A New Mathematical Model for Closed-loop Supply Chains Considering Product Pricing, Fleet of Hetero Geneous Vehicles, and Inventory Costs

Isa Nakhai kamal Abadi [a*], Mohammad Mohammadnejad[b], Ramin Sadeghian[c], Fardin Ahmadizar[d]

[a] Professor, Department of Industrial Engineering, University of kurdistan, Iran
[b] Ph.D Candidate, Department of Industrial Engineering, Payame Noor University, Tehran ,Iran.
[c] Assistant Professor, Department of Industrial Engineering, Payame Noor University, Tehran, Iran
[d] Associate Professor, Department of Industrial Engineering, University of kurdistan, Iran

Received 16 September 2015; Revised 02 July 2016; Accepted 09 July 2016

Abstract

Mathematical models are used in many areas of supply chain management. Here, we present a mixed-integer non-linear programming (MINLP) model to solve a multi-period, closed-loop supply chain (CLSC) with two echelons of producers and customers. To satisfy the customers' demands, the manufacturer must produce the product,; so, they have to order materials at the beginning of each period for the current or later periods. A fleet of heterogeneous vehicles is routed to deliver the products from the producers to the customers and to pick up defective products from the customers to transport to the collection-repair center. The objective function maximizes the profit, which is equal to total cost minus income. The income is divided into two parts (selling products and wastes), and the total cost consists of the cost of defective product, ordering, holding in producers and collection-repair center, transportation, and assigning place for the collection-repair center. Two numerical examples with their computational results are discussed, and then a solution approach is presented which is analyzed by applying the examples to show the efficiency of the proposed method. The results demonstrate that the approach performances are faster than the MINLP model with negligible gap.

Keywords: Closed-loop supply chain, Mathematical modeling, Heterogeneous vehicle routing, Inventory, Pricing.

1. Introduction

In today's world, supply chain management (SCM) has an effective role in the success of companies and the satisfaction of customers (Shukla et al., 2010; Chopra & Meindl, 2001). It further plays essential roles in societies, cultural evolutions, and medical missions. The rapid development of new technologies such as the internet and connective product marking has led to the transformation of a basic supply chain (SC) into a supply chain network (SCN). In both chains, goods are moved from supplier(s) to customer(s), and materials and information are moved by the linking companies together for servicing the end customers. In SCN, "chain" represents a sequential set of links, and "network" shows a more complex structure with cross-links and two-way exchanges between organizations (Harland et al. 2001). In both SC and SCN, the main objective is optimization of the under-study system. In SCN, optimization decisions include every single part of a supply chain like production facilities, distribution centers, suppliers and customers, and every type of flow and link between the existing nodes in the network. In recent decades, many researchers have focused on the issues of environmental protection and economic advantages of using returned goods and achieved significant successes in designing and executing reverse logistics networks and closed-loop supply chain (CLSC) networks (Meade et al., 2007 and Petek et al. 1996). In forward supply chain, production and distribution planning are performed while reverse supply chain deals with operations such as product recycling planning, separation of defective products, and repair or disposal. Many companies found that mixing the activities of reverse logistics not only reduces the environmental impact, but also leads to reduction of cost and an increase in productivity by providing new services and products out of the recycled products.

* Corresponding author Email address: I.nakhai@uok.ac.ir

2. Literature Review

Stochastic mixed integer-programming model was first presented by Listes and Dekker in a sand recycling industry (2005) with the main aim of maximizing the total profit. The model was developed for different situations regarding a number of options. Salema et al. (2007) studied a problem of capacitated CLSC network design under demand uncertainty, and formulated it as a stochastic mixed-integer optimization model. El-Sayed et al. (2010) formulated a stochastic mixed-integer programming model for forward-reverse logistics network design under demand and return uncertainty to maximize the total profit. Pishvaee and Torabi (2010) considered a CLSC network design under uncertainty and risk in such networks. They formulated the problem as a bi-objective probabilistic mixed integer-programming model. In addition, they presented an interactive fuzzy solution method for solving the proposed model in which several efficient solution methods were combined. A multi-objective fuzzy optimization model for environmental supply chain network design under inherent uncertainty of input data was provided by Pishvaee and Razmi (2012 a). The objectives minimized the multiple environmental impacts and the traditional costs. They used a life cycle assessment-based method for assessing and quantifying the environmental impact of different scenarios. Moreover, an interactive fuzzy solution approach was presented for solving the proposed model. Pishvaee et al. (2012b) considered a problem of socially responsible supply-chain network design under uncertain conditions. They formulated the problem as a bi-objective mathematical programming model, aiming to minimize the total cost and maximize the supply chain's social responsibility. For coping with uncertain parameters, the authors introduced a robust probabilistic programming approach. Pishvaee et al. (2014) considered the problem of sustainable medical supply-chain network design under epistemic uncertainty of input data. They presented a multi-objective probabilistic programming model for the proposed problem with conflicting economic, environmental, and social objectives. To solve the proposed problem, an accelerated Benders' decomposition algorithm was used with three acceleration mechanisms. The model was conducted on a medical needle and a syringe supply chain in Iran. Paucar-Caceres and Espinosa (2011) performed a review on environmental management and sustainability applying management science approaches. Seven different types of CLSC were investigated by Ostlin et al. (2008) to detect the kinds of existing relationships between remanufacturers and customers/suppliers, to figure out how to manage these relationships, and how the customer/supplier relationships can support product take-back for remanufacturing. The effects of RFID on CLSC were re-investigated by Visich et al. (2007). The aim was to implement an RFID active closed-loop system to increase value recovery. Chunga et al. (2008) presented a CLSC multi-echelon inventory system with remanufacturing capability. They formulated the proposed problem as a mathematical programming model to optimize the production and replenishment policy for maximizing the joint profit. Shia et al. (2011) studied a closed-loop system with uncertain demand and return where the demand was sensitive to the selling price, and the return was sensitive to the purchase price of used products. A mathematical programming model with the aim of maximizing the expected overall profit was presented. Using the proposed model, the return flow, production/inventory planning, and pricing were simultaneous optimized. They presented a solution approach for solving the model. A dynamic pricing problem in a closed-loop hybrid manufacturing system under a multi-period scenario was studied by Chen and Chang (2013). It was assumed that demand is price-dependent and interchangeable between the new and reproduced goods. Applying Lagrange, relaxation, and dynamic programming methods, they designed an unconstrained static model and two constrained dynamic models, in which the pricing policy was considered in a multi-period dual-channel setting. Seuring (2013) summarized the research on quantitative models for forward supply chains, and thereby contributed to the further substantiation of the field. While different kinds of models are applied, it is evident that the social side of sustainability is not taken into account. On the environmental side, life-cycle assessment-based approaches and impact criteria clearly dominate. On the modeling side, there are three dominant approaches: equilibrium models, multi-criteria decision-making, and analytic hierarchy process. To the best of our knowledge, there is the sacristy of limited empirical research in this field. Wei et al. (2013) considered a CLSC with symmetric and asymmetric information structures in which pricing and collecting decisions were made using the Game Theory. The authors considered optimal retail price, optimal wholesale price, and optimal collection rate as the decision scenarios with symmetric information. El bounjimi et al. (2014) presented a literature review for green CLSC network design in order to clarify the different concepts, the points of synergy, and the difference between the traditional supply chain and green supply chain. They further aimed to review the relevant mathematical models for green closed-loop supply-chain network design, and eventually proposed some research avenues. Rezapoura et al. (2014) presented a CLSC network design operating in a competitive environment in which demand function is price-dependent. They investigated the impacts of strategic facility location decisions of the under-study supply chain on the tactical/operational transport and inventory decisions. They formulated the proposed problem as a bi-level mathematical model. The upper level was to optimize the reverse network design, and the lower one was to present a network equilibrium model for manufacturing and distribution planning. A modified projection solution approach was presented for solving the proposed problem. Govindan, et al. (2014) gave a comprehensive review of reverse logistics and CLSC. They considered the first order conditions that the optimal retail price, the optimal wholesale price, and the optimal collection rate are satisfied as the decision scenarios with asymmetric information. The objective was to make optimal decisions about wholesale price, retail price, and collection rate for both the manufacturer and retailer under symmetric and asymmetric information conditions. They provided four different game decision scenarios to study the strategies of each company and the role of the manufacturer and retailer in these scenarios. Hong et al. (2015) studied a CLSC with joint decisions on local advertising, pricing in centralized and decentralized chains, and used-product collection. For achieving the optimal decisions, they made Stackelberg game models. He (2015) focused on acquisition pricing and remanufacturing decisions in a CLSC with a manufacturer and recycles and reliable supply channels. The author considered the proposed problem for two cases: with deterministic demand and with stochastic demand. He also examined two recycling channels for the CLSC: centralized (integrated) recycle channel and decentralized recycle channel.

3. Problem Definition and Mathematical Model

In this paper, we consider a multi-period CLSC with two echelons consisting of producers and customers. To fulfill the demands, the manufacturers produce the product by ordering the materials at the beginning of each period for one or more periods. A fleet of heterogeneous vehicles is routed in order to deliver the products from producers to customers. They also pick up defective products from the customers and move them to the collection-repair center. The objective is the maximization of the profit, which comes from subtracting the costs from the incomes. The problem is elaborated by the following assumptions and indices, parameters, variables, and the mathematical model presented in this section. Figure 1 illustrates components material flows in the closed-loop supply chain.

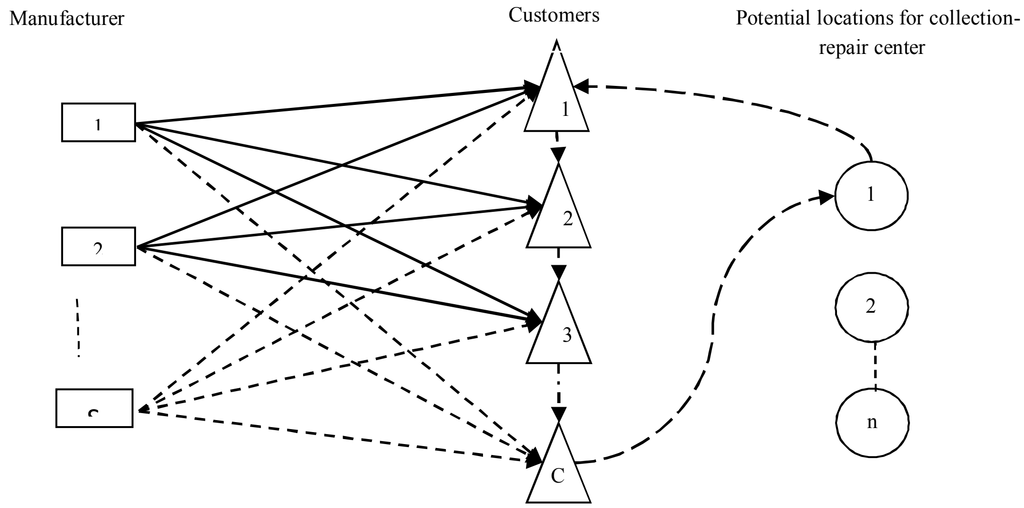

Fig. 1. Illustration of the proposed closed-loop supply chain.

Assumptions:

- The problem is planned for the horizon of several periods.
- The chain consists of two echelons of producers and customers.
- The manufacturer can order materials at the beginning of each period.
- Defective products are picked up from the customers and transported to the collection-repair center.
- The collection - repair center is placed in one of the several potential locations.
- Defective products might be repaired and return to the distribution system in the next period or sold as a waste.
- Demand of customers depends on the price of the product.
- The rate of defective products is related to the price; in other words, the more expensive the products, the less the rate of defect.
- Transportation between the manufacturer and the customer is directed, and at most, on the vehicle is assigned to each pair of them.
- A vehicle is routed from collection-repair center for the customers in order to pick up defective products and to deliver a part of their demands by the repaired products.

Indices, parameters, and decision variables are presented as follows:

Indices:

S: Set of the manufacturers

N: Set of locations containing collection-repair potential centers (Set L, *n* first nodes of N nodes) and customers (Set C, *N-n* remaining nodes)
V: Set of vehicles
T: Set of planning periods

Parameters:

n: Number of candidate potential locations for establishing the collection-repair center.

γ: Rate of defective products; it is dependent on the price.

λ : Price of products with the highest quality

φ : Rate of defective products that can be repaired.

FP_j : Fixed cost for establishing the collection-repair center in node j

FV_l : Fixed cost of vehicle l

DV_l : Unit shipping cost of vehicle l

CV_l : Capacity of vehicle l

Pt_t : Price of un-repairable defective product in period t

CR_t : Unit cost of repair in period t

α_{jt} : Mean demand, which is independent from price for customer j in period t

β_{jt} : Rate of price-dependent demand of customer j in period t

h'_{kt} : Unit storage cost in location k in period t

A_{it} : Ordering cost for manufacturer i in period t

Pb_{it} : Material cost of each unit of product for manufacturer i in period t

h_{it} : Unit storage cost in the warehouse of manufacturer i in period t

$Dist_{ij}$: Distance between nodes i and j

M : An arbitrary large number

Decision variables:

LR_t : Amount of loading of the vehicle when it leaves the collection-repair center

Ps_t : Sales price of the product in period t

RW_t: Number of products repaired in period t

W_{jt} : Auxiliary variables in order to eliminate sub-tours in node i in period t

DE_{jt} : Demand of customer j in period t

D_{jt} : Amount of the repaired products delivered to customer j in period t

P_{jt} : Amount of backward defective products from customer j in period t

LC_{jt} : Amount of vehicle loading at the time before leaving customer j in period t

I'_{kt} : Amount of warehouse inventory in location k in period t

I_{it} : Amount of warehouse inventory in manufacturer i in period t

Q_{it} : Amount of ordering of manufacturer i in period t

X_{ijlt} : Amount of the product transferred from manufacturer i to customer j by vehicle l in period t

$: \begin{cases} 1, \text{if the collection} - \text{repair center is opened in location } j \\ 0 \end{cases}$

$B_{jt} : \begin{cases} 1, \text{if costomer i recieves repaired product or has backward in period t} \\ 0 \end{cases}$

$O_{it} : \begin{cases} 1, \text{if manufacturer i orders material at period t} \\ 0 \end{cases}$

$V_{ijlt} : \begin{cases} 1, \text{if vehicle } l \text{ is used for transfering the products from manufacturer } i \text{ to customer } j \text{ in period t} \\ 0 \end{cases}$

$V'_{lt} : \begin{cases} 1, \text{if vehicle } l \text{ is used in the collection} - \text{repair center in period } t \\ 0 \end{cases}$

$Y_{jklt} : \begin{cases} 1, \text{if vehicle } l \text{ traveles from node } j \text{ to } k \text{ in period } t \\ 0 \end{cases}$

Mathematical model:

$$
\begin{aligned}
MaxZ = & \sum_{t\in T}\sum_{j\in C} DE_{jt} Ps_t - \left(\sum_{t\in T}\sum_{j\in C} P_{jt} - \sum_{t\in T} RW_t\right)(Ps_t - Pt_t) \\
& - \sum_{t\in T}\sum_{i\in S} O_{it} A_{it} - \sum_{t\in T}\sum_{i\in S} Pb_{it} Q_{it} - \sum_{t\in T}\sum_{i\in S} I_{it} h_{it} - \sum_{t\in T}\sum_{k\in L} I'_{kt} h'_{kt} \\
& - \sum_{t\in T}\sum_{l\in V}\sum_{i\in S}\sum_{j\in C} V_{ijlt} FV_l - \sum_{t\in T}\sum_{l\in V} V'_{lt} FV_l \\
& - \sum_{t\in T}\sum_{l\in V}\sum_{i\in S}\sum_{j\in C} X_{ijlt} DV_l Dist_{ij} - \sum_{t\in T}\sum_{l\in V}\sum_{j\in N}\sum_{k\in N} Y_{jklt} DV_l Dist_{jk} \\
& - \sum_{k\in L} Z_k FP_l - \sum_{t\in T}\sum_{j\in C} RW_{jt} RC_t
\end{aligned} \quad (1)
$$

Subject to:

$$DE_{jt} \geq \alpha_{jt} - \beta_{jt} Ps_t \quad j \in C,\ t \in T \quad (2)$$

$$DE_{jt} \le \alpha_{jt} - \beta_{jt} Ps_t + 1 \quad j \in C,\, t \in T \tag{3}$$

$$D_{jt} + \sum_{l \in V} \sum_{i \in S} X_{ijlt} \ge DE_{jt} \quad j \in C,\, t \in T \tag{4}$$

$$I_{it} = I_{it-1} + Q_{it} - \sum_{l \in V} \sum_{j \in C} X_{ijlt} \quad i \in S,\, t \in T \tag{5}$$

$$Q_{it} \le O_{it} M \quad i \in S,\, t \in T \tag{6}$$

$$X_{ijlt} \le V_{ijlt} CV_l \quad i \in S, j \in C,\, l \in V,\, t \in T \tag{7}$$

$$P_{jt} \ge \left(\gamma + (1 - \frac{Ps_t}{\lambda}) \right) \sum_{l \in V} \sum_{i \in S} X_{ijlt} \quad j \in C,\, t \in T \lim_{x \to \infty} \tag{8}$$

$$P_{jt} \le \left(\gamma + (1 - \frac{Ps_t}{\lambda}) \right) \sum_{l \in V} \sum_{i \in S} X_{ijlt} + 1 \quad j \in C,\, t \in T \tag{9}$$

$$P_{jt} \le B_{jt} M \quad j \in C,\, t \in T \tag{10}$$

$$D_{jt} \le B_{jt} M \quad j \in C,\, t \in T \tag{11}$$

$$\sum_{l \in V} \sum_{j \in N} Y_{jklt} = B_{kt} \quad k \in C,\, t \in T \tag{12}$$

$$\sum_{j' \in N} Y_{j'jlt} = \sum_{j'' \in N} Y_{jj''lt} \quad j \in N,\, l \in V,\, t \in T \tag{13}$$

$$\sum_{t \in T} \sum_{l \in V} \sum_{k \in C} Y_{j(k+n)lt} \le Z_j M \quad j \in L \tag{14}$$

$$\sum_{j \in N} \sum_{k \in N} Y_{jklt} \le V'_{lt} M \quad l \in V,\, t \in T \tag{15}$$

$$\sum_{j \in L} Z_j = 1 \tag{16}$$

$$w_{kt} > w_{jt} - (1 - Y_{jklt}) M \quad j \in N,\, k \in C,\, l \in V,\, t \in T \tag{17}$$

$$\sum_{l \in V} \sum_{k \in C} Y_{j(k+n)lt} \le 1 \quad j \in L,\, t \in T \tag{18}$$

$$RW_t \ge \varphi \sum_{j \in C} P_{jt} \quad t \in T \tag{19}$$

$$RW_t \le \varphi \left(\sum_{j \in C} P_{jt} \right) + 1 \quad t \in T \tag{20}$$

$$I'_{kt} = I'_{kt-1} + RW_t - \sum_{j \in C} D_{jt} \quad k \in L,\, t \in T \tag{21}$$

$$\sum_{j \in C} D_{jt} \le I'_{kt-1} + (1 - Z_k) M \quad k \in L,\, t \in T \tag{22}$$

$$LR_t = \sum_{j \in C} D_{jt} \quad t \in T \tag{23}$$

$$LC_{kt} \ge LR_t - D_{kt} + P_{kt} - (1 - Y_{j(k+n)lt}) M \quad k \in C,\, j \in L,\, t \in T \tag{24}$$

$$LC_{kt} \ge LC_{kt} - D_{kt} + P_{kt} - (1 - Y_{j(k+n)lt}) M \quad j, k \in C,\, t \in T \tag{25}$$

$$LR_t \le CV_l + (1 - V'_{lt}) M \quad l \in V,\, t \in T \tag{26}$$

$$LC_{kt} \le CV_l + (1 - V'_{lt}) M \quad k \in C,\, l \in V,\, t \in T \tag{27}$$

$$Z_j \in \{0,1\} \quad j \in L \tag{28}$$

$$Y_{jklt} \in \{0,1\} \quad j, k \in N,\, l \in V,\, t \in T \tag{29}$$

$$B_{jt} \in \{0,1\} \quad j \in C \tag{30}$$

$$O_{jt} \in \{0,1\} \quad j \in C \tag{31}$$

$$V_{ijlt} \in \{0,1\} \quad i \in S,\, j \in C,\, l \in V,\, t \in T \tag{32}$$

$$V'_{lt} \in \{0,1\} \quad l \in L,\, t \in T \tag{33}$$

$$DE_{jt}, P_{jt}, RW_t \text{ are integer} \quad j \in C,\, t \in T \tag{34}$$

Describes the objective function and constraints

The objective function of the proposed mathematical model is to maximize profit. Profit is calculated from the total costs subtracted from the income which comes from selling the products. The costs include nine parts: the waste cost (including the products that are sold as waste), the cost of the order, the cost of purchasing material, cost of storage in the manufacturers' places, the cost of storage in the collection-repair center, fixed cost of employing vehicles, travelling cost that is dependent on distance, establishing the cost of the collection-repair center, and repair cost of the defective products, respectively. Constraints (2) and (3) represent the amount of customers' demand at the beginning of each period, which is dependent on the price. Constraint (4) ensures that the demand of each customer should be satisfied in each period. Constraint (5) calculates the amount of inventory in the warehouse of manufacturers at the end of each period. Constraint (6) ensures that a manufacturer can only order materials when the ordering has been completed. Constraint (7) ensures that vehicle load no more than their capacities. Constraints (8) and (9) calculate the number of backward products for each customer in every period that is dependent on the price. Constraints (10) and (11) determine if each customer requires a vehicle from the collection-repair center. Constraint (12) guarantees that the vehicles from the collection-repair center only meet the customers that need repaired products to be delivered or defective products to be picked up. Constraint (13) guarantees that if a vehicle enters a node in a period, it should leave it immediately after its mission is finished. Constraint (14) ensures that at the first of each period, a vehicle only comes out of the location in which the collection-repair center has been established. Constraint (15) illustrates the types of vehicle used at the collection-repair center in every period. Constraint (16) ensures that only one location must be selected as the collection-repair center. Constraint (17) eliminates sub-tours. Constraint (18) ensures that each vehicle cannot travel more than one time in every period. Constraints (19) and (20) determine the number of repaired products in each period. Constraint (21) calculates the inventory level in the collection-repair center at each period. Constraint (22) shows the number of backwards for every customer in each period. Constraint (23) calculates the loading amount of each vehicle when it is leaving the collection-repair center in each period. Constraint (24) calculates the loading amount of every vehicle when it is leaving the first customer after the collection-repair center through its route. Constraint (25) calculates the load of each vehicle when it is leaving each customer in its route. Constraints (26) and (27) ensure that the loading level never exceeds the capacity of the vehicle. Finally, constraints (28) and (34) show the kind of variables used in the model.

4. Numerical Examples

We have generated two numerical examples in order to illustrate the performance of the proposed mathematical model. The examples are applied in LINGO 9 on the computer with an Intel Core i5 processor and 4GB RAM. In this section, the information of the numerical examples is presented. The data have been adopted from Soleimani and Kennan. (2014) as reference, and modified in order to illustrate the application of the multi-period model. The model should determine the design and planning of a CLSC considering the routes of vehicles.

The network super-structure in the first numerical example is composed of 3 suppliers (S1 to S3), 3 manufacturers (MF1 to MF3), 5 customers (C1 to C5), 2 collection-repair centers (CC1 and CC2), and 2 vehicles (V1 and V2). The parameters are presented in Tables 1-9.

Table1
Distance between nodes (customers and collection-repair center) in the first numerical example

Node/Node	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Node 7
Node 1	0	233	185	168	268	209	182
Node 2	233	0	314	307	393	24	307
Node 3	185	314	0	17	83	292	201
Node 4	168	307	17	0	100	283	194
Node 5	268	393	83	100	0	375	198
Node 6	209	24	292	283	375	0	289
Node 7	182	307	201	194	198	289	0

Table2
Distance between supplier and Manufactureres in the first numerical example

Manufacturers/ Customer	Customer 1	Customer 2	Customer 3	Customer 4	Customer 5
Manufacturere1	42	35	123	252	159
Manufacturere2	352	345	349	342	151
Manufacturere3	340	357	293	632	343

Table 3
Mean price-independent demand of each customer in each period in the first numerical example

Customer/Period	Period 1	Period 2
Customer 1	1196	2133
Customer 2	2094	1020
Customer 3	2244	1461
Customer 4	1283	2530
Customer 5	1618	2678

Table 4
Rate of price-dependent demand of each customer in each period in the first numerical example

Customer/Period	Period 1	Period 2
Customer 1	0.2392	0.4266
Customer 2	0.4188	0.2040
Customer 3	0.488	0.2922
Customer 4	0.2566	0.5060
Customer 5	0.3236	0.5356

Table 5
Ordering cost of each manufacturer in each period in the first numerical example

Manufacturer/Period	Period 1	Period 2
Manufacturer1	45548	53183
Manufacturer2	67118	77692
Manufacturer3	70605	5617

Table 6
Unit purchasing cost of each manufacturer in each period in the first numerical example

Manufacturer/Period	Period1	Period2
Manufacturer1	868	1022
Manufacturer2	1377	1466
Manufacturer3	687	824

Table 7
Unit storage cost in warehouse of each manufacturer in every period in the first numerical example

Manufacturers/Period	Period1	Period2
Manufacturer1	49	82
Manufacturer2	44	27
Manufacturer3	41	54

Table 8
Unit storage cost in each location in every period in the first numerical example

Candidate location/Period	Period1	Period2
Candidate1	50	74
Candidate2	68	78

Table 9
Data based on each vehicle in the first numerical example

	Vehicle 1	Vehicle2
Capacity	8246	6285
Fixed cost	90000	130000
Unit shipping cost	30	35

We assume that γ, λ, and φ are equal to 0.05, 5000, and 0.7, respectively.

By solving the problem, the objective function of maximizing profit is equal to 14772430, and we have 2508.36 and 2973.99 for the prices of the products in the periods 1 and 2, respectively. The results also show that the number of repaired products is equal to 1619 and 756 during periods 1 and 2, respectively. Tables 10-12 illustrate other results obtained from the numerical example in detail.

Table 10
Demand of each customer at each period in the first numerical example

Customer/Period	Period 1	Period 2
Customer1	597	865
Customer2	1044	411
Customer3	1119	593
Customer4	640	1026
Customer5	807	1086

Table 11
Ordering amount of each manufacturer at the beginning of every period in the first numerical example

Manufacturer/Period	Period 1	Period 2
Manufacturer1	-	-
Manufacturer2	-	-
Manufacturer3	6574	-

Table 12
Amount of products that are traveled from every manufacturer to each customer by each vehicle in each period in the first numerical example

Name	Value	Name	Value
X(3,1,2,1)	597	X(3,5,1,1)	807
X(3,2,1,1)	1044	X(3,1,2,2)	865
X(3,3,1,1)	1121	X(3,2,2,2)	414
X(3,4,1,1)	640	X(3,5,1,2)	1086

In addition, the collection-repair center is located in the second place. For further imagination, the vehicle routing is illustrated graphically in Figures 2 and 3.

In the period 1, vehicle 2 serves in the collection-repair center and begins its route from customer 2 to customers 1, 5, 3, 2, and 4, respectively, and finally returns to the collection-repair center 2. In this period, the initial loading of the vehicle is zero because there is not any repaired item at the first period in the center.

In period 2, again potential location 2 and vehicle 2 are applied. The vehicle begins its route by loading some 1619 repaired items. It moves through customers 1, 5, 4, 3, and 2, and finally returns back to the collection-repair center in location 2. The vehicle delivers 593 and 1026 repaired items to customers 3 and 4 and picks up 189, 394, and 495 defective products from customers 2, 1, and 5, respectively.

Moreover, the network super-structure in the second numerical example is composed of four manufacturers (MF1 to MF4), 12 customers (C1 to C12), two collection-repair centers (CC1 and CC2), and four vehicles (V1 to V4). Moreover, the parameters are presented in Tables 13-21.

We assume that γ, λ, and φ are equal to 0.1, 5000, and 0.9, respectively.

The results illustrate that the price of the products is equal to 3227.9759 and 3940.8559, and the amount of repaired products is equal to 3335 and 585 during periods 1 and 2, respectively. Other results are presented in Tables 22-24.

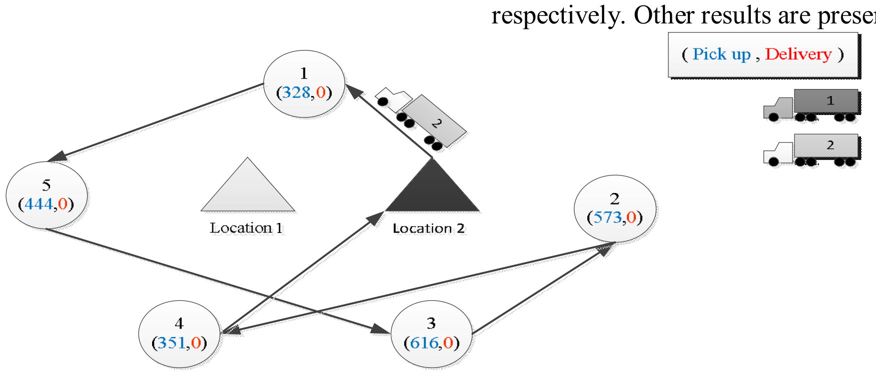

Fig. 2. Illustration of vehicle routing in period 1

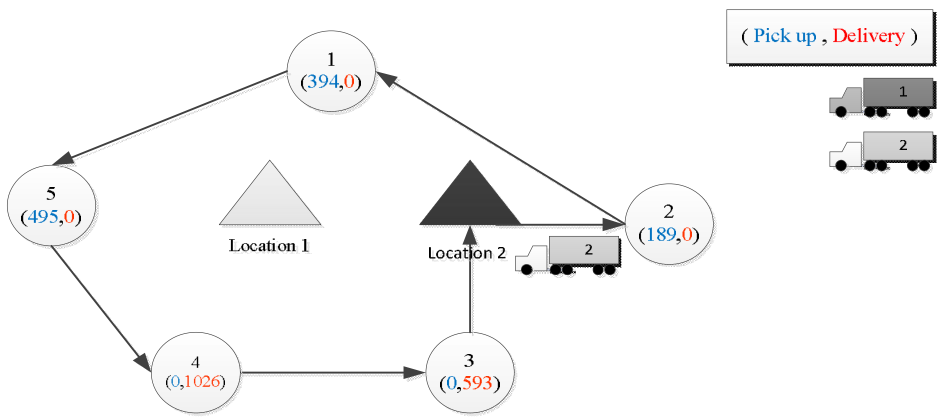

Fig. 3. Illustration of vehicle routing in period 2

Table 13
Distance between the nodes (customers and collection-repair center) in the second numerical example

Node/Node	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Node 7	Node 8	Node 9	Node 10	Node 11	Node 12	Node 13	Node 14	Node 15
Node1	0	105	153	58	56	125	80	93	76	155	87	29	86	94	127
Node2	105	0	182	125	123	80	185	160	143	50	192	96	85	199	232
Node3	153	182	0	211	209	202	147	246	229	232	138	182	171	91	72
Node4	58	125	211	0	34	79	126	35	74	127	133	29	40	140	173
Node5	56	123	209	34	0	113	92	37	40	143	99	27	74	118	139
Node6	125	80	202	79	113	0	205	94	153	82	212	96	39	219	252
Node7	80	185	147	126	92	205	0	111	82	235	9	109	166	56	75
Node8	93	160	246	35	37	94	111	0	59	162	118	64	75	155	174
Node9	76	143	229	74	40	153	82	59	0	183	91	57	114	138	157
Node10	155	50	232	127	143	82	235	162	183	0	242	126	87	249	282
Node11	87	192	138	133	99	212	9	118	91	242	0	116	173	47	66
Node12	29	96	182	29	27	96	109	64	57	126	116	0	57	123	156
Node13	86	85	171	40	74	39	166	75	114	87	173	57	0	180	213
Node14	94	199	91	140	118	219	56	155	138	249	47	123	180	0	33
Node15	127	232	72	173	139	252	75	174	157	282	66	156	213	33	0

Table 14
Distance between the supplier and customers in the second example

	Customer 1	Customer 2	Customer 3	Customer 4	Customer 5	Customer 6	Customer 7	Customer 8	Customer 9	Customer 10	Customer 11	Customer 12
Manufacturer1	114	219	139	160	126	239	34	145	90	269	27	143
Manufacturer2	48	115	201	38	8	117	88	45	36	147	95	21
Manufacturer3	35	120	118	93	91	140	65	128	111	170	72	64
Manufacturer4	30	135	161	76	48	155	50	85	68	185	57	59

Table 15
Mean demand, which is independent from the price for each customer in each period in the second example

	Customer 1	Customer 2	Customer 3	Customer 4	Customer 5	Customer 6	Customer 7	Customer 8	Customer 9	Customer 10	Customer 11	Customer 12
Period1	2739	2159	2100	1290	2706	2244	1702	2027	1804	1152	1480	1246
Period2	1367	1480	1834	1099	2806	2890	1982	1987	1675	2801	1738	1222

Table16
Rate of price-dependent demand of each customer in each period in the second example

	Customer 1	Customer 2	Customer 3	Customer 4	Customer 5	Customer 6	Customer 7	Customer 8	Customer 9	Customer 10	Customer 11	Customer 12
Period 1	0.5221	0.4116	0.4003	0.2459	0.5158	0.4278	0.3244	0.3864	0.3439	0.2196	0.2821	0.2375
Period 2	0.2606	0.2821	0.3496	0.2095	0.5349	0.5509	0.3778	0.3771	0.3193	0.5339	0.3313	0.2329

Table 17
Ordering cost of each manufacturer in each period in the second example

	Manufacturer 1	Manufacturer 2	Manufacturer 3	Manufacturer 4
Period1	471706	239993	178638	200433
Period2	346418	289316	240664	432332

Table 18
Unit purchasing cost of each manufacturer in each period in the second example

	Manufacturer 1	Manufacturer 2	Manufacturer 3	Manufacturer 4
Period1	2985	2050	2418	1786
Period2	2257	2254	1880	2068

Table 19
Unit storage cost in warehouse of each manufacturer in each period in the second example

	Manufacturer 1	Manufacturer 2	Manufacturer 3	Manufacturer 4
Period1	53	52	77	89
Period2	97	56	79	73

Table 20
Unit storage cost in each location in each period in the second example

Period/ Candidate location	Candidate 1	Candidate 2	Candidate 3
Period1	172	188	276
Period2	188	272	186

Table 21
Data based on each vehicle in the second example

	Vehicle 1	Vehicle 2	Vehicle 3	Vehicle 4
Capacity	13658	9883	12403	7135
Fixed cost	275003	93163	154689	245940
Unit shipping cost	488	490	247	492

Table 22
Demand of each customer in each period in the second example

	Customer 1	Customer 2	Customer 3	Customer 4	Customer 5	Customer 6	Customer 7	Customer 8	Customer 9	Customer 10	Customer 11	Customer 12
Period1	1054	831	808	497	1042	864	655	780	694	444	570	480
Period2	341	369	457	274	699	719	494	492	417	697	433	305

Table 23
Ordering amount of each manufacturer in each period in the second example

	Manufacturer 1	Manufacturer 2	Manufacturer 3	Manufacturer 4
Period1	-	-	-	11081
Period2	-	-	-	-

Table 24
Amount of products sent from each manufacturer to each customer by each vehicle in each period in the second example

Name	Value	Name	Value
X(4,1,2,1)	1054	X(4,6,2,2)	677
X(4,2,2,1)	831	X(4,7,2,1)	655
X(4,3,2,1)	808	X(4,7,2,2)	494
X(4,4,2,1)	497	X(4,8,2,1)	780
X(4,5,2,1)	1042	X(4,8,2,2)	492
X(4,5,2,2)	699	X(4,9,2,1)	694
X(4,6,2,1)	864	X(4,10,2,1)	444
X(4,11,2,1)	570	X(4,12,2,1)	480

According to the proposed mathematical model, the objective value reaches 20162760. The collection-repair center is located in the second place. In addition, a third vehicle is used in both periods in order to collect the defective products from the customers and deliver the repaired ones to them. The vehicle travels through customers 9, 7, 3, 10, 1, 5, 2, 11, 12, 8, 4, and 6, and finally returns back to the collection-repair center in the first period. In this period, as there is not any repaired product in the center, obviously, the vehicle does not deliver anything to the customers. Nonetheless, it picks up 295, 279, 343, 189, 446, 442, 353, 243, 204, 332, 212, and 367 defective products, respectively, depending on which customers it passes through along the route. In the second period, the vehicle begins with customer 10, and then goes to customers 5, 2, 9, 6, 8, 12, 11, 4, 1, 3, and 7. Finally, it returns back to the collection-repair center established in the second location. On the route, the vehicle delivers 697, 0, 369, 417, 42, 0, 305, 433, 274, 341, 457, and 0 repaired products, and picks up 0, 192, 186, 0, 0, 135, 0, 0, 0, 0, 0, and 136 defective products from the customers according to their predefined priority.

5. Solution Approach

As the proposed model is a Mixed Integer Non-linear programming (NILP), it takes a long time to achieve the global best solution, even in small cases. However, in this section, a proposed solution approach is presented that catches a solution extremely close to the global best solution. In this method, the price of the product, which is the variable that makes the model non-linear, is determined according to a procedure and inserted into the mathematical model as a parameter. Therefore, the model is transformed to a Mixed Integer Linear Programming (MILP) that is solvable considerably in shorter time than MINLP does. The objective function contains non-linear part of the mathematical model because of multiplying of the price of the product (Ps_t) to the demand of cure turned (DE_{jt}) and amount of returned defective products (P_{jt}) which both depend on the price that are used in the first and second parts of the objective function, respectively. The other parts have at most one factor of the price indirectly like the forth part that amount of orders changes based on demand which is dependent on the price. Therefore, the objective performances like a quadratic function which means its second derivative would be a real number, and so, the objective function is a convex or concave function. According to this concept, the heuristic solution approach is presented whose pseudo-code is illustrated in the figure 4.

The proposed method is applied to both presented examples in the previous section. Figure 5. illustrates the iterations of the solution approach for the first example that is found the best price for the first and second periods after six and five iterations, respectively. Each iteration in average needed about 4 minutes on average that means the proposed approach took approximately 24 minutes in total that is many times less than the MILP model. In addition, the gap between the global best solution and the solution achieved by the proposed method is lower about 0.001 percent that illustrates the quality of the method. According to the proposed method, the second test problem terminated after six iterations for each of the period. Each iteration approximately needs 10 minutes to be solved that is a short time related to solving MINLP model. The obtained objective value of the proposed method is 20162010 for the second example that is greatly close to the objective value of the global best solution which is equal to 20162760. In other words, the gap for the second test problem is close to zero.

```
L = 0
U =
for t = 1 to T
        Ps_t = (U-L)/2
end for
for t=1 to T
        while <= 0.0001
                P^- = Ps_t - ((U-L)/4)
                P^+ = Ps_t + ((U-L)/4)
                if MILP(Ps_t =P^-) > MILP(Ps_t= Ps_t)
                        L = P^- , U = Ps_t
                elseif MILP(Ps_t =P^+) > MILP(Ps_t= Ps_t)
                        L = Ps_t, U = P+
                else
                        L = Ps_t - ((U-L)/4) , U = Ps_t + ((U-L)/4)
                end if
                Ps_t = (U-L)/2
        end while
end for
*MILP(Ps,Ps_t=a) is the objective value for the mathematical model in which the prices are equal to Ps and price of period t is equal to a.
```

Fig. 4. Pseudo-code for the proposed solution approach

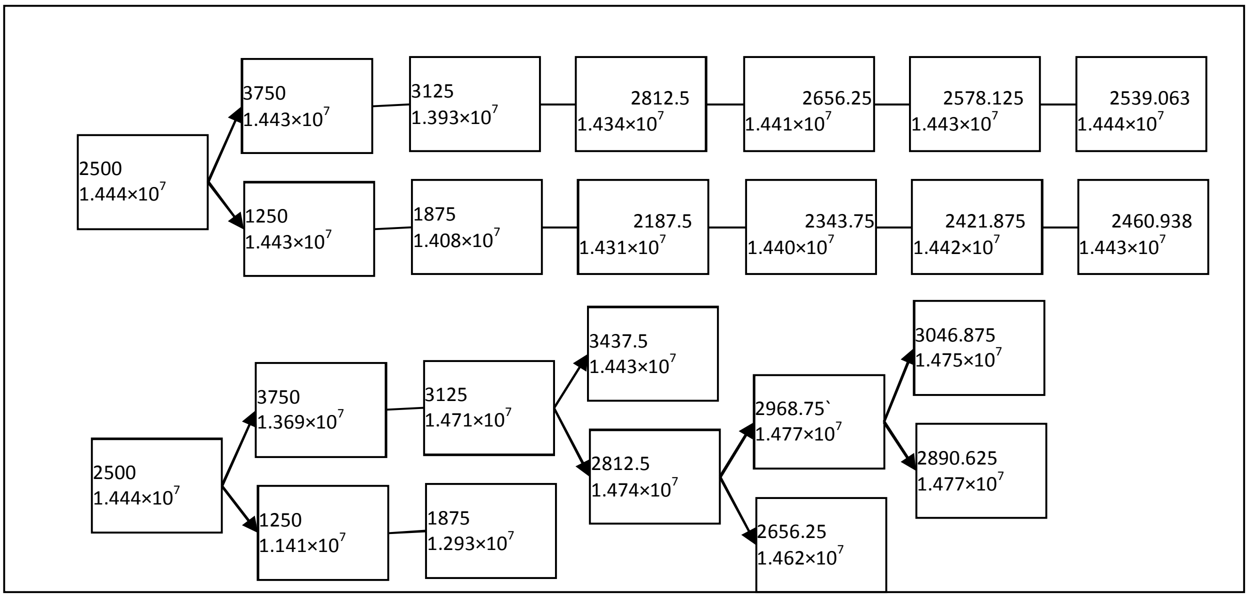

Fig. 5. Iterations of the proposed approach for the first test problem

6. Conclusion

In this paper, a multi-period closed-loop supply chains (CLSCs) model is presented, in which there are two echelons consisting of manufacturers and customers. In order to produce customers' demand, the manufacturers can order materials at the beginning of each period for one or more periods. In addition, products might be defective that are refunded by the customers and collected in a collection-repair center. Some of these products can be repaired, and so, returned to the supply chain, and others are sold as waste. A fleet of heterogeneous vehicles is used for transportation of products in the supply chain. Demands and the number of defective products depend on the price of the product. The more expensive product, the more demands and less defective products, and vice versa. Thus, the proposed MINLP model determines the best price that establishes a trade-off between costs and incomes. The objective is to determine the price of the product, the amount of orders for every manufacturer in each period, vehicles and their routing that are used in order to maximize the profits. Two examples are generated in small and medium scales to illustrate the performance of the proposed. Each of them is applied by the MINLP model in LINGO 9, and their results consequently are presented through tables and figures. In addition, as the proposed mathematical model is non-linear programing and extremely time consuming, a solution approach which converts the non-linear model to a linear one is presented. Then, two test numerical examples are applied by the approach, and the results demonstrate that the proposed method can achieve a high-quality solution in a considerably shorter time. The extremely little gap between the global best solution obtained from the MINLP model and solution achieved by the proposed heuristic method demonstrates agreeable performance of the heuristic approach.

References

Chen, J.-M. & Chang, C.-I., (2013). Dynamic pricing for new and remanufactured products in a closed-loop supply chain. *International Journal of Production Economics*, http://dx.doi.org/10.1016/j.ijpe.2013.06.017i

Chopra, S., & Meindl, P. (2001). Supply chain management: Strategy, planning and operation. Englewood Cliffs: *Prentice Hall.*

El bounjimi, M Abdulnour, G & Daoud A.K.(2014). Green Closed-loop Supply Chain Network Design: A Literature Review. *International Journal of Operations and Logistics Management. 4,275-286.*

El-Sayed. M, N. Afia,& N. El-Kharbotly, (2010). A stochastic model for forward–reverse logistics network design under risk, *Computers & Industrial Engineering* 58, 423–431.

Harland, C.M., Knight, L.A. & Sutton, R.Y. (2001). "Information for supply interventions: sector, network and organization opportunities from network and organization opportunities from E-commerce", paper presented at the 10th International Annual Conference of International Purchasing and Supply Education and Research Association, Jo¨ nko¨ ping, 8-11 April.

Jianmai Shia, Guoqing Zhang, & Jichang Sha, (2011). Optimal production and pricing policy for a closed loop system, Resources, *Conservation and Recycling* 55, 639–647.

Kannan Govindan, Hamed Soleimani, Devika Kannan, (2014). Reverse logistics and closed-loop supply chain: A comprehensive review to explore the future. *European Journal of Operational Research.*

Listes. O, & Dekker. R, (2005). A stochastic approach to a case study for product recovery network design, *European Journal of Operational Research* 160, 268–287.

Meade. L, J. Sarkis & A. Presley, (2007). The theory and practice of reverse logistics, *International Journal of Logistics Systems and Management* 3, 56–84.

M.I.G. Salema, A.P. Barbosa-Povoa, & A.Q. Novais, (2007). An optimizationmodel for the design of a capacitatedmulti-product reverse logistics network with uncertainty, *European Journal of Operational Research* 179, 1063–1077.

Ostlin, J., Sundin, E., & Bjorkman, M. (2008). Importance of closed-loop supply chain relationships for product remanufacturing. *International Journal of Production Economics,* 115 (2), 336–348.

Paucar-Caceres, A. & Espinosa, A., (2011). Management science methodologies in environmental management and sustainability: Discourses and applications. *Journal of the Operational Research Society*, 62(9), 1601–1620.

Petek. J, & P. Glavic, (1996). An integral approach to waste minimization in process industries, *Resource, Conservation and Recycling* 17, 169–188.

Pishvaee, M.S., & Torabi, S.A., (2010). A possibilistic programming approach for closed-loop supply chain network design under uncertainty. Fuzzy Sets Syst. 161(20), 2668–2683.

Pishvaee, M.S., Razmi, J. & Torabi, S.A., (2012a). Robust possibilistic programming for socially responsible supply chain network design: A new approach. Fuzzy Sets Syst. 206, 1–20.

Pishvaee, M.S., Torabi, S.A., & Razmi, J., (2012b). Credibility-based fuzzy mathematical programming model for green logistics design under uncertainty. *Comput. Indust. Eng*. 62, 624–632.

Pishvaee, M.S. Razmi, M.S. & Torabi S.A., (2014). An accelerated Benders decomposition algorithm for sustainable supply chain network design under uncertainty: A case study of medical needle and syringe supply chain, Transportation Research Part E 67, 14–38.

Rezapour S, Farahani RZ, Fahimnia B, Govindan K, & Mansouri Y, (2014). Competitive Closed-Loop Supply Chain Network Design with Price-Dependent Demands, Journal of Cleaner Production.

Seuring, S. (2013). A review of modeling approaches for sustainable supply chain management, Decision Support Systems, 54 , 1513–1520.

Shen-Lian Chunga, Hui-Ming Weeb, Po-Chung Yangc, (2008). Optimal policy for a closed-loop supply chain inventory system with remanufacturing, Mathematical and Computer Modelling 48, 867–881.

Shukla, S. K., Tiwari, M. K., Wan, H.-D., & Shankar, R. (2010). Optimization of the supply chain network: Simulation, Taguchi, and Psychoclonal algorithm embedded approach. *Computers & Industrial Engineering*, *58*(1), 29–39.

Soleimani H, Kannan G., A hybrid particle swarm optimization and genetic algorithm for closed-loop supply chain network design in large-scale networks, *Applied Mathematical Modelling* 39 (14), 3990-4012.

Visich, J. K., Li, S., & Khumawala, B. M. (2007). Enhancing product recovery value in closed-loop supply chains with RFID. Journal of Managerial Issues, 436–452.

Wei J, Govindan K, Li Y, & Zhao J, (2013). Pricing and collecting decisions in a closed-loop supply chain with symmetric and asymmetric information. *Computers and Operations Research.*

Xianpei Hong, Lei Xu, Peng Du, & Wenjuan Wang, (2015). Joint advertising, pricing and collection decisions in a closed-loop supply chain, *Int. J. Production Economics*, 167,12-22.

Yuanjie He, (2015). Acquisition pricing and remanufacturing decisions in a closed-loop supply chain, *Int. J. Production Economics.*

This article can be cited: Nakhai Kamal Abadi, I., Mohammadnejad, M., Sadeghian, R. & Ahmadizar, F. (2017). A New Mathematical Model for Closed-loop Supply Chains Considering product pricing, fleet of heterogeneous vehicles, and Inventory Costs. *Journal of Optimization in Industrial Engineering.* 10(21), 29-40.

URL: http://qjie.ir/article_258_37.html