

# A Chance Constraint Approach to Multi Response Optimization Based on a Network Data Envelopment Analysis

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Received 6 October, 2011; Revised 19 August, 2012; Accepted 27 December, 2012

## Abstract

In this paper, a novel approach to multi response optimization is presented. In the proposed approach, response variables in treatments combination occur with a certain probability. Moreover, we assume that each treatment has a network style. Due to the probabilistic nature of the treatment combination, the suggested approach can compute the efficiency of each treatment under the desirable reliability. The approach has been constructed based on a network data envelopment analysis and the chance constraint method. Finally, a numerical example shows applicability of the proposed method to a multi response problem. The results indicate that the proposed approach is a capable method for analyzing the best treatment and compared to other existing methods, it performs better in efficient treatment determination.

*Keywords:* Multi Response Optimization; Chance Constraint; Network Data Envelopment Analysis; Knapsack Approach.

## 1. Introduction

Multi response optimization (MRO) is a useful and well-studied method in industrial problems. In multiple response problems, response variables must be considered generally in complex product/process designs. In this regard, some previous studies were done based on the MADM (multi attribute decision-making) approaches such as data envelopment analysis (DEA) and TOPSIS. In most of them, the aim was to aggregate several criteria into a single criterion. Yet, there are some approaches which identify the non-dominated solution by pareto solution search methods. On the other hand, in multi response problems, there are treatments with controllable factors and response variables in a single stage; it means that we have many controllable factors as inputs and many responses as experimental results. In this structure, the desirability of each treatment is checked to find optimum factors levels among all of the treatments. Nevertheless, the experiment can be more complex than one stage and even with network structure for experiments.

In other words, in addition to relations between input/s and output/s of each stage, it is possible for the output/s of intermediate stage to be consumed as input for the

neighborhood stage or to play the role of final response in the experiment. So, in this situation the experimental style is more complex. This condition is illustrated in Figure 1.

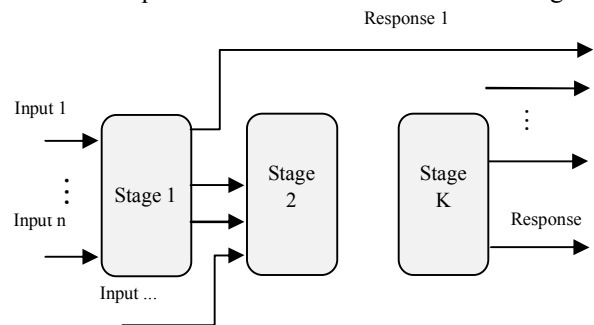


Fig. 1. The network structure of the multiple response problem

The deterministic aspect of experiment design is a special case for the probabilistic view with 100% confidence level. Moreover, in some problems, we accept a considered risk to forecast future events and we can disregard the scenarios according to the risk interval. Therefore, we are interested in determining the most efficient and reliable treatment with a considered confidence level. In this study, to find the most efficient treatment an approach is proposed based on the Network DEA (Lewis and Sexton, 2004) and chance constraint method (Charnes and Cooper, 1959). The first one helps to solve the complexity of structure and the second one is

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considered the desired confidence level for the MRO problem.

The rest of this paper is organized as follows: the related literature on multi response optimization is reviewed in the next section. Section 3 involves the statement of problem and its application. Section 4 focuses on the proposed approach for determining the most efficient treatment/s with a specific reliability level. In Section 5, a numerical example is given and the comparison results between the proposed method and other existing approaches are reported. Finally, some concluding remarks are presented in the last section.

## 2. Literature Review

As an interesting subject, multiple response problems have attracted the attention of many researchers. Existing methods in this area include both simple weighted-sum approaches (Gauri and Pal, 2010) and more complex regression and mathematical programming approaches (Al-Refaei et al., 2009), Principal Components Analysis (Fung and Kang, 2005), Fuzzy Logic (Tarng, 2000), Grey Rational Analysis (Manivannan et al., 2011) and multi criteria decision making (MCDM) approaches (Refaie, 2010; Lan, 2009).

Caporaletti et al. (1999) proposed an input DEA model for NTB characteristics using  $(\bar{y}_{ij} - y_i)^2$  and  $S_{ij}^2$  as inputs. The authors only considered their experiment to judge about controllable factors status efficiency. Liao and Chen (2002) proposed an input-oriented model (CCR DEA model) that uses the normalized mean of responses as inputs (if STB or NTB) or outputs (if LTB). Like the aforementioned study, only the data of the actual experiments were used. Liao (2004) also used a Neural Network to estimate the mean responses of all factor level combinations and then used a CCR DEA model to select the best level based on Normalized S/N ratios (as outputs) (Charnes et al, 1978).

In another study, Gutiérrez and Lozano (2010) used an Artificial Neural Network to train the relation between the responses and all factor level combinations. Since then, Data Envelopment Analysis (DEA) is used to determine the efficient factor level combinations, and then to select among factors levels, mean square deviations of the quality characteristics are used as DEA inputs. Tsai et al. (2008) presented an optimization procedure, which uses DEA to analyze multiple inputs/outputs and costs for experiments with mixtures. The authors assessed the method in a real case.

The survey on DEA extensions shows that Lewis and Sexton (2004) suggested a novel model that applies to DMUs and consists of a network of Sub-DMUs. They considered a method which yields the efficiency scores of both DMU and sub-DMU.

Camanho and Dyson (2005) presented an application of stochastic DEA with chance programming. Also, Bruni et

al. (2009) proposed a stochastic model for data envelopment analysis (DEA) based on the theory of joint probabilistic constraints. The authors evaluated their proposed method in a supplier selection case study.

Some studies have also been done on MRO. For example, Chiao and Hamada (2001) considered experiments with correlated multiple responses. The means, variances, and correlations depend on experimental factors. Diaz Garcia et al. (2005) studied the application of some stochastic programming approaches to response surface optimization. They assumed a normal distribution for response surface coefficients (response surface methodology, RSM) and compared their approach with the results of some methods such as E- Model, V- Model, and Lexicography. The RSM is used to find a suitable approximation of the true relationship between response variables and controllable factors.

In another study, Hejazi et al. (2011) proposed goal programming for multi response problems with stochastic parameters  $(\mu, \sigma^2)$ . In the study, the mean and variance were considered as goals and goal programming led to tend the characteristics to the desirable values.

Sometimes the main problem which occurs in multi response optimization problems is that when the mean square error (MSE) of the regression model is high, the ability of the model to describe the relationship of the response variable and the controllable factors is poor (Kim et al. , 2001). To overcome this problem, the ANN can be used as a proper substitute method for response estimation. Some authors compared the response surface and regression models with ANN in model building and the preciseness of ANN was verified in their results (Namvar-Asl et al. , 2008; Tsao, 2008).

Table 1 shows a summary of existing approaches classified according to the DEA technique and its applications. In addition, deterministic and uncertainty aspects of MRO are another important aspect considered to survey the characteristics of published works and the novelty of proposed methods. It is obvious that because of the stochastic nature of experimentation, the probabilistic view of treatment is an essential and attractive issue. Therefore, the stochastic view of the MRO problem leads to the identification of more reliable levels of controllable factors. Moreover, the network structure of some experiments and its complexity structure have not been studied in previous studies.

Table 1  
The summary of related published works in the literature based on DEA

Published works	DEA		Problem Status		Application		
	Simple	Network	Deterministic	uncertainty	MRO		Other
					Simple Structure	Network structure	
Ref. [1]		*	*				*
Ref.[7]	*		*		*		
Ref.[10,11,12,14]	*		*		*		
Ref.[13]	*		*		*		
Ref[16,17]	*			*			*
Ref[18,19,20]				*	*		
The proposed method		*		*		*	

### 3. Statement of the Problem

In this section, more details of multi response problem which is discussed in this paper are explained. First, the network structure of experiments is evaluated.

Most of multiple response problems existing in the literature have a single stage structure so that in each treatment, controllable factors have the role of inputs and response variables are considered as outputs, while structure of experiments may be more complex.

According to Figure 1 and its network structure, formal approaches designed for simple experiments cannot determine the optimum treatment accurately because of the structure of the experiment which involves more than one stage. For more explanation, suppose that we have a refining process. The core refining process is simple distillation. Because crude oil is made up of a mixture of hydrocarbons, this first basic refining process is aimed at separating the crude oil into its fractions, the broad categories of its component hydrocarbons. Crude oil is heated and put into a still (a distillation column) and different products boil off and can be recovered at different temperatures. After that, some products of the previous stage are added to the unfinished product and new inputs are used as inputs to a vacuum distillation unit. It is worth noting that consideration of a single stage for this problem is wrong because the inputs of the second stage do not have effect on the first stage and entering the second stage inputs to the first stage leads to mistakes in the analysis of experiment. In this real case and many

other similar processes, some of input/s, output/s and intermediate products (unfinished products) exist in a multi stage experimental design. Therefore, it seems that the network structure of Figure 1 is more suitable for the mentioned problems.

In this paper, in addition to the network style of experiments, uncertainty situation increases the problem complexity. Paying attention to the probabilistic aspect of MRO in this study lets us to consider the deterministic situation as a special case of probabilistic approach with 100% confidence level. For defining the probabilistic situation of the problem, suppose that according to the collected experimental data, there are lots of different results during a special period of time and the analysis of them should be done considering the stochastic nature. One of the popular approaches to stochastic analysis is the scenario-based approach and we have used the clustered data according to the scenarios. Therefore, there are some treatments with their occurrence probability in every scenario. The problem in the form of probabilistic scenarios is presented in Table 2. This table shows the simple kind of experiment (one stage) in a predefined scenario.

Let us define the problem parameters for each treatment  $j$ ,  $j=1, \dots, n$ :

$i=1, \dots, m$  controllable factor index.

$r=1, \dots, q$  response index.

$p=1, \dots, P$  scenario index.

$Y_j^p = (y_{1j}^p, \dots, y_{qj}^p)^T$  responses vector in the  $j$ th Trt and the  $p$ th scenario.

Table 2  
Characteristics of the treatments in each scenario

Scenario	Trt	Probability of scenario occurrence	Factors			Responses		
$p^{th}$	1	$Pr^p$	$x_{11}$	...	$x_{m1}$	$y_{11}^p$	...	$y_{q1}^p$
	2		$x_{12}$	...	$x_{m2}$	$y_{12}^p$	...	$y_{q2}^p$
	$\vdots$		$\vdots$	$\vdots$		$\vdots$		$\vdots$
	$n$		$x_{1n}$	...	$x_{mn}$	$y_{1n}^p$	...	$y_{qn}^p$

For this problem, identification of the best treatment with the minimum risk and best ranking is the main objective.

#### 4. The Proposed Approach

In this section, to determine the efficiency of each treatment, Data Envelopment Analysis and its extension are represented for the deterministic situation. Then by adding the probabilistic approach to the efficiency model, the proposed stochastic efficiency model for the network MRO problem is presented. Finally, the steps of the proposed method are described.

##### 4.1. Data Envelopment Analysis

DEA is one of the useful MADM techniques. DEA measures the efficiency of organizations with multiple inputs and outputs. These organizations are called decision-making units (DMUs). Also, by applying the DEA to the related problems, the rank of each DMU is specified based on its efficiency score. In this paper, DEA is used for multi response problems so that after experimental designs, each treatment is considered as a DMU. Then, the efficiency score of each treatment helps us in the optimization stage.

In this study, the DMU and Sub-DMU are the equivalent of treatment (trt) and sub-treatment (sub-trt) respectively in the design of experiment, respectively. We assume that each trt is comprised of a set of sub-trts. Each input to a Sub-trt is either exogenous to the trt or is the output of another Sub-trt. Similarly, each output from a Sub-trt either leaves the trt as a final response variable or is an input to another Sub-trt.

The network DEA model (output-oriented) can be illustrated in this way:

Let  $k$  be the index of the trt in the experiments. For  $d=1, \dots, D$ , and  $s, t=1, \dots, S$ ,  $s \neq t$ ,

$X_{dsi}$  = The level of input  $i$  consumed by Sub-trt  $s$  in trt  $d$ , for  $i = 1, \dots, I$ ,

$Y_{dsto}$  = The level of intermediate product  $o$  produced by Sub-trt  $s$  and consumed by Sub-trt  $t$  in trt  $d$ , for  $o=1, \dots, O$ ,

$Z_{dsr}$  = The value of response  $r$  produced by Sub-trt  $s$  in trt  $d$  for  $r = 1, \dots, R$ ,

$\lambda_{dsk}$  = The weight placed on Sub-trt  $s$  in trt  $d$  by Sub-trt  $s$  in trt  $k$

$\theta_{sk}$  = The inverse efficiency of Sub-trt  $s$  in trt  $k$ .

For Sub-trt  $s$  at trt  $k$ , we let,

$$\text{Max } \theta_{sk} \quad (1)$$

S.t.

$$\sum_{d=1}^D \lambda_{dsk} X_{dsi} \leq X_{ksi}, \quad i = 1, \dots, I \quad (2)$$

$$\sum_{d=1}^D \lambda_{dsk} \left( \sum_{t=1}^S Y_{dsto} \right) \leq \sum_{t=1}^S Y_{ktso}, \quad o = 1, \dots, O \quad (3)$$

$$\sum_{d=1}^D \lambda_{dsk} \left( \sum_{t=1}^S Y_{dsto} \right) \geq \theta_{sk} \sum_{t=1}^S Y_{ktso}, \quad o = 1, \dots, O \quad (4)$$

$$\sum_{d=1}^D \lambda_{dsk} Z_{dsr} \geq \theta_{sk} Z_{ksr}, \quad r = 1, \dots, R \quad (5)$$

$$\lambda_{dsk} \geq 0; \quad d = 1, \dots, D$$

$$\theta_{sk} \geq 0$$

$\theta_{sk}$  is the inverse efficiency value of sub-trt  $s$  at trt  $k$ . Also, Inequalities (2) and (3) ensure that the reference Sub-trt consumes no more of each input and each intermediate product as does Sub-trt  $s$  at Trt  $k$ . Inequalities (4) and (5) guarantee that the reference Sub-trt produces at least as much as each intermediate product and each response as does Sub-trt  $s$  at Trt  $k$ .

For each sub-trt, by calculating the value of  $\theta_{sk}$ ,  $\lambda_{dsk}^*$ , we can compute the efficient value of output ( $Z^*$ ) or intermediate products ( $Y^*$ ) according to Equations (6) and (7).

$$Z_{ksr}^* = \sum_{d=1}^D \lambda_{dsk}^* Z_{dsr}, \quad r = 1, \dots, R \quad (6)$$

$$Y_{ksr}^* = \sum_{d=1}^D \lambda_{dsk}^* \sum_{t=1}^S Y_{dsto}, \quad r = 1, \dots, R \quad (7)$$

where  $\lambda_{dsk}^*$  is the optimal value of  $\lambda_{dsk}$  for the reference sub-trt and  $Z_{ksr}^*$  is the level of response variable  $r$  produced by sub-trt  $s$  at trt  $k$  if it would be efficient. Suppose that we have a structure with two stages and we have computed all of efficiency scores in the first stage of first Trt. To calculate the overall efficiency score, we need to replace the optimal value of outputs or intermediate product (according to Equations (6) and (7)) in the next sub-trt. Hence, for the second sub-trt, the inverse efficiency score is computed as follows:

$$\text{Max } \theta_{21} \quad (8)$$

S.t.

$$\sum_{d=1}^D \lambda_{dsk} X_{dsi} \leq X_{ksi}, \quad i = 1, \dots, I \quad (9)$$

$$\sum_{d=1}^D \lambda_{dsk} \left( \sum_{t=1}^S Y_{dsto} \right) \leq \sum_{t=1}^S Y_{ktso}^*, \quad o = 1, \dots, O \quad (10)$$

$$\sum_{d=1}^D \lambda_{dsk} Z_{dsr} \geq \theta_{sk} Z_{ksr}, \quad r = 1, \dots, R \quad (11)$$

$$\lambda_{dsk} \geq 0; \quad d=1, \dots, D \quad (12)$$

$$\theta_{sk} \geq 0 \quad (13)$$

To obtain the overall inverse efficiency for each treatment, we have:

$$\theta_k = \min_{r=1, \dots, R} \left( \frac{\sum_{s=1}^S Z_{ksr}^{**}}{\sum_{s=1}^S Z_{ksr}} \right) \quad (14)$$

where  $Z_{ksr}^{**}$  is the level of output  $r$  that Sub-trt  $s$  in trt  $k$  would produce under these conditions if it were efficient.

Notice that  $\sum_{d=1}^D \lambda_{dsk} = 1$  because variables are returns to the scale. By calculating the inverse efficiency scores, the treatment rank can be determined (for more explanation, see [1]).

#### 4.2. The probabilistic approach

In this section, the probabilistic aspect of the problem is added to the proposed model. We have some scenarios with certain occurrence probability and in each scenario, a set of treatments network structure are tested. Knapsack approach is used to compute  $(1-\alpha)\%$  confidence level to determine the efficiency of each trt.

Definition 1: (Scenario efficiency). Trt  $k$  is efficient with respect to scenario  $p$  if it is impossible to find a feasible solution for the following problem:

$$\sum_{d=1}^D \lambda_{dsk} X_{dsi}^p \leq X_{ksi}^p, \quad i=1, \dots, I \quad (15)$$

$$\sum_{d=1}^D \lambda_{dsk} \left( \sum_{t=1}^S Y_{dts}^p \right) \geq \sum_{t=1}^S Y_{kts}^p, \quad o=1, \dots, O \quad (16)$$

$$\sum_{d=1}^D \lambda_{dsk} Z_{dsr}^p \geq Z_{ksr}^p, \quad r=1, \dots, R \quad (17)$$

We have  $1-\alpha\%$  reliability for selecting the best treatment. In the stochastic framework, for each trt, uncertainty parameters are represented by the random variables. These variables can be considered as the

approximation of the known distribution function or the set of scenarios.

Definition 2:  $1-\alpha\%$  confidence level is performed if:

$$\text{Max } \theta_{sk} \quad (18)$$

S.t.

$$p \left( \begin{array}{l} \sum_{d=1}^D \lambda_{dsk} \tilde{X}_{ds}(\omega) \leq \tilde{X}_{ks}(\omega), i=1, \dots, I \\ \sum_{d=1}^D \lambda_{dsk} \left( \sum_{t=1}^{S\sim} Y_{dts}(\omega) \right) \leq \sum_{t=1}^{S\sim} Y_{kts}(\omega), o=1, \dots, O \\ \sum_{d=1}^D \lambda_{dsk} \left( \sum_{t=1}^{S\sim} Y_{dts}(\omega) \right) \geq \theta_{sk} \sum_{t=1}^{S\sim} Y_{kts}(\omega), o=1, \dots, O \\ \sum_{d=1}^D \lambda_{dsk} \tilde{Z}_{dsr}(\omega) \geq \theta_{sk} \tilde{Z}_{ksr}(\omega), r=1, \dots, R \end{array} \right) \geq 1-\alpha \quad (19)$$

where input/s, output/s and intermediate product are represented by the random variable defined on a given probability space. For example,  $\tilde{Z}_{ksr}(\omega)$  is the  $r$ th response of sub-trt  $s$  at trt  $k$  under the stochastic situation.

Definition 3: (probabilistic knapsack problem) Knapsack approach is characterized by the allocation of limited resources to competing items. The items are associated with resource requirements as well as rewards. In this paper, we have a knapsack with  $1-\alpha\%$  volume. Each scenario has a certain volume and an individual effect on the maximization of the inverse efficiency score of sub-trt and treatment. Therefore, this approach adds big  $M$  and  $0-1$  decision variable to decide which constraints can be removed from the knapsack based on disregarding  $\alpha\%$  (as the maximum risk-value) of the overall knapsack volume.

#### 4.3. The proposed multiple response optimization approach

In this section, the multi response optimization method is proposed. The proposed approach is illustrated in Figure 2 and its explanation is provided below:

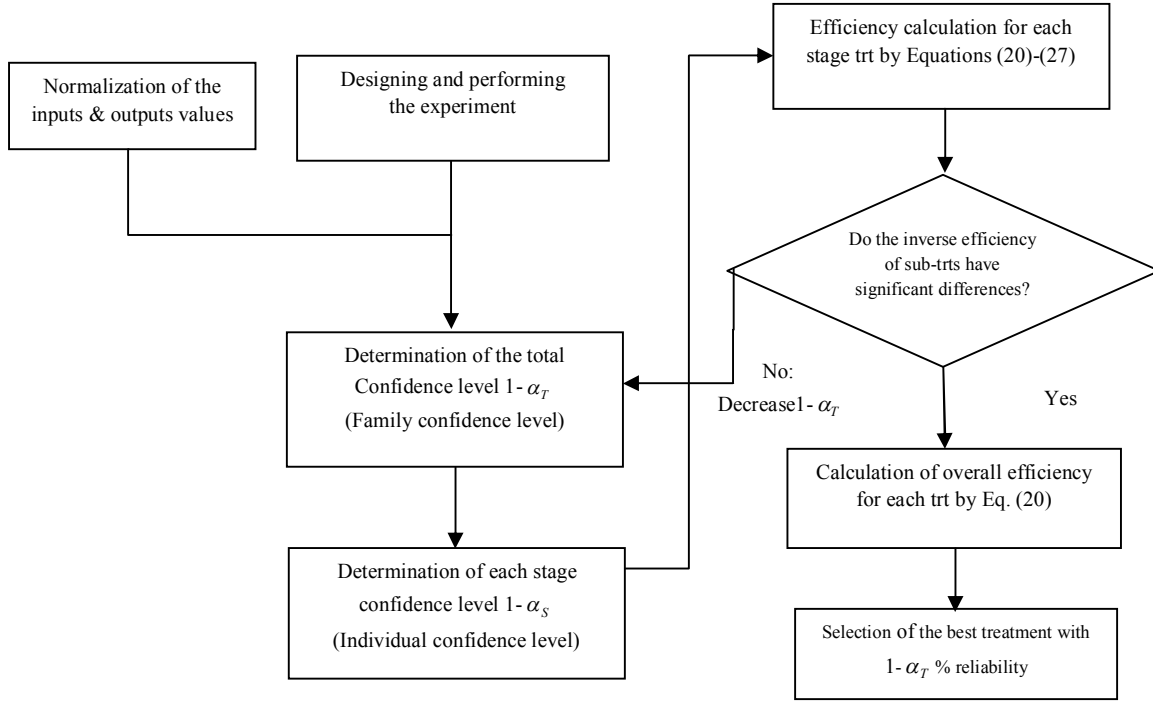


Fig. 2. The proposed multiple response optimization approach

Step1. Experiment information

Determine the controllable factors, response variables and quality characteristics (i.e., larger-the-better (LTB), nominal-the-best (NTB) or smaller-the-better (STB)) of the response variables and collect the experiment data.

Step2. Normalization of the gathered data Normalize all of the results to reduce the effects of various measuring units of the response variables on efficiency analysis in the Network DEA.

Note that in most of MRO approaches, the analyzer does not attend to cost considerations of factors levels and the controllable factors' values are not considered. However, in the DEA approach we assume that the analyzer is interested in decreasing the input variable value.

We can use one of the available normalization formulas (according to the STB, LTB and NTB).

Step3. Efficiency calculation for each sub-trt Compute the inverse efficiency score for each sub-trt at each trt according to following model:

$$\text{Max } \theta_{sk} \tag{20}$$

S.t.

$$\sum_{d=1}^D \lambda_{dsk} X_{dsi}^p \leq X_{dsi}^p, \tag{21}$$

$$i = 1, \dots, I, p = 1, \dots, P$$

$$\sum_{d=1}^D \lambda_{dsk} \left( \sum_{t=1}^S Y_{dstp}^p \right) \leq \sum_{t=1}^S Y_{ktsop}^p + M\delta^p, \tag{22}$$

$$o = 1, \dots, O; p = 1, \dots, P$$

$$\sum_{d=1}^D \lambda_{dsk} \left( \sum_{t=1}^S Y_{dstp} \right) + M\delta^p \geq \theta_{sk} \sum_{t=1}^S Y_{kstp}, \tag{23}$$

$$p = 1, \dots, P$$

$$\sum_{d=1}^D \lambda_{dsk} Z_{dsr} + M\delta^p \geq \theta_{sk} Z_{ksr}, \tag{24}$$

$$r = 1, \dots, R$$

$$\sum_{p=1}^P \text{Pr}^p \cdot \delta^p \leq (\alpha_S) \tag{25}$$

$$\lambda_{dsk} \geq 0; \quad d = 1, \dots, D \tag{26}$$

$$\theta_{sk} \geq 0, \delta^p = 0, 1 \tag{27}$$

where  $1 - \alpha_S = \sqrt[S]{1 - \alpha_t}$ . In this relation,  $S$  is the number of stages (sub-trt) in the experiment structure and  $\alpha_S$  is the complement of the imposed reliability level (risk-value) for each stage and  $\alpha_t$  has the same concept for the overall inverse efficiency calculation.

Equation (20) is the objective function for determining the efficiency score of the  $sth$  sub-trt at the  $kth$  treatment and Equation (25) defines a binary knapsack constraint, which guarantees the violation of the constraints for a subset of scenarios whose cumulative probability is less than the complement of the imposed confidence level. Also,  $\delta^p$  is a binary variable for eliminating the  $pth$  scenario in the complement of the imposed confidence level interval.

Other equations have the same concepts mentioned in section 4.1 with a scenario treatment.

Step 4. Treatment selection based on efficiency scores  
 Determine the best treatment among all based on the overall efficiency score of all treatments. To this end, get the overall efficiency in accordance with Equation (8). Notice that  $\theta_k$  is the inverse of efficiency in  $k^{\text{th}}$  treatment.

5. A Numerical Example

In this section, a hypothetical example is presented. The structure of experiments at each treatment is given in Figure 3.

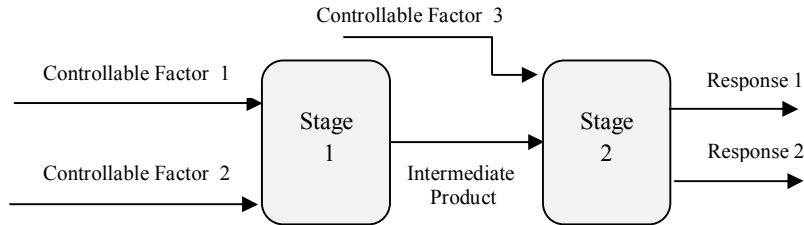


Fig. 3. The network structure of the example

As it is shown in the figure, each treatment consists of two stages. The first stage has two controllable factors, each at three levels, and two outputs. Other controllable factors are an input for the second stage.

The detailed information about the numerical example is given in Table 3. In this regard, suppose that we experiment with 9 treatments. The aggregated data show that we have 4 scenarios with the probabilities of 0.1, 0.35, 0.5 and 0.05, respectively.

In this example, 80% overall confidence level ( $1-\alpha_T$ ) is defined by the decision maker. Thus, giving attention to

the individual confidence level for each sub-trt shows that 90% confidence level is suitable for running the proposed MILP model and computing each efficiency score.

We used a classic optimization software (Lingo 8) for calculating the efficiency of each treatment. The results, summarized in Table 4, show that the 4<sup>th</sup> treatment has the highest efficiency score among the others ( $\theta_4 = 1$ ).

To further clarify the proposed approach, the computed variables for the 7th treatment are reported in Table 5.

Table 3  
 Information about the numerical example

Scenario (occurrence probability Pr)	Trt	Controllable Factors			Intermediate product	Response variables	
		Factor 1	Factor 2	Factor 3		Response1	Response2
1 (Pr=0.1)	1	0.45	0.1	0.15	0.5	0.4	0.3
	2	0.45	0.5	0.12	0.6	0.2	0.2
	3	0.45	0.9	0.15	0.4	0.3	0.3
	4	0.5	0.1	0.19	0.7	0.8	0.8
	5	0.5	0.5	0.15	0.6	0.4	0.4
	6	0.5	0.9	0.18	0.5	0.6	0.6
	7	0.55	0.1	0.15	0.5	0.2	0.2
	8	0.55	0.5	0.12	0.4	0.3	0.3
	9	0.55	0.9	0.16	0.4	0.4	0.4
2 (Pr=0.35)	1	0.45	0.1	0.19	0.55	0.42	0.3
	2	0.45	0.5	0.16	0.6	0.22	0.54
	3	0.45	0.9	0.19	0.3	0.3	0.3
	4	0.5	0.1	0.1	0.65	0.78	0.7
	5	0.5	0.5	0.2	0.58	0.35	0.32
	6	0.5	0.9	0.11	0.3	0.62	0.65
	7	0.55	0.1	0.2	0.5	0.2	0.2
	8	0.55	0.5	0.15	0.3	0.35	0.3
	9	0.55	0.9	0.13	0.3	0.4	0.6
3 (Pr=0.5)	1	0.45	0.1	0.2	0.55	0.3	0.35
	2	0.45	0.5	0.17	0.53	0.15	0.4
	3	0.45	0.9	0.2	0.4	0.2	0.4
	4	0.5	0.1	0.11	0.65	0.8	0.8
	5	0.5	0.5	0.21	0.5	0.38	0.33
	6	0.5	0.9	0.12	0.38	0.6	0.7
	7	0.55	0.1	0.21	0.5	0.2	0.15
	8	0.55	0.5	0.15	0.4	0.3	0.5
	9	0.55	0.9	0.14	0.4	0.6	0.52
4 (Pr=0.05)	1	0.45	0.1	0.21	0.4	0.4	0.45
	2	0.45	0.5	0.18	0.5	0.17	0.47
	3	0.45	0.9	0.21	0.4	0.5	0.2
	4	0.5	0.1	0.12	0.57	0.7	0.68
	5	0.5	0.5	0.22	0.5	0.38	0.3
	6	0.5	0.9	0.13	0.35	0.6	0.7
	7	0.55	0.1	0.22	0.55	0.25	0.25
	8	0.55	0.5	0.16	0.35	0.55	0.3
	9	0.55	0.9	0.15	0.5	0.5	0.6

Table 4  
Efficiency results of the problem for each treatment

	Treatments								
	1	2	3	4	5	6	7	8	9
efficiency score	0.73	0.857	0.68	1	0.54	0.97	0.37	0.79	0.89
Rank	6	4	7	1	8	2	9	5	3

Table 5.  
Efficiency calculation for the 7th treatment considering 81% confidence level

Treatment	$\theta_{17}$	$\lambda_{417}^*$	Inverse Efficiency score	Efficiency Score
7	1.3	1	2.7	0.37

Table 5 reveals that the 7<sup>th</sup> treatment has got 0.37 efficiency score with 0.81% confidence level (two stages, each with 90% confidence level).

5.1. Method analysis

The proposed approach emphasizes cost consideration. Most of multi response optimization approaches overlook controllable factors values but in some cases the amount of controllable factors (with different costs) are not equal in different treatments. In such cases, using the proposed approach is, therefore, more appropriate. Suppose two treatments with two factors, each at two levels, according to Table 6. Most of existing approaches select the first

treatment; however, it seems that the second one is appropriate too.

Table 6  
Regarding or disregarding of costs in MRO

Trial No.	Factors		Responses	
	A	B	Response 1	Response 2
1	10	20	11	21
2	1	2	10	20

To check the good performance of the proposed approach, we generated random data for all of the treatments. By increasing the LTB responses in a certain trt and scenarios, the reasonable behavior existed for its efficiency value. This concept is presented in Table 7.

Table 7  
The effect of response changes on the inverse efficiency score (the first response of the 7th treatment)

Scenario number	Current value of the first response	Changed values of the first response					
		0.2	0.3	0.22	0.1	0.18	0.2
1	0.2	0.3	0.22	0.1	0.18	0.22	
2	0.2	0.3	0.22	0.1	0.18	0.24	
3	0.25	0.35	0.275	0.15	0.225	0.31	
4							
Current value of the inverse efficiency score	2.73	-	-	-	-	-	
Modified value of the inverse efficiency score	-	2	2.54	2.8	2.77	2.25	

To evaluate the efficiency of the proposed method, we implement two existing approaches and compare the results. We solve the mentioned multi response problem by means of the deterministic DEA and Network DEA. The results are reported in Table 8. All of the surveyed approaches can be used for determining the efficient trt but the main problem is which method works with more accuracy and capability. As it clear in Table 8, the mean of inverse efficiency score for Network DEA (1.197) is greater than DEA (1.079); therefore the Network DEA is more capable of identifying sources of inefficiency in complex models, thereby potentially yielding greater analysis insights into experimental design improvements. Moreover, the comparison of DEA and Network DEA shows that the number of efficient trt is less for Network DEA and ranges and standard deviations of inverse efficiencies for Network DEA are more spread. The defined criteria show that the Network DEA can propose better solutions for analyzer so that the analyzer can specify the source of inefficiency by considering the sub-trt efficiencies in the network structure. By decreasing the confidence level from 100%, the model can determine the resources of inefficiencies better. For example, considering 81% reliability level for the numerical

example leads to a greater mean for the probabilistic kind of Network DEA compared with the deterministic DEA and Network DEA. Table 8 shows that the probabilistic Network DEA works better than the others so that it has a high mean of inverse efficiency scores (1.445) and rang of differences (1.73). Moreover, the proposed method suggests only one trt as the optimal solution whereas the others have more than one solution. In other words, the proposed approach helps analyzer to identify the pareto optimal solution by accepting the risk value in a network structure of experiments.

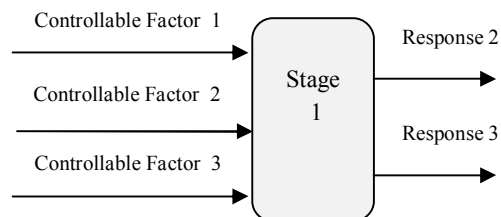


Fig. 4. The simplified structure of the numerical example for using in the simple DEA



In this section, behavior of inefficiencies with respect to the risk value is evaluated. It is worth noting that the inverse efficiency score increases or (in the worst case) remains stable with growing the risk values. This illustrates the importance of choosing appropriate threshold probability levels to avoid incorrect

classifications of treatments. Table 9 presents the risk value and its effect on the inverse efficiency scores. This table is related to the inverse efficiency score of the first stage in the numerical example.

Table 8  
Comparison results obtained for the DEA, Network DEA and the proposed approach

Trt No.	Inverse Efficiency Score		
	DEA	Network DEA	(the proposed approach)
1	1	1.138	1.362
2	1	1.077	1.167
3	1	1.186	1.464
4	1	1	1
5	1.21	1.286	1.84
6	1	1	1.03
7	1.5	2.09	2.73
8	1	1	1.27
9	1	1	1.14
No. 1	7	4	1
Mean	1.079	1.197	1.445
SD	0.163	0.33	0.515
Rang	0.5	1.09	1.73

Table 9  
Effects of confidence level changes on the inverse efficiency score in the example

Trt	Confidence level % or 100-(percent of risk-value)			
	50	70	90	100
1	1.037	1	1	1
2	1	1	1	1
3	1.375	1.375	1.354	1.25
4	1	1	1	1
5	1.3	1.12	1.12	1.12
6	1.4	1.3	1.3	1.3
7	1.3	1.3	1.3	1.036
8	1.628	1.625	1.625	1.625
9	1.625	1.625	1.625	1.14

We can analyze the stability of efficiency scores and ranks according to risk intervals. For example, a risk value is suggested by the analyzer, but the results would be stable for a higher confidence level. So, we can check the neighborhood interval of the threshold probability level to report the most reliable level.

In Figure 5, the behavior of the inverse efficiency score with respect to the risk-values is evaluated. In this figure the first stage of numerical example is used to interpret the results of Table 8. The figure demonstrates that increasing the risk leads to cutting the overlap scenarios or constraints in the proposed model.

To evaluate the validity of the proposed approach, we generated five random samples according to the mentioned structure. The results show the capability of the proposed method through appropriate scores in comparison to the other approaches related to DEA. All of the scores are reported in Table 10.

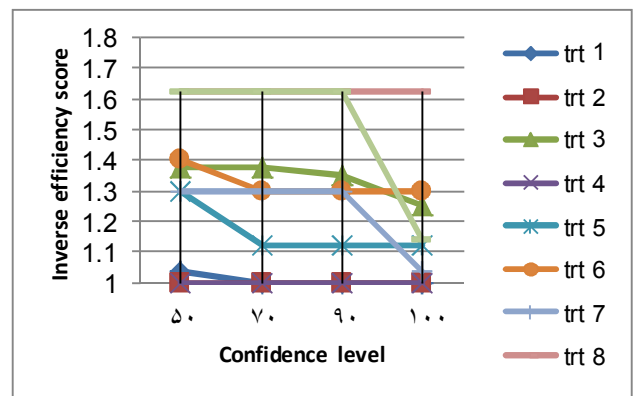


Fig. 5. Behavior of the inverse efficiency score for the first stage with respect to the confidence level

### 6. Conclusion

In this paper, an attempt was made to propose an efficient approach to find the best controllable factors levels in a joint probabilistic situation and network form of

experiments. First, an MILP model based on the network DEA and knapsack approach was presented. Then the efficiency score of each sub-trt was obtained by the proposed model according to the individual desirable confidence level. After that, the overall efficiency score for each treatment was calculated. Finally, the mentioned approach was discussed in numerical examples. The results show that the suggested approach not only proposes a unique treatment to opt, but it also can rank the treatments according to a wide range of efficiency scores.

Table. 10

Defined criteria for comparing the methods' capability based on inverse efficiency scores in five samples

Samples	No.1			Mean			SD			Rang		
	1*	2**	3***	1	2	3	1	2	3	1	2	3
1	6	3	1	1.12	1.29	1.77	0.22	0.41	0.52	0.23	1.11	1.69
2	6	2	1	1.09	1.38	1.46	0.26	0.34	0.61	0.39	1.16	1.71
3	4	1	1	1.19	1.43	1.47	0.21	0.39	0.43	0.41	1.24	1.67
4	5	3	1	1.06	1.31	1.64	0.19	0.29	0.48	0.51	1.08	1.53
5	3	2	1	1.11	1.39	1.57	0.18	0.33	0.52	0.57	1.18	1.64

\*DEA , \*\*Network DEA, \*\*\* The proposed approach

### 7. References

[1] Al-Refaei, A., Al-Durgham, L. and Bata, N. (2009). Optimal Parameter Design by Regression Technique and Grey Relational Analysis. Proceedings of World Congress on Engineering. London, U.K., 2091-2095.

[2] Al-Refaei, A. (2010). Grey-DEA approach for solving the multi-response problem in Taguchi method. Proceeding of the Institution of Mechanical Engineering-Part B. Journal of Engineering Manufacture, 224, 147-158.

[3] Bruni, M.E., Conforti, D., Beraldi, P. and Tundis, E. (2009). Probabilistically constrained models for efficiency and dominance in DEA. International Journal of Production Economics, 117, 219-228.

[4] Camanho, A.S. and Dyson, R.G. (2005). Cost efficiency measurement with price uncertainty: a DEA application to bank branch assessments. European Journal of Operational Research, 161, 432-446.

[5] Caporaletti, L.E., Dul, J.H. and Womer, N.K. (1999). Performance evaluation based on multiple attributes with nonparametric frontiers. Omega, 27, 637-645

[6] Charnes, A. and Cooper, W.W. (1959). Chance-constrained programming. Management Science, 15, 73-79.

[7] Charnes, A., Cooper, W.W. and Rhodes, E. (1978). Measuring the efficiency of decision making units. Eur. J. Operat. Res., 2, 429-444.

[8] Chiao, C. and Hamada, M. (2001). Analyzing experiments with correlated multiple responses. Journal of Quality Technology, 33, 451-465.

[9] Diaz-Garcia, J. A., Ramos-Quiroga, R. and Cabrera-Vicencio, E. (2005). Stochastic programming methods in the response surface methodology", Computational Statistics & Data Analysis, 49, 837-848.

[10] Fung, C-P. and Kang, P-C. (2005). Multi-response optimization in friction properties of PBT composites using Taguchi method and principle component analysis. Journal of Materials Processing Technology, 170, 602-610.

[11] Gauri, S.K. and Pal, S. (2010). Comparison of performances of five prospective approaches for the multi-response optimization, International Journal of Advanced Manufacturing Technology, 48, 1205-1220.

[12] Gutiérrez, E. and Lozano, S. (2010). Data Envelopment Analysis of multiple response experiments. Applied Mathematical Modeling, 34, 1139-1148.

[13] Hejazi, T.H., Bashiri, M., Diaz-Garcia, J. A. and Noghondarian, K. (2011). Optimization of Probabilistic Multiple Response Surfaces, Applied Mathematical Modeling, 36, 1275-1285.

[14] Kim K.J., Byun J.H., Min D. I.J. Jeong. (2001), Multiresponse surface optimization: concept, methods, and future directions, Tutorial; Korea Society for Quality Management.

[15] Lan, T.S. (2009). Taguchi optimization of multi-objective CNC Machining using TOPSIS. Information Technology Journal, 8, 917-922.

[16] Lewis, H.F. and Sexton, T.R. (2004). Network DEA: efficiency analysis of organizations with complex internal structure. Computers & Operations Research, 31, 1365-1410.

[17] Liao, H.C. and Chen, Y.K. (2002). Optimizing multi-response problem in the Taguchi method by DEA based ranking method. International Journal of Quality & Reliability Management, 19, 825-837.

[18] Liao, H.C. (2004). A data envelopment analysis method for optimizing multi-response problem with censored data in the Taguchi method. Computers and Industrial Engineering, 46, 817-835.

[19] Manivannan, S., Devi, S.P., Arumugam, R. and Sudharsan, N.M. (2011). Multi-objective optimization of flat plate heat sink using Taguchi-based Grey relational analysis. International Journal of Advanced Manufacturing Technology, 52, 739-749.

[20] Tarnq, Y.-S., Yang, W.-H. and Juang, S.C. (2000). The use of fuzzy logic in the Taguchi method for the optimization of the submerged arc welding process. Int. J. Adv. Manuf. Technol., 16, 688-694.

[21] Tsai, Chih-Wei., Tong, Lee-Ing. and Wang, Chung-Ho. (2008). Using DEA to optimize a mixture design with multiple response incorporating cost considerations. Journal of Scientific & Industrial Research, 67, 479-485.

- [22] Tsao C. (2008), Comparison between response surface methodology and radial basis function network for core-center drill in drilling composite materials; *The International Journal of Advanced Manufacturing Technology*, 37, 1061-1068.

