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A Ratio-Based Efficiency Measurement for Ranking Multi-Stage Production Systems in DEA

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Abstract

Conventional data envelopment analysis (DEA) models are used to measure efficiency score of production systems when they are considered as black boxes and their internal relationship is ignored. This paper deals with a common special case of network systems called multi-stage production system and can be generalized to many organizations. A multi-stage production system has some stages in which the outputs of each stage are used as the inputs of the next stage to produce the final outputs of the system. Most of the approaches handling multi-stage systems in DEA evaluate efficiency measure of a production system considering the interrelationship between its stages; however, they do not present their ranking or impact of each stage on ranking of a special multi-stage system through comparison with the others. In this paper, considering the series internal structure of the multi-stage systems and their efficiency measures, we propose some new ratio-based DEA models to determine the best and worst ranks of the multi-stage systems over all sets of feasible weights. In order to improve the performance of the whole system, the proposed models are used to recognize the stages with the most important role in the system's inefficiency. Some numerical examples are presented to illustrate the approach.

Keywords: Data envelopment analysis, Multi-stage production systems, The best and worst ranks, Inefficiency sources.

1. Introduction

Data envelopment analysis (DEA) is a mathematical programming technique introduced by Charnes et al. (1978) for measuring the relative performance of a set of decision-making units (DMUs) which exploit multiple inputs to produce multiple outputs. Conventional DEA models deal with efficiency evaluation of the production units without considering the operations of sub-processes. As a result, they may generate inaccurate information about the performance and inefficient sources of the systems with network structures.

Series structure is one of the most common cases of the network production systems in which outputs of each stage, named intermediate products, are used as inputs of the next one to produce the final outputs of the system. In real world, there are many systems, such as industrial, educational, agricultural, etc., which can be shown in series structure to have accurate information about their performance. In recent DEA literature, many studies are devoted to two-stage production systems as a special case of series systems in both modeling and applications. The first attempt for handling two-stage systems is reported by Charnes et al. (1986) in which army recruitment was analyzed. Wang et al. (1997) proposed two independent models for each stage of two-stage systems. Seiford and Zhu (1999) presented a two-stage system to measure the profitability and marketability of US commercial banks. Chen and Zhu (2004) claimed that dependence of two stages is not considered in Wang et al. (1997); so, they solve the problem with the assumption that intermediate products are unknown variables. Kao and Hwang (2008) took into account a series relationship of two-stage systems and developed a different approach to estimate the overall efficiency of the units. Kao (2009) generated the model of Kao and Hwang (2008) for systems with multi stages which are connected in series. Azizi and Kazemi Matin (2010) analyzed two-stage systems under variable returns to scale technology. Kao (2014) considered general multi-stage systems as the systems in which exogenous inputs are consumed in addition to intermediate products. Kazemi Matin and Azizi (2015) introduced a unified general model for efficiency evaluation of network production systems when arbitrary relations between individual sub-processes are allowed. Kazemi Matin and Azizi (2016) used a unified general model for performance analysis of Iran's provinces in producing wheat. All of the above approaches evaluate efficiency score of systems with network structure, but they do not present their ranking.

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Having information about the rank of a production system helps a decision-maker to make logical decisions about the ways to improve system's performance. In traditional DEA, many approaches were presented to discriminate and rank units. For example, super efficiency approach was introduced by Andersen and Petersen (1993) in order to distinguish efficient units. Cross efficiency method, proposed by Sexton et al. (1986) and generalized by Doyle and Green (1994), is also used to rank units. Both approaches have some deficiencies which were analyzed and improved in some other ranking techniques (Rodder and Reucher, 2011; Lee and Zhu, 2012; Chen, 2013; Yang et al. 2013). Recently, Salo and Punkka (2011) introduced a novel ranking interval method based on the efficiency measure of a DMU with considering all feasible input/output weights in evaluation.

All the mentioned studies for ranking units treat the system as a black-box without considering internal sub-processes. To the best of our knowledge, there are a few studies in DEA literature that deal with ranking network production systems. Liu et al. (2009) and Liu and Lu (2010) proposed a discriminate method for network production systems applied to rank the performance of R&D organizations. As the most recent work, Liu and Lu (2012) introduced a network ranking approach to increase discrimination among efficient two-stage production systems. Most of these approaches are based on optimal efficient input/output weights in which production unit has its best performance under evaluation. As a result, since they failed to make a complete comparison between the units, they could not be considered as complete ranking methods.

In this paper, we generalize the proposed models of Salo and Punkka (2011) to discriminate multi-stage systems to have accurate information about the rank, performance, and inefficiency sources of systems to make proper decisions in order to improve systems' performance. The new ratio-based DEA models consider all feasible input/output weights in the evaluation not just the self-appraisal optimal weights. As a result, we can derive a ranking interval for each unit (efficient and inefficient) in which the best and worst rankings of multi-stage systems are determined. Simultaneously, considering the interrelationship of the stages, we can also evaluate the performance of each stage of a production unit through comparing the corresponding stage of the other units. This helps decision-makers to recognize the stages responsible for the inefficiency of the whole system. The contributions of this paper are as follows:

- For the first time, we determine the best and worst ranks of DMUs with a multi-stage structure.
- We determine the stages with more effect on inefficiency of the corresponding system in comparison with that of another system.
- A numerical example shows the applicability and efficiency of the proposed model in agriculture industry.

The rest of this paper is organized as follows. Section 2 is devoted to giving a brief review of basic DEA models for multi-stage systems. Section 3 presents the provided ratio-based ranking technique including a simple algorithm and some new DEA models for multi-stage production systems. Section 4 includes an illustrative example. Conclusions are given in section 5.

2. Efficiency Measure of Multi-Stage Production Units

In DEA, each observed DMU_l ($l=1,\dots,n$) is specified by some non-negative inputs and outputs. Throughout this paper, inputs and outputs vectors are denoted by $\boldsymbol{x}_l=(x_{1l},\dots,x_{ml})$ and $\boldsymbol{y}_l=(y_{1l},\dots,y_{sl})$, respectively. Regarding these notations, the ratio form of CCR model is presented by Charnes et al. (1978) to measure the efficiency score of DMU_k as follows:

$$\begin{aligned} E_k &= Max\ \boldsymbol{uy}_k/\boldsymbol{vx}_k \\ s.t.\quad & \boldsymbol{uy}_l/\boldsymbol{vx}_l \le 1,\ \ l=1,\dots,n \\ & \boldsymbol{u}\ge 0, \boldsymbol{v}\ge 0 \end{aligned} \tag{1}$$

Now, suppose that all production units are composed of q stages connected to sub-processes in series relation as depicted in Fig 1. In these systems, intermediate products $z_l=(z_{1l},\dots,z_{dl})$ are the outputs of each stage as well as the inputs of the next stage.

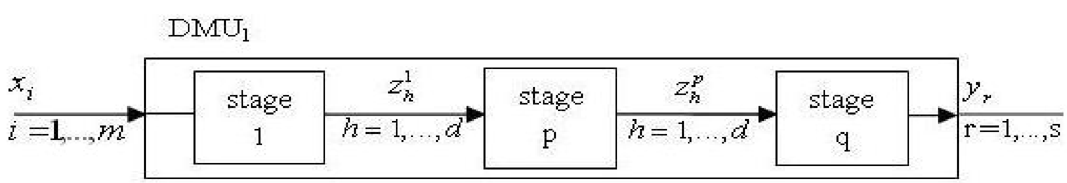

Fig. 1. Multi – stage (series) production units.

Note that the conventional DEA models do not take intermediate products into account in estimating the efficiency of multi-stage production units. To obtain the ratio CCR efficiency score of the first, p^{th} (p=2,…,q-1) and the last stage of DMU_k, models (2), (3) and (4) are used, respectively. Besides, model (1) estimates the efficiency score of the whole system without considering its interrelationship.

$$
\begin{aligned}
& E_k^1 = Max\, \boldsymbol{w}^1 \boldsymbol{z}_{\boldsymbol{k}}^1 / \boldsymbol{v}\boldsymbol{x}_k \\
& s.t. \quad \boldsymbol{w}^1 \boldsymbol{z}_{\boldsymbol{l}}^1 / \boldsymbol{v}\boldsymbol{x}_l \leq 1 \qquad l = 1, \dots, n \\
& \boldsymbol{w}^1 \geq 0, \boldsymbol{v} \geq 0
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
& E_k^p = Max\, \boldsymbol{w}^p \boldsymbol{z}_{\boldsymbol{k}}^p / \boldsymbol{w}^{p-1} \boldsymbol{z}_{\boldsymbol{k}}^{p-1} \qquad p = 2, \dots, q-1 \\
& s.t. \quad \boldsymbol{w}^p \boldsymbol{z}_{\boldsymbol{l}}^p / \boldsymbol{w}^{p-1} \boldsymbol{z}_{\boldsymbol{l}}^{p-1} \leq 1 \quad l = 1, \dots, n,\ p = 2, \dots, q-1 \\
& \boldsymbol{w}^p \geq 0 \qquad p = 2, \dots, q-1
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
& E_k^q = Max\ \boldsymbol{u}\boldsymbol{y}_k / \boldsymbol{w}^{q-1} \boldsymbol{z}_{\boldsymbol{k}}^{q-1} \\
& s.t. \quad \boldsymbol{u}\boldsymbol{y}_l / \boldsymbol{w}^{q-1} \boldsymbol{z}_{\boldsymbol{l}}^{q-1} \leq 1 \qquad l = 1, \dots, n \\
& \boldsymbol{u} \geq 0, \boldsymbol{w}^{q-1} \geq 0
\end{aligned} \tag{4}
$$

To present the precise overall efficiency score for a multi-stage production unit, the operation of the stages was embedded into model (1) by adding constraints of models (2), (3), and (4). In this way, the following ratio model was proposed by Kao (2009).

$$
\begin{aligned}
& E_k = Max\ \boldsymbol{u}\boldsymbol{y}_k / \boldsymbol{v}\boldsymbol{x}_k \\
& s.t. \quad \boldsymbol{u}\boldsymbol{y}_l / \boldsymbol{v}\boldsymbol{x}_l \leq 1, \ l = 1, \dots, n \\
& \boldsymbol{w}^1 \boldsymbol{z}_{\boldsymbol{l}}^1 / \boldsymbol{v}\boldsymbol{x}_l \leq 1 \qquad l = 1, \dots, n \\
& \boldsymbol{w}^p \boldsymbol{z}_{\boldsymbol{l}}^p / \boldsymbol{w}^{p-1} \boldsymbol{z}_{\boldsymbol{l}}^{p-1} \leq 1 \quad l = 1, \dots, n,\ p = 2, \dots, q-1 \\
& \boldsymbol{u}\boldsymbol{y}_l / \boldsymbol{w}^{q-1} \boldsymbol{z}_{\boldsymbol{l}}^{q-1} \leq 1 \qquad l = 1, \dots, n \\
& \boldsymbol{u} \geq 0, \boldsymbol{v} \geq 0, \boldsymbol{w}^p \geq 0 \qquad p = 2, \dots, q-1
\end{aligned} \tag{5}
$$

In this model, $\boldsymbol{u}$, $\boldsymbol{v}$, and $\boldsymbol{w}^p$, $p = 2, \dots, q-1$ are non-negative vectors of inputs weights, outputs weights, and intermediate product weights, respectively.

Since the same weights are exploited for intermediate products of the stages whose intermediate products are produced as outputs and are consumed as inputs, decomposition relationship $E_k = E_k^1 \times \dots \times E_k^p \times \dots \times E_k^q$ can be established. That is, the efficiency of q-stage production unit is the product of the efficiencies of q stages. Note that aggregating of the last three constraints of model (5) equals to the first constraint, so the first set of constraints is redundant and can be omitted.

In the next section, the introduced model of Kao (2009) is used to present a complete ranking method based on the best and worst efficiency scores of multi-stage production units.

3. Presenting a Ratio-Based Ranking Interval

The stages of a series system and the whole production system can be ranked for all feasible input, output, and intermediate product weights. We aim to present ranking interval for each multi-stage production system by computing the best and worst rankings of each system.

Following the proposed definition in the paper of Salo and Punkka (2011), the worst ranking of DMU_k indicates the *maximum* number of DMUs which have at least as high efficiency score as the under evaluated one and the best ranking of DMU_k shows the *minimum* number of DMUs with higher efficiency score than DMU_k. Using the presented decomposition of Kao (2009) for efficiency scores, it is obvious that $E_l(\boldsymbol{u}, \boldsymbol{w}, \boldsymbol{v}) > E_k(\boldsymbol{u}, \boldsymbol{w}, \boldsymbol{v})$ may happen when at least the efficiency score of one of the stages of DMU_l is strictly higher than the efficiency score of the corresponding stage of DMU_k. Furthermore,

$E_l(\boldsymbol{u},\boldsymbol{w},\boldsymbol{v}) \geq E_k(\boldsymbol{u},\boldsymbol{w},\boldsymbol{v})$ may happen when at least the efficiency score of one of the stages of DMU_l is at least as high as the efficiency score of the corresponding stage of DMU_k.

Now, we want to attain the best ranking of each multi-stage system through comparing the other ones. To simplify our method which will be presented further, the following algorithm is introduced for a set of feasible weights. Our method will analyze all sets of feasible weights.

Input: an arbitrary set of feasible weights and n DMUs with q stages, m inputs, d intermediate products, and s outputs.

Initialization: set i:=1, and l:=1

Step 1: Solve $\tilde{\theta}_{1l} = \frac{\boldsymbol{w}^1 \boldsymbol{z}_k^1/\boldsymbol{v}\boldsymbol{x}_k}{\boldsymbol{w}^1 \boldsymbol{z}_l^1/\boldsymbol{v}\boldsymbol{x}_l} \quad l \neq k$

If $\tilde{\theta}_{1l} > 1$, *set* $b_l^1 = 1$.

Else if, set $b_l^1 = 0$.

Step p:for p=2,…,q-1 Solve $\tilde{\theta}_{pl} = \frac{\boldsymbol{w}^p \boldsymbol{z}_k^p/\boldsymbol{w}^{p-1}\boldsymbol{z}_k^{p-1}}{\boldsymbol{w}^p \boldsymbol{z}_l^p/\boldsymbol{w}^{p-1}\boldsymbol{z}_l^{p-1}} \quad l \neq k$

If $\tilde{\theta}_{pl} > 1$, *set* $b_l^p = 1$.

Else if, set $b_l^p = 0$.

Step q: Solve $\tilde{\theta}_{ql} = \frac{\boldsymbol{u}\boldsymbol{y}_k/\boldsymbol{w}^{q-1}\boldsymbol{z}_k^{q-1}}{\boldsymbol{u}\boldsymbol{y}_l/\boldsymbol{w}^{q-1}\boldsymbol{z}_l^{q-1}} \quad l \neq k$

If $\tilde{\theta}_{ql} > 1$, *set* $b_l^q = 1$.

Else if, set $b_l^q = 0$.

Step q+1: Solve $\tilde{\theta}_l = \frac{\boldsymbol{u}\boldsymbol{y}_k/\boldsymbol{v}\boldsymbol{x}_k}{\boldsymbol{u}\boldsymbol{y}_l/\boldsymbol{v}\boldsymbol{x}_l} \quad l \neq k$

If $\tilde{\theta}_l > 1$, *set* $t_l = 1$, $i := i + 1$

Else if, set $t_l = 0$.

Step q+2: if $l \leq n - 1$, *set l:=l+1. Go to step 1.*

Else if, stop.

Output: i, b_l^p *and* t_l $(p = 1, \dots, q, l = 1, \dots, n \; ; l \neq k)$.

In the above $q+2$ step procedure, i shows ranking of DMU_k and b_l^p (p=1,…,q), and t_l show the performance of each stage of DMU_l and the whole system compares the corresponding DMU_k ones, respectively. For instance, $b_l^p = 1$ means that the efficiency score of p^{th} stage of DMU_l is higher than that of DMU_k, and $b_l^p = 0$ means that the efficiency score of p^{th} stage of DMU_k is at least as high as that of DMU_l.

In step 1, the efficiency score of the first stage of DMU_k and DMU_l will be compared and 1 will be set for b_l^1 if DMU_l performs better than DMU_k in its first stage. The same case will be done for p^{th} (p=2,…q-1) stage of the DMUs, and $b_l^p = 1$ is set if the ratio efficiency score of p^{th} stage of DMU_l is strictly higher than p^{th} stage of DMU_k. In step q, the last stage will be analyzed and $b_l^q = 1$ shows that the ratio efficiency score of the last stage of DMU_l is strictly higher than the one of DMU_k. In step $q+1$, the same comparison will be done for the overall efficiency score of the DMUs and $t_l = 1$ shows that the whole ratio efficiency score of DMU_l is strictly higher than that of DMU_k. Also, this step accounts for the number of DMUs with higher efficiency score than DMU_k. If $t_l = 1$, 1 will be added to the counter. The last step is the final step of the algorithm. If there is still a DMU which is not compared with DMU_k, it will be under scrutiny from the first step to the end. If all the DMUs are considered, the algorithm will be stopped.

Note that to attain the worst ranking of each multi-stage system considering an arbitrary set of feasible weights, it is sufficient to convert (>) to (≥) in the above procedure.

Using this algorithm, not only the ranking of the individual production systems will be determined, but also the status of each stage of DMU_l $(l = 1, \dots, n \; ; l \neq k)$ will be achieved by comparing it to the corresponding stages of DMU_k. This is used to determine the stages which need performance improvement to boost the performance of the whole system.

Now, we exemplify how the algorithm works. Let us run the algorithm with six two-stage DMUs, which have two outputs and a single input and a single intermediate product, as given in Table 1. We use the new procedure to obtain the best rank of DMU_F. The optimal weights of model (5) for DMU_C are considered as a set of arbitrary feasible weights.

Table1
Data of 6 two-stage production systems

Output 1	Output 2	Input	Intermediate product
2	4	3	3
5	7	3	4
6	5	5	3
7	3	4	5
3	6	5	4
5	4	4	3

The computed results of the algorithm are shown in Table 2. In the first step, we compare the efficiency score of DMU_A with DMU_F in their first stage by calculating $\tilde{\theta}_{1l}$. Table 2 shows that achieved result is more than 1, so we set $b_l^1 = 1$. In the next step, we do the same for the second stage. So, $b_l^2 = 0$ because $\tilde{\theta}_{2l} < 1$. In the third step, comparison is done for the efficiency score of the whole DMU_A and DMU_F. We set $t_l = 0$ due to $\tilde{\theta}_l < 1$. Finally, in step 3, i remains 1 and does not increase because the whole efficiency score of DMU_A is less than the whole efficiency score of DMU_F. Step 4 makes the same procedure done for other units except DMU_F until all of them are analyzed.

Table 2
The results of the implementation of the algorithm

b_l^1	Second stage	b_l^2	Whole system	t_l	i
1	0.333	0	0.25	0	1
1	0.625	0	0.625	1	2
0	1	1	0.45	0	2
1	0.670	0	0.656	1	3
1	0.375	0	0.225	0	3
-	0.8338	-	0.469	-	-

So, the same procedure is performed for DMU_B and the other DMUs to obtain the ranking of DMU_F. In conclusion, the best rank of DMU_F for the used feasible weights is 3. If we run the algorithm with the same feasible weights to obtain the worst rank of DMU_F, it will be equal to 3. In general, DMUs B and D have better performance than that of DMU_F because they perform better than DMU_F in their first stages. So, performance improvement of the first stage of DMU_F is needed to upgrade its best ranking. The same best rank is obtained for DMU_F by the model which will be subsequently provided. In addition, the new model estimates 5 as the worst rank of DMU_F because it analyzes all sets of feasible weights, and not just the optimal weights of (5). Thus, the ranking interval of DMU_F is [3, 5]. This suggests that there is at least one set of feasible weights which makes DMU_F as the fourth DMU and at least one set of feasible weights which makes DMU_F as the fifth DMU over six.

Now, to achieve the best and worst rankings of multi-stage systems considering all feasible weights, we present models (6) and (7), respectively. In these models, applying cone ratio method presented by Charnes et al. (1989) or assurance region method introduced by Thompson et al., (1986), we assume that the weights belong to special sets $(U, V, W) \subseteq (\Re^s_{++}, \Re^m_{++}, \Re^d_{++})$ to impose the preference information of decision-makers on inputs, outputs, and intermediate products.

$$\min \quad \sum_{l \neq k} \sum_p b_l^p + t_l \tag{6}$$

$$\begin{aligned}
s.t. \quad & w^1 z_l^1 - v x_l \leq M b_l^1 && l \neq k && (6.1)\\
& w^p z_l^p - w^{p-1} z_l^{p-1} \leq M b_l^p && l \neq k && (6.2)\\
& && p = 2, \dots, q-1 &&\\
& u y_l - w^{q-1} z_l^{q-1} \leq M b_l^q && l \neq k && (6.3)\\
& u y_l - v x_l \leq M t_l && l \neq k && (6.4)\\
& b_l^p, t_l \in \{0,1\} \quad p = 1, \dots, q && l \neq k &&\\
& v x_k = u y_k = w^p z_k^p = 1 && p = 1, \dots, q-1 &&\\
& (u, v, w^p) \in (U, V, W) &&&&
\end{aligned}$$

$$\max \quad \sum_{l \neq k} \sum_p b_l^p + t_l \tag{7}$$

$$\begin{aligned}
s.t. \quad & -w^1 z_l^1 + v x_l \leq M(1 - b_l^1) && l \neq k && (7.1)\\
& -w^p z_l^p + w^{p-1} z_l^{p-1} \leq M(1 - b_l^p) && l \neq k && (7.2)\\
& && p = 2, \dots, q-1 &&\\
& -u y_l + w^{q-1} z_l^{q-1} \leq M(1 - b_l^q) && l \neq k && (7.3)\\
& -u y_l + v x_l \leq M(1 - t_l) && l \neq k && (7.4)\\
& b_l^p, t_l \in \{0,1\} \quad p = 1, \dots, q && l \neq k &&\\
& v x_k = u y_k = w^p &&&&
\end{aligned}$$

In these models, the weight sets are considered to be closed and bounded by the constraints $\boldsymbol{v x_k} = \boldsymbol{u y_k} = \boldsymbol{w^p z_k^p} = 1,\ p = 1, \dots, q-1$. The first, second, third, and fourth constraints are related to the first, pth, last stage, and the whole production unit, respectively. M is a sufficiently large positive constant.

By means of the following theorems, we access the best and worst rankings of observed multi-stage production units based on their efficiency scores for all feasible weights.

Theorem 1: The best rank of DMU_k is $i = 1 + \sum_{l \neq k} t_l$ based on the optimal solution of model (6).

Proof: constraints (6.1)-(6.4) allow model (6) to evaluate all the optimal DEA weights in the series model of all the DMUs. Besides, (6.1) identifies the performance of the first stage of DMU_l in comparison with DMU_k. It means that it determines all DMU_l (l=1,...,n; $l \neq k$) that have efficiency score lower than, higher than, or equal to DMU_k in the first stage. Therefore, for every choice of weights, if $E_l^1 \leq E_k^1$ for some DMU_l ($l \neq k$), then b_l^1 will be necessarily 0 due to the minimizing model. While if $E_l^1 > E_k^1$ for some DMU_l ($l \neq k$), then b_l^1 will be necessarily 1 associated with M establishes the feasible constraint. The same procedure is established for constraints (6.2), (6.3), and (6.4) for pth stage (*p=2,...,q-1*), the last one, and the whole system, respectively. Finally, $i = 1 + \sum_{l \neq k} t_l$ is 1+ minimum number of DMUs with higher efficiency score than DMU_k.

Theorem 2: The worst rank of DMU_k is $i = 1 + \sum_{l \neq k} t_l$ based on the optimal solution of model (7).

Proof: The procedure of proof in this theorem is similar to that of theorem 1.

Note that the existence of the first, second, and third constraints in both models is necessary to preserve the series relationship between the stages and also help DMs to simply decide which stages need improvement to boost the overall efficiency of the system.

4. An Illustrative Example

One of the main primary foods of humans of the world, such as the Iranian people, is wheat. Wheat production is so important in terms of income. Hence, performance analysis and comparison of provinces in which wheat is produced are so important to increase the amount of production and to provide the needs of Iranian people and even to export.

This section presents the best and worst rankings of 20 provinces of Iran in wheat production in 2008-2009 crop year which is started on 22 September, 2008 and ended on 22 September, 2009. Each province is considered as a series system with three stages: preparing-sowing, growing, and harvesting, respectively.

Table 3.
Descriptive statistics on a data set of 20provinces

Cosumed seed	Cultivated area	Harvested area	Wheat production
1009	7700	7656	11611
160755	821189	612064	1396649
40240.9	233844.8	192809	408474.4

Table 3 summarizes such descriptive statistics on a data set of 20 under evaluated provinces on Iran wheat farming in 2008-2009 crop year. In preparing-sowing process, suitable land for wheat production is prepared with ploughing, clods crushing, and manuring. Manuring is done in two parts, first in preparing step, and second in growing step. In the preparing-sowing step, all the phosphate fertilizer and half of the nitrogen fertilizer are used. The inputs of the system are consumed fertilizer (based on kilogram) and consumed seed (based on ton). There is one intermediate product in the system which is the output of preparing-sowing process as well as the input of growing process and the output of the growing process as well as the input of the harvesting process. The intermediate product (land based on hectare) which is produced by preparing-sowing process is from a cultivated area and the one which is produced by growing process is from a harvested area. The output of the system is wheat production (based on ton). As it is shown in table 4, the best ranking of DMUs 9 and 20 is 1. It means that in the best position of these DMUs, there are no other DMUs which perform better than them. The worst ranking of DMUs 6 and 15 is 20 which denotes that there is at least one set of feasible weights that makes the efficiency score of all other units as high as the mentioned ones. The best rank position is assigned to province 9 because its best rank is 1 and its worst rank is better than the other ones. It means that there is at least one set of feasible weights for which the minimum number of DMUs with strictly higher efficiency score than DNU_9 is 0. Besides, there is at least one set of feasible weights for which the maximum number of DMUs with as high efficiency score as DMU_9 is 5.

Now, we use these results to analyze each stage's impact on the performance of the whole system. As a result, we can determine the improvements which should be done in a three-stage unit to upgrade its rank.

Table 4
Shows the best and worst rankings of provinces achieved by models (6) and (7).

DMU	Best rank	Worst rank
1. Azerbaijan, East	6	19
2. Azerbaijan, West	4	15
3. Ardabil	2	8
4. Isfahan	6	11
5. Ilam	10	19
6. Bushehr	18	20
7. Chahar Mahaal and Bakhtiari	12	16
8. Khorasan, South	3	16
9. Khorasan, Razavi	1	6
10. Khuzestan	2	18
11. Sistan and aluchestan	6	11
12. Fars	2	14
13. Qom	2	19
14. Kerman	8	15
15. Gilan	12	20
16. Lorestan	7	16
17.Mazandaran	2	12
18. Markazi	3	9
19.Hormozgan	5	17
20. Yazd	1	16

To illustrate, consider a specific province, say DMU_2. The optimal solution computed by solving model (6) is:

$$\{l; b_l^1 = 1\} = \{1,9\}, \{l; b_l^2 = 1\} = \{7,8,9,11,13,16,17,18,19,20\},$$
$$\{l; b_l^3 = 1\} = \{3,4,7,8,9,10,11,12,13,14,17,18,19,20\}, \{l; t_l = 1\} = \{3,9,12\}.$$

Results show that the minimum number of DMUs with better performance than DMU_2 is 3. This number is 2 for the first stage, 10 for the second, and 14 for the third one. Also, the best ranking of DMU_2 is 4 because 3 other DMUs (DMUs 3,9 and 12) have better performance than DMU_2. So, to upgrade the best ranking of DMU_2, it should perform better than DMUs 3, 9, and 12. DMUs 3 and 12 have better performance than DMU_2 in the third stage leading to have better performance for the whole system, so DMU_2 should them. DMU_9 has better performance than DMU_2 in its all stages, so DMU_2 should improve the performance of its all stages to overcome DMU_9.

5. Conclusion

This paper provides a ratio-based efficiency method for ranking network production units in DEA framework. In this paper, two DEA models are introduced to compute ranking intervals for multi-stage production systems considering all feasible input, output, and intermediate product weights. The lower bound of the achieved interval shows the minimum number of the units with strictly higher efficiency score than the under evaluated system. On the other side, the computed upper bound shows the maximum number of the systems with as high efficiency score as the under evaluated system. These provide required information to make a complete ranking of multi-stage systems. In addition, the results provide accurate information about the inefficient stages. Thus, they help DMs to improve the performance of their DMUs.

To address more practical problems, we suggest further research to be conducted for determining ranking of multi-stage units in variable returns to scale technology. It is also worth determining the best and worst rankings of network systems without series structure.

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