by exactly one DC and also guarantee that each customer's demand on each edge between a customer and a DC is transferred by a vehicle type and exactly one of it, and capacity constraints (Eq.6) ensure that the total demand assigned to a DC cannot exceed its capacity. The constraints (Eq.7) ensure that the total demand transferred by a vehicle cannot exceed its capacity. In this paper, capacity constraints of DCs have been relaxed considering penalty function.

In general, a penalty function approach is as follows. Given an optimization problem:

$$\min f(X) \tag{9}$$

$$\text{s.t. } X \in A$$

$$X \in B$$

where  $X$  is a vector of decision variables, the constraints " $X \in A$ " are relatively easy to satisfy, and the constraints " $X \in B$ " are relatively difficult to satisfy, the problem can be reformulated as:

$$\min f(X) + p(d(X,B)) \tag{10}$$

$$\text{s.t. } X \in A$$

where  $d(X, B)$  is a metric function describing the distance of the solution vector  $X$  from the region  $B$ , and  $p(0)$  is a monotonically non-decreasing penalty function such that  $p(0) = 0$ . Furthermore, any optimal solution of (Eq.10) will provide an upper bound on the optimum for (Eq.9), and this bound will in general be tighter than that obtained by simply optimizing  $f(X)$  over  $A$ .

In this paper, the objective functions are as follows:

$$\min \widehat{z}_1 = z_1 + \delta_1 \cdot Vi \tag{11}$$

$$\min \widehat{z}_2 = z_2 + \delta_2 \cdot Vi$$

where  $\delta_1 \cdot Vi$  and  $\delta_2 \cdot Vi$  are penalty functions.  $\delta_1$  and  $\delta_2$  are two positive coefficients where usually are considered greater than  $\max(z_1)$  and  $\max(z_2)$ , respectively. Also,  $Vi$  represents relatively violation value of capacity constraints related to DCs (Eq.6):

$$Vi = (\sum_{v=1}^V \sum_{i=1}^m d_i x_{ij}^v - s_j y_j) / s_j y_j \quad \text{if } \sum_{v=1}^V \sum_{i=1}^m d_i x_{ij}^v > s_j y_j \quad \forall j=1, \dots, p$$

And also

$$Vi = 0 \quad \text{if } \sum_{v=1}^V \sum_{i=1}^m d_i x_{ij}^v \leq s_j y_j \quad \forall j=1, \dots, p \tag{12}$$

Besides fulfilling other constraints (Eq. 5 and Eq. 7), the solutions with  $Vi = 0$  are feasible and otherwise the solutions are infeasible.

#### 4. Solution Approach

In this paper, MATLAB platform, along with two evolutionary algorithms, NSACO and NSGA-II are used as the optimization tools in extracting the solution of the bi-objective capacitated multi-vehicle allocation of customers to distribution centers problem. In order to more validation of the proposed algorithms, the LP-metric method is used. In this section, the LP-metric approach and the NSACO algorithm are described to solve the problem.

##### 4.1. LP-metric method

LP-metric method which is usually discussed in multi-objective decision making (MODM) references such as (Hwang & Masud, 1979) is among optimization techniques that combine multiple objectives into a single objective. In this approach, the decision maker must define the reference point  $z$  to attain. Then, a distance metric between the referenced point and the feasible region of the objective space is minimized. The aspiration levels of the reference point are introduced into the formulation of the problem, transforming it into a mono-objective problem. For instance, the objective function can be defined as a weighted norm that minimizes the deviation from the reference point. Using the LP-metric, the problem can be formulated in the following way:

$$\text{MOP } (\lambda, z) = \min (\sum_{j=1}^n \lambda_j |f_j(x) - z_j|^p)^{1/p} \tag{13}$$

$$\text{s.t. } x \in S$$

where  $1 \leq p \leq \infty$ ,  $\lambda_j$  is the weight of  $j$ th objective function and  $z$  is the reference point. When  $p = 1$  is used, the resulting problem reduces to a weighted sum of the deviations. When  $p = 2$  is used, a weighted Euclidean distance of any point in the objective space from the referenced point is minimized. When  $p = \infty$  is considered, the largest deviation is minimized. Chankong and Haimes (1983) showed that when LP method is used then all solutions corresponding to  $1 \leq p \leq \infty$  and  $\lambda_j > 0$  are efficient solutions.

##### 4.2. Non-dominated sorting ant colony optimization (NSACO) algorithm

Ant Colony Optimization (ACO) algorithms are the most successful and widely recognized algorithmic techniques based on real ant behaviors (Dorigo & Stutzle, 2004). Several papers proposed to extend the ant colony optimization (ACO) method in order to handle a multi-objective optimization problem, (e.g. Chen & Ting, 2006;

Chan & Kumar, 2009; Kalhor et al. 2011; Mostafavi & Afshar, 2011).

In this paper, an evolutionary algorithm named Non-dominated Sorting Ant Colony Optimization (NSACO) is proposed to tackle the bi-objective capacitated multi-vehicle allocation of customers to distribution centers problem. NSACO algorithm is based on the same non-dominated sorting concept used in NSGA-II (Deb et al. 2000). The proposed methodology is based on a new variant of ant colony optimization (ACO) specialized in multi-objective optimization problem. Steps of the NSACO are as follows:

In the first step, for a better search of the solution space, a colony of ants with the size of  $2 \times nAnt$  ( $nAnt$  is the original population size) is considered. Then, ACO parameters such as  $\alpha$ ,  $\beta$ ,  $\rho$ , etc. are initialized, which  $\alpha$  and  $\beta$  are parameters used for controlling the exponential weight of the pheromone trail and the heuristic exponential weight and  $\rho$  is evaporation rate (Dorigo & Stutzle, 2004). Also in this step, the value of the initial pheromone trail,  $\tau_0$ , is determined and the Tabu lists of all ants are constructed, which contain all the unvisited nodes for each ant and the list of optimal paths traversed by the ants. The initial pheromone intensity,  $\tau_{ij}$  or the path from nodes  $i$  to  $j$  is set equal to  $\tau_0$ , that is  $\tau_{ij} = \tau_0$  and  $\Delta\tau_{ij} = 0$ .

In the second step, for each ant of the colony, a new solution using ACO probabilistic rule is created. It means that, for each ant a DC vector, an allocation matrix and a vehicle vector are assigned. The DC vector is a binary vector that indicates the opening or not opening DCs, the allocation matrix is a binary matrix that indicates the allocation of customers to the located DCs and the vehicle vector is an integer vector that indicates the vehicle type for transferring customers' demand. The allocation matrix and vehicle vector form a three-dimensional decision variable named  $x_{ij}^v$ . Then objective values for this solution are calculated and evaluated.

In order to construct the solution, ant  $k$  currently at node  $i$  determines the next node to visit, node  $j$ , by applying the sampling approach known as the Roulette Wheel Selection (Xia, 2012). For this purpose, first, movement probability for ant  $k$  from node  $i$  to other nodes including the neighbors of the node  $i$ , must be calculated.  $S_k(i)$  is a Tabu list, to avoid creating a loop, containing those unvisited nodes for ant  $k$  currently at node  $i$ . Therefore, node  $j \in S_k(i)$  is the node randomly chosen from the list  $S_k(i)$  according to the pseudo random proportional distribution rule Eq.14 and the roulette wheel selection:

$$P_{ij}^k = \tau_{ij}^\alpha \eta_{ij}^\beta / \sum_{u \in S_k(i)} (\tau_{iu}^\alpha \eta_{iu}^\beta) \quad \text{if } j \in S_k(i) \text{ and otherwise}$$

$$P_{ij}^k = 0 \tag{14}$$

where  $P_{ij}^k$  is the probability that ant  $k$  chooses to move from node  $i$  to node  $j$  and  $\eta_{ij}$  is a heuristic value which equals to the inverse of the length from node  $i$  to node  $j$ ,  $\tau_{ij}$  is the amount of pheromone trail of the path from node  $i$  to node  $j$ ,  $\alpha$  and  $\beta$  are two parameters used for controlling

the exponential weight of the pheromone trail and the heuristic value. Then, after calculating probability values, the roulette wheel selection is used to select next node among these existing neighbor nodes (Xia, 2012). In this paper, this process is occurred three times for constructing the DC vector, the allocation matrix and the vehicle vector.

In the third step, after all the ants of the colony traversed their paths, the non-dominated sorting method is applied, where the entire population is sorted into various non-domination fronts. In a minimization problem, a vector  $x^{(1)}$  is partially less than another vector  $x^{(2)}$ , ( $x^{(1)} < x^{(2)}$ ) when no value of  $x^{(2)}$  is less than  $x^{(1)}$  and at least one value of  $x^{(2)}$  is strictly greater than  $x^{(1)}$  (Tamura and Miura, 1979). A solution which is not partially less is a dominated solution and a solution which cannot be dominated throughout an existing solution set is called a non-dominated solution or Pareto front. The first front being completely a non-dominant set in the current population and the second front being dominated by the individuals in the first front only and the front goes so on. Each individual in each front is assigned fitness values or based on front in which they belong to. Individuals in the first front are given a fitness value of 1 and individuals in the second are assigned a fitness value of 2 and so on. Therefore, in addition to the fitness value, a parameter called crowding distance is calculated for each ant to ensure the best distribution of the non-dominated solutions. The crowding distance is an important concept proposed by Deb et al. (2000) in his algorithm NSGA-II. It serves for getting an estimate of the density of solutions surrounding a particular solution in the population. Fig 2 shows the calculation of the crowding distance of point  $i$  which is an estimate of the size of the largest cuboid enclosing  $i$  without including any other points. In fact, the crowding distance is a measure of how close an individual is to its neighbors. Consequently, all ants of a colony are sorted based on quality and discipline factors, simultaneously.

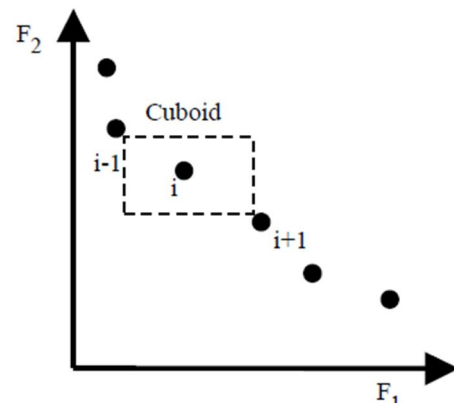


Fig 2. Crowding distance computation (Deb et al. 2000)

Then, the population with the size of  $2 \times nAnt$  is truncated to a population with the size of  $nAnt$ . At the end of each iteration, pheromone trails are updated and evaporation process occurs according to non-dominated



solutions. In this paper, three pheromone trails matrix are designed for DC vector, allocation matrix and vehicle vector. The pheromone trails matrix for DC vector is a  $2 \times p$  dimensions matrix, in which 2 is identified as open or closed state of the each DC, which the first row and the second row are considered for closing and opening the DCs, respectively, and  $p$  is identified as the number of DCs (Eq. 15). The pheromone trails matrix for allocation matrix is a  $p \times m$  dimensions matrix, in which  $p$  and  $m$  are identified as number of DCs and number of customers, respectively (Eq. 16) and the pheromone trails matrix for vehicle vector is a  $V \times m$  dimensions matrix, in which  $V$  and  $m$  are identified as types of vehicles and number of customers, respectively (Eq. 17).

$$\tau_{au1} = \begin{bmatrix} \tau_{11} & \dots & \tau_{1p} \\ \tau_{21} & \dots & \tau_{2p} \end{bmatrix} \quad (15)$$

$$\tau_{au2} = \begin{bmatrix} \tau_{11} & \dots & \tau_{1m} \\ \vdots & \ddots & \vdots \\ \tau_{p1} & \dots & \tau_{pm} \end{bmatrix} \quad (16)$$

$$\tau_{au3} = \begin{bmatrix} \tau_{11} & \dots & \tau_{1m} \\ \vdots & \ddots & \vdots \\ \tau_{v1} & \dots & \tau_{vm} \end{bmatrix} \quad (17)$$

The heuristic information matrix for DC vector is a  $2 \times p$  dimensions matrix, in which 2 is identified as closed or open state of the each DC, which the first row and the second row are considered for fixed cost for opening potential DCs and inverse of fixed cost for opening potential DCs, respectively and  $p$  is identified as the number of DCs (Eq. 18). The heuristic information matrix for allocation matrix is a  $p \times m$  dimensions matrix, in which  $p$  and  $m$  are identified as number of DCs and number of customers, respectively, which it contains inverse of distance values between customers and DCs (Eq. 19) and the heuristic information matrix for vehicle vector is a  $V \times m$  dimensions matrix, in which  $V$  and  $m$  are identified as types of vehicles and number of customers, respectively which it contains inverse of shipping cost from DCs to customers. There is one heuristic information matrix  $\forall j=1, \dots, p$  (Eq. 20).

$$\eta_{au1} = \begin{bmatrix} f_1 & \dots & f_p \\ 1/f_1 & \dots & 1/f_p \end{bmatrix} \quad (18)$$

$$\eta_{au2} = \begin{bmatrix} 1/d_{11} & \dots & 1/d_{1m} \\ \vdots & \ddots & \vdots \\ 1/d_{p1} & \dots & 1/d_{pm} \end{bmatrix} \quad (19)$$

$$\eta_{au3} = \begin{bmatrix} 1/c_{1j1} & \dots & 1/c_{mj1} \\ \vdots & \ddots & \vdots \\ 1/c_{1jv} & \dots & 1/c_{mjv} \end{bmatrix} \quad (20)$$

The pheromone trails are updated according to the non-dominated solutions in the Pareto Front and in order to prevent unlimited accumulation of the pheromone trails and help the algorithm to forget bad decisions of formers, evaporation process is applied on pheromone trails. This updating process affects selection of new solutions using ACO probabilistic rule in the next iteration. This cycle is repeated for a predefined number of iterations known as Cycle Iteration. At the end of running this algorithm, the present non-dominated solutions in the last iteration are the optimal solutions of the multi-objective problem. Fig 3, shows a graphical representation of NSACO.

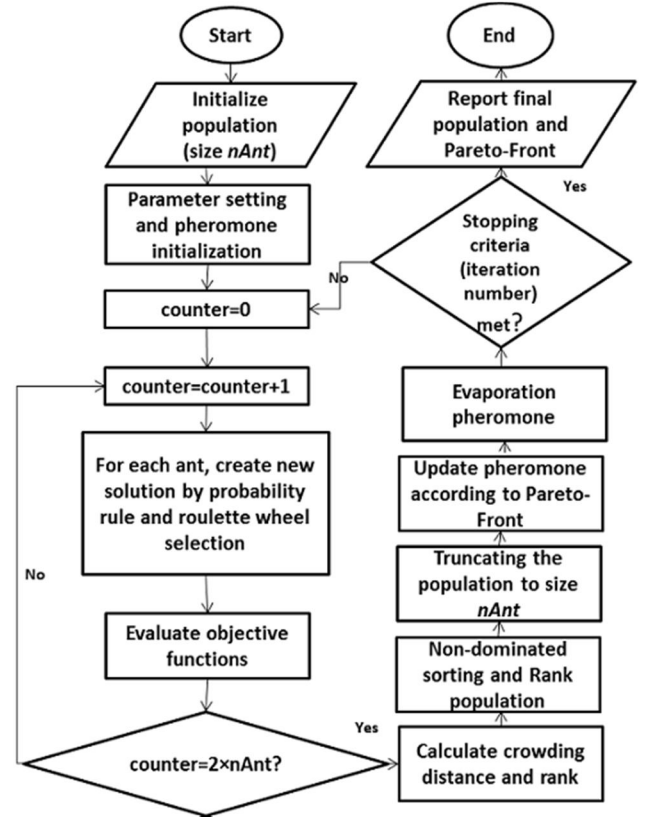


Fig 3. Graphical representation of NSACO

#### 4.3. Best compromise solution

Once the Pareto optimal set is obtained, it is possible to choose one solution from all solutions that satisfy different goals to some extent (Niimura & Nakashima, 2003). Due to the imprecise nature of the decision maker's (DM) judgment, it is natural to assume that the DM may have fuzzy or imprecise nature goals of each objective function (Dhillon et al. 1993). Hence, the membership functions are introduced to represent the goals of each objective function; each membership function is defined by the experiences and intuitive knowledge of the decision maker (Bo and Yi-jia, 2005). In this study, a simple linear membership function is considered for each of the objective functions. The membership function is defined as follows:

$$\begin{aligned} \mu_i &= 1 && \text{if } F_i \geq F_i^{max} \\ \mu_i &= (F_i^{max} - F_i)/(F_i^{max} - F_i^{min}) && \text{if } F_i^{min} < F_i < F_i^{max} \\ \mu_i &= 0 && \text{if } F_i \leq F_i^{min} \end{aligned} \quad (21)$$

where  $F_i^{min}$  and  $F_i^{max}$  are the minimum and the maximum value of the  $i$ th objective function among all non-dominated solutions, respectively. The membership function  $\mu$  is varied between 0 and 1, where  $\mu=0$  indicates the incompatibility of the solution with the set, while  $\mu=1$  means full compatibility (Dhillon et al. 1993). For each non-dominated solution  $k$ , the normalized membership function  $\mu_k$  is calculated as

$$\mu_k = \sum_{i=1}^{N_{obj}} \mu_i^k / \sum_{k=1}^M \sum_{i=1}^{N_{obj}} \mu_i^k \quad (22)$$

where  $M$  is the number of non-dominated solutions and  $N_{obj}$  is the number of objective functions. The function  $\mu_k$  can be considered as a membership function of non-dominated solutions in a fuzzy set, where the solution having the maximum membership in the fuzzy set is considered as the best compromise solution.

#### 4.4. Parameter tuning

In this paper, an evolutionary algorithm, NSACO is proposed as the optimization tool. NSACO algorithm is coded in MATLAB software and tested on a Core 2 Duo/2.66 GHz processor. As shown in Table 2, eight numerical cases in small scale and eight numerical cases in large scale are provided to demonstrate the application of this method. To obtain the best parameters, an auto tuning approach is used. First, some numbers, for example, 10 numbers in the range 0.8 to 1.8 are selected randomly for  $\alpha_1$ , *pheromone exponential weight for DC vector*, and the program runs for each value of  $\alpha_1$ . Then by observing the best answer, we tried the next random number to be close to the  $\alpha_1$  related to the best answer. In fact, the Beginning and the end of the range is updated according to the  $\alpha_1$  corresponding to the best answer in each iteration (see Fig 4). Exactly the same procedure in the range 0.05 to 0.6 is repeated for  $\beta_1$ , *heuristic exponential weight for DC vector*. These initial ranges are considered according to both existing literature in the field of ACO algorithm and some tentative running of NSACO program. This procedure is repeated for other parameters, simultaneously. Simultaneous tuning of all parameters causes considering the interaction effect of the parameters on each other. This process is performed by an external NSACO program for auto tuning parameters. Fig 5 shows auto tuning of two parameters among all, for instance. As shown in Fig 5, these two parameters are tuned in 10<sup>th</sup> iteration approximately. It means that if we consider 10 random numbers in each range in each iteration, these parameters are tuned with considering 100 times running of algorithm.

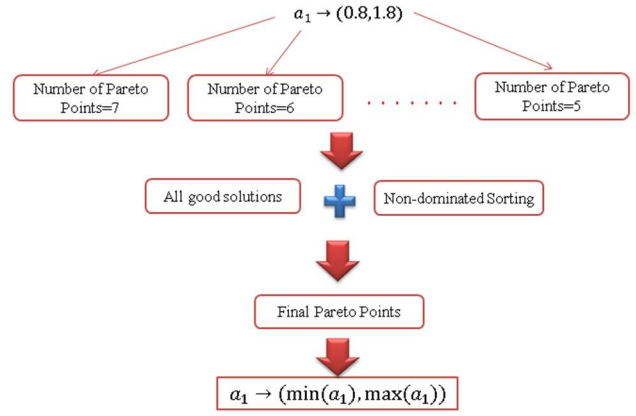


Fig 4. Schematic of updating  $\alpha_1$  in each iteration of auto tuning parameter process

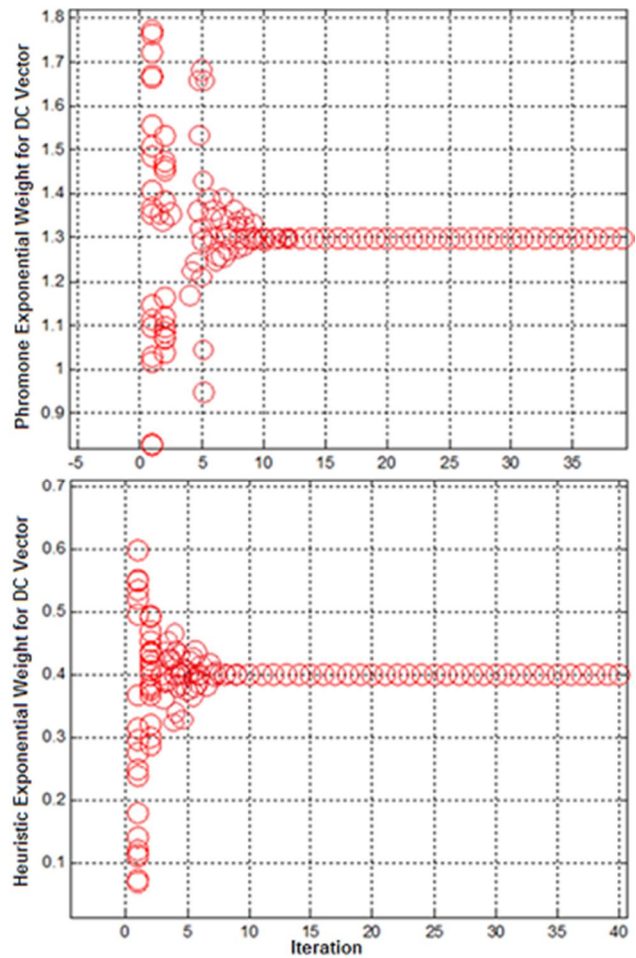


Fig 5. Auto tuning of two parameters of NSACO algorithm (Pheromone exponential weight for DC vector, Heuristic exponential weight for DC vector)

As mentioned above, the parameters of NSACO for all optimization cases are summarized in Table 1.

Table1  
NSACO parameters

$\alpha_1$ (Pheromone exponential weight for DC vector)	1.30
$\beta_1$ (Heuristic exponential weight for DC vector)	0.40
$\alpha_2$ (Pheromone exponential weight for allocation matrix)	1.58
$\beta_2$ (Heuristic exponential weight for allocation matrix)	0.33
$\alpha_3$ (Pheromone exponential weight for vehicle vector)	1.34
$\beta_3$ (Heuristic exponential weight for vehicle vector)	0.52
$\rho$ (Evaporation rate)	0.05

Table 2  
Some specification of small and large problems

Small Scale Cases				Large Scale cases			
Problems	Number of customers	Number of DCs	Types of Vehicles	Problems	Number of customers	Number of DCs	Types of Vehicles
Problem1	21	7	3	Problem1	32	7	3
Problem2	8	3	2	Problem2	40	11	3
Problem3	15	3	2	Problem3	34	8	2
Problem4	10	4	2	Problem4	70	9	3
Problem5	12	5	2	Problem5	62	9	3
Problem6	14	6	3	Problem6	80	7	3
Problem7	20	5	2	Problem7	68	11	3
Problem8	26	4	2	Problem8	60	10	3

In this paper, initial population size  $n_{Ant}$  is assumed 100 and 200 for small and large scales, respectively.

### 5. Performance Evaluation of the Algorithms

To illustrate the performance of the used procedures to optimize the proposed model, problem 1 in small scale is considered, (see Figs 6 and 7). Fig 8, presents the Pareto front of problem 8 in small scale by NSACO and NSGA-II for instance.

To check the quality of solutions obtained by these algorithms, five performance metrics including: (1) number of Pareto solutions (NOS), (2) diversity metric ( $\Delta$ ) (Zitzler et al. 2000), (3) mean ideal distance (MID) metric measuring convergence, (4) hypervolume indicator (HVI) and (5) CPU time have been used. The  $\Delta$  and MID metrics are formulated as follows:

$$\Delta = \sqrt{\sum_{j=1}^m (\max_n f_n^j - \min_n f_n^j)^2} \quad (23)$$

$$MID = \sum_{i=1}^n C_i / n \quad (24)$$

where in Eq.23,  $m$  is the number of objectives,  $n$  is the number of Pareto solutions and in Eq.24,  $n$  is the number of Pareto solutions and  $C_i$  is the distance of  $i$ th Pareto solution from ideal point ((0,0) in bi-objective minimization). Fig 9 shows MID metrics comparison for problem 5 in small scale. For better display, MID axis is considered under Logarithmic scale. As shown in Figure 9, in the first iterations, there are more infeasible solutions and they cause adding large penalty functions to objective values, but during the process of algorithm, the infeasible solutions because of great objective values are discarded and objective values are more real and then convergence process goes smoothly. Also, to view the output of the decision variables, one Pareto member of problem 6 in small scale is given in the appendix.

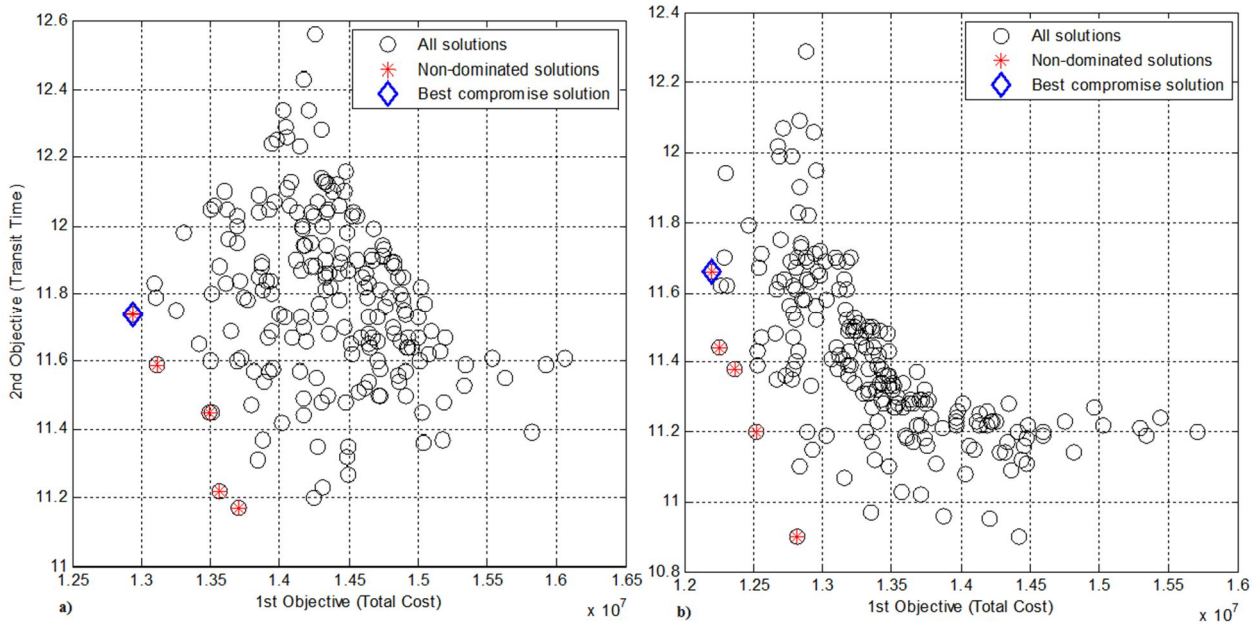


Fig 6. Pareto front of problem 1 in small scale by NSGA-II a) 3<sup>rd</sup> and b) 100<sup>th</sup> iteration with  $n_{Pop}=200$



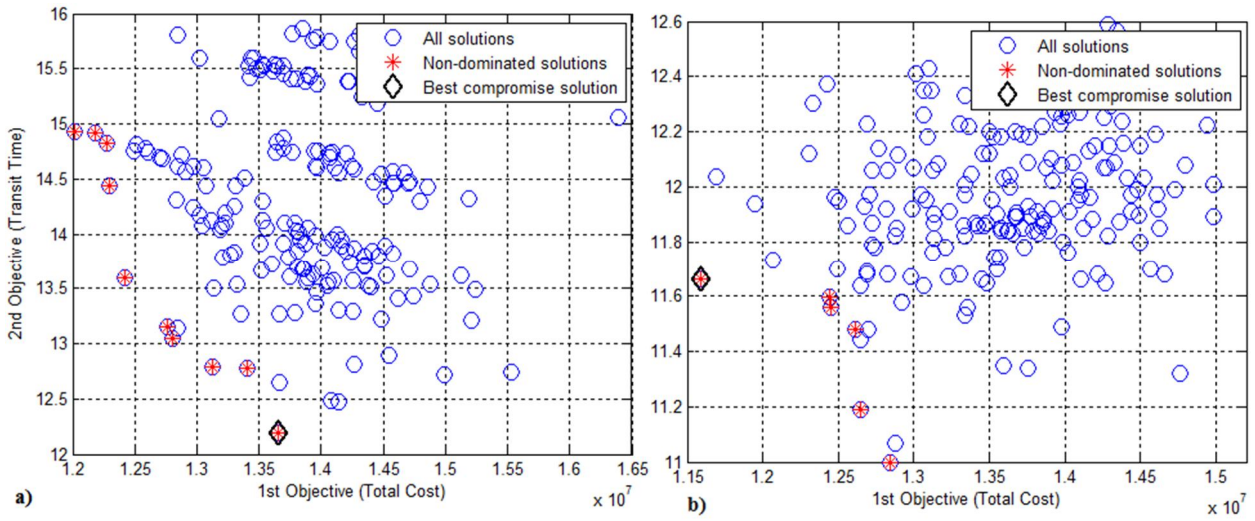


Fig 7. Pareto front of problem 1 in small scale by NSACO a) 3<sup>rd</sup> and b) 100<sup>th</sup> iteration with  $n_{Ant}=200$

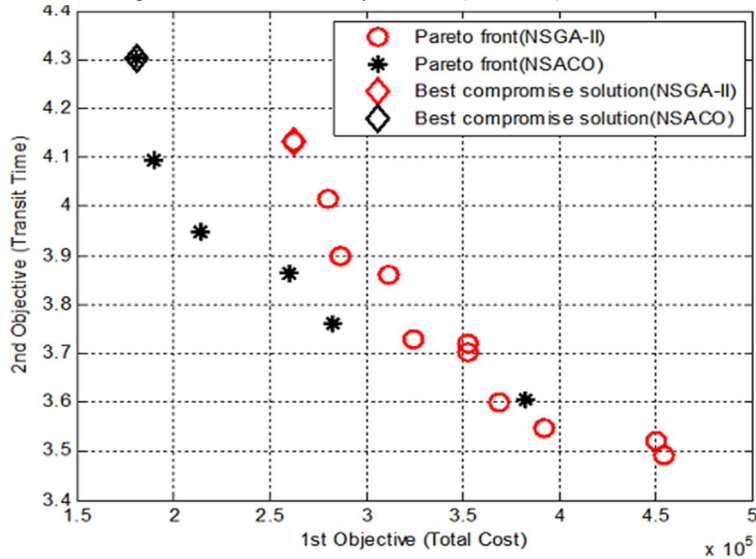


Fig 8. NSACO and NSGA-II comparison of Pareto front of problem 8 in small scale ( $n_{Ant}= 200$  and 100<sup>th</sup> iteration)

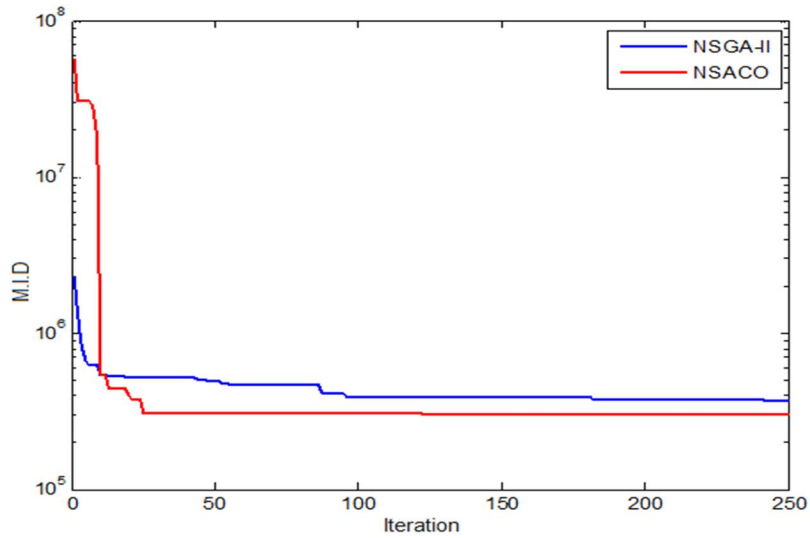


Fig 9. MID metric comparisons for problem 5 in small scale

The hypervolume indicator measures the volume of the dominated portion of the objective space. The hypervolume was first introduced for performance assessment in multi-objective optimization by Zitzler and Thiele (1999). One key point is that a set of solutions achieving the maximum hypervolume for a specific problem covers the entire Pareto front (Fleischer, 2003). Since the computation of the HVI has been widely studied, there is a range of algorithms that compute the exact hypervolume and others that compute an

approximation (Bader & Zitzler, 2011). In this paper, we use the MATLAB codes that are written by Kruisselbrink (2011) for computing an approximation hypervolume by means of a Monte-Carlo approximation method.

Tables 3 and 4 show the algorithms comparison results for some small and large scale cases with iteration number of 200. In this paper, in order to evaluate the performance of the proposed algorithms, the *Mann-Whitney* test is done by Statistical Package for the Social Sciences (SPSS 16.0) software, as shown in Table 5.

Table 3  
Algorithms comparison results for small scale cases

	NSACO with 200 iterations ( $nAnt=100$ )					NSGA-II with 200 iterations ( $nPop=100$ )				
	NOS	$\Delta$	MID	HVI	Time (min)	NOS	$\Delta$	MID	HVI	Time (min)
<b>Problem1</b>	8	3.98e+05	1.81e+07	1.84e+21	8.07	11	1.96e+06	2.21e+07	4.06e+42	12.51
<b>Problem2</b>	7	978.60	5.32e+04	3.67e+09	4.77	9	1.38e+04	6.61e+04	1.88e+19	13.17
<b>Problem3</b>	7	2.73e+04	3.45e+05	1.15e+17	6.59	10	3.56e+04	4.74e+05	2.39e+28	12.36
<b>Problem4</b>	6	1.23e+04	2.14e+05	2.03e+07	5.18	9	6.60e+04	3.07e+05	8.78e+21	12.21
<b>Problem5</b>	6	3.11e+04	2.90e+05	1.52e+11	6.52	7	4.96e+04	3.27e+05	9.99e+12	12.64
<b>Problem6</b>	7	2.68e+04	2.48e+05	4.06e+16	9.63	7	8.55e+04	2.98e+05	8.85e+17	12.74
<b>Problem7</b>	7	4.18e+03	9.13e+04	8.40e+14	8.60	8	4.76e+03	9.66e+04	9.01e+14	14.32
<b>Problem8</b>	6	1.28e+04	2.75e+05	1.83e+10	10.43	11	2.93e+05	4.29e+05	5.19e+33	15.47
<b>Average</b>	6.57	6.41e+04	2.45e+06	2.3e+20	7.47	9	3.31e+05	3.01e+06	5.8e+41	13.17

Table 4  
Algorithms comparison results for large scale cases

	NSACO with 200 iterations ( $nAnt=200$ )					NSGA-II with 200 iterations ( $nPop=200$ )				
	NOS	$\Delta$	MID	HVI	Time (min)	NOS	$\Delta$	MID	HVI	Time (min)
<b>Problem1</b>	8	1.98e+04	4.29e+05	3.40e+17	47.19	9	5.31e+04	5.32e+05	1.26e+19	52.73
<b>Problem2</b>	8	1.20e+05	2.04e+06	1.68e+25	47.53	12	3.52e+05	2.53e+06	6.58e+44	51.63
<b>Problem3</b>	6	3.79e+03	5.01e+05	4.36e+11	29.16	7	4.58e+03	5.66e+05	3.63e+14	47.69
<b>Problem4</b>	6	6.13e+03	4.91e+05	2.41e+11	44.52	8	9.29e+03	5.71e+05	1.86e+17	48.78
<b>Problem5</b>	6	1.64e+04	1.02e+06	2.04e+12	46.44	7	6.86e+03	1.54e+06	2.09e+13	52.08
<b>Problem6</b>	7	1.59e+04	2.04e+06	9.39e+12	48.53	7	4.33e+04	2.62e+06	6.19e+14	52.10
<b>Problem7</b>	7	2.01e+05	1.26e+06	2.25e+12	47.39	8	2.52e+05	1.76e+06	2.44e+13	55.41
<b>Problem8</b>	6	2.79e+04	1.39e+06	2.93e+12	50.03	8	2.39e+05	1.78e+06	5.81e+19	55.51
<b>Average</b>	6.75	5.13e+04	1.15e+06	2.1e+24	45.09	8.25	1.20e+05	1.48e+06	8.2e+43	51.99

Table 5  
Statistical comparison results of NSACO and NSGA-II ( $\alpha=5\%$ )

	Mann-Whitney Test			
	Small scale cases		Large scale cases	
	P-Value	results	P-Value	results
<b>NOS</b>	0.006	NSGA-II is preferred to NSACO	0.036	NSGA-II is preferred to NSACO
<b>Diversity</b>	0.074	Both algorithms are similar	0.345	Both algorithms are similar
<b>MID</b>	0.238	Both algorithms are similar	0.248	Both algorithms are similar
<b>HVI</b>	0.028	NSGA-II is preferred to NSACO	0.064	Both algorithms are similar
<b>Time</b>	0.001	NSACO is preferred to NSGA-II	0.002	NSACO is preferred to NSGA-II

For further validation of the proposed method, some small cases are solved with LP-metric by Lingo 13.0. Table 6 shows the results of LP-metric method for problems 2, 4 and 5 in small scale with  $p=1, 2$  and 3. The weights of objectives are assumed identical. In Table 7 the best solution of Pareto front (best compromise solution according to section 4.3) are shown for these small scale cases. Results show the average of error percentages of objective functions related to NSACO in comparison to the LP-metric method are 0.13% and 0.01%, respectively and the average of error percentages of objective functions related to NSGA-II in comparison to the LP-metric method are 1.05% and 0.05%, respectively. Furthermore, it can be seen that with increasing size of the problems, while the time of problem solving increases

exponentially by using the LP-metric method, the running time of NSACO and NSGA-II are more stable.

Table 6  
Computational results of LP-metric method for some small scale cases

Some problems in small cases	$p$	First objective	Second objective
Problem 2	1	58740	1.894
	2	59791	1.879
	3	59791	1.879
Problem 4	1	297399.63	1.850
	2	299489.48	1.825
	3	302801.48	1.817
Problem 5	1	237972.92	1.439
	2	252391.48	1.390
	3	252391.48	1.390

Table 7  
Comparison of NSACO, NSGA-II and LP-metric method for solving some small scale cases

Some problems in small cases	Number of variables	LP-metric method			Best compromise solution by NSACO (500 iterations)			Best compromise solution by NSGA-II (500 iterations)			Error of NSACO (%)		Error of NSGA-II (%)	
		First objective	Second objective	Time (min)	First objective	Second objective	Time (min)	First objective	Second objective	Time (min)	Z1	Z2	Z1	Z2
Problem 2	51	58740.0	1.89	2.26	58963.7	1.88	3.96	59147.0	1.88	5.25	0.38	0	0.69	0
Problem 4	84	297399.6	1.82	3.08	299016.3	1.83	4.14	300419.5	1.85	5.53	0.06	0.54	1.01	0
Problem 5	125	237972.9	1.44	5.07	238105.4	1.44	4.61	241403.4	1.45	5.76	0.00	0	1.44	0.14
Average	-	198037.5	1.72	3.47	198695.1	1.72	4.2	200323.3	1.72	5.51	0.13	0.01	1.05	0.05

## 6. Discussion and conclusion

The importance of a quick and efficient service towards the customers has been identified in the competitive business environment during the past few decades. Distribution centers (DCs) play an important role in maintaining the uninterrupted flow of goods and materials between the manufacturers and their customers. In this paper, a bi-objective optimization model for capacitated multi-vehicle allocation of customers to distribution centers is proposed. The optimization objectives are to minimize transit time and total cost including opening cost, assumed for opening a potential DC and shipping cost per unit from DC to the customers. Results show the trade-off between total transit time and total cost, since considering heterogeneous vehicles lead to a more realistic model and cause more conflicts in the two objectives.

In this paper, an evolutionary algorithm named non-dominated sorting ant colony optimization (NSACO) is presented as the optimization tool to solve this model. The proposed methodology is based on a new variant of ant colony optimization (ACO) specialized in multi-objective optimization problem. The crowding distance technique is used to ensure the best distribution of the non-dominated solutions. The computational Results of NSACO in comparison to NSGA-II show that while both algorithms are efficient to solve the model and the distribution of solutions in the trade-off surface of both algorithms does

not differ significantly, NSACO algorithm is more efficient than NSGA-II in terms of optimality, convergence and running time saving (See Tables 5 and 7). Also, for better validation of the proposed method, the computational results in some small cases are compared with those obtained by LP-metric method. As shown in Table 7, results show the error percentages of objective functions in comparison to the LP-metric method are less than 2%. Furthermore, it can be seen that with increasing the size of the problems, while the time of problem solving increases exponentially by using the LP-metric method, the running time of NSACO and NSGA-II are more stable.

Future research may develop hybrid approaches based on the used algorithm and other approaches available in the literature. Additionally, may be modeled location allocation for non-deterministic condition, such as stochastic demand. Moreover, given the successful application of NSACO to the bi-objective warehouse allocation problem, the used algorithm can be modified to obtain non-dominated solutions for warehouse allocation problems with more than two objectives.

**Appendix**

One Pareto member for problem 6 in small scale in 200th iteration by NSACO approach is as follows (where number of customers = 14, number of DCs = 6, types of vehicles = 3):

Number of Pareto front Members = 3

Pareto front:

For 1st element of Pareto front, depot vector is:

y =  
1 1 1 1 1 1

For 1st element of Pareto front, Allocation matrix is:

x =

	DC 1	DC 2	DC 3	DC 4	DC 5	DC 6	Types of vehicles
customer1	0	0	0	0	0	1	3
customer2	0	0	1	0	0	0	3
customer3	0	0	0	1	0	0	3
customer4	0	0	0	1	0	0	2
customer5	0	0	0	0	1	0	2
customer6	0	1	0	0	0	0	3
customer7	1	0	0	0	0	0	1
customer8	0	1	0	0	0	0	3
customer9	0	0	0	1	0	0	1
customer10	0	0	1	0	0	0	3
customer11	0	0	0	0	1	0	2
customer12	0	0	0	0	1	0	2
customer13	0	0	1	0	0	0	2
customer14	1	0	0	0	0	0	1

Final objective values:

For 1st element of Pareto front, objective values are:

Total Cost = 3.1560e+005

Transit Time = 2.8400

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