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Research Article

# Hierarchical Group Compromise Ranking Methodology Based on Euclidean–Hausdorff Distance Measure under Uncertainty: an Application to Facility Location Selection Problem

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**Abstract**

Proposing a hierarchical group compromise method can be regarded as a one of major multi-attributes decision-making tool that can be introduced to rank the possible alternatives among conflict criteria. Decision makers' (DMs') judgments are considered as imprecise or fuzzy in complex and hesitant situations. In the group decision making, an aggregation of DMs' judgments and fuzzy group compromise ranking is more capable and powerful than the classical compromise ranking. This research extends a new hierarchical group compromise ranking methodology under a hesitant fuzzy (HF)environment to handle uncertainty, in which for the margin of error, the DMs could assign the opinions in several membership degrees for an element. The hesitant fuzzy set (HFS)is taken into account for the process of the proposed hierarchical group compromise ranking methodology, namely HFHG-CR, and for avoiding the data loss, the DMs' opinions with risk preferences are considered for each step separately. Also, the Euclidean–Hausdorff distance measure is utilized in a new proposed index for calculating the average group score, worst group score and compromise measure regarding each DM. A new ranking index is presented for final compromise solution for the evaluation. Proposed HFHG-CR methodology is applied to a practical example for a facility location selection problem, i.e. cross-dock location problem, to show the validation and application.

***Keywords*:** Compromise ranking; Group decision-making; Last aggregation; Euclidean–Hausdorff distance measure; Hesitant fuzzy sets; Facility location selection problem.

## 1. Introduction

Decision-making problem is a very significant problem that could obtain the best alternatives among selected potential alternatives. In real-world applications, it is not possible to regard all aspects of a problem by single decision maker (DM) (Gitinavard et al., 2017a,b; Xu, 2000). For these reasons, some DMs should be considered in different fields (Hashemi et al., 2013; Mousavi et al., 2016,2019). Regarding this issue, the multi-criteria group decision-making (MCGDM) problem can be established. Many researchers studied on solving the decision problems by considering MCGDM situations(Hashemi et al., 2014; Mojtahedi et al., 2010; Mousavi et al., 2014, 2015; Tavakkoli-Moghaddam et al., 2011; Yu and Lai, 2011; ; Vahdani et al., 2014a,b;Yue, 2012; Mohagheghi et al., 2017).

When the complexity of real-life decisions is increased, the information can be incomplete, and the DMs might assign their judgments by imprecise or fuzzy information rather than precise (e.g., Foroozesh et al., 2017a,b; Vahdani et al., 2017; Ghaderi et al., 2017; Dorfeshan et al., 2018). For these reasons,some studies focused on MCGDM methods under fuzzy preference relations (Chiclana et al., 2013; Meng and Pei, 2013; Yue, 2011, 2014).Chen and Niou(2011)via fuzzy preference relation presented an approach under (MCGDM) problems. Viedma et al. (2002)by different preference structure for multi-person decision making presented a consensus model. Kacprzyk et al. (1992)designed a method for group decision making (GDM) under fuzzy majority and preferences. Mata et al. (2009)regarded an adoptive consensus support model for the GDM under multi-granular fuzzy linguistic variables. Xu(2008)introduced a GDM method via multiple types of linguistic terms relations.

In hesitant situations, DMs have expressed their opinions in some values under a set. An appropriate solution is the hesitant fuzzy set (HFS) introduced by Torra and Narukawa(2009) and Torra(2010). In recent years, the HFS theory widely used in decision-making problems and received more attention (Chen et al., 2013; Rodríguez et al., 2013; Yu et al., 2012; Zhang et al., 2014; Tavakkoli-Moghaddam et al.,2015).In addition, by widely utilizing HFSs concepts, this theory is developed and some operators, such as basic operators, distance measure operators and aggregation operators, are introduced. Xia et al.(2013) proposed some aggregation operators for hesitant information, and also they discussed about relations between proposed aggregation operators. Some other studies, which are focused on distance measure, similarity measure, and aggregation operators, are

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mentioned as follows (Gitinavard et al., 2016a,b; Wei, 2012; Xia et al., 2013;Xu and Xia, 2011). Farhadinia (2014) developed the HFS to a higher order type and extended distance measure and similarity measure for them under some assumptions.

As mentioned above, the HFS could be a useful concept for expressing uncertain information. Zhang and Wei (2013)developed VIKOR (from Serbian, Vise Kriterijum ska Optimizacija I Kompromisno Resenje) and technique for order preference by similarity to ideal solution (TOPSIS) method under HF environment. Liao and Xu(2013b)regarded a new HF VIKOR method Xu and Zhang (2013)provided maximizing deviation and TOPSIS method by considering the criteria' weights as incomplete.Yu et al. (2016)extended a decision-making method based on induced HF Hamacher ordered weighted geometric to solve evaluation problem in closed-loop logistics systems. Wibowo and Grandhi (2016) developed a GDM method via preference index and HF information for selecting the best high-technology project.

The survey of the literature shows that considering some appropriate characteristics as hierarchical structure, last aggregation approach, risk' preferences of each expert and determining experts' weights are not considered simultaneously to enhance the developed approaches in field of decision-making methodologies-based hesitant fuzzy. In fact, the hierarchical structure in defining the criteria could lead to a precise solution by evaluating all aspects of the GDM problem. Moreover, collecting experts' judgments in last step of the procedure, called last aggregation approach, could avoid the data loss. In addition, determining and considering the weight of each expert in proposed HF hierarchical group compromise ranking method, namely HFHG-CR, ensure that the obtained results are reliable by decreasing the judgments' errors. Hence, this paper proposes a new hierarchical group compromise ranking methodology based on last aggregation approach and HF information to solve a facility location selection problem, i.e. cross-dock location problem. In summary, main contributions of this paper are mentioned as follows: (1) Considering the DMs' opinions with risk preferences as the HFSs in the process of classical compromise ranking method and developing the hierarchical group compromise ranking method under HFSs; (2) Proposing a new index for computing the average group score and worst group score by HF Euclidean–Hausdorff distance measure; (3) Proposing a new ranking index for calculating the compromise measure by regarding the DMs' judgments; (4) Aggregating the DMs' opinions for the prevention of the data loss at the end of the proposed methodology to obtain final compromise measure; and (5) Introducing a new CR index without aggregation operator.

The rest of paper is organized as follows; the preliminary is defined in section 2. Proposed HFHG-CR method under HF-situations is presented in section 3. In section 4, an adopted practical example is provided. In section 5, by some remarkable conclusions our paper ends.

## 2. Preliminary

**Definition 1**(Torra and Narukawa, 2009). Let $X$ be a discourse universe, then $E$ a HFS on $X$ is described by function $h_E(\mathrm{x})$ that is applied to $X$ returns to subset of [0, 1].

$$E = \{< x, h_E(\mathrm{x}) > | \, \mathrm{x} \in \mathrm{X}\} \tag{1}$$

Where $h_E(\mathrm{x})$ is describing as set of some membership degrees for an element in subset of [0,1].

**Definition 2**(Atanassov, 1989, 2000). Let reference set be $X$, $E$ on $X$ that is intuitionistic fuzzy set (IFS) demonstrated as $E = \langle x_i, \mu_E(x_i), \nu_E(x_i) \rangle$ for $x_i \in X$ .Regarding this concept, the membership degree has been indicated by $\mu_E(\mathrm{x}_i)$ and the non-membership degree has been indicated by $\nu_E(\mathrm{x}_i)$ . Also, the following constraint should be satisfied; $0 \le \mu_E(x_i) + \nu_E(x_i) \le 1 \text{ for } x_i \in X$

**Definition 3**(Xia and Xu, 2011).By considering the above-mentioned definitions (i.e., definitions 1 and 2),and by considering the correlation between the IFV and HFE, some basic operations are defined as follows:

$$h_1 \oplus h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1 + \gamma_2 - \gamma_1 . \gamma_2\} \tag{2}$$

$$h_1 \otimes h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1 . \gamma_2\} \tag{3}$$

$$h^\lambda = \cup_{\gamma \in h} \{\gamma^\lambda\} \tag{4}$$

$$\lambda h = \cup_{\gamma \in h} \{1 - (1-\gamma)^\lambda\} \tag{5}$$

**Definition 4**(Liao and Xu, 2013a).Regarding a correlation between IFV and HFS and respecting to the subtraction and division operators of IFSs, the subtraction and division relations of HFS are defined as follows:

$$h_1 - h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \begin{Bmatrix} \dfrac{\gamma_1 - \gamma_2}{1-\gamma_2} & if\ \gamma_1 \ge \gamma_2 \ and\ \gamma_2 \ne 1; \\ 0 & otherwise \end{Bmatrix} \tag{6}$$

$$\frac{h_1}{h_2} = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \begin{Bmatrix} \dfrac{\gamma_1}{\gamma_2} & if\ \gamma_1 \le \gamma_2 \ and\ \gamma_2 \ne 0; \\ 1 & otherwise \end{Bmatrix} \tag{7}$$

**Definition 5**(Xu and Xia, 2011). Hamming distance represented by Eq. (8), the Euclidean distance measure indicated by Eq. (9), Hamming–Hausdorff and Euclidean–Hausdorff distance measure are shown by Eqs. (10)-(11), respectively. Let $h_M$ and $h_N$ two HFEs, then above-mentioned distance measures are as follows:

$$d_{hh}\left(h_M,h_N\right)=\frac{1}{l_{x_i}}\sum_{\lambda=1}^{l_{x_i}}\left|h_M^{\sigma(\lambda)}\left(x_i\right)-h_N^{\sigma(\lambda)}\left(x_i\right)\right| \tag{8}$$

$$d_{he}(h_M,h_N)=\sqrt{\frac{1}{l_{x_i}}\sum_{\lambda=1}^{l_{x_i}}\left|h_M^{\sigma(\lambda)}(x_i)-h_N^{\sigma(\lambda)}(x_i)\right|^2} \tag{9}$$

$$d_{hhh}\left(h_M,h_N\right)=\max_{\lambda}\left|h_M^{\sigma(\lambda)}\left(x_i\right)-h_N^{\sigma(\lambda)}\left(x_i\right)\right| \tag{10}$$

$$d_{heh}(h_M,h_N)=\sqrt{\max_{\lambda}\left|h_M^{\sigma(\lambda)}(x_i)-h_N^{\sigma(\lambda)}(x_i)\right|^2} \tag{11}$$

The $\lambda th$ largest value in $h_M$ and $h_N$ are denoted as $h_M^{\sigma(\lambda)}$ and $h_N^{\sigma(\lambda)}$.

**Definition 6** (Xia and Xu, 2011). Some aggregation operators are described for the HFSs. The hesitant fuzzy weighted geometric (HFWG) and the hesitant fuzzy weighted averaging (HFWA) operator are indicated by Eqs. (12)-(13), respectively. Let $h_j\left(j=1,2,...,n\right)$ be some of the HFEs, then:

$$HFWA\left(h_1,h_2,...,h_n\right)=\overset{n}{\underset{j=1}{\oplus}}\left(w_jh_j\right)=\cup_{\gamma_1\in h_1,\gamma_2\in h_2,...,\gamma_n\in h_n}\left\{1-\prod_{j=1}^{n}\left(1-\gamma_j\right)^{w_j}\right\} \tag{12}$$

$$HFWG\left(h_1,h_2,...,h_n\right)=\overset{n}{\underset{j=1}{\otimes}}\left(h_j\right)^{w_j}=\cup_{\gamma_1\in h_1,\gamma_2\in h_2,...,\gamma_n\in h_n}\left\{\prod_{j=1}^{n}\left(\gamma_j\right)^{w_j}\right\} \tag{13}$$

where $w=\left(w_1,w_2,...,w_n\right)^T$ the weight vector of $h_j\left(j=1,2,...,n\right)$.

### 3. Introduced HF-hierarchical Group Compromise Ranking Methodology

Let $A=\left\{A_1,A_2,...,A_m\right\}$ be a set of alternatives, and $C=\left\{C_1,C_2,...,C_{n'}\right\}$ be a set of criteria that are in the first level; $SC=\left\{SC_1,SC_2,...,SC_{n''}\right\}$ is a set of sub-criteria in the second level and $MC=\left\{MC_1,\mathrm{MC}_2,...,MC_{n'''}\right\}$ is a set of main criteria in the third level. Properties of each available alternative respecting to each main criteria are indicated by $A_i^{MCk}$, where index *MC* and *k* represented the main criteria level and the number of the DMs, respectively; also, the results provided by HFSs are as follows:

$$A_i^{MCk}=\left\{\left(\mu_{i1}^{MC1},\mu_{i1}^{MC2},...,\mu_{i1}^{MCK}\right),\left(\mu_{i2}^{MC1},\mu_{i2}^{MC2},...,\mu_{i2}^{MCK}\right),...,\left(\mu_{in''}^{MC1},\mu_{in''}^{MC2},...,\mu_{in''}^{MCK}\right)\right\}\quad\forall i \tag{14}$$

After expressing the steps of the proposed methodology, the structure of proposed HFHG-C Runder HF environment is depicted in Figure 1, and the steps are provided as below:

**Step 1.** Determine the weight of each DM

$$\lambda_k=\frac{\sum_{i}^{m}\sum_{j}^{n}\mu_{ij}^k}{\sum_{k}^{K}\sum_{i}^{m}\sum_{j}^{n}\mu_{ij}^k} \tag{15}$$

$$\sum_{k=1}^{K}\lambda_k=1 \tag{16}$$

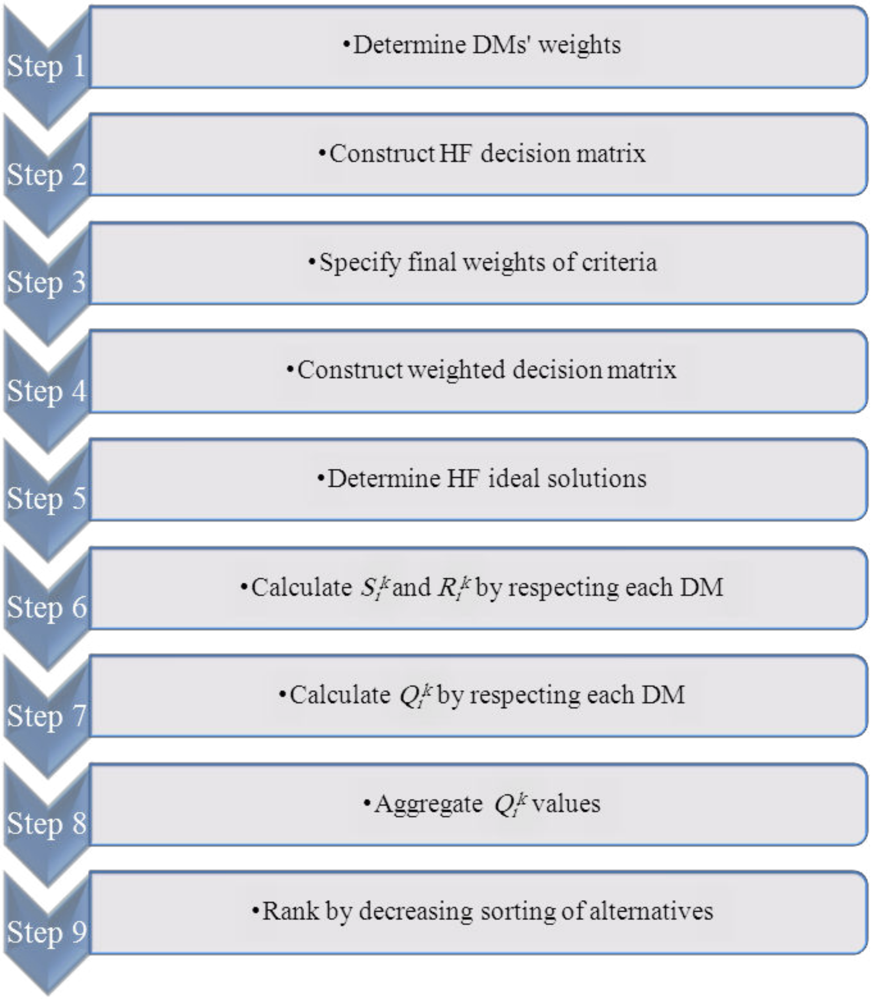


Fig. 1. Procedure of proposed HFHG-CR methodology

**Step 2.**ConstructHF decision matrix by DMs' judgments.

$$R_k = \begin{bmatrix} \mu_{A_1}^k(x_1) & \mu_{A_1}^k(x_2) & \cdots & \mu_{A_1}^k(x_n) \\ \mu_{A_2}^k(x_1) & \mu_{A_2}^k(x_2) & \cdots & \mu_{A_2}^k(x_n) \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{A_m}^k(x_1) & \mu_{A_m}^k(x_2) & \cdots & \mu_{A_m}^k(x_n) \end{bmatrix} \qquad \forall k \tag{17}$$

**Step 3.**Specify the final weight of each criterion by respecting each level.

$$w_j^{lk} = \varpi_j^{(l-1)k} \varpi_j^{lk} \qquad \forall l, k, j \tag{18}$$

**Step 4.**Construct weighted hesitant fuzzy decision matrix.

$$R_k^F = \begin{bmatrix} w_1^{Fk}\mu_{A_1}^k(x_1) & w_2^{Fk}\mu_{A_1}^k(x_2) & \cdots & w_n^{Fk}\mu_{A_1}^k(x_n) \\ w_1^{Fk}\mu_{A_2}^k(x_1) & w_2^{Fk}\mu_{A_2}^k(x_2) & \cdots & w_n^{Fk}\mu_{A_2}^k(x_n) \\ \vdots & \vdots & \ddots & \vdots \\ w_1^{Fk}\mu_{A_m}^k(x_1) & w_2^{Fk}\mu_{A_m}^k(x_2) & \cdots & w_n^{Fk}\mu_{A_m}^k(x_n) \end{bmatrix} \qquad \forall k \tag{19}$$

**Step 5.**Estimate hesitant fuzzy ideal solutions $\left(r_j^*\right)$ for all main criteria. Also, consider $J_1$ and $J_2$ as benefit criterion and cost criterion, respectively. Then, $r_j^{*k}$ is achieved:

$$r_j^{*k} = \left(\left(\max_i \mu_{R_k^F}\left(x_j\right) \mid j \in J_1\right), \left(\min_i \mu_{R_k^F}\left(x_j\right) \mid j \in J_2\right)\right) \quad (20)$$

**Step 6.**Compute a hesitant fuzzy average group score value $S_i^k$ and hesitant fuzzy worst group score value $R_i^k$ for each alternative $A_i$.

$$S_i^k = \sum_{j=1}^{n} w_j^{Fk} d\left(R_k^F, r_j^{*k}\right), \ \forall i$$

$$S_i^k = \sum_{j=1}^{n} w_j^{Fk} \sqrt{\max_\lambda \left|R_k^{F\sigma(\lambda)}(x_i) - r_j^{*k\sigma(\lambda)}(x_i)\right|^2}, \ \forall i \quad (21)$$

$$R_i^k = \max_j \left(w_j^{Fk} d\left(R_k^F, r_j^{*k}\right)\right) \quad \forall i$$

$$R_i^k = \max_j \left( w_j^{Fk} \sqrt{\max_\lambda \left|R_k^{F\sigma(\lambda)}(x_i) - r_j^{*k\sigma(\lambda)}(x_i)\right|^2} \right) \quad \forall i \quad (22)$$

where $w_j$ is final weight of main criteria *j* that are assigned by DM *k*.

**Step 7.**Compute the index $Q_i^k$ as follows:

$$Q_i^k = \nu d\left(S_i^k, \max\left(S_i^k\right)\right) + (1-\nu) d\left(R_i^k, \max\left(R_i^k\right)\right) \quad (23)$$

$$\nu = \frac{d\left(S_i^k, \max\left(S_i^k\right)\right)}{d\left(S_i^k, \max\left(S_i^k\right)\right) + d\left(R_i^k, \max\left(R_i^k\right)\right)} \quad (24)$$

$$Q_i^k = \frac{d\left(S_i^k, \max\left(S_i^k\right)\right)^2 + d\left(R_i^k, \max\left(R_i^k\right)\right)^2}{d\left(S_i^k, \max\left(S_i^k\right)\right) + d\left(R_i^k, \max\left(R_i^k\right)\right)} \quad (25)$$

$$Q_i^k = \frac{\max_\alpha \left|S_i^{k\sigma(\alpha)}(x_i) - \left(\max\left\{S_i^{k\sigma(\alpha)}(x_i)\right\}\right)\right|^2 + \max_\alpha \left|R_i^{k\sigma(\alpha)}(x_i) - \left(\max\left\{R_i^{k\sigma(\alpha)}(x_i)\right\}\right)\right|^2}{\sqrt{\max_\alpha \left|S_i^{k\sigma(\alpha)}(x_i) - \left(\max\left\{S_i^{k\sigma(\alpha)}(x_i)\right\}\right)\right|^2} + \sqrt{\max_\alpha \left|R_i^{k\sigma(\alpha)}(x_i) - \left(\max\left\{R_i^{k\sigma(\alpha)}(x_i)\right\}\right)\right|^2}} \forall i, k \quad (26)$$

**Step 8.** Aggregate the $Q_i^k$ value and estimate the final $Q_i$ value.

$$Q_i = HFWG\left(\mathrm{Q}_i^1, \mathrm{Q}_i^2, ..., \mathrm{Q}_i^k\right) = \bigotimes_{k=1}^{K}\left(\mathrm{Q}_i^k\right)^{\lambda_k} = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2, ..., \gamma_n \in h_n} \left\{\prod_{k=1}^{K}\left(\mathrm{Q}_i^k\right)^{\lambda_k}\right\} \ \forall i \quad (27)$$

$$Q_i = \min_k \left\{Q_i^k\right\} \ \forall i \quad (28)$$

**Step 9.**Rankoptions by decreasing sorting of $Q_i$ value.

### 4. Practical Example for Facility Location Selection problem

A practical example, that is adopted from Mousavi and Vahdani (2016),is presented for the selection problem of facility location selection problem, i.e., cross-docking location selection problem. In Figure 2, the hierarchical of the practical example is depicted. The attribute of application example is expressed as follows: Costs ($C_1$),markets ($C_2$), governments influence ($C_3$), infrastructure ($C_4$), and labor resource ($C_5$).

In our decision problem, the DMs' risk preferences are considered in three levels. The risk preferences are defined pessimist, moderate, and optimist. In this practical example, $DM_1$ is pessimist, $DM_2$ is moderate and $DM_3$ is optimist. Hesitant linguistic terms for estimating the weight of selected criteria and for rating possible alternatives are defined by the DMs in Tables 1 and 2.

As presented in Tables 3 and 5, the opinions of three DMs are linguistic terms. Also, these tables are converted to the hesitant fuzzy values that are given in Tables 4 and 6.

Table 1
Hesitant linguistic variables for rating the importance of criteria and DMs.

| | | DMs' risk preferences | | |
|---|---|---|---|---|
| Hesitant linguistic variables | Hesitant interval-valued fuzzy sets | Pessimist | Moderate | Optimist |
| Very important (VI) | [0.90, 0.90] | 0.90 | 0.90 | 0.90 |
| Important (I) | [0.75, 0.80] | 0.75 | 0.775 | 0.80 |
| Medium (M) | [0.50, 0.55] | 0.50 | 0.525 | 0.55 |
| Unimportant (UI) | [0.35, 0.40] | 0.35 | 0.375 | 0.40 |
| Very unimportant (VUI) | [0.10, 0.10] | 0.10 | 0.10 | 0.10 |

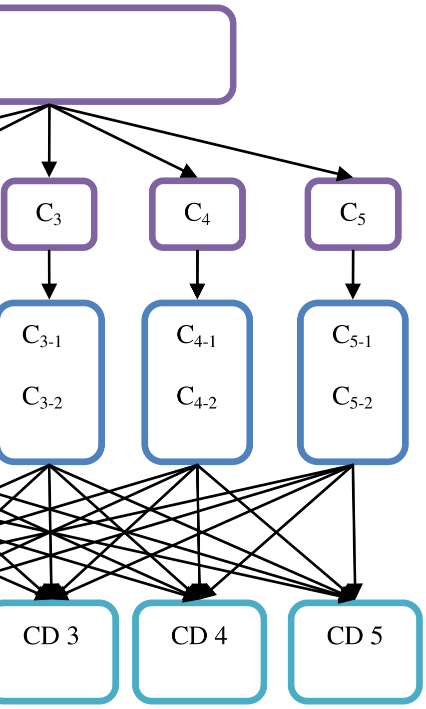


Fig. 2. Hierarchy structure of cross-dock location problem

Table 2
Hesitant linguistic variables for rating possible alternatives.

| | | DMs' risk preferences | | |
|---|---|---|---|---|
| Hesitant linguistic variables | Hesitant interval-valued fuzzy sets | Pessimist | Moderate | Optimist |
| Extremely good (EG)/extremely high (EH) | [1.00, 1.00] | 1 | 1 | 1 |
| Very very good (VVG)/very very high (VVH) | [0.90, 0.90] | 0.90 | 0.90 | 0.90 |
| Very good (VG)/very high (VH) | [0.80, 0.90] | 0.80 | 0.85 | 0.90 |
| Good (G)/high (H) | [0.70, 0.80] | 0.70 | 0.75 | 0.80 |
| Medium good (MG)/medium high (MH) | [0.60, 0.70] | 0.60 | 0.65 | 0.70 |
| Fair (F)/medium (M) | [0.50, 0.60] | 0.50 | 0.55 | 0.60 |
| Medium bad (MB)/medium low (ML) | [0.40, 0.50] | 0.40 | 0.45 | 0.50 |
| Bad (B)/low (L) | [0.25, 0.40] | 0.25 | 0.325 | 0.40 |
| Very bad (VB)/very low (VL) | [0.10, 0.25] | 0.10 | 0.175 | 0.25 |
| Very very bad (VVB)/very very low (VVL) | [0.10, 0.10] | 0.10 | 0.10 | 0.10 |

Table 3
Performance ratings of some alternatives in linguistic variables.

| Criteria | Alternatives | Decision makers | | |
|---|---|---|---|---|
| | | $DM_1$ | $DM_2$ | $DM_3$ |
| C1-1 | $A_1$ | VVB | VVB | VVB |
| | $A_2$ | VVB | VVB | VB |
| | $A_3$ | VB | VB | VB |
| | $A_4$ | B | VB | VB |
| | $A_5$ | VB | VB | VB |
| … | | | | |
| C5-2 | $A_1$ | MG | MG | G |
| | $A_2$ | VG | VG | EG |
| | $A_3$ | G | VG | VG |
| | $A_4$ | MG | F | G |
| | $A_5$ | G | F | F |

Table 4
Performance ratings of the alternatives in hesitant fuzzy values.

| Criteria | Alternatives | Decision makers | | |
|---|---|---|---|---|
| | | $DM_1$ | $DM_2$ | $DM_3$ |
| C1-1 | $A_1$ | 0.10 | 0.10 | 0.10 |
| | $A_2$ | 0.10 | 0.10 | 0.25 |
| | $A_3$ | 0.10 | 0.175 | 0.25 |
| | $A_4$ | 0.25 | 0.175 | 0.25 |
| | $A_5$ | 0.10 | 0.175 | 0.25 |
| … | | | | |
| C5-2 | $A_1$ | 0.60 | 0.65 | 0.80 |
| | $A_2$ | 0.80 | 0.85 | 1 |
| | $A_3$ | 0.70 | 0.85 | 0.90 |
| | $A_4$ | 0.60 | 0.55 | 0.80 |
| | $A_5$ | 0.70 | 0.55 | 0.60 |

Table 5
Weights of the main criteria and sub-criteria by linguistic variables.

| DMs / Sub-criteria | $DM_1$ | $DM_2$ | $DM_3$ |
|---|---|---|---|
| C1 | UI | UI | VUI |
| C2 | UI | M | VI |
| … | | | |
| C5-1 | VUI | UI | UI |
| C5-2 | VI | I | VI |

Table 6
Weights of the main criteria and sub-criteria by hesitant fuzzy values.

| DMs / criteria | $DM_1$ | $DM_2$ | $DM_3$ |
|---|---|---|---|
| C1 | 0.35 | 0.375 | 0.10 |
| C2 | 0.35 | 0.525 | 0.90 |
| … | | | |
| C5-1 | 0.10 | 0.375 | 0.40 |
| C5-2 | 0.90 | 0.775 | 0.90 |

By utilizing step 3, final weights of main criteria are determined and the results are shown in Table 7. In addition, the weighted decision matrix for each DM is constructed and represented in Tables 8 to 10 (Step 4). By using steps 5 to 6, $S_i$ and $R_i$ are computed for each possible alternative respecting to each DM. The results are reported in Table 11.

Table 7
Final weights of the main criteria by hesitant fuzzy values.

| DMs / Criteria | $DM_1$ | $DM_2$ | $DM_3$ |
|---|---|---|---|
| C1-1 | 0.1225 | 0.140625 | 0.01 |
| C1-2 | 0.1225 | 0.140625 | 0.01 |
| … | | | |
| C5-1 | 0.035 | 0.140625 | 0.22 |
| C5-2 | 0.315 | 0.290625 | 0.495 |

Table 8
Weighted decision matrix for the first DM.

| Alternatives / Criteria | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $r_J^*$ |
|---|---|---|---|---|---|---|
| C1-1 | 0.01225 | 0.012250 | 0.01225 | 0.030625 | 0.01225 | 0.01225 |
| C1-2 | 0.01225 | 0.030625 | 0.01225 | 0.030625 | 0.01225 | 0.01225 |
| … | | | | | | |
| C5-1 | 0.01400 | 0.00350 | 0.00350 | 0.00350 | 0.00875 | 0.01400 |
| C5-2 | 0.18900 | 0.25200 | 0.22050 | 0.18900 | 0.22050 | 0.25200 |

Table 9
Weighted decision matrix for the second DM.

| Alternatives / Criteria | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $r_J^*$ |
|---|---|---|---|---|---|---|
| C1-1 | 0.0140625 | 0.0140625 | 0.0246094 | 0.0246094 | 0.0246094 | 0.0140625 |
| C1-2 | 0.0140625 | 0.0246094 | 0.0140625 | 0.0457031 | 0.0246094 | 0.0140625 |
| … | | | | | | |
| C5-1 | 0.0457031 | 0.0140625 | 0.0246094 | 0.0246094 | 0.0246094 | 0.0457031 |
| C5-2 | 0.1889063 | 0.2470313 | 0.2470313 | 0.1598438 | 0.1598438 | 0.2470313 |

Table 10
Weighted decision matrix for the third DM.

| Alternatives / Criteria | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $r_J^*$ |
|---|---|---|---|---|---|---|
| C1-1 | 0.0010 | 0.0025 | 0.0025 | 0.0025 | 0.0025 | 0.001 |
| C1-2 | 0.0025 | 0.0010 | 0.0025 | 0.0025 | 0.0010 | 0.001 |
| … | | | | | | |
| C5-1 | 0.0550 | 0.0220 | 0.0550 | 0.1100 | 0.0550 | 0.110 |
| C5-2 | 0.3960 | 0.4950 | 0.4455 | 0.3960 | 0.2970 | 0.495 |

Table 11
The $S_i$ and $R_i$ values based on three DMs' opinions.

| DMs / $S_i$ and $R_i$ | DM1 | DM2 | DM3 |
|---|---|---|---|
| $S_1$ | 0.4180069 | 0.4208724 | 0.5199300 |
| $S_2$ | 0.2436403 | 0.3274648 | 0.4829088 |
| $S_3$ | 0.4183744 | 0.2297836 | 0.5051163 |
| $S_4$ | 0.1681078 | 0.2237408 | 0.6417525 |
| $S_5$ | 0.4693763 | 0.3457895 | 0.6971725 |
| $R_1$ | 0.1312200 | 0.1312200 | 0.1555200 |
| $R_2$ | 0.1312200 | 0.1443002 | 0.2624400 |
| $R_3$ | 0.1968300 | 0.0721501 | 0.1968300 |
| $R_4$ | 0.0656100 | 0.1312200 | 0.2073600 |
| $R_5$ | 0.1968300 | 0.1312200 | 0.1555200 |
| Max$\{S_i\}$ | 0.4693763 | 0.4208724 | 0.6971725 |
| Max$\{R_i\}$ | 0.1968300 | 0.1443002 | 0.2624400 |

The $Q_i^k$ values are computed by using step 7, and by utilizing step 8 the alternatives could be ranked by considering the $\min_k \{Q_i^k\} \forall i$ or the alternatives ranked by decreasing sorting the aggregated of $Q_i^k \ \forall i$. For calculating the aggregation of $Q_i^k \ \forall i$, the DMs' weights should be considered. The results of above-mentioned are presented in Tables 12 to 14.

As indicated in Table 14, the best and the worst candidates based on the proposed HFHG-CR methodology is selected as the fourth and fifth cross-docking location candidates, respectively. Moreover, the obtained ranking results from the proposed approach (i.e., $A_4 > A_1 > A_3 > A_2 > A_5$) is compared with the obtained ranking results from Mousavi and Vahdani (2016)' approach (i.e., $A_4 > A_1 > A_2 > A_3 > A_5$) to validate the proposed HFHG-CR methodology. The comparative analysis shows that the outcomes could be similar regarding both fuzzy decision approaches. Minor variations in the outcomes could be from the consideration of last aggregation approach for preventing data loss and also the structure of each decision approach regarding the uncertainty modeling and the logic of each decision methodology.

Table 12
$Q_i^k$ values provided by each DM and ranked by decreasing sort.

| DMs / $Q_i^k$ | DM$_1$ | DM$_2$ | DM$_3$ | $\min_k \{Q_i^k\} \forall i$ |
|---|---|---|---|---|
| $Q_1^k$ | 0.059356500 | 0.013080156 | 0.150782704 | 0.01308015600 |
| $Q_2^k$ | 0.189676200 | 0.093407598 | 0.214263750 | 0.09340759800 |
| $Q_3^k$ | 0.051001900 | 0.158489408 | 0.159859024 | 0.05100187500 |
| $Q_4^k$ | 0.249674600 | 0.185679295 | 0.055250523 | 0.05525052300 |
| $Q_5^k$ | 0.00000000001 | 0.065884010 | 0.1069200 | 0.00000000001 |

Table 13
Relative importance of each DM.

| DMs / DMs' weights | DM$_1$ | DM$_2$ | DM$_3$ |
|---|---|---|---|
| Weight of each decision maker | 0.2843517 | 0.3371088 | 0.3785395 |

Table 14
Aggregated $Q_i$ values and comparative results

| DMs / $Q_i$ | Final $Q_i$ | Ranked by the proposed HFHG-CR methodology | Ranked by Mousavi and Vahdani (2016)based on fuzzy COPRAS method |
|---|---|---|---|
| $Q_1$ | 0.0507342 | 2 | 2 |
| $Q_2$ | 0.1564394 | 4 | 3 |
| $Q_3$ | 0.1151850 | 3 | 4 |
| $Q_4$ | 0.1276617 | 1 | 1 |
| $Q_5$ | 0.0001278 | 5 | 5 |

Although we have used the proposed HFHG-CR methodology to the facility location selection problem, i.e., cross-docking location selection problem, it can be employed for evaluating and making a best decision in other logistics fields, such as warehousing location selection, distribution center selection and plant location selection problems to handle uncertainty.

## 5. Concluding Remarks and Future Suggestions

The HFSis a powerful and effective tool in expressing uncertain assessment information. In this respect, we have investigated the developments of the canonical compromise ranking methodology under hesitant fuzzy situations in hierarchical form, namely HFHG-CR. By considering HFE uclidean–Hausdorff distance measure, we have developed several new indexes for computing the average group score, worst group score, and the compromise measure. Then, the procedure of proposed HFHG-CR methodology has been expressed in detail as depicted in Figure 1. All decision makers (DMs)' opinions have been considered in each step of the proposed methodology. We have aggregated the DMs' judgments in final step for the prevention of the data loss. Furthermore, as demonstrated in Tables 1 and 2, the risk preferences of the DMs' opinions for evaluating the criteria' weights and candidates were defined in three categories, including pessimist, moderate, and optimist, to decrease the judgments' errors. The weight of each DM has been computed based on a hesitant fuzzy index to determine the expertise of each DM. In addition, as depicted in Figure 2, the proposed HFHG-CR methodology has been provided based on hierarchical structure to evaluate more aspects of cross-docking location selection problem. Finally, to illustrate the procedure of the proposed HFHG-CR and to show its application and validation, an application example to facility location selection problem has been given in the logistics management for the selection problem of cross-docking location. As indicated in Table 14, the results have indicated that the fourth and fifth candidates have been selected for locating the cross-docking centers as the best and worst alternatives, respectively. Also, the obtained ranking results have been compared with a recent study from the literature to confirm the results from the proposed HFHG-CR methodology. Although the comparative analysis has demonstrated that the obtained ranking results from both decision approaches are similar in somewhat and they have selected the same candidates for best and worst alternatives, the small variations in the ranking results could be from the last aggregation approach, structure of the proposed methods, and uncertainty modeling. The results and compromise analysis have indicated that the proposed HFHG-CR methodology is powerful in solving the complex hierarchical decision-making problems regarding hesitant fuzzy information. In future studies, the methodology can be developed by utilizing an optimization model via maximizing deviation method for criteria' weights. Moreover, developing a new procedure to determine the criteria' weights regarding hierarchical structure can enhance the proposed HFHG-CR. It is appreciated to note that preparing an evaluation approach based on expert system can facilitate an assessment of cross-docking location candidates with hierarchical criteria.

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