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Research Article

Production Constraints Modeling: A Tactical Review Approach

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Abstract

A constraint is a limitation or a restriction that poses a threat to the performance and efficiency of a system. This paper presented a tactical review approach to production constraints modeling. It discussed the theory of constraints (TOC) as a thinking process and continuous improvement strategy to curtail constraints in other to constantly increase the performance and efficiency of a system. It also x-rayed the working process of implementing the TOC concept which consists of five steps called "Process of On-Going Improvement". Furthermore, it talked about constraints programming and constraints-based models which were explained to some details. Finally, production constraints model formulation procedures for linear programming and non-linear programming scenarios were extensively discussed with reference to published literature as instances of production constraints modeling were also cited.

Keywords: Constraints; Production, Models; Linear; Non-linear; Programming; TOC

1. Introduction

A constraint is something that limits or restricts something else. It must be binding and has no room for "sort of", "kind of" or partiality. It is not a bias or a preference but can take the form of a rule or physical constraints (Blackstone, 2001; M. Gupta, 2003; Naor, Bernardes, & Coman, 2013). If there is a suitable alternative, then there is no constraint. Theory of Constraints (TOC) is a philosophy of management put forth by Eliyahu M. Goldratt, which claims that each system has at least one constraint or bottleneck which can be defined as any kind of situation that impedes the system to reach high-performance level in terms of its purposes (Cyplik, Hadaś, & Domański, 2009; Goldratt, 1990; Okutmuş, Kahveci, & Kartašova, 2015; Şimşit, Günay, & Vayvay, 2014). The performance of any system is limited by the rate of throughput of the system's constraint. Identifying the system's constraint as the weakest link of the chain and eliminating it is the main idea behind the TOC (Izmailov, 2014; Şimşit et al., 2014; Utku, Cengiz, & Ersoy, 2011). Every company operates within a large set of constraints, including annual budgets, material purchase contracts, resource capacity, hospital ward space, environmental regulations, customer order contracts, financial reporting regulations etc (Cyplik et al., 2009; Hazem & Farouk, 2016; RONEN & SPECTOR, 1992; Spector, 2011). According to Cyplik et al (2009), TOC philosophy is applied in many functional areas of companies, ranging from production flow management, marketing, accounting or project management to be a tool of logical reasoning. The importance of taking major understood and managed constraints, such as budgets, normally trickle down to various decision-making levels.

The need to reduce system constraints to the barest minimum or to possibly eliminate them in totality gave rise to some mathematical models and/or programs. Mathematical programming is a technique for solving certain unique problems, notably: maximizing profits and minimizing costs; subject to constraints on resources, capacities, supplies, demands etc. (Coman & Ronen, 2000; Ellis, 2011; Hossain, n.d.; Okutmuş et al., 2015; Puche, Ponte, Costas, Pino, & de la Fuente, 2016; SPENCER & COX, 1995). Constraints programming or constraint-Based models are subsets of mathematical programming and models.

"Constraint programming is the study of computational systems based on constraints. The idea of constraint programming is to solve problems by stating constraints (requirements) about the problem area and, consequently, finding a solution satisfying all the constraints. i.e. assigning a value to each unknown from the respective domain"(Bartak, 1998; Barták, n.d.). It is possible to state constraints over various domains; however, probably more than 95% of all constraint applications deal with finite domains currently (Tsang & Tsang, 1993b). The scheduling problems are the most successful application area of constraints with a finite domain (Lever, Wallace, & Richards Ic-Parc, 1995; Wallace, 1994).

Constraint-based modeling is a scientifically-proven mathematical approach, in which the outcome of each decision is constrained by a minimum and maximum range of limits (+/- infinity is allowed) (Barták, n.d.; Spector, 2011; Tsang & Tsang, 1993a; Umble, Umble, & Murakami, 2006). Decision variables having a common constraint must also have their solution values fall within

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that constraint's bounds. A constraint-based modelling approach is most commonly and effectively used with optimization techniques, such as the use of linear and mixed-integer programming to maximize an objective function. Constraint-based modelling is not hypothesis-driven; hence it is a probabilistic model and not a deterministic model.

In this paper, the focus is on production constraints modelling in which linear programming and non-linear programming of production constraints are discussed to some details and tactically reviewed with examples given.

2. Literature Review

2. 1 Theory of Constraints (TOC)

The Theory of Constraints (TOC) is a way of thinking that targets at constantly increasing the efficiency of a System (Cyplik et al., 2009). TOC has a wide range of implementation scale that can be applied in production, logistics, supply chain, distribution, project management, accounting, research and development, sales and marketing and so on (Şimşit et al., 2014). The goal of every company is to maximize profit and minimize cost. However, there is at least one inherent constraint that is preventing the company from achieving its desired aim. To this end, anything that deviates from the way to making more profit is a constraint. The company must do whatever it takes to handle constraints in their system and manage these constraints, by so doing; they would have a continuous improvement management system that could guarantee the achievement of higher profits. Consequently, to increase the efficiency of the whole system, TOC provides management tools in the form of a five-step procedure which, provided that it is followed consistently, will yield beneficial outcomes (Aguilar-Escobar, Garrido-Vega, & González-Zamora, 2016; Cyplik et al., 2009; M. Gupta, 2003; S. Gupta & Starr, 2014; Huang, Chen, Chiu, & Chen, 2014; Kujawińska, Rogalewicz, & Diering, 2016; Reid, 2007; Sereshti, Bijari, & Moslehi, 2012; Trojanowska & Dostatni, 2017). The working process of implementing the TOC concept which consists of five focusing steps is called Process of On-Going Improvement (Şimşit et al., 2014). The steps according to Goldratt and Cox (1992) are:

(a) identify the system's constraint,
(b) decide how to exploit the system's constraint,
(c) subordinate everything else to the above decision,
(d) elevate the system's constraint and
(e) if in any of the previous steps a constraint is broken, go back to step 1.

Ronen and Spector (1992) extended the process of continuous improvement by adding two preliminary steps, (i) define the system's goal and (ii) determine the global performance measures; so as to redefine it as a seven-step method. Cyplik et al (2009) explain the five steps as follows:

2.1.1 Identification of System Constraint

Identification of constraints that may occur both within and outside the production system is of vital importance. Production constraints markedly reduce the throughput of production. Consequently, managers or decision-makers should concentrate their efforts on removing the constraints, since the outcome of production as a whole depends on impact exerted by the constraints.

2.1.2 Decision to Exploit the Constraint

This stage comes down to deciding what actions to be taken to pool out the maximum benefits out of the constraint. In other words, how to exploit the constraint to ensure a maximum capital flow for the company is decided. This means optimization of the capacity of the constraint in such a way as to reduce its effects on the production system to the barest minimum. The constraint should be subject to ongoing monitoring to make sure there is no downtime. At the same time, all activities should be scheduled to bring maximum gains: the focus should be on those products which, in the bottleneck analysis, generate the highest profits.

2.1.3 Subordination of the Entire System to the Constraint

The scheme of operation of all activities within the system is adjusted to the existing constraint.

2.1.4 Reinforcement of the constraint

This step consists of increasing the constraint's capacity, i.e. improving its action. For example, the manufacturer can increase the capacity of the constraint by redesigning products, increasing stock or enhancing production capacity, which enables a quick response to change customers' demands.

2.1.5 Return to step one

If a previously identified constraint has been eliminated, the system needs to be reassessed to verify whether the constraint has not relocated to another link.

According to Simsit et al (2014), TOC is an effective approach that needs to be endorsed with the performance measurement system. The underlying reason is that every company wants to measure the effect of improvements on their system. This main idea leads to the development of a process-focused performance measurement system. This system focuses the organization on actions that improve overall financial performance. In literature, this framework is called "Throughput Accounting". According to TOC, there are financial and operational measurements. Financial performance measures are net profit, return on investment (ROI) and cash flow which are global performance measurements. TOC uses this measurement system but states that they are not enough in the subsystem level. So, there must be operational measurements as well which are: throughput, inventory and operating expense.

Generally, production constraints can be a linear programming model or a non-linear programming model.

3. Methodology

3.1 Production Constraint linear Programming Models

A general and concise way of expressing production constraints linear programming models is by using mathematical notations. The general form of the problem is described using algebraic notations for the objective and the constraints. The following depicts a production constraints problem. It is a symbolic linear programming model and its components are fundamental to all models.

- **Sets**, e.g. products
- **Parameters**, e.g. production and profit rates
- **Variables**, whose values the solver is to determine
- an **Objective**, to be maximized or minimized
- **Constraints** that the solution must satisfy.

For instance;

Given: P, a set of products
a_j = tons per hour of product j, for each $j \in P$
b = hours available at the mill
c_j = profit per ton of product j, for each $j \in P$
u_j = maximum tons of product j, for each $j \in P$

Define variables: X_j = tons of product j to be made, for each $j \in P$

Maximize: $\sum c_j X_j$ for each $j \in P$

- Subject to: $\sum (1/a_j) X_j \leq b$
$0 \leq X_j \leq u_j$ for each $j \in P$

The model describes an infinite number of related optimization problems. If we provide specific values with them for data, however, the model becomes a specific problem, or *instance* of a model, that can be solved. Each different collection of data values defines a different instance; the example below is one such instance.

The model above will be best understood using case scenarios as follows:

1. A steel company must decide how to allocate next week's time on a rolling mill. The mill takes unfinished slabs of steel as input and can produce either of two semi-finished products, which are called bands and coils. The mill's two products come off the rolling line at different rates:

Tons per hour:	Bands	200
	Coils	140

and they also have different Profitabilities:

Profit per ton:	Bands	$25
	Coils	$30

To further complicate matters, the following weekly production amounts are the most that can be justified in light of the currently booked orders:

Maximum tons:	Bands	6,000
	Coils	4,000

The question facing the company is as follows: If 40 hours of production time are available this week, how many tons of bands and how many tons of coils should be produced to bring in the greatest total profit?

While the numeric values for production rates and per-unit profits are given, the tons of bands and of coils to be produced are yet unknown. These quantities are the decision *variables* whose values must be determined to maximize profits. The purpose of the linear program is to specify the profits and production limitations as explicit formulas involving the variables so that the desired values of the variables can be determined systematically.

In an algebraic statement of a linear program, it is customary to use mathematical shorthand for the variables. Thus, let XB the number of tons of bands to be produced and XC for tons of coils. The total hours to produce all these tons is then given by:

(hours to make a ton of bands) × XB + (hours to make a ton of coils) × XC

This number cannot exceed the 40 hours available. Since hours per ton are the reciprocal of the tons per hour given above, we have a *constraint* on the variables:

$$(1/200)\, XB + (1/140)\, XC \leq 40.$$

There are also production limits:

$$0 \leq XB \leq 6000 \quad \text{and} \quad 0 \leq XC \leq 4000$$

In the statement of the problem above, the upper limits were specified, but the lower limits were assumed — it was obvious that a negative production of bands or coils would be meaningless.

By analogy with the formula for total hours, the total profit must be

(profit per ton of bands) × XB + (profit per ton of coils) × XC

That is, our objective is to maximize 25 XB + 30 XC. Putting this all together, we have the following linear program:

Maximize $25\, XB + 30\, XC$

Subject to constraints $(1/200)\, XB + (1/140)\, XC \leq 40$

$0 \leq XB \leq 6000$

$0 \leq XC \leq 4000$

This is a very simple linear program, and can be solved easily in a couple of ways:

First, by multiplying profit per ton times tons per hour, we can determine the profit per hour of mill time for each product:

Profit per hour:	Bands	$5,000
	Coils	$4,200

Bands are clearly a more profitable use of mill time, so to maximize profit we should produce as many bands as the production limit will allow — 6,000 tons, which takes 30 hours. Then we should use the remaining 10 hours to make coils — 1,400 tons in all. The profit is $25 times 6,000 tons plus $30 times 1,400 tons, for a total of $192,000.

Alternatively, since there are only two variables, we can show the possibilities graphically. If *XB* values are plotted along the horizontal axis, and *XC* values along the vertical axis, each point represents a choice of values, or solution, for the decision variables:

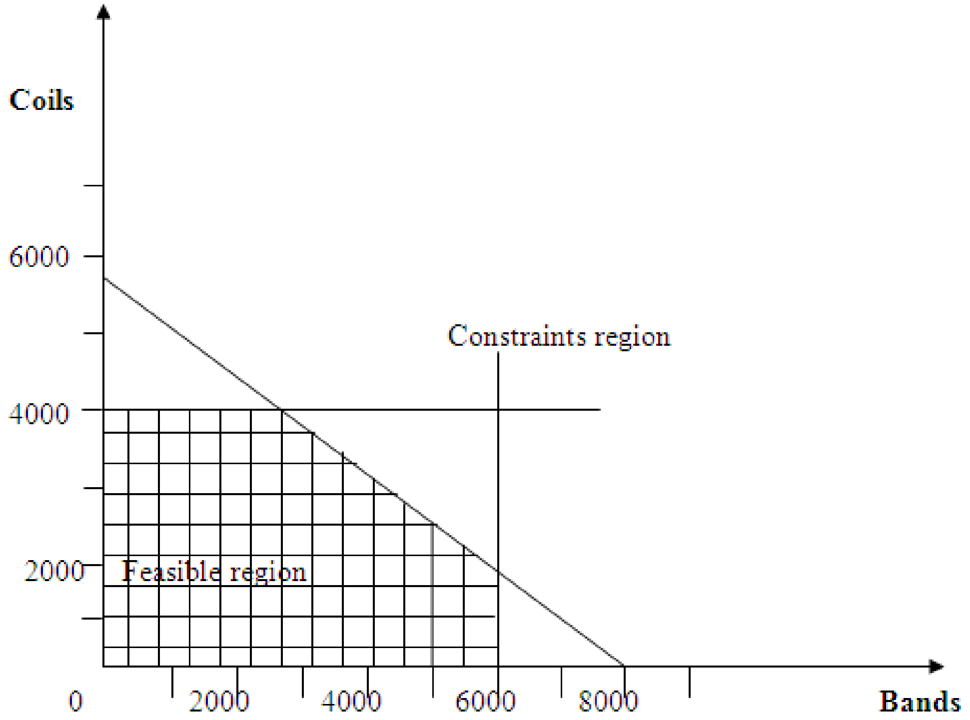

Fig. 1. Plot of Production limit for coils vs. bands

The horizontal line represents the production limit on coils, the vertical on bands. The diagonal line is the constraint on hours; each point on that line represents a combination of bands and coils that requires exactly 40 hours of production time and any point downward and to the left requires less than 40 hours.

The shaded region bounded by the axes and these three lines corresponds exactly to the *feasible* solutions — those that satisfy all three constraints. Among all the feasible solutions represented in this region, we seek the one that maximizes the profit.

Among all profit lines that intersect the feasible region, the highest and furthest to the right is the answer and it is the middle line, which just touches the region at one of the corners. When drawn appropriately it is found that this point corresponds to 6,000 tons of bands and 1,400 tons of coils, and a profit of $192,000 — the same as we found before.

Example 2. A furniture manufacturing company manufactures chairs and tables. The data in the table shows the resources consumed and unit profit. Further, it is assumed that wood and labour are the two resources which are consumed in manufacturing furniture. The owner of the firm wants to determine how many chairs and tables should be made to maximize the total profit.

Table 1
Total profit

	Table	chair	**Availability**
Wood (sq. ft)	30	20	**300**
Labour (Hrs)	5	10	**110**
profit	**6**	**8**	

Following the customary use of mathematical shorthand for the variables;

let X_1 be the number of tables and

X_2 be the number of chairs.

So that the objective is to maximize the total profit is given as:

Max. $Z = 6X_1 + 8X_2$

Subject to; $30X_1 + 20X_2 \leq 300$

$$5X_1 + 10X_2 \leq 110$$

Where X_1 and $X_2 \geq 0$

Now in order to plot the constraints on a graph, the inequalities have to be converted into equations temporarily to obtain the solution as follows:

$$30X_1 + 20X_2 = 300 \qquad \text{(a)}$$

$5X_1 + 10X_2 = 110$ (b)

In equation (a) $30X_1 + 20X_2 = 300$

If $X_1 = 0$; $X_2 = \frac{300}{20} = 15$ or $(0, 15)$

If $X_2 = 0$; $X_1 = \frac{300}{30} = 10$ or $(10, 0)$

Similarly, in equation (b) $5X_1 + 10X_2 = 110$

If $X_1 = 0$; $X_2 = \frac{110}{10} = 11$ or $(0, 11)$

If $X_2 = 0$; $X_1 = \frac{110}{5} = 22$ or $(22, 0)$

X_2

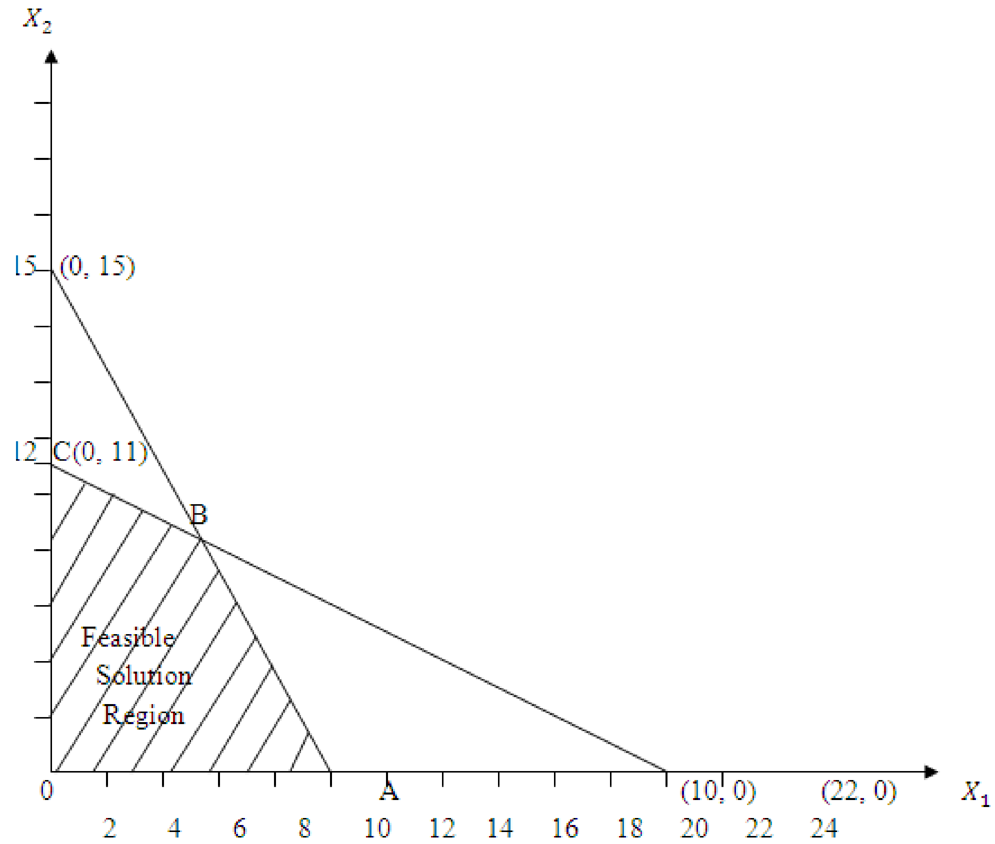

Fig. 2. Plot of Production limit on chairs and tables

Any combination of the values of X and which satisfies the given constraints is known as a feasible solution. The area 0ABC in figure 2 satisfied by constraints is shown by the shaded area and is known as a feasible solution region.

The coordinate of the points on the corners of the region can be obtained by solving the two equations of the lines intersecting on point B.

Table 2
Coordinate Points

Corner Points	Coordinate Points	$Z = 6X_1 + 8X_2$
0	(0, 0)	6 x 0 + 8 x 0 = 0
A	(10, 0)	6 x 10 + 8 x 0 = 60
B	(4, 9)	6 x 4 + 8 x 9 = 96
C	(0, 11)	6 x 0 + 8 x 11 = 88

Hence, $Z = 96$, $X_1 = 4$ *and* $X_2 = 9$
Therefore, to maximize profit which is relative 96 unit, for every four (4) tables produced, the owner should also produce nine (9) chairs.

3.2 Production Constraints Non-Linear Programming Models

3.2.1 General Non-linear Programming Problem

Definition – Let Z be a real-valued function of *n* variables defined by (Verma, 2013):

(a) $Z = f(x_1, x_2, \ldots, x_n)$
Let $(b_1, b_2, \ldots, b_n)$ be a set of constants such

(b) $\left\{\begin{array}{l} g^1(x_1, x_2, \dots, x_n)[\leq\geq \ or = b_1] \\ g^2(x_1, x_2, \dots, x_n)[\leq\geq \ or = b_2] \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ g^m(x_1, x_2, \dots, x_n)[\leq\geq \ or = b_m] \end{array}\right\}$

where g^i's are real valued functions of variables $x_1, x_2, \dots, x_n$

(c) $x_j \geq 0,\ j = 1, 2, \dots, n$ If either $f(x_1, x_2, \dots, x_n)$ or some $g^i(x_1, x_2, \dots, x_n)$ $i = 1, 2, \dots, m$; or both are non-linear, then the problem of determining the n-type $(x_1, x_2, \dots, x_n)$ which makes Z a maximum or minimum and satisfies (b) and (c) is called a general non-linear programming problem (GNLPP).

3.2.2 Constrained Maxima and Minima Problems (Equality Constraints)

Necessary Conditions: Let us take a simple problem of Maximize $Z = f(x_1, x_2)$ subject to the constraints $g(x_1, x_2) = C$ and $x_1, x_2 = 0$, where C is a constant.
We assume that $f(x_1, x_2)$ and $g(x_1, x_2)$ are differentiable w.r.t. $x_1,\ and\ x_2$. Let us introduce a differentiable function $h(x_1, x_2)$ differentiable w.r.t. $x_1\ and\ x_2$ and defined by $h(x_1, x_2) = g(x_1, x_2)$ -C
The problem is then Maximize $Z = f(x_1, x_2)$ s.t. constraints

$$h(x_1, x_2) = 0 \text{ and } x_1,\ x_1 \geq 0$$

To find the **necessary condition** for a maximum (or minimum) value of Z, a new function is formed by introducing a multiplier λ known as **Lagrange multiplier** as $L(x_1, x_2, \lambda) = f(x_1, x_2) - \lambda h(x_1, x_2)$. The number λ is an unknown constant, and the function $L(x_1, x_2, \lambda)$ is called the *Lagrange function* with *Lagrange multiplier λ*. The necessary conditions for a maximum or minimum of $f(x_1, x_2)$ subject to

$h(x_1, x_2) = 0$ are given by:

$$\frac{\partial L(x_1, x_2, \lambda)}{\partial x_1} = 0, \frac{\partial L(x_1, x_2, \lambda)}{\partial x_2} = 0\ and\ \frac{\partial L(x_1, x_2, \lambda)}{\partial \lambda} = 0$$

The partial derivatives are given by:

$$\frac{\partial L}{\partial x_1} = \frac{\partial f}{\partial x_1} - \lambda \frac{\partial h}{\partial x_1};\ \frac{\partial L}{\partial x_2} = \frac{\partial f}{\partial x_2} - \lambda \frac{\partial h}{\partial x_2};\ and\ \frac{\partial L}{\partial \lambda} = -h$$

where L is the function $L(x_1, x_2, \lambda)$, f is the function $f(x_1, x_2)$ and h is the function $h(x_1, x_2)$.

Sufficient Conditions for Extrema of Objective Function

Let the Lagrange function be $L(x, \lambda) = f(x) - \lambda h(x)$. The necessary conditions also become sufficient for a maximum or minimum are:

$$\frac{\partial L}{\partial x_i} = \frac{\partial f}{\partial x_i} - \lambda \frac{\partial h}{\partial x_i} = 0\ (j = 1, 2, \dots, n)$$
$$\frac{\partial L}{\partial \lambda} = -h(x) = 0$$

Hence;

$$\lambda = \frac{\partial f / \partial x_j}{\partial h / \partial x_j}\ (for\ j = 1, 2, \dots, n)$$

The sufficient conditions for an extremum require the evaluation of at each stationary point (n-1) principal minors of the determinant as shown below:

$$\Delta_{n+1} = \begin{vmatrix} 0 & \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} \cdots \\ \frac{\partial h}{\partial x_1} & \frac{\partial^2 f}{\partial x_1^2} - \lambda \frac{\partial^2 h}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} - \lambda \frac{\partial^2 h}{\partial x_1 \partial x_2} \cdots \\ \frac{\partial h}{\partial x_2} & \frac{\partial^2 f}{\partial x_2 \partial x_1} - \lambda \frac{\partial^2 h}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} - \lambda \frac{\partial^2 h}{\partial x_2^2} \cdots \\ & \vdots & \end{vmatrix}$$

where Δ is a minor of the determinant
If $\Delta_3 > 0, \Delta_4 < 0, \Delta_5 > 0 \dots$ the signs pattern being alternate; the stationary point is a local maximum
If $\Delta_3 < 0, \Delta_4 < 0, \Delta_5 < 0 \dots$ the signs being always negative; the stationary point is a local minimum
For better understanding, let us consider a scenario as follows:

1. A manufacturing concern operates its three available machines to polish its metal products. The three machines are equally efficient, although their maintenance costs are different. The daily cost and maintenance of the machines in dollars are formulated into a non-linear programming problem as:

$$Z = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$$

Subject to the constraints
$$x_1 + x_2 + x_3 = 20,\ \ x_1, x_2, x_3 \geq 0$$

Minimize the function by using the Lagrange multipliers to solve the non-linear programming problem.
The above production constraint model can be summarized as:
Minimize;

$$Z = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$$

Subject to the constraints

$$x_1 + x_2 + x_3 = 20,\ \ x_1, x_2, x_3 \geq 0$$

The Langrange function is given as

$$L(x_1, x_2, \lambda) = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100 - \lambda(x_1 + x_2 + x_3 - 20)$$

The necessary conditions for the stationary points are:

$$\frac{\partial L}{\partial x_1} = 0 = 4x_1 + 10 - \lambda;\ \frac{\partial L}{\partial x_2} = 0 = 2x_2 + 8 - \lambda$$
$$\frac{\partial L}{\partial x_3} = 0 = 6x_3 + 6 - \lambda;\ \frac{\partial L}{\partial \lambda} = -(x_1 + x_2 + x_3 - 20) = 0$$

Solving the above equations we have $x_1 = 5,\ x_2 = 11,\ x_3 = 4\ and\ \lambda = 30$

The **sufficient conditions** for the stationary point to be a minimum is that the minors $\Delta_3\ and\ \Delta_4$ be both negative.

Now $$\Delta_4 = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 4 & 0 & 0 \\ 1 & 0 & 2 & 0 \end{vmatrix} = -44 < 0$$

and $$\Delta_3 = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 4 & 0 \\ 1 & 0 & 2 \end{vmatrix} = -6 < 0$$

1 0 0 6

Since Δ_3 and Δ_4 are both negative, hence the solution $x_0 = (5, 11, 4)$ is the minimum with $Z = 281$.

2. The running costs of production of three different items in a cosmetic industry have been described with the following non-linear programming function;

$$Z = 2x_1^2 - 24x_1 + 2x_2^2 - 8x_2 + 2x_3^2 - 12x_3 + 200$$

Subject to the constraints

$$x_1 + x_2 + x_3 = 11,\ \ x_1, x_2, x_3 \geq 0$$

The Lagrange function is given as

$$L(x_1, x_2, x_3, \lambda) = 2x_1^2 - 24x_1 + 2x_2^2 - 8x_2 + 2x_3^2 - 12x_3 + 200 - \lambda(x_1 + x_2 + x_3 - 11)$$

The necessary conditions for the stationary points are:

$$\frac{\partial L}{\partial x_1} = 0 = 4x_1 - 24 - \lambda;\ \frac{\partial L}{\partial x_2} = 0 = 4x_2 - 8 - \lambda$$
$$\frac{\partial L}{\partial x_3} = 0 = 4x_3 - 12 - \lambda;\ \frac{\partial L}{\partial \lambda} = -(x_1 + x_2 + x_3 - 11) = 0$$

Solving the above equations we have $x_1 = 6,\ x_2 = 2,\ x_3 = 5\ and\ \lambda = 0$

The **sufficient conditions** for the stationary point to be a minimum is that the minors $\Delta_3\ and\ \Delta_4$ be both negative.

Now $$\Delta_4 = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 4 & 0 & 0 \\ 1 & 0 & 4 & 0 \end{vmatrix} = -48 < 0$$

and $$\Delta_3 = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 4 & 0 \\ 1 & 0 & 4 \end{vmatrix} = -8 < 0$$

1 0 0 4

Since Δ_3 and Δ_4 are both negative, thus $x_0 = (6, 2, 3)$ provides a solution to the above non-linear function with minimum $Z = 102$.

4. Conclusion

The goal of any production system is to make money, maximize profits and minimize losses. However, constraints in the form of bottles or hindrances stand as impediments to reaching the desired goal. Therefore, identifying constraints and eliminating them is the main idea behind TOC. As we have seen, TOC actually focuses on continuous system improvement by dealing with constraints as a thinking process and logical reasoning, while adopting continuous improvement strategies to curtail constraints so as to constantly increase the performance efficiency of a system. The theory can be implemented in almost every sector and almost every size of the company. In this paper, we have examined how to model production system in order to eliminate constraints, minimize risks, costs, losses etc resulting from constraints and maximizing profits by formulating the problem into either linear or non-linear constraint models.

In the formulation of the linear programming models as shown in figures 1 and 2, managers or decision makers now know exactly what to do or the right decisions to take to eliminate constraints or to reduce its effect to the barest minimum when faced with similar cases. While formulating non –linear programming models, one must be well abreast with the knowledge of differential equations and determinants to appreciate and handle appropriately each and every type of the scenario that may come along.

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