# Centralized Supply Chain Network Design: Monopoly, Duopoly, and Oligopoly Competitions under Uncertainty 

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#### Abstract

This paper presents a competitive supply chain network design problem in which one, two, or three supply chains are planning to enter the price-dependent markets simultaneously in uncertain environments and decide to set the prices and shape their networks. The chains produce competitive products either identical or highly substitutable. Fuzzy multi-level mixed integer programming is used to model the competition modes, and then the models are converted into an integrated bi-level one to be solved, in which the inner part sets the prices in dynamic competition and the outer part shapes the network cooperatively.Finally, a real-world problem is investigatedto illustrate how the bi-level model works and discuss how price, market share, total income, and supply chain network behave with respect to key marketing activities such as advertising, promotions, and brand loyalty.


Keywords:Competitive supply chain network design, Fuzzy multi-level mixed integer programming, Bi-level programming, Nash equilibrium.

## 1. Introduction

Today's competition is promoted form "firms against firms" to "supply chains versus supply chains". Taylor (2003)maintained that "the traditional company VS company competition is replacedby SC VS SC model, and success is based on assembling a team of competitors that can rise above the win/loss negotiations of conventional business relationships and work together to deliver the best product at the best price." In addition, according to the Deloitte consulting (1999), "no longer will companies compete against other companies, but total supply chainscompete against each other".Therefore, for each chain, designing a competitive supply chain network can be a weapon against rival chains. Supply chain(SC) is a network of suppliers, manufacturers, warehouses, and retailers organized to produce and distribute merchandise at the right quantities, to the right locations, and at the right time in order to minimize the total costs, while satisfying the service level requirements (Simchi-Levi, Simchi-Levi et al., 1999). Designing the physical structure of a SC is called Supply Chain Network Design (SCND) with huge effect on performance and cost reduction of a SC. According to the SCND literature (Beamon 1998; Meixell and Gargeya, 2005;Shen 2007), a great numberof papers (e.g., Altiparmak, Gen et al., 2006;Torabi and Hassini, 2008;Pishvaee and Rabbanil 2011;Badri, Bashiri et al.l 2013;Shankar, Basavarajappa et al., 2013;Özceylan, Paksoy et al., 2014;Vahdani and Mohammadi, 2015;Yang, Liu et al., 2015;Ardalan, Karimi et al., 2016;Keyvanshokooh, Ryan et

[^0]al.,2016;Özceylan, Demirel et al., 2016;Jeihoonian, Zanjani et al., 2017;Varsei and Polyakovskiy, 2017) have considered monopoly assumptions. Competitive SCND (CSCND) considers the impact of competitive markets ondesigning the network structure of a chain to improve its future competitiveness (see Farahani, Rezapour et al., 2014for a review on CSCND).
Players and customers are two basic elements in CSCND, and the number of existing and incoming rivals reacting to the entry of newcomers is an important factor, which can result in monopoly competition (if no rival exists), duopoly competition (in case there is one rival), and oligopoly competition (when more than one SC exists). Based on their reactions, three kinds of competition have been examined in the literature:
Players and customers are the basic factors in CSCND. If no rival exists, monopolistic game takes place;if just one rival exists, the game is duopolistic; moreover,in case of existence of more than one rival, oligopolistic game takes place. Based on the reactions of rivals, three different kinds of competition have been examined in the literature:

1) Static competition: In this competition, a new entrant makes decisions regarding the fact that no rival will show any reaction to his entry (Berman and Krass, 1998;Aboolian, Berman et al., 2007;Aboolian, Berman et al., 2007;ReVelle, Murray et al., 2007). Plastria (2001)did a review of this kind of competition.
2) Dynamic competition: If rivals show simultaneous reactions, this type of competition takes place. This kind of competition happens
primarily in the context of operational characteristics and is modeled using unconstrained programming, which is solved using differential systems (Zhang, 2006;Xiao and Yang, 2008; Godinho and Dias, 2010; Sinha and Sarmah, 2010; Friesz, Lee et al., 2011; Godinho and Dias, 2013; Jain, Panchal et al., 2014; Chen, Liang et al., 2015; Nagurney, Saberi et al., 2015; Santibanez-Gonzalez and Diabat, 2016; Hjaila, Puigjaner et al., 2017; Lipan, Govindan et al., 2017).
3) Competition with foresight: This type of competition occurs when there is a time sequence between the reactions of the players and is mostly related to strategic characteristics. It is modeled using bi-level or multi-level programing. It was named Stackelberg or leaderfollower game(Drezner and Drezner, 1998;Plastria and Vanhaverbeke, 2008;Küçükaydin, Aras et al., 2011;Küçükaydın, Aras et al., 2012;Zhang and Liu, 2013;Yue and You, 2014;Drezner, Drezner et al., 2015;Yang, Jiao et al., 2015;Zhu, 2015;Aydin, Kwong et al., 2016;Genc and De Giovanni, 2017).For reviews, see (Eiselt and Laporte 1997, Kress and Pesch 2012).

Customer utility function is another important consideration that is categorized mostly into random utility function and deterministic one. In deterministic utility model (Hotelling, 1990),customers only visit a facility which gives them the highest utility, whereas in random utility, each facility has a certain chance to be chosen by the customers. Gravity-based model(Huff, 1964;Huff, 1966) is the most used random utility model in the literature.In this model, the probability ofa customer patronizing a facility is proportional to attractiveness of the facility and inversely proportional to thefunction of the distance between the customer and facility.
Basically, there are three types of competitions in SC models: competition among firms of one tier of a SC (Nagurney, Dong et al., 2002;Dong, Zhang et al., 2004;Cruz 2008, Zhang and Zhou, 2012;Qiang, Ke et al., 2013;Hsueh, 2015; Li and Nagurney, 2015;Qiang, 2015;Nagurney, Flore et al., 2016); competition among different tiers (Bernstein and Federgruen, 2005;Anderson and Bao, 2010;Chen, Fang et al., 2013;Wu, 2013;Zhao and Wang, 2015;Bai, Xu et al., 2016;Chaab and RastiBarzoki, 2016;Huang, Ke et al., 2016;Li, Hou et al., 2016;Li and Li, 2016;Zhang, Tang et al., 2016;Genc and De Giovanni, 2017;Wang, Song et al., 2017); competition between SCs (Boyaci and Gallego, 2004;Zhang, 2006;Xiao and Yang, 2008;Li, Zhou et al., 2013;Chung and Kwon, 2016). Those competitions are usually related to planning phase of SC, and there are only a few papers that consider competition in designing phase of SC. Rezapour and Farahani (2010)developed an equilibrium model to design a centralized SCN under deterministic price-dependent demands and duopoly competition. They established finite-dimensional variation inequality formulation and solved it by using a modified projection
method. Rezapour, Farahani et al. (2011) modeled a duopolistic SCND problem with sequential acting under deterministic price-dependent demand. Rezapour and Farahani (2014)proposed a bi-level model for competitive SCND in the market under stochastic price and service level; the inner level determines equilibrium retail price and service level and the outer level determines the network structure. Rezapour, Farahani et al. (2014)presented a bi-level model for competitive SCND in the market where demand is elastic with respect to price and distance and customer behavior is probabilistic based on these factors.Rezapour, Farahani et al. (2015)presented a bi-level model for closed-loop SCND in price-dependent market demand with an existing SC, which only has a forward direction, but the new chain is a closed-loop SC. Fallah, Eskandari et al. (2015)presented a competitive closed-loop SCND problem in a pricedependent market under uncertainty and investigated the impact of simultaneous and Stackelberg competition between the chains. Fahimi, Seyedhosseini et al. (2017) presented a simultaneous competitive supply chain network design problem with continuous attractiveness variables and proposed an algorithm based on the Lemke and Howson algorithm and variational inequality formulation with the help of bi-level programming, the modified projection method, and the possibility theory to solve the problem.
This paper presents CSCND in which one, two, or three SCs are planning to enter the price-dependent markets and decide to set the price and design their network simultaneously.The chains encounter lack of knowledge and imprecise information to predict their required parameters. Fuzzy mathematical programming is used to cope with this uncertainty. Each SC has its own model that is converted into a one integrated bi-level model in which the inner part specifies the equilibrium prices in simultaneous competition and the outer part sets the locations of the SC's facilities in cooperative game with respect to the given prices; up to our knowledge, such a model has not been previously appeared in the literature. Our main contributions lay the groundwork for developing a fuzzy bi-level model in which the inner level specifies the equilibrium prices and the outer level sets the equilibrium locations cooperatively. The proposed model also isadopted for monopoly, duopoly and oligopoly competitions. The rest of this paper is organized as follows: Section 2 is dedicated to problem definition, Section 3 presents the solution approach, Section 4 is for numerical study and discussions and Section 5 is concludes the paper.

## 2. Problem Definition

This study presents an environment in which a two-tire SC, including plant and distribution center (DC), (named by SC1), is planning to enter a virgin and price-dependent market and set the prices and locations of its plants and DCs to maximize its profits, while one or two more SCs, named by SC2 and SC3, may want to enter the market by the same decisions and goalsat the same time [Fig 1]. The chains produce either identical or highly substitutable
products. According to the described situation, SC1 may encounter monopoly (if not any SC is entering at the same time), duopoly (in the case of one simultaneous entering SC), or oligopoly competitions (for more than one simultaneous entering SCs exist) with respect to the fact that SCs are entering the market at the same time to design their networks; consequently, simultaneous SCND game happening between them is rare in the literature. Pricing and location decisions are two main decisions that affect the SC's profit functions.Actually, pricing is an operational decision, whereas location is a strategic one. Therefore, two different games will occur between the chains:one is related to pricing strategy and the second one is related to location decisions.
On the other hand, the chains are newcomers and have no imprecise information about the market parameters that
prevent them fromobtaining random distribution functions for uncertain parameters. Liu and Iwamura (1998)mentioned that uncertainty is classified into two main types: probability and possibility. If the distribution function is available or is found by experiments, we encounter a probabilistic case, and stochastic programming approaches are used to model this situation. However, if not enough information is available to find the distribution functions, we are faced with some kind of ill-known parameters in which possibilistic theory and fuzzy mathematical programming are used to model the situation. According to the described circumstance, we used fuzzy multi-level mixed integer programming to model the situations.


Fig.1. Competitive SCND

The followings assumptions, indexes, parameters, and variables are used to model the introduced problem:

## Assumptions

> The candidate locations of plants are known in advance.
> The candidate locations of the DCs are known in advance.
> There are no common potential locations between the chains
$>$ The demand of each customer market is concentrated ondiscrete points.
$>$ Demand is elastic and price-dependent.
$>$ Customer utility function is based on price.
$>$ The products are either identical or highly substitutable.

## Indexes

$i \quad$ Index of candidate location of plants for SC1
$j \quad$ Index of candidate location of DCs for SC1
$i^{\prime} \quad$ Index of candidate location of plants for SC2
$j^{\prime} \quad$ Index of candidate location of DCs for SC2
$i^{\prime \prime} \quad$ Index of candidate location of plants for SC3
$j^{\prime \prime} \quad$ Index of candidate location of DCs for SC3
$k$ Index of customer location

## Parameters

| $f \Upsilon_{i}$ | Fixed cost of opening a plant at location $i$ for <br> SC1 <br> Fixed cost of opening a DC at location $j$ for <br> SC1 |
| :--- | :--- |
| $g \Psi_{j}$ | Fixed cost of opening a plant at location $i^{\prime}$ <br> for SC2 |
| Fixed cost of opening a DC at location $j^{\prime}$ |  |
| for SC2 |  |


| $c H^{\prime \prime} j^{\prime \prime}$ | Unit transportation cost between plant $i^{\prime}$ and DC $j^{\prime}$ for SC3 |
| :---: | :---: |
| $\varepsilon \Gamma_{j^{\prime \prime k}}$ | Unit transportation cost between DC $j^{\prime}$ and customer k for SC3 |
| $\tilde{d}_{k}$ | Demand of customer k |
| Єр $\Upsilon_{i}$ | Capacity of plant i |
| Єр $\Psi_{j}$ | Capacity of DC $j$ |
| Єр $\chi_{i^{\prime}}$ | Capacity of plant $\mathrm{i}^{\prime}$ |
| $C_{p \mathrm{E}_{j^{\prime}}}$ | Capacity of plant $j^{\prime}$ |
| Сp $\mathrm{H}_{i^{\prime \prime}}$ | Capacity of plant $\mathrm{i}^{\prime \prime}$ |
| Єр $\Gamma_{j \prime}$ | Capacity of plant $j^{\prime \prime}$ |
| $h \Psi_{j}$ | Unit holding cost at DC $j$ in SC1 |
| $h \mathrm{E}_{j^{\prime}}$ | Unit holding cost at DC $j^{\prime}$ in SC2 |
| $h \Gamma_{j^{\prime \prime}}$ | Unit holding cost at DC $j^{\prime \prime}$ in SC3 |
| $P \Upsilon$ | Number of opened plants in SC1 |
| $P \chi$ | Number of opened plants in SC2 |
| PH | Number of opened plants in SC3 |
| $P \Psi$ | Number of opened DCs in SC1 |
| PE | Number of opened DCs in SC2 |
| $P \Gamma$ | Number of opened DCs in SC3 |
| $n$ | Maximum number of plants in SC1 |
| $m$ | Maximum number of DCs in SC1 |
| $n^{\prime}$ | Maximum number of plants in SC2 |
| $m^{\prime}$ | Maximum number of DCs in SC2 |
| $n^{\prime \prime}$ | Maximum number of plants in SC3 |
| $m^{\prime \prime}$ | Maximum number of DCs in SC3 |
| 1 | Number of available customers |

Decision variables
$r_{i}$
$\Psi_{j} \quad\left\{\begin{array}{l}1 \text { if SC1 opens a DC in location } j \\ 0 \text { else }\end{array}\right.$
$\chi_{i^{\prime}} \quad\left\{\begin{array}{l}1 \text { if SC2 opens a plant in location } i^{\prime} \\ 0 \text { else }\end{array}\right.$
$\mathrm{E}_{j^{\prime}} \quad\left\{\begin{array}{l}1 \text { if SC2 opens a DC in location } j^{\prime} \\ 0 \text { else }\end{array}\right.$
$\mathrm{H}_{i^{\prime}} \quad\left\{\begin{array}{l}1 \text { if SC3 opens a plant in location } i^{\prime \prime} \\ 0 \text { else }\end{array}\right.$
$\Gamma_{j^{\prime \prime}} \quad\left\{\begin{array}{l}1 \text { if SC3 opens a DC in location } j^{\prime \prime} \\ 0 \text { else }\end{array}\right.$
$y_{i j k k^{\prime} j^{\prime}} \quad\left\{\begin{array}{l}1 \text { if path } i j \text { is opened to serve market } k \text { in monopoly } \\ 0 \text { otherwise }\end{array}\right.$
$y_{i j k l^{\prime} j^{\prime}} \quad\left\{\begin{array}{l}1 \text { if path iji'j' } \text { is opened to serve market } k \text { in doupoly } \\ 0 \text { otherwise }\end{array}\right.$

| $y_{i j k i^{\prime} j^{\prime} j^{\prime \prime}{ }^{\prime \prime}}$ | $\left\{\begin{array}{l}1 \text { if path } i j i^{\prime} j^{\prime} i^{\prime \prime} j^{\prime \prime} \text { is opened to serve market } k \text { in oligpc } \\ 0 \text { otherwise }\end{array}\right.$ |
| :---: | :---: |
| $x \Upsilon_{i j}$ | Quantity of product shipped from plant i to DC j for SC1 |
| $x \Psi_{j k}$ | Quantity of product shipped from DC j to customer k for SC1 |
| $x \chi_{i j^{\prime}}$ | Quantity of product shipped from plant $i^{\prime}$ to DC $j^{\prime}$ for SC2 |
| $x \mathrm{E}_{j^{\prime} k}$ | Quantity of product shipped from DC $j^{\prime}$ to customer k for SC2 |
| $x \mathrm{H}^{\prime}{ }^{\prime} j^{\prime \prime}$ | Quantity of product shipped from plant $i^{\prime \prime}$ to DC $j^{\prime \prime}$ for SC3 |
| $x \Gamma_{j^{\prime \prime} k}$ | Quantity of product shipped from DC $j^{\prime \prime}$ to customer k for SC3 |
| $P \Psi_{i j k}$ | Retail price of SC1 by path ijk in monopoly competition |
| $P \Psi_{i j k i^{\prime} j^{\prime}}$ | Retail price of SC1 by path $i j k i^{\prime} j^{\prime}$ in duopoly competition |
| $P \mathrm{E}_{i j k i^{\prime} j^{\prime}}$ | Retail price of SC2 by path $i j k i^{\prime} j^{\prime}$ in duopoly competition |
| $P \Psi_{i j k i^{\prime} j^{\prime} j^{\prime \prime}}$ | Retail price of SC1 by path $i j k i^{\prime} j^{\prime} i^{\prime \prime} j^{\prime \prime}$ in oligopoly competition |
| $P \mathrm{E}_{i j k i^{\prime} j^{\prime} i^{\prime}{ }^{\prime \prime}}$ | Retail price of SC2 by path $i j k i^{\prime} j^{\prime} i^{\prime \prime} j^{\prime \prime}$ in oligopoly competition |

Monopoly demand:

$$
D \Upsilon_{j k}=\tilde{\alpha}_{S C 1} \tilde{d}_{k}-\tilde{\delta} P \Upsilon_{j k}
$$

Duopoly demand:

$$
\begin{aligned}
& D \Upsilon_{j k}=\tilde{\alpha}_{S C 1} \tilde{d}_{k}-\tilde{\delta} P \Upsilon_{j k}+\tilde{\beta}\left(P \chi_{j k}-P \Upsilon_{j k}\right) \\
& Đ \chi_{j^{\prime} k}=\tilde{\alpha}_{S C 2} \tilde{d}_{k}-\tilde{\delta} P \chi_{j^{\prime} k}+\tilde{\beta}\left(P \Upsilon_{j k}-P \chi_{j^{\prime} k}\right)
\end{aligned}
$$

Oligopoly demand:

$$
\begin{aligned}
& D \Upsilon_{j k}=\tilde{\alpha}_{S C 1} \tilde{d}_{k}-\tilde{\delta} P \Upsilon_{j k}+\tilde{\beta}\left(P \chi_{j k}+P \Gamma_{j^{\prime k}}-P \Upsilon_{j k}\right) \\
& Ð \chi_{j^{\prime} k}=\tilde{\alpha}_{S C 2} \tilde{d}_{k}-\tilde{\delta} P \chi_{j^{\prime k}}+\tilde{\beta}\left(P \Upsilon_{j k}+P \Gamma_{j^{\prime \prime}}-P \chi_{j^{\prime k}}\right) \\
& D H_{j^{\prime \prime}}=\tilde{\alpha}_{S C 3} \tilde{d}_{k}-\tilde{\delta} P \Gamma_{j^{\prime k}}+\tilde{\beta}\left(P \Upsilon_{j k}+P \chi_{j^{\prime} k}-P \Gamma_{j^{\prime \prime} k}\right)
\end{aligned}
$$

$\tilde{d}_{k}$ is the potential market size (if price was set to zero), $\tilde{\alpha}_{S C 1}, \tilde{\alpha}_{S C 2}, \tilde{\alpha}_{S C 3}$ are related to SC1, SC2, and SC3 brand reputations, $\tilde{\alpha}_{S C 1} \tilde{d}_{k}, \tilde{\alpha}_{S C 2} \tilde{d}_{k}, \tilde{\alpha}_{S C 3} \tilde{d}_{k}$ are related to based demand for SC1, SC2, and SC3 if all the prices were set to zero. If a SC reduces its price in market $k$, the related demand will increase. In addition, there are two types of customers taken by the chains as switching and marginal customers. Switching customers are those who will
$P \Gamma_{i j k i i^{\prime} i^{\prime} j^{\prime \prime}} \quad$ Retail price of SC3 by path $i j k i^{\prime} j^{\prime} i^{\prime \prime} j^{\prime \prime}$ in oligopoly competition
$x \Upsilon_{i j k} \quad$ Quantity of product shipped by path $i j k$ in monopoly competition for SC1
$x \Upsilon_{i j k i j^{\prime} j^{\prime}} \quad$ Quantity of product shipped by path $i j k i^{\prime} j^{\prime}$ in duopoly competition for SC1
$x \chi_{i j k i j^{\prime}} \quad$ Quantity of product shipped by path $i j k i^{\prime} j^{\prime}$ in duopoly competition for SC2
$x \Upsilon_{i j k i^{\prime} i^{\prime} i^{\prime \prime \prime}} \quad \begin{aligned} & \text { Quantity of } \\ & i j k i^{\prime} \prime^{\prime} i^{\prime \prime} j^{\prime \prime}\end{aligned}$ product shipped by path $i j k i^{\prime} j^{\prime} i^{\prime \prime} j^{\prime \prime}$ in oligopoly competition for SC1
$x \chi_{i j k i^{\prime} j^{\prime} j^{\prime} j^{\prime \prime}}$ Quantity of product shipped by path $i j k i^{\prime} j^{\prime} i^{\prime \prime} j^{\prime \prime}$ in oligopoly competition for SC2
$x \mathrm{H}_{i j k i^{\prime} j^{\prime} i^{\prime} j^{\prime \prime}}$ Quantity of product shipped by path $i j k i^{\prime} j^{\prime} i^{\prime \prime} j^{\prime \prime}$ in oligopoly competition for SC3

The following terms show the demand functions ofDC $j^{\prime} j^{\prime}, j^{\prime \prime}$ of SC1, SC2, and SC3 in market $k$ in accordance with monopoly, duopoly, and oligopoly (three players) competitions,similar to the description provided byTsay and Agrawal (2000) and Anderson and Bao (2010):
definitely buy the products, but try to find the one with the lowest price;marginal customers will buy the product only if the price is below a certain level. $\tilde{\delta}$ is related to the switching customers and $\tilde{\beta}$ is related to the marginal customers; also, a unit reduction of price increases the demand function by $(\tilde{\delta}+\tilde{\beta})$.

Now, we can formulate the models of the SCs as follows:
Model of SC1:

$$
\begin{align*}
& P_{S C 1}: \max Z_{1}=\sum_{j} \sum_{k} x \Psi_{j k} P \Psi_{j k}-\left(\sum_{i} f \Upsilon_{i} \Upsilon_{i}+\sum_{j} 马_{j} \Psi_{j}+\sum_{i} \sum_{j} \bigcap_{i} x \Upsilon_{i j}\right. \\
& \left.+\sum_{i} \sum_{j} \mathbb{E} \Upsilon_{i j} X \Upsilon_{i j}+\sum_{i} \sum_{j}\left(\frac{h \Psi_{j}}{2}\right) x \Psi_{j k}+\sum_{k} \sum_{j} C^{j} \Psi_{j k} X \Psi_{j k}\right)  \tag{7}\\
& \text { s.t } \\
& \sum_{j} x r_{i j} \leq E p r_{i} r_{i}  \tag{8}\\
& \sum_{k} x \Psi_{j k} \leq \Xi p \Psi_{j} \Psi_{j}  \tag{9}\\
& \sum_{i} \Upsilon_{i}=P \Upsilon  \tag{10}\\
& \sum_{j} \Psi_{j}=q \Psi  \tag{11}\\
& \sum_{k} x \Psi_{j k}=\sum_{j} x \Upsilon_{i j}  \tag{12}\\
& \sum_{j} x \Psi_{j k}=D \Upsilon_{j k}  \tag{13}\\
& x \Psi_{j k}, x \Upsilon_{i j}, P \Psi_{j k}^{S C 1} \geq 0, \Upsilon_{i}, \Psi_{j}=\{0,1\} \tag{14}
\end{align*}
$$

Model of SC2:
$P_{S C 2}: \max Z_{2}=\sum_{j^{\prime}} \sum_{k} x \mathrm{E}_{j^{k} k} P E_{j^{\prime} k}-\left(\sum_{i} \mathrm{~F}_{x_{i}} x_{i}+\sum_{j} g \mathrm{E}_{j^{\prime}} E_{j^{\prime}}\right.$
$\left.+\sum_{i^{\prime}} \sum_{j^{\prime}} s \chi_{i^{\prime}} x \chi_{i j^{\prime}}+\sum_{i^{\prime}} \sum_{j^{\prime}} \varepsilon \chi_{i j^{\prime}} x \chi_{i j^{\prime}}+\sum_{i^{\prime}} \sum_{j^{\prime}}\left(\frac{h \mathrm{E}_{j^{\prime}}}{2}\right) x \mathrm{E}_{j^{\prime} k}+\sum_{k} \sum_{j^{\prime}} c \mathrm{E}_{j^{\prime} k} x \mathrm{E}_{j^{k} k}\right)$
s.t
$\sum_{j^{\prime}} x \chi_{i j^{\prime}} \leq \boxminus p \chi_{i{ }^{\prime}} x_{i^{\prime}}$
$\sum_{k} x \mathrm{E}_{j_{k}} \leq E_{p \mathrm{E}_{j^{\prime}} \mathrm{E}_{j^{\prime}}}$
$\sum_{i^{\prime}} \chi_{i^{\prime}}=P \chi$
$\sum_{j^{\prime}} \mathrm{E}_{j^{\prime}}=q \mathrm{E}$
$\sum_{k} x \mathrm{E}_{j{ }^{\prime} \mathrm{k}}=\sum_{j^{\prime}} x \chi_{i j^{\prime}}$
$\sum_{j^{\prime}} x \mathrm{E}_{j^{\prime k}}=\Phi \chi_{j^{k}}$
$\forall k$
$x \chi_{i j^{\prime}}, x \mathrm{E}_{j^{\prime} k}, P \mathrm{E}_{j^{\prime} k}, \geq 0, \chi_{i}, \mathrm{E}_{j^{\prime}}=\{0,1\}$

Model of SC3:

$$
\begin{align*}
& P_{S C 3}: \max Z_{3}=\sum_{j^{\prime \prime}} \sum_{k} x \Gamma_{j^{\prime \prime} k} P H_{j^{\prime \prime} k}-\left(\sum_{i} \not \mathrm{H}_{i} H_{i^{\prime \prime}}+\sum_{j} g \Gamma_{j} \Gamma_{j^{\prime \prime}}\right.  \tag{23}\\
& \left.+\sum_{i^{\prime \prime}} \sum_{j^{\prime \prime}} s \mathrm{H}_{i^{\prime \prime} j^{\prime \prime}} x \mathrm{H}_{i^{\prime \prime j^{\prime \prime}}}+\sum_{i^{\prime \prime}} \sum_{j^{\prime \prime}} c \mathrm{H}_{i^{\prime j^{\prime \prime}}} x \mathrm{H}_{i^{\prime} j^{\prime \prime}}+\sum_{i^{\prime \prime}} \sum_{j^{\prime \prime}}\left(\frac{h \Gamma_{j^{\prime}}}{2}\right) x \Gamma_{j^{\prime \prime} k}+\sum_{k} \sum_{j^{\prime \prime}} c \mathrm{~F}_{j^{\prime \prime} k} x \Gamma_{j^{\prime} k}\right) \\
& \text { s.t } \\
& \sum_{j^{\prime \prime}} x \mathrm{H}_{i j^{\prime \prime}} \leq \Xi_{p \mathrm{H}_{i}} \mathrm{H}_{i{ }^{\prime \prime}} \\
& \sum_{k} x \Gamma_{j^{\prime k}} \leq \notin p \Gamma_{j^{\prime}} \Gamma_{j j^{\prime \prime}} \\
& \sum_{i^{\prime \prime}} \mathrm{H}_{i^{\prime \prime}}=\mathrm{PH}  \tag{26}\\
& \sum_{i^{\prime}} \Gamma_{j^{\prime \prime}}=q \Gamma  \tag{27}\\
& \sum_{k} x \Gamma_{j^{\prime k}}=\sum_{j^{*}} x H_{i j^{\prime \prime}}  \tag{28}\\
& \sum_{j^{\prime}} x \Gamma_{j^{\prime} k}=D H_{j^{k} k}  \tag{29}\\
& \forall i^{\prime \prime}  \tag{24}\\
& \forall j "  \tag{25}\\
& x \Gamma_{j^{\prime} k}, x \mathrm{H}_{i j^{\prime}}, P \mathrm{H}_{j^{\prime} k}, \geq 0, \mathrm{H}_{i i^{\prime}}, \Gamma_{j^{\prime}}=\{0,1\} \tag{30}
\end{align*}
$$

Terms (7),(15),(23) represent the objective functions of SCs that include revenue captured by selling the product to the customers minus fixed cost of opening plants and DCs, production cost at plants, transportation cost between plants and DCs, holding cost at DCs, and transportation cost between DCs and customers. Constraints $(8,9),(16,17)$, and $(24,25)$ ensure that in each chain, only opened plants and DCs can satisfy their related demands up to their capacity; constraints $(10,11)$ ,$(18,19)$, and $(26,27)$ ensure that only specified amounts of plants and DCs will get opened in each chain;Constraints (12), (20),(28) arerelated to flow balance; Constraints (13), (21), (29) ensure that all customer demand of each chain is satisfied and constraints (14),(22), (30) are related to the binary and non-negativity restrictions on the corresponding decision variables.

## 3. Solution Approaches

To the best of ourknowledge, there is no solution procedure in the literature to be able to solve our proposed problem, so this section presents our solution method tothe proposed problem. It is worth noting that as each chain has its own model, we encountersingle-level (in the case of monopoly competition), bi-level (in the case of duopoly competition), and multi-level (in the case of oligopoly competition) programming. Most of multi-level
and bi-level models are converted into a single-level one by KKT conditions to be solved in the literature (Colson, Marcotte et al.,(2007); Küçükaydin, Aras et al.,(2011); Rezapour, Farahani et al.,(2011); Küçükaydın, Aras et al.,(2012)). However, this procedure is very hard and time-consuming due to the Lagrangian terms that are resulted from KKT conditions, and it changes the model into nonlinear, non-convex one even for small-scale problems.
Therefore, we use the following procedure (is approximately similar to that of Rezapour and Farahani, 2014) in which our presented problem is solved without any requirement to convert the multi-level model into a single one; also, in each step, we write the equivalent crisp level of the models according to Appendix 1.
It is a realistic assumption to assume that location decision will betaken"once and for all" because it is a strategic decision, but pricing decision is an operational decision and can be adjusted in short-time basis; also, these two intrinsically different decisions have integral effect on each other (Rezapour and Farahani, 2014), thus the introduced models are broken into a bi-level formulation to solve the pricing and location step individually by considering their corresponding effects on each other. Now, we are able to introduce our bi-level programming as follows:

### 3.1. Pricing decision

This step deals with the inner part of the bi-level model, which determines the equilibrium prices for the SCs; in fact, the market prices of the chains are exactly related to
their variable costs including: production cost at plant, transportation cost between plant and DC, holding cost at DC and transportation cost between DC and customer.In other words, according to each possible path (combination of one plant and one DC of each chain), the market prices are calculated and the best structure of each chain in the next step will be selected by the outer part of the modelaccording to the computed prices.

In pricing strategy, the plants simultaneously decide the market prices that maximize the SCs profits, and then by the determined prices for each path, they select the best locations for the plants and DCs to be opened cooperatively

### 3.1.1. Monopoly competition

The following model should be maximized here to obtain Nash prices in market k:

$$
\begin{align*}
& \pi_{S C 1}^{k}=\left(P \Upsilon_{i j k}-E V\left(C \Upsilon_{i j k}\right)\right)\left(E V\left(\tilde{\alpha}_{S C 1}\right) E V\left(\tilde{d}_{k}\right)-E V(\tilde{\delta}) P \Upsilon_{j k}\right) \\
& \max \left\{\pi_{S C 1}^{k}\right\} \tag{31}
\end{align*}
$$

where $E V\left(E \Upsilon_{i j k}\right)=E V\left(s \Upsilon_{i}\right)+E V\left(E \Upsilon_{i j}\right)+\frac{E V\left(\hbar \Psi_{j}\right)}{2}+E V\left(e \Psi_{j k}\right)$

### 3.1.2. Duopoly competition

In this competition mode, the following models should be maximized in order to achieve the equilibrium prices in market k :
$\pi_{P l a n t}^{S C 1}=\left(P \Psi_{i j k i i^{\prime}}-E V\left(C \Upsilon_{i j k}\right)\right)\left(E V\left(\tilde{\alpha}_{S C 1} \tilde{d}_{k}\right)-E V(\tilde{\delta}) P \Psi_{i j k^{\prime} j^{\prime}}+E V(\tilde{\beta})\left(P \mathrm{E}_{i j k k^{\prime} j^{\prime}}-P \Psi_{i j k k^{\prime j}}\right)\right)$
$\max \left\{\pi_{\text {Plant }}^{\text {SC1 }}\right\}$
$\pi_{P l a n t}^{S C 2}=\left(P \mathrm{E}_{i j k i^{\prime} j^{\prime}}-E V\left(E_{\chi_{i j^{\prime} k}}\right)\right)\left(E V\left((1-\tilde{\alpha}) \tilde{d}_{k}\right)-E V(\tilde{\delta}) P \mathrm{E}_{i j k i^{\prime} j^{\prime}}+E V(\tilde{\beta})\left(P \Psi_{i j k i^{\prime} j^{\prime}}-P \mathrm{E}_{i j k i^{\prime} j^{\prime}}\right)\right)$
$\max \left\{\pi_{\text {Plant }}^{\text {SC2 }}\right\}$
That $E V\left(E \chi_{i^{\prime} j^{\prime}}\right)=E V\left(s \chi_{i^{\prime}}\right)+E V\left(E \chi_{i^{\prime} j^{\prime}}\right)+\frac{E V\left(h \mathrm{E}_{j^{\prime}}\right)}{2}+E V\left(C \mathrm{E}_{j^{\prime} k}\right)$

### 3.1.3. Oligopoly competition

following models should be maximized here to achieve the equilibrium prices in market k :
This competition mode is shown for three players and clearly can be extended to more players similarly; the

$\max \left\{\pi^{S C 1}\right\}$

$\max \left\{\pi^{\mathrm{SC} 2}\right\}$
$\pi^{S C 3}=\left(P \Gamma_{i j k i^{\prime} i^{\prime} j^{\prime \prime}}-E V\left(E H_{i j^{\prime \prime} j^{\prime \prime}}\right)\right)\left(E V\left(\tilde{\alpha}_{S C 3} \tilde{d}_{k}\right)-E V(\tilde{\delta}) P \Gamma_{i j k^{\prime} i^{\prime} i^{\prime} j^{\prime \prime}}+E V(\tilde{\beta})\left(P \Psi_{i j k k^{\prime} j^{\prime} i^{\prime \prime}}+P E_{i j k i^{\prime} i^{\prime} j^{\prime \prime}}-P \Gamma_{i j k i^{\prime} i^{\prime} i^{\prime \prime}}\right)\right)$
$\max \left\{\pi^{\text {SC3 }}\right\}$
where $E V\left(C H_{i^{\prime j} j^{\prime \prime}}\right)=E V\left(s H_{i^{\prime}}\right)+E V\left(C H_{i^{\prime \prime} j^{\prime \prime}}\right)+\frac{E V\left(\hbar \Gamma_{j^{\prime \prime}}\right)}{2}+E V\left(c \Gamma_{j^{\prime \prime} k}\right)$

Differentiating the terms and solving them simultaneously will result in equilibrium prices for the SCs in market k in each competition mode.

### 3.2. Location decision

This step deals with the outer part of the bi-level model, addressing network design of the chains cooperatively in
which the chain in cooperative game; by the following mathematical model and with respect to the given prices from the inner part shape their networks, the following model represents the outer level:

### 3.2.1. Monopoly competition

$$
\begin{equation*}
P_{\text {monopoly }}: \max Z_{4}=\sum_{i} \sum_{j} \sum_{k}\left(P \Psi_{i j k}^{*}-E V\left(C \Upsilon_{i j k}\right)\right) x \Upsilon_{i j k}-\left(\sum_{i} E V\left(f \Upsilon_{i}\right) \Upsilon_{i}+\sum_{j} E V\left(g \Psi_{j}\right) \Psi_{j}\right) \tag{37}
\end{equation*}
$$

s.t
$(10,11)$
$\begin{array}{ll}x \Upsilon_{i j}=D \Upsilon_{i j k}^{*} y_{i j k} & \forall k, i, j \\ \sum_{i} \sum_{j} \sum_{k} y_{i j k}=1 & \forall \mathrm{k} \\ y_{i j k} \leq \Upsilon_{i} \Psi_{j} & \forall k, i, j \\ \sum \sum_{i} & \forall i\end{array}$
$\sum_{j} \sum_{k} x \Upsilon_{i j k} \leq\left(\varphi C p \Upsilon_{(i i)}+(1-\varphi) C p \Upsilon_{(2 i)}\right) \Upsilon_{i}$
$\forall j$

Term 37 represents the objective function of SC1. Constraint 38 is related to the demand satisfaction. Constraint 39 ensures that only one path is assigned to each customer. Constraint 40 ensures that a path could not be opened unless the related plants and DCs of the chain are opened. Terms 41, 42 are related to the capacity
constraints of the SC, changed to the crisp mode according to Appendix 1. Term 43 is related to the binary and non-negativity restrictions on the corresponding decision variables.

### 3.2.2. Duopoly competition

$P_{\text {Doupoly }}: \max Z_{2}=\sum_{i} \sum_{j} \sum_{i^{\prime}} \sum_{j^{\prime}} \sum_{k}\left(P \Psi_{i j k i^{\prime} j^{\prime}}^{*}-E V\left(C \Upsilon_{i j k i j^{\prime}}\right)\right) x \Upsilon_{i j k i^{\prime} j^{\prime}}+\left(P \mathrm{E}_{i j k i^{\prime} j^{\prime}}^{*}-E V\left(\mathrm{C} \chi_{i^{\prime} j^{\prime} k}\right)\right) x \chi_{i j k i j^{\prime} j^{\prime}}$
$-\left(\sum_{i} E V\left(f \Upsilon_{i}\right) \Upsilon_{i}+\sum_{j} E V\left(g \Psi_{j}\right) \Psi_{j}+\sum_{i} E V\left(f \chi_{i}\right) \chi_{i}+\sum_{j} E V\left(g E_{j}\right) E_{j}\right)$
s.t
(10,11,18,19)

$$
\begin{array}{ll}
x \Upsilon_{i j k i^{\prime} j^{\prime}}=D \Upsilon_{i j k k^{\prime} j^{\prime}}^{*} y_{i j k^{\prime} j^{\prime}} & \forall k, i, j, i^{\prime}, j^{\prime} \\
x \chi_{i j k i^{\prime} j^{\prime}}=D \chi_{i j k i^{\prime} j^{\prime}}^{*} y_{i j i^{\prime} j^{\prime}} & \forall k, i, j, i^{\prime}, j^{\prime} \\
\Gamma \Gamma \Gamma & \forall \mathrm{k} \tag{46}
\end{array}
$$

$$
\begin{equation*}
\sum_{j^{\prime}} \sum_{i^{\prime}} \sum_{j} \sum_{i} y_{i j k^{\prime} j^{\prime}}=1 \tag{47}
\end{equation*}
$$

$$
\begin{equation*}
y_{i j k^{\prime} j^{\prime}} \leq \Upsilon_{i} \Psi_{j} \chi_{i^{\prime}} \mathrm{E}_{j^{\prime}} \tag{48}
\end{equation*}
$$

$$
\forall k, i, j, i^{\prime}, j^{\prime}
$$

$$
\begin{equation*}
\sum_{j} \sum_{i^{\prime}} \sum_{j^{\prime}} x \Upsilon_{i j k i j^{\prime}} \leq\left(\varphi C p \Upsilon_{(i i)}+(1-\varphi) C p \Upsilon_{(2 i)}\right) \Upsilon_{i} \tag{49}
\end{equation*}
$$

$$
\forall i
$$

$$
\begin{equation*}
\sum_{i} \sum_{i^{\prime}} \sum_{j^{\prime}} \sum_{k} x \Upsilon_{i j k k^{\prime} j^{\prime}} \leq\left(\varphi C p \Psi_{(1 j)}+(1-\varphi) C p \Psi_{(2 j)}\right) \Psi_{j} \tag{50}
\end{equation*}
$$

$$
\forall j
$$

$$
\begin{equation*}
\sum_{i} \sum_{j} \sum_{j^{\prime}} \sum_{k} x \chi_{i j k k^{\prime} j^{\prime}} \leq\left(\varphi C p \chi_{\left(i^{\prime}\right)}+(1-\varphi) C p \chi_{\left(i^{\prime}\right)}\right) \chi_{i^{\prime}} \tag{51}
\end{equation*}
$$

$$
\forall i^{\prime}
$$

$$
\begin{equation*}
\sum_{i} \sum_{j} \sum_{i^{\prime}} \sum_{k} x \chi_{i j k i^{\prime} j^{\prime}} \leq\left(\varphi C p \mathrm{E}_{\left(1 j^{\prime}\right)}+(1-\varphi) C p \mathrm{E}_{\left(2 j^{\prime}\right)}\right) \mathrm{E}_{j^{\prime}} \tag{52}
\end{equation*}
$$

$$
\forall j^{\prime}
$$

$x \Upsilon_{i j k k^{\prime} j^{\prime}}, x \chi_{i j k k^{\prime} j^{\prime}} \geq 0, y_{i j k k^{\prime} j^{\prime}}, \Upsilon_{i}, \Psi_{j}, \chi_{i^{\prime}}, \mathrm{E}_{j^{\prime}}=\{0,1\}$

Term 45 represents the objective functions of SC1 and SC2. Constraints 46, 47 are related to the demand satisfaction. Constraint 48 ensures that only one path is assigned to each customer. Constraint 49 ensures that a path could not be opened unless the related plants and DCs of the chains are open. Terms 50-52 are related to the capacity constraints of the SCs, changed to the crisp mode according to Appendix 1. Term 53 is related to the binary
and non-negativity restrictions on the corresponding decision variables.

### 3.2.3. Oligopoly competition

Term 54 represents the objective functions of SC1, SC2, and SC2. Constraints $55-57$ are related to the demand satisfaction. Constraint 58 ensures that only one path is assigned to each customer. Constraint 59 ensures that a path could not be opened unless the related plants and DCs of the chains are opened. Terms 60-65 are related to the capacity constraints of the SCs, changed to the crisp mode according to Appendix 1. Term 66 is related to the binary and non-negativity restrictions on the corresponding decision variables.

## 4. Numerical Example and Discussion

In this section, we use a real-world example in which one investor (SC1) is planning to produce a specific kind of oil seal in the capital city of Iran, Tehran. This product is classified into different classes according to the chemical material used to produce it and its water resistance; with respect to these specifications, the market is virgin for the

Iranian brands, although there are some imported brands like TTO, most of which arecategorized into different classes. The investor also considers a situation in which one or more investor(s) (SC2 and SC3), at the same time, decide(s) to enter to the market, so it may encounter monopoly, duopoly, and oligopoly competitions.
According to the modeling framework, the prices will be specified at first.Then, location decision will be made and the network s structure will be shaped with respect to the achievable market shares and costs of the paths and by the cooperation between the entities of the chains. The following distributions are used to extract the required parameters. The parameters are assumed to be trapezoidal fuzzy numbers, and four prominent values of the trapezoidal numbers are generated by uniform distributions.
In addition,discussion of the results is provided in Section 4-2.

$$
f \Upsilon_{i}, f x_{i^{\prime}}, f \mathrm{H}_{i^{\prime \prime}} \square(u(1500,2000), u(2000,2500), u(2500,3000), u(3000,4000))
$$

$$
g \Psi_{j}, g \mathrm{E}_{j^{\prime}}, g \AA_{j^{\prime \prime}} \square(u(900,1500), u(1500,2000), u(2000,2500), u(2500,3000))
$$

$$
\begin{align*}
& \text { s.t } \\
& \text { (10,11,18,19,26,27) } \\
& x \Upsilon_{i j k i^{\prime} j^{\prime} j^{\prime \prime}}=D \Upsilon_{i j k i^{\prime} j^{\prime} i^{\prime \prime} j^{\prime}}^{*} y_{i j k^{\prime} j^{\prime} i^{\prime} j^{\prime \prime}} \\
& x \chi_{i j k i^{\prime} j^{\prime} i^{\prime} j^{\prime \prime}}=D \chi_{i j k^{\prime} j^{\prime} i^{\prime} j^{\prime \prime}}^{*} y_{i j k i i^{\prime} i^{\prime} j^{\prime \prime}} \\
& x \mathrm{H}_{i j k^{\prime} i^{\prime} i^{\prime} j^{\prime \prime}}=D H_{i j k i i^{\prime} i^{\prime} j^{\prime} y^{\prime \prime}}^{*} y_{i j k^{\prime} j^{\prime} i^{\prime} j^{\prime \prime}} \\
& \forall k, i, j, i^{\prime}, j^{\prime}, i^{\prime \prime}, j^{\prime \prime}  \tag{55}\\
& \forall k, i, j, i^{\prime}, j^{\prime}, i^{\prime \prime}, j^{\prime \prime}  \tag{56}\\
& \forall k, i, j, i^{\prime}, j^{\prime}, i^{\prime \prime}, j^{\prime \prime}  \tag{57}\\
& \forall \mathrm{k}  \tag{58}\\
& \sum_{j^{\prime}} \sum_{i^{\prime}} \sum_{j} \sum_{i} \sum_{i^{\prime \prime}} \sum_{j^{\prime \prime}} y_{i j k k^{\prime} j^{\prime} j^{\prime \prime} j^{\prime \prime}}=1 \\
& y_{i j k j^{\prime}} \leq \Upsilon_{i} \Psi_{j} \chi_{i^{\prime}} \mathrm{E}_{j^{\prime}} \mathrm{H}_{i^{\prime}} \Gamma_{j^{\prime \prime}}  \tag{59}\\
& \forall k, i, j, i^{\prime}, j^{\prime}, i^{\prime \prime}, j^{\prime \prime} \\
& \sum_{j} \sum_{i^{\prime}} \sum_{j^{\prime}} \sum_{i^{\prime \prime}} \sum_{j^{\prime \prime}} \sum_{k} x \Upsilon_{i j k k^{\prime} i^{\prime} i^{\prime \prime}} \leq\left(\varphi C p \Upsilon_{(1 i)}+(1-\varphi) C p \Upsilon_{(2 i)}\right) \Upsilon_{i}  \tag{60}\\
& \sum_{i} \sum_{i^{\prime}} \sum_{j^{\prime}} \sum_{i^{\prime \prime}} \sum_{j^{\prime \prime}} \sum_{k} x \Upsilon_{i j k i i^{\prime} i^{\prime} j^{\prime \prime}} \leq\left(\varphi C p \Psi_{(1 j)}+(1-\varphi) C p \Psi_{(2 j)}\right) \Psi_{j}  \tag{61}\\
& \sum_{i} \sum_{j} \sum_{j^{\prime}} \sum_{i^{\prime \prime}} \sum_{j^{\prime \prime}} \sum_{k} x \chi_{i j k i^{\prime} i^{\prime} i^{\prime \prime} j^{\prime}} \leq\left(\varphi C p \chi_{\left(i^{\prime}\right)}+(1-\varphi) C p \chi_{\left(2 i^{\prime}\right)}\right) \chi_{i^{\prime}}  \tag{62}\\
& \sum_{i} \sum_{j} \sum_{i^{\prime}} \sum_{i^{\prime \prime}} \sum_{j^{\prime \prime}} \sum_{k} x \chi_{i j k^{\prime} i^{\prime} i^{\prime} j^{\prime \prime}} \leq\left(\varphi C p \mathrm{E}_{\left(1 j^{\prime}\right)}+(1-\varphi) C p \mathrm{E}_{\left(2 j^{\prime}\right)}\right) \mathrm{E}_{j^{\prime}}  \tag{63}\\
& \sum_{i} \sum_{j} \sum_{j^{\prime}} \sum_{i^{\prime}} \sum_{j^{\prime \prime}} \sum_{k} x \mathrm{H}_{i j k k^{\prime} j^{\prime} i^{\prime} j^{\prime \prime}} \leq\left(\varphi C p \mathrm{H}_{\left(1^{\prime}\right)}+(1-\varphi) C p \mathrm{H}_{\left(2 i^{\prime}\right)}\right) \mathrm{H}_{i^{\prime}}  \tag{64}\\
& \sum_{i} \sum_{j} \sum_{j^{\prime}} \sum_{i^{\prime}} \sum_{i^{\prime}} \sum_{k} x \mathrm{H}_{i j k i^{\prime} i^{\prime} j^{\prime \prime}} \leq\left(\varphi C p \Gamma_{\left(1 j^{\prime \prime}\right)}+(1-\varphi) C p \Gamma_{\left(2 j^{\prime \prime}\right)}\right) \Gamma_{j^{\prime \prime}}  \tag{65}\\
& \forall j^{\prime \prime} \\
& x \Upsilon_{i j k k^{\prime} j^{\prime} j^{\prime \prime}}, x \chi_{i j k i^{\prime} j^{\prime} j^{\prime \prime}}, x \Gamma_{i j k k^{\prime} j^{\prime} i^{\prime j} j^{\prime \prime}} \geq 0, y_{i j k k^{\prime} j^{\prime} i^{\prime \prime}}, \Upsilon_{i}, \Psi_{j}, \chi_{i^{\prime}}, \mathrm{E}_{j^{\prime}}, \mathrm{H}_{i^{\prime \prime}}, \Gamma_{j^{\prime \prime}}=\{0,1\} \tag{66}
\end{align*}
$$

$$
\begin{aligned}
& s \Upsilon_{i}, s \chi_{i}, \square(u \square(2,2.5), u \square(2.5,2.75), u \square(2.75,3), u \square(3,3.5)) \\
& \overparen{ } \subset \Upsilon_{i j}, E \chi_{i j^{\prime}}, \square(u(0.9,1.5), u(1.5,2.1), u(2.1,2.5), u(2.5,3.12)) \\
& { }_{c} \Psi_{j^{\prime} k}, C \mathrm{E}_{j^{\prime} k}, c \Gamma_{j^{\prime} k} \square(u(1.5,2), u(2,2.5), u(2.5,3), u(3,3.5)) \\
& h \Psi_{j}, h \mathrm{E}_{j^{\prime}}, h \Gamma_{j^{\prime \prime}} \square(u(1.25,1.5), u(1.5,1.75), u(1.75,2), u(2,2.25)) \\
& d_{k} \square(u(9000,10000), u(10000,11000), u(11000,12000), u(12000,13000)) \\
& E_{p \Upsilon_{i},}, \Xi_{p \chi_{i}}, \Xi_{p \Psi} \Psi_{j}, \Xi_{p E_{j^{\prime}}}, \Xi_{p H_{j}}, \Xi_{p \Gamma_{j^{\prime}}} \square(u(18000,20000), u(20000,22000), u(22000,24000), u(24000,26000))
\end{aligned}
$$

### 4.1.Numerical study

Example1. Monopoly competition
In this example, only SC1 exists and wants to enter two available markets.It has 2 potential locations for opening plants and 2 for opening DCs and wants to open one plant and one DC to satisfy to markets; the elements of demand functions are as the follows:

$$
E V\left(d_{1}\right)=115605 ; E V\left(d_{2}\right)=107795 ; \delta=0.03 E V\left(d_{k}\right) ; \alpha=1
$$

According to table 1, DC price is equal to 20.15 in the first market that leads to 45740.71 market share;correspondingly, 20.43 and 41725.85 are the DC price and market share in market 2.In addition, the first location for plant and the second location for DC areopened that lead to the opened path named by $(1,2)$.

## Table 1

Monopoly competition

|  | Market 1 |  | Market 2 |  | Objective <br> function |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Market <br> share 1 | assigned <br> path | DC price | Market <br> share 2 | assigned <br> path | DC price | Total SC |  |
| SC1 | 45740.71 | $(1,2)$ | 20.15 | 41725.85 | $(1,2)$ | 20.43 | 1136925 |

## Example 2.Duopoly competition

In this example, two SCs enter the market simultaneously; they have two potential locations for plants and two for

DCs and want to open one plant and one DC to capture the demand of the two markets by the following parameters:
$E V\left(d_{1}\right)=115605 ; E V\left(d_{2}\right)=107795 ; E V(\delta)=0.03 E V\left(d_{k}\right) ; E V(\beta)=0.05 E V\left(d_{k}\right) ; E V\left(\alpha_{1}\right)=0.55 ; E V\left(\alpha_{2}\right)=0.45$

Table 2 shows the obtained results by solving bi-level model using the presented solution method.According to this table, SC1 opens plant 1 and DC2 to serve markets 1 and 2 by 9.9 and 9.55 as DC prices in those markets;SC2
opens plant 1 and DC2 and sets the DC prices to 10.3 and 9.9 for the corresponding markets. The total incomes of SC1 and SC2 are equal to 141370.3and 82995.3.

Table 2
Duopoly competition

|  | Market 1 |  |  | Market 2 |  | Objective function |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Market <br> share 1 | assigned <br> path | DC price | Market <br> share 2 | assigned <br> path | DC price | Total SC |  |
| SC1 | 27215.27 | $(1,2,1,2)$ | 9.9 | 23858.11 | $(1,2,1,2)$ | 9.55 | 141370.3 |
| SC2 | 20954.01 | $(1,2,1,2)$ | 10.3 | 18640.17 | $(1,2,1,2)$ | 9.90 | 82995.3 |

## Example3. Oligopoly competition

In this competition, three SCs are considered to enter the markets simultaneously. They have two potential locations for plants and two for DCs and want to open one
plant and one DC to capture the demands of two markets by the following parameters:
$E V\left(d_{1}\right)=115605 ; E V\left(d_{2}\right)=107795 ; E V(\delta)=0.03 d_{k} ; E V(\beta)=0.05 d_{k} ; E V\left(\alpha_{1}\right)=0.30 ; E V\left(\alpha_{2}\right)=0.37 ; E V\left(\alpha_{3}\right)=0.33$
Table 3
Oligopoly competition

|  | Market 1 |  | Market 2 | Objective <br> function |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Market <br> share 1 | assigned path | DC price | Market <br> share 2 | assigned <br> path | DC price | Total SC |
| SC1 | 13372.57 | $(1,2,1,2,1,2)$ | 7.85 | 8493.737 | $(1,2,1,2,1,2)$ | 8.13 | 12325.09 |
| SC2 | 9963.964 | $(1,2,1,2,1,2)$ | 8.21 | 13935.32 | $(1,2,1,2,1,2)$ | 8.45 | 14376.28 |
| SC3 | 8951.974 | $(1,2,1,2,1,2)$ | 8.50 | 3128.029 | $(1,2,1,2,1,2)$ | 8.28 | -20243.5 |

Table 3 shows the result of this competition mode.

### 4.2.Discussion

The former examples present CSCND in price-dependent market in which no rival exists and one, two, or three SCs are planning to enter the market simultaneously, set the price competitively, and shape their network cooperatively. Following the fact that price can be adjusted in the short run, location is set "once and for all" and the demand of the customers, market shares, and total SC incomes are not only related to SC own price and location, but also to the rivals, respectively. In the realworld competitions, producers often use different marketing activities to improve their market shares such as advertising and promotions. Such modifications influence the parameter values of the demand

Table 4
The change of the optimal price, market share, SCN structure and total income with respect to $\tilde{\beta}$ in monopoly competition

|  | Market 1 |  |  |  | Market 2 |  |  | Objective <br> function |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Market share <br> 1 | assigned <br> path | DC price | Market share 2 | assigned <br> path | DC price | Total SC |  |
| SC1 | 57762.29403 | $(1,2)$ | 5003.477875 | 53856.92785 | $(1,2)$ | 5003.763825 | 557687789.2 | $\tilde{\delta}=0.001 E V\left(\tilde{d}_{k}\right)$ |
| SC1 | 57681.88208 | $(1,2)$ | 1670.144542 | 53775.78355 | $(1,2)$ | 1670.430492 | 185355040.9 | $\tilde{\delta}=0.003 E V\left(\tilde{d}_{k}\right)$ |
| SC1 | 57601.47013 | $(1,2)$ | 1003.477875 | 53694.63924 | $(1,2)$ | 1003.763825 | 110888959.3 | $\tilde{\delta}=0.005 E V\left(\tilde{d}_{k}\right)$ |
| SC1 | 57521.05818 | $(1,2)$ | 717.7635893 | 53613.49494 | $(1,2)$ | 718.0495393 | 78975258.7 | $\tilde{\delta}=0.007 E V\left(\tilde{d}_{k}\right)$ |
| SC1 | 57440.64623 | $(1,2)$ | 559.0334306 | 53532.35064 | $(1,2)$ | 559.3193806 | 61245685.04 | $\tilde{\delta}=0.009 E V\left(\tilde{d}_{k}\right)$ |
| SC1 | 53781.90261 | $(1,2)$ | 53.477875 | 49840.28484 | $(1,2)$ | 53.763825 | 4801750.428 | $\tilde{\delta}=0.01 E V\left(\tilde{d}_{k}\right)$ |
| SC1 | 45740.71 | $(1,2)$ | 20.15 | 41725.85 | $(1,2)$ | 20.43 | 1136925 | $\tilde{\delta}=0.03 E V\left(\tilde{d}_{k}\right)$ |
| SC1 | 37699.51303 | $(1,2)$ | 13.477875 | 33611.42421 | $(1,2)$ | 13.763825 | 450765.5598 | $\tilde{\delta}=0.05 E V\left(\tilde{d}_{k}\right)$ |

Table 5
The change of the optimal price, market share, SCN structure and total income with respect to $\tilde{\beta}$ in duopoly competition

| $\tilde{\delta}=0.03 E V\left(\tilde{d}_{k}\right)$ | Market 1 |  |  | Market 2 |  |  | Objective | $\tilde{\beta}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Market share 1 | assigned path | DC price | Market share 2 | assigned path | $\begin{gathered} \hline \mathrm{DC} \\ \text { price } \end{gathered}$ | Total SC |  |
| SC1 | 20433.98908 | (1,2,1,2) | 12.31 | 18075.34973 | (1,2,1,2) | 12.61 | 196574.0274 | $\tilde{\beta}=0.003 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 14259.36 | (1,2,1,2) | 11.02 | 12533.53 | (1,2,1,2) | 11.26 | 92687.06621 |  |
| SC1 | 20879.36895 | (1,2,1,2) | 12.12 | 18457.21905 | (1,2,1,2) | 12.42 | 193316.6019 | $\tilde{\beta}=0.005 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 14784.43 | (1,2,1,2) | 10.93 | 13007.85 | (1,2,1,2) | 11.18 | 94098.05369 |  |
| SC1 | 21305.38679 | (1,2,1,2) | 11.94 | 18822.5954 | (1,2,1,2) | 12.25 | 190228.4949 | $\tilde{\beta}=0.007 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 15270.92 | (1,2,1,2) | 10.85 | 13447.55 | (1,2,1,2) | 11.11 | 95088.66832 |  |
| SC1 | 21712.57404 | (1,2,1,2) | 11.77 | 19171.8257 | (1,2,1,2) | 12.09 | 187272.3627 | $\tilde{\beta}=0.009 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 15723.34 | (1,2,1,2) | 10.77 | 13856.73 | (1,2,1,2) | 11.03 | 95735.31955 |  |
| SC1 | 21909.33849 | (1,2,1,2) | 11.69 | 19340.56179 | (1,2,1,2) | 12.01 | 185835.7851 | $\tilde{\beta}=0.01 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 15937.95 | (1,2,1,2) | 10.73 | 14050.94 | (1,2,1,2) | 10.99 | 95949.15955 |  |
| SC1 | 25064.72852 | (1,2,1,2) | 10.57 | 22038.24938 | (1,2,1,2) | 10.94 | 160944.6136 | $\tilde{\beta}=0.03 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 19090.45 | (1,2,1,2) | 10.03 | 16918.50 | (1,2,1,2) | 10.35 | 92026.6197 |  |
| SC1 | 27215.27 | (1,2,1,2) | 9.9 | 23858.11 | (1,2,1,2) | 9.55 | 141370.3 | $\tilde{\beta}=0.05 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 20954.01 | (1,2,1,2) | 10.3 | 18640.17 | (1,2,1,2) | 9.90 | 82995.3 |  |
| SC1 | 28795.59988 | (1,2,1,2) | 9.45 | 25177.79024 | (1,2,1,2) | 9.86 | 125811.7664 | $\tilde{\beta}=0.07 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 22152.68 | (1,2,1,2) | 9.20 | 19772.31 | (1,2,1,2) | 9.57 | 73945.90873 |  |

Table 6
The change of the optimal price, market share, SCN structure and total income with respect to $\tilde{\delta}$ in duopoly competition

| $\tilde{\beta}=0.05 E V\left(\tilde{d}_{k}\right)$ | Market 1 |  |  | Market 2 |  |  | Objective | $\tilde{\delta}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Market share 1 | assigned path | $\begin{gathered} \hline \mathrm{DC} \\ \text { price } \end{gathered}$ | Market share 2 | assigned path | $\begin{gathered} \hline \mathrm{DC} \\ \text { price } \end{gathered}$ | Total SC |  |
| SC1 | 59987.77 | (1,2,2,1) | 17.13 | 53664.73 | (1,2,2,1) | 17.29 | 1129479.88 | $\tilde{\delta}=0.001 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 51688.83 | (1,2,2,1) | 16.85 | 50478.06 | (1,2,2,1) | 16.59 | 911665.07 |  |
| SC1 | 54989.545 | (1,2,1,2) | 15.93 | 50898.16655 | (1,2,1,2) | 16.44 | 942251.7 | $\tilde{\delta}=0.003 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 49749.60 | (1,2,1,2) | 15.40 | 46450.54 | (1,2,1,2) | 15.87 | 776843.1629 |  |
| SC1 | 51877.89428 | (1,2,1,2) | 15.11 | 47893.2472 | (1,2,1,2) | 15.61 | 805445.4 | $\tilde{\delta}=0.005 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 46550.38 | (1,2,1,2) | 14.60 | 43377.74 | (1,2,1,2) | 15.05 | 653409.5101 |  |
| SC1 | 51877.89428 | $(1,2,1,2)$ | 14.40 | 47893.2472 | (1,2,1,2) | 14.88 | 805445.3665 | $\tilde{\delta}=0.007 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 43643.94 | (1,2,1,2) | 13.90 | 40583.08 | (1,2,1,2) | 14.34 | 653409.5101 |  |
| SC1 | 46474.27532 | (1,2,1,2) | 13.77 | 42662.34918 | (1,2,1,2) | 14.24 | 598120.143 | $\tilde{\delta}=0.009 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 40978.76 | (1,2,1,2) | 13.29 | 38017.58 | (1,2,1,2) | 13.71 | 468686.8047 |  |
| SC1 | 45259.89667 | (1,2,1,2) | 13.48 | 41484.36859 | (1,2,1,2) | 13.94 | 556686.4526 | $\tilde{\delta}=0.01 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 39723.75 | (1,2,1,2) | 13.01 | 36808.53 | (1,2,1,2) | 13.71 | 442754.2922 |  |
| SC1 | 27215.26868 | (1,2,1,2) | 9.90 | 23858.10589 | (1,2,1,2) | 10.29 | 141370.2626 | $\tilde{\delta}=0.03 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 20954.01 | (1,2,1,2) | 9.55 | 18640.17 | (1,2,1,2) | 9.90 | 82995.30417 |  |
| SC1 | 14543.14754 | $(1,2,1,2)$ | 8.21 | 11338.70453 | (1,2,1,2) | 8.58 | 25500.1468 | $\tilde{\delta}=0.05 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 7668.81 | (1,2,1,2) | 7.94 | 5680.33 | (1,2,1,2) | 8.26 | 3309.196008 |  |

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Table 7
The change of the optimal price, market share, SCN structure and total income with respect to $\tilde{\beta}$ in oligopoly competition

| $\tilde{\delta}=0.03 E V\left(\tilde{d}_{k}\right)$ | Market 1 |  |  | Market 2 |  |  | Objective function Total SC |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Market share 1 | assigned path | DC price | Market share 2 | assigned path | DC price |  |  |
| SC1 | 6680.49 | (1,2,1,2,1,2) | 8.40 | 4977.20 | (1,2,1,2,1,2) | 8.68 | 10674.35 | $\tilde{\beta}=0.005 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 8052.83 | (1,2,1,2,1,2) | 9.38 | 9694.49 | (1,2,1,2,1,2) | 9.60 | 30216.88 |  |
| SC3 | 6247.11 | (1,2,1,2,1,2) | 9.38 | 5030.51 | (1,2,1,2,1,2) | 9.09 | -10020.58 |  |
| SC1 | 7155.24 | (1,2,1,2,1,2) | 8.36 | 5292.38 | (1,2,1,2,1,2) | 8.64 | 11248.43 | $\tilde{\beta}=0.007 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 8257.08 | (1,2,1,2,1,2) | 9.25 | 10017.43 | (1,2,1,2,1,2) | 9.48 | 28929.33 |  |
| SC3 | 6509.18 | (1,2,1,2,1,2) | 9.30 | 5100.61 | (1,2,1,2,1,2) | 9.01 | -10577.93 |  |
| SC1 | 7594.32 | (1,2,1,2,1,2) | 8.32 | 5575.65719 | (1,2,1,2,1,2) | 8.61 | 11679.66 | $\tilde{\beta}=0.009 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 8440.30 | (1,2,1,2,1,2) | 9.14 | 10316.48 | (1,2,1,2,1,2) | 9.37 | 27749.10 |  |
| SC3 | 6744.03 | (1,2,1,2,1,2) | 9.22 | 5141.27 | (1,2,1,2,1,2) | 8.94 | -11155.38 |  |
| SC1 | 7802.39 | (1,2,1,2,1,2) | 8.31 | 5707.03 | (1,2,1,2,1,2) | 8.59 | 11852.82 | $\tilde{\beta}=0.01 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 8524.79 | (1,2,1,2,1,2) | 9.09 | 10457.94 | (1,2,1,2,1,2) | 9.32 | 27193.18 |  |
| SC3 | 6852.55 | (1,2,1,2,1,2) | 9.19 | 5151.93 | (1,2,1,2,1,2) | 8.91 | -11447.15 |  |
| SC1 | 11013.19 | (1,2,1,2,1,2) | 8.014 | 7482.47 | (1,2,1,2,1,2) | 8.30 | 12706.43 | $\tilde{\beta}=0.03 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 9562.11 | (1,2,1,2,1,2) | 8.49 | 12552.66 | (1,2,1,2,1,2) | 8.72 | 19137.34 |  |
| SC3 | 8256.65 | (1,2,1,2,1,2) | 8.73 | 4528.85 | (1,2,1,2,1,2) | 8.49 | -16656.70 |  |
| SC1 | 13372.57 | (1,2,1,2,1,2) | 7.845555 | 8493.737 | (1,2,1,2,1,2) | 8.133768 | 12325.09 | $\tilde{\beta}=0.05 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 9963.964 | (1,2,1,2,1,2) | 8.207401 | 13935.32 | (1,2,1,2,1,2) | 8.446884 | 14376.28 |  |
| SC3 | 8951.974 | (1,2,1,2,1,2) | 8.504538 | 3128.029 | (1,2,1,2,1,2) | 8.276818 | -20243.5 |  |

The following managerial insights can be derived from the numerical results:
$\checkmark$ According to the tables, it is clear that $\tilde{\beta}, \tilde{\delta}$ have negative effect on equilibrium prices and the total incomes of the chains, and the prices and total incomes increaseby decreasing the amount of parameters.
$\checkmark$ Increasing the competition intensity forces the SCs to reduce their prices, and consequently their profits will decrease.
$\checkmark$ Changing the competition from monopoly to oligopoly leads to huge amount of decrease on the SCs profits.
$\checkmark$ Developing brand loyalty leads to decreasing the effect of $\tilde{\delta}$ and leads to more profits.

Table 8
The change of the optimal price, market share, SCN structure and total income with respect to $\tilde{\delta}$ in oligopoly competition

| $\tilde{\beta}=0.05 E V\left(\tilde{d}_{k}\right)$ | Market 1 |  |  | Market 2 |  |  | Objective function Total SC |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Market share 1 | assigned path | DC price | Market share 2 | $\begin{gathered} \hline \text { assigned } \\ \text { path } \\ \hline \end{gathered}$ | DC price |  |  |
| SC1 | 30871.72 | (2,1,1,2,1,2) | 11.46 | 29945.50 | (2,1,1,2,1,2) | 11.21 | 159121.38 | $\tilde{\delta}=0.001 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 37669.45 | (2,1,1,2,1,2) | 11.12 | 44885.34 | (2,1,1,2,1,2) | 11.20 | 295338.59 |  |
| SC3 | 36579.65 | (2,1,1,2,1,2) | 11.37 | 35921.90 | (2,1,1,2,1,2) | 11.00 | 209125.55 |  |
| SC1 | 38871.87 | (1,2,1,2,1,1) | 10.22 | 33621.99 | (1,2,1,2,1,1) | 10.56 | 223991.37 | $\tilde{\delta}=0.003 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 35276.37 | (1,2,1,2,1,1) | 10.62 | 39819.19 | (1,2,1,2,1,1) | 10.91 | 239710.83 |  |
| SC3 | 28373.98 | (1,2,1,2,1,1) | 11.15 | 25816.18 | (1,2,1,2,1,1) | 11.07 | 100328.05 |  |
| SC1 | 36636.92 | (1,2,1,2,1,1) | 9.97 | 31457.68 | (1,2,1,2,1,1) | 10.31 | 193287.78 | $\tilde{\delta}=0.005 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 33100.12 | (1,2,1,2,1,1) | 10.37 | 37557.50 | (1,2,1,2,1,1) | 10.66 | 207477.30 |  |
| SC3 | 26104.16 | (1,2,1,2,1,1) | 10.91 | 23342.87 | (1,2,1,2,1,1) | 10.82 | 76871.77 |  |
| SC1 | 34073.28 | (1,2,1,2,2,2) | 9.71 | 28605.54 | (1,2,1,2,2,2) | 10.01 | 160079.51 | $\tilde{\delta}=0.007 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 30235.72 | (1,2,1,2,2,2) | 10.11 | 34966.78 | (1,2,1,2,2,2) | 10.36 | 172361.94 |  |
| SC3 | 25764.66 | (1,2,1,2,2,2) | 10.57 | 21976.19 | (1,2,1,2,2,2) | 10.37 | 70286.88 |  |
| SC1 | 32034.09 | (1,2,1,2,2,2) | 9.50 | 26630.90 | (1,2,1,2,2,2) | 9.79 | 137074.50 | $\tilde{\delta}=0.009 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 28248.50 | (1,2,1,2,2,2) | 9.89 | 32900.19 | (1,2,1,2,2,2) | 10.14 | 148082.14 |  |
| SC3 | 23727.03 | (1,2,1,2,2,2) | 10.36 | 19722.20 | (1,2,1,2,2,2) | 10.15 | 52385.67 |  |
| SC1 | 31045.33 | (1,2,1,2,2,2) | 9.40 | 25672.59 | (1,2,1,2,2,2) | 9.69 | 126653.71 | $\tilde{\delta}=0.01 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 27283.80 | (1,2,1,2,2,2) | 9.79 | 31897.63 | (1,2,1,2,2,2) | 10.04 | 137062.11 |  |
| SC3 | 22737.22 | (1,2,1,2,2,2) | 10.26 | 18626.07 | (1,2,1,2,2,2) | 10.05 | 44442.82089 |  |
| SC1 | 13372.57 | (1,2,1,2,1,2) | 7.845555 | 8493.73 | (1,2,1,2,1,2) | 8.133768 | 12325.09 | $\tilde{\delta}=0.03 E V\left(\tilde{d}_{k}\right)$ |
| SC2 | 9963.964 | (1,2,1,2,1,2) | 8.207401 | 13935.32 | (1,2,1,2,1,2) | 8.446884 | 14376.28 |  |
| SC3 | 8951.974 | (1,2,1,2,1,2) | 8.504538 | 3128.029 | (1,2,1,2,1,2) | 8.276818 | -20243.5 |  |

## 5. Conclusion

In this paper, three fuzzy multi-level mixed integer programming models are presented for CSCND in monopoly, duopoly, and oligopoly competitions in uncertain environments.Then, they are converted into one integrated bi-level model, and the inner part of the model sets equilibrium prices in dynamic competition, while the outer part specifies the equilibrium network structure by cooperation between the chains.Finally, one numerical
example is used to illustrate and discuss the effect of $\tilde{\beta}, \tilde{\delta}$ parameters on market shares, equilibrium prices, total incomes, and SCN structures. We also conclude that for increasing the profits of SCs,brand loyalty should be increasedand the competition intensity should be decreased. This paper can be improved by considering stochastic programming, sustainable, closed loop or robust SCND as the future research.

## Appendix A

Assume that we have a fuzzy number by the following membership function (Dubois and Prade, 1987;Pishvaee, Razmi et al., 2012):
$\mu_{\tilde{k}}(x)=\left\{\begin{array}{lll}l_{\tilde{k}^{\prime}}(x) & \text { if } & k_{(1)(2)} x \leq k, \\ 1 & \text { if } & k_{(2)} \leq x \leq k_{(3)}, \\ h_{\tilde{k}}(x) & \text { if } & k_{(3)} \leq x \leq k_{(4)}, \\ 0 & \text { if } & k_{(4)} \prec x \text { or } k \prec a_{(1)}\end{array}\right.$

Then, the upper and lower expected values of $\left(E^{*}(\tilde{k}), E_{*}(\tilde{k})\right)$ by means of Choquet integral are defined $E^{*}(\tilde{k})=k_{(3)}+\int_{k(3)}^{k_{(4)}} h_{\tilde{k}}(x) d x$,
$E_{*}(\tilde{k})=k_{(2)}-\int_{k^{(1)}}^{k_{(2)}} l_{\tilde{k}}(x) d x$

Also, the expected value [EV] and expected interval [EI]
$E I=\left[k_{(2)}-\int_{k_{(1)}}^{k_{(2)}} I_{\hat{k}}(x) d x, k_{(3)}-\int_{k_{(e)}}^{k_{(4)}} h_{\tilde{k}}(x) d x\right]$,
$E V=\frac{1}{2}\left[k_{(2)}-\int_{k_{(1)}}^{k_{(2)}} l_{\tilde{k}}(x) d x+k_{(3)}-\int_{k_{(e)}}^{k_{(4)}} h_{\tilde{k}}(x) d x\right]$

Imagine that $\tilde{k}$ has the trapezoidal membership function,
$E I(\tilde{k})=\left[\frac{k_{(1)}+k_{(2)}}{2}, \frac{k_{(3)}+k_{(4)}}{2}\right]$,
$E V(\tilde{k})=\left[\frac{k_{(1)}+k_{(2)}+k_{(3)}+k_{(4)}}{4}\right]$

Now, assume that $w$ is a real number, the possibility (Pos) and necessity (Nec) of $\tilde{k} \leq w$ can be defined as
$\operatorname{Pos}(\tilde{k} \leq w)=\left\{\begin{array}{l}1 \\ \frac{w-k_{(1)}}{k_{(2)}-k_{(1)}} \\ 0\end{array}\right.$
if $k_{(2)} \leq w$,
$\operatorname{Nec}(\tilde{k} \leq w)=\left\{\begin{array}{lll}1 & \text { if } & k_{(4)} \leq w, \\ \frac{w-k_{(3)}}{k_{(4)}-k_{(3)}} & \text { if } & k_{(3)} \leq w \leq k_{(4)}, \\ 0 & \text { if } & k_{(3)} \geq w\end{array}\right.$
as follows(Dubois and Prade, 1987;Pishvaee, Razmi et al., 2012):
of $\tilde{k}$ can be defined as follows(Dubois and Prade, 1987;Pishvaee, Razmi et al., 2012):
then (Dubois and Prade, 1987;Pishvaee, Razmi et al., 2012)
follows (Dubois and Prade, 1987;Pishvaee, Razmi et al., 2012):

It can be shown that for $\alpha \succ 0.5$, we have (Inuiguchi and
$\operatorname{Pos}(\tilde{k} \leq w) \geq \alpha \Leftrightarrow w \geq(1-\alpha) k_{(1)}+(\alpha) k_{(2)}$,
$\operatorname{Nec}(\tilde{k} \leq w) \geq \alpha \Leftrightarrow w \geq(1-\alpha) k_{(3)}+(\alpha) k_{(4)}$
(76),(77) are directly applied to convert our fuzzy constraints into their equivalent crisp one.It is worth noting that as the necessity measure is more meaningful to satisfy the constraints, we applied this measure in the
$\min =f y+c x$
$s^{7}$
$A x \geq d$
$S x \leq N y$
$x \geq y \in\{0,1\}$
where vectors $f, c, d$ are related to the imprecise parameters (like fixed costs, variable costs, and demands); matrixes $A, S, N$ are the coefficient matrixes of the constraints; $x, y$ are the vector of continuous and binary variables. Then, consider that $f, c, d, N$ are imprecise parameters with trapezoidal possibility distributions, and the rest of the parameters are crisp
$\min \mathrm{Z}=\left(\frac{f_{(1)}+f_{(2)}+f_{(3)}+f_{(4)}}{4}\right) y+\left(\frac{c_{(1)}+c_{(2)}+c_{(3)}+c_{(4)}}{4}\right) x$
s.t
$A x \geq(1-\alpha) d_{(3)}+\alpha d_{(4)}$
$S x \leq\left((1-\alpha) N_{(2)}+\alpha N_{(1)}\right) y$
$x \geq 0, y \in\{0,1\}$

The proposed method is easy to handle, requires less computations, and matches our described environment; also, it is a powerful tool for handling uncertainty.Therefore, we use the procedure for handling uncertainty and writing equivalent crisp amounts of parameters in the objective functions and constraints of the problem in this paper.

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Ramık, 2000;Pishvaee, Razmi et al., 2012):
paper to cope with fuzzy constraints(Inuiguchi and Ramık, 2000;Pishvaee, Razmi et al., 2012).
Let us consider the following impact form:
according to possibility chance-constrained programming (Dubois and Prade, 1987;Inuiguchi and Ramık, 2000;Pishvaee, Razmi et al., 2012); we are able to replace the expected values of the imprecise parameters and the necessary measure in the objective function and constraints as follows:

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