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A way to Reduce Effects of Packet Loss in Video Streaming Using Multiple Description Coding

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Abstract

Multiple description (MD) coding has appeared to be an attractive technique to decrease impact of network failures and increase the robustness of multimedia communications. Very common model of this technique is multiple-description lattice vector quantization, which is the best choice for robust data transmission over the unreliable network channels. However, MD coinciding lattice vector quantizer (MDCLVQ) is not considered discrete network conditions, so in this scheme, all videos are received or are not received. In this paper, this scheme is implemented in real network environment. So, raw video will be send in various packet, packets send independently and packets lose independently. The possibility of lossing all packets together is close to zero. Our object for increasing of resistance transfer in error- prone communication channels are used. This technique has been tested for standard videos "Akiyo", "Carphone", "Miss-America" and "Foreman". This results show that the quality of the reconstructed videos from the average PSNR values of the central decoder and the side decoders has been reached to grate degree, so increases error resilience over error-prone communication channels.

Keywords: *Multiple description coding, Real-Time video, Lattice, Sub-lattice hexagonal, Quantization*.

1. Introduction

In recent years according to the widespread use of smart-phones, tablet and personal computers, the demand for multimedia communication applications has been increased[1]. Usage of multimedia services has increased over wired and wireless network[2]. multimedia services are classified can be classified into two categories: real-time or live and on demand[3]. Today Real-time video stream transmission over wireless network has received a wide welcome, so that; it is becoming part of our daily life[1, 4]. Video streaming traffic is rising significantly. Thus supporting high quality video on wireless devices is very important[5].

Real-time services are very sensitive. The transmission of raw services is not feasible due to the very large bandwidth required. Video compression refers to the process of reducing the amount of video data used to represent digital videos; it is a combination of spatial image compression and temporal motion compensation[1, 6].

The transmission of real-time video stream in environments like drone-hat that there is no strong processor and there is no encoding techniques like MPEG, so raw video will be send. Video transmission over noisy channels has been a challenging problem for engineers and researchers in this field.

To counter the effect of data loss for video transmission over noisy networks, there are three categories of approaches: Automatic Repeat Request (ARQ), Forward Error Correction (FEC), Error Resilient Coding (ERC)[7].

ARQ approach for delay is not efficient way to deal with packet loss. ARQ's applications are mostly presentational ones such as YouTube type of video distribution and on-demand services, this method is not suitable for real-time communications due to the

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time it takes to ask for retransmission and then to receive it[8].

FEC is more suitable for real-time streaming video because of its shorter transmission delay time and better transmission reliability due to its redundancy packet. this technique suffers from redundancy and still may not work for packet loss or errors in bursts[8, 9].

The thired category is Error Resilient Coding (ERC). An example of third category is Layered Coding (LC), in this method, which is also called Scalable Video Coding (SVC), video is coded into a base layer and one or more enhancement layers. The base layer provides a basic quality of video and enhancement layers improve the quality of the base layer. Layers in SVC are hierarchical which means that a given layer cannot be decoded unless all of its lower layers have been received correctly. This limits the error resiliency and resistance. However, This technique is suitable for low loss rate situations[10].

Another example of third category is Multiple Description Coding (MDC). MDC can be used as either source level protection[7]. It has appeared to be an attractive technique to decrease impact of network failures and increase the robustness of multimedia communications [11-13].

Multiple description coding is a technique that encodes a single video stream into two or more equally important sub-streams, the sub-streams called descriptions. these multiple descriptions are sent to the destination through different channels, resulting in much less probability of losing the entire video stream (all the descriptions), where the packet losses of all the channels are assumed to be independently and identically distributed. At the receiver, if all the descriptions are correctly received, the original data can be accurately reconstructed. However, if some of the descriptions fail to reach the destination, the rest of the descriptions are used in the side decoders to find an estimate of the original data [7, 11, 12, 14, 15].

The MDC process, i.e. generating and splitting the information into the descriptions, can be carried out in several domains. The domains are spatial, temporal and frequency, in which the descriptions are generated by partitioning the pixels, the frames and the transformed data respectively[7].

The MDC has variety of model. Very common model of this technique is multiple-description lattice vector quantization (MDLVQ), which is the best choice for robust data transmission over the unreliable network channels[1, 11]. It is an MD coding scheme that uses a combination of a lattice vector quantizer and a labeling function[16]. The lattice vector quantization is a well-known technique for data compression, which offers less computational complexity due to the regular structure of the lattice[17]. The design of an MD coding scheme based on lattice vector quantization (MDCLVQ) is first described in [18]. In this kind of work, the input stream is encoded by a lattice vector quantizer (LVQ), rather than two scalar quantizers. The quantized stream is then transformed into two descriptions using the labeling function. Different schemes for MDLVQ have been presented so far, the main difference between these schemes is the labeling function being used[16, 18].

MD coinciding lattice vector quantizer (MDCLVQ) is presented in [1] includes a new labeling function based on the coinciding similar sub-lattices of the root lattices. The coinciding sub-lattices are special sub-lattices because they have the same index, although they are generated by different generator matrices. This scheme utilizes two different sub-lattices, rather than a single sub-lattice. The MDCLVQ is different from schemes presented in terms of sub-lattice utilization, the partitioning scheme, and the labeling function used.

The proposed scheme in[13] and [1] does not rely on specific characteristics of the video coding standard being used and does not make any assumption about the state of the channel or probability distribution of error. It is not targeted for any specific video coding scheme and does not rely on the properties of the video encoding being used. Thus, it can effectively increase the performance of the encoding system in terms of compression efficiency and reliability, regardless of the encoding scheme being used. Although running parallel H.264/AVC or Motion JPEG2000 encoders requires more computational resources than the original encoder, the total computation time of the MD encoders is less than that of the original because the entropies of the side videos are significantly decreased by the MDCLVQ and thus, the time required by of the arithmetic coding is decreased. However, this scheme is not considered discrete network conditions, so in this scheme, all videos are received or are not received. In this paper, this challenge will be evaluate.

The our objective of this research is to implement a generic MD video coding scheme in [1] in the real network environment. We will be evaluate performance scheme in real environment. If network simulated properly; so that, packets will be send independently and packets will be lose independently. The possibility of losing all packets together is close to zero.

2. Elementary of lattices

A lattice is considered as a subset of points in the Euclidean space that share a common property[1]. The lattice points are generated using a generator matrix. The generator matrix is composed of the basis vectors of the lattice[17], in [1] is described. Therefore, the lattice A_n can be defined by equation (1).

$$A_n = \{(x_0, x_1, ..., x_n) \in Z^{n+1} : x_0 + x_1 + ... + x_n = 0\} \quad (1)$$

In equation (1), the lattice A_n is a subset of points with n+1 coordinates, such that the sum of these coordinates is zero. For example, the A_2 lattice is a subset of the complex space $\mathbb{C}$ and at unit scale it is generated by the basis vectors $\{1, \omega\} \subset \mathbb{C}$, where $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$ [18].therefore, the A_2 lattice at unit scale is generated by equation (2).

$$G_{2\times 2} = \begin{pmatrix} \mathrm{Re}(1) & \mathrm{Im}(1) \\ \mathrm{Re}(\omega) & \mathrm{Im}(\omega) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \quad (2)$$

In an n-dimensional lattice Λ, the Voronoi region of a lattice point is defined as the union of all non-lattice points within $\mathbb{R}^n$ that are closer to this particular lattice point than any other lattice point. Thus, the Voronoi region of $\lambda \in \Lambda$ is defined by equation (**3**)[18].

$$V(\lambda) \triangleq \{x \in \mathbb{R}^n : \|x - \lambda\| \le \|x - \lambda'\|, \forall \lambda' \in \Lambda\} \quad (3)$$

As a consequence, all the points within $V(\lambda)$ must be quantized to λ. The Voronoi regions of the points in the A_2 are hexagons; therefore, it is called the hexagonal lattice. The Voronoi region of a sub-lattice point λ' is the set of all lattice points that are closer to λ' than any other sub-lattice points. Thus, the Voronoi region of $\lambda' \in \Lambda'$ is defined as equation (4)[18].

$$V(\lambda') \triangleq \{\lambda \in \Lambda : \|\lambda - \lambda'\| \le \|\lambda - \lambda''\|, \forall \lambda'' \in \Lambda'\} \quad (4)$$

Assume that Λ is an L-dimensional lattice with the generator matrix G. A sub-lattice $\Lambda' \subset \Lambda$ with generator matrix G' is said to be geometrically similar to Λ if and only if $G' = cUGB$, for nonzero scalar c, an integer matrix U with det $U = \pm 1$, and a real orthogonal matrix B (with $BB^t = I$)[17].

The rest of this paper is organized as follows. In Section 2, the definitions of the elementary of lattices are presented. The proposed scheme are presented in Section 3. The experimental results are included in Section 4. Finally section 5 concludes the paper.

The index *N* is defined as the ratio of the fundamental volume of the sub-lattice Λ' to the fundamental volume of the lattice Λ. The fundamental volume of the lattice () is equal to the determinant of the generator. Thus, *N* is calculated by equation (5)[16].

$$N = \frac{vol'}{vol} = \sqrt{\frac{\det \Lambda'}{\det \Lambda}} = \frac{\det G'}{\det G} \quad (5)$$

For the hexagonal lattice, Λ' is similar to Λ if *N* is of the form equation (6)[18].

$$N = \alpha^2 - \alpha\beta + \beta^2 \quad For\ \alpha, \beta \in \mathbb{Z} \quad (6)$$

Sub-lattice $\Lambda' \subset \Lambda$ is considered as a clean sublattice if all the points of the Λ reside only inside the Voronoi region of the sub-lattice points rather than on the boundary of the Voronoi region. It means that the lattice points are not shared between the Voronoi regions of adjacent sub-lattice points[19].

The sub-lattices of A_2 are clean, if and only if, α and β are relatively primes. It follows that A_2 has a clean similar sub-lattice of index *N* if and only if *N* is a product of primes congruent to 1 (mod 6). The commonly used values of *N* are *7*, *13*, *19*, and *37*. In other words, α and β are selected such that the value of *N* satisfies these conditions and hence a clean similar sub-lattice of the hexagonal is generated[19].

In order to find the suitable values of α and β, all twofold combinations of integers [-10, -10] …[+10, +10] have been examined in [13] and only the combinations that generate clean sub-lattices of the hexagonal lattice with the desired indices are selected. The values of α and β that generate clean sub-lattices with *N=7*, *13*, *19*, and *37* combinations are provided in[1].

As shown in The coinciding similar sub-lattices are defined as geometrically similar sub-lattices of a root lattice with the same index *N* but generated by different generator matrices. In other words, the similar sub-lattices are coinciding with each other in a regular manner.

Thus, these sub-lattices are called coinciding similar sub-lattices of the hexagonal lattice. These sub-lattices are called coinciding similar sub-lattices of the hexagonal lattice[1].

It is seen in Fig. 1 , that there are only two distinct sub-lattices for index N=7. This occurs because the sub-lattices form two groups, each group consists of 6 sub-lattices which are coinciding with each other. The same scenario happens for other values of N, there are only two distinct coinciding sub-lattices for every value of N. the two coinciding sub-lattices overlap with each other in a regular pattern. That is, they overlap with each other every *N* lattice points in every direction. These points are shown by big red circles with black border. The lattice, the first group of sub-lattices coincide and the second group coincide and are shown with triangles, blue squares and red circles respectively [1, 13].

, There are 12 combinations of integer values that satisfy these conditions for N =7, 13[1, 13]. Thus, 12 generator matrices are calculated using different values. The generator matrices corresponding to N=7 are provided in [13].

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Table 1
Different values of a and B for different values of N(7,13,19,37)[1,13]

i	N	α	β	N	α	β
1		-3	-2		-4	-3
2		-3	-1		-4	-1
3		-2	-3		-3	-4
4		-2	1		-3	1
5		-1	-3		-1	-4
6		-1	2		-1	3
7	7	1	-2	13	1	-3
8		1	3		1	4
9		2	-1		3	-1
10		2	3		3	4
11		3	1		4	1
12		3	2		4	3

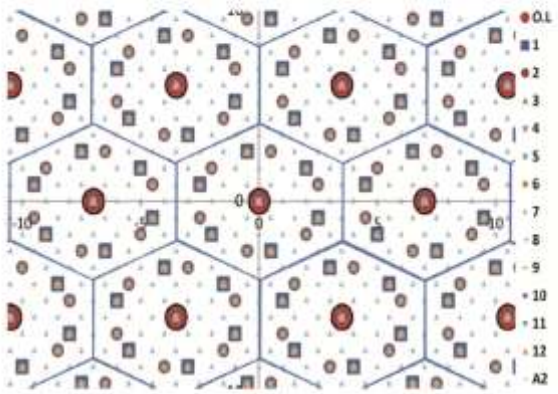

Fig. 1. The coinciding similar sub-lattices of hexagonal lattice with index N=7 but in a limited range[1].

The Super-Voronoi set of an overlapping point is the set of all lattice points that are closer to this point than any other overlapping point. The Super-Voronoi regions of the overlapping sub-lattice points with N=7 is shown with blue hexagon in Fig. 1. This symmetry is used in construction of the partitions. Partitions are used to define the new equivalence class, and the new shift property that can be used to simplify the labeling function. The new equivalence class and the new labeling function are presented in[1, 13].

3. Proposed Method

The proposed multiple-description coinciding lattice vector quantization (MDCLVQ) scheme A_2 lattice points are used as the

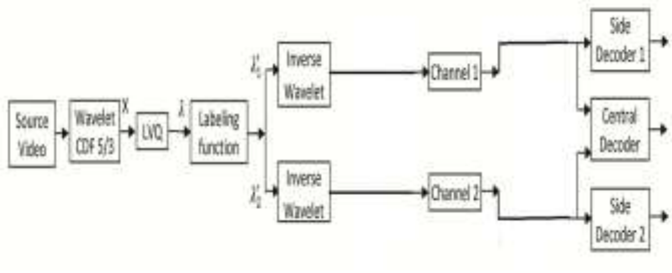

Fig. 2. MDCLVQ-A_2 video coding scheme applied to raw video[13]

At first, the input videos with the QCIF format (144×176 pixels) are read to save data (For data storage). Then, in the wavelet module every video is decomposed into four video (72×88 pixels). Finally, In the fifth stage, MDCLVQ-A_2 scheme is implemented in real network environment. Network environment includes: two senders, A FIFO queue, one receiver, two similar channels are implemented.

codebooks of the quantizer and the coinciding similar sub-lattice points are used as the labels MDCLVQ is a generic scheme, it applies to almost every video coding standard[11]; however, in this research, it has only been applied to raw video. Proposed scheme diagram is shown in Fig. 2.

As shown in Fig. 2, the proposed scheme includes: Wavelet decomposition module, Lattice Vector Quantizer (LVQ) module, Coinciding Labeling Function module, Inverse Wavelet Transforms module, Video Decoders module. This modules are described in[1, 13].

the video decomposed are formed into two-dimensional vectors with $\frac{144\times176}{2}$ raw and streamed to the LVQ module.

In the second stage, the two-dimensional input vectors are mapped into 3D vectors by a transformation matrix. Then, the 3D vectors are mapped to the lattice points by simple arithmetic manipulations. Procedure LVQ module is described in [1].

In the third stage, the labeling function maps each lattice point into two coinciding sub-lattice points that form a label. At the end of this stage, two independent descriptions of the video stream are produced[1]. The next stage, the inverse wavelet transform on the streams generated by the labeling function is done. The original data (144×176 pixels) has been restored.

It's assumed, that, sender (like drone-hat) has Weak processor, there is no encoding techniques like MPEG, and raw video will be send. So, limitation bit rate is not considered. Variable bit error rate is considered. So that, packets send independently and packets lose independently. Also, the FIFO queue is used as an essential part of a discrete event simulator. So that, the packets be in the queue.

4. Experimental Results

In this section, the experimental results of the MDCLVQ-A_2 scheme proposed, that, has been implemented in the real network environment are presented. The standard videos of "Akiyo", "Miss-America", "Carphone", and "Foreman" with the QCIF format (144×176 pixels) are chosen. It's assumed, that, sender has Weak processor, there is no encoding techniques like MPEG, and raw video will be send. So, limitation bit rate is not considered. The bit rate of 10MbpS is considered. Variable bit error rate of 10^0 to 10^-10 is considered. So that, packets send independently and packets lose independently. We will be evaluate performance scheme in real environment.

The MDCLVQ-A_2 scheme is a multivariable systems, because it is controlled by several factors, such as the wavelet threshold, the fundamental area of the hexagonal lattice and the index of the coinciding similar sub-lattices. Fundamental area of the hexagonal lattice is changed by sigma. The variable sigma is used by the fast quantizing algorithm to affect the generator matrix of the lattice being used. The fundamental area of the hexagonal lattice is changed by sigma = 0.1, 0.2…, 1, while maintaining the shape of the hexagonal lattice. Thus, the granularity of the quantization is adapted[1]. In this experiment the labeling function with index *N=7* has been used constantly and the wavelet threshold is constantly zero. Therefore, the analyses provided below are based on different values of sigma and Variable bit error rate. The performance of the MD coding scheme is evaluated based on the average required bit rate and reconstruction fidelity in the decoders.

A common measure to evaluate the quality of the video encoding is to compare the reconstructed video with the original video through the average peak signal to noise ratio (PSNR) of the luminance (Y) components[1, 13]. In the following results, the PSNR is measured for the luminance component. The average PSNR values of the central decoder and the side decoders of the proposed scheme are provided in Table 1

The average PSNR values of the central decoder and side decoders are improved as sigma is increased. In other words, the qualities of the reconstructed videos are directly proportional to the value of sigma.

As for example, PSNR values of the central decoder for the all videos corresponding to *sigma*=0.2 and Variable error bit rate is shown in Fig. 3.

The experimental results for the "Foreman" video sequences are shown in Fig. 4, that average PSNR values of the central decoder and the side decoders of the proposed scheme are higher than the corresponding values of the MDCLVQ-A_2 provided in [1]. So, this is because of better reconstruction quality of the proposed scheme as compared with the MDCLVQ-A_2 in[1]. However, the general pattern is the same and all the PSNR values increase as the sigma is increased because reconstruction fidelity is more affected by the performance of the MDCLVQ-A_2.

According to

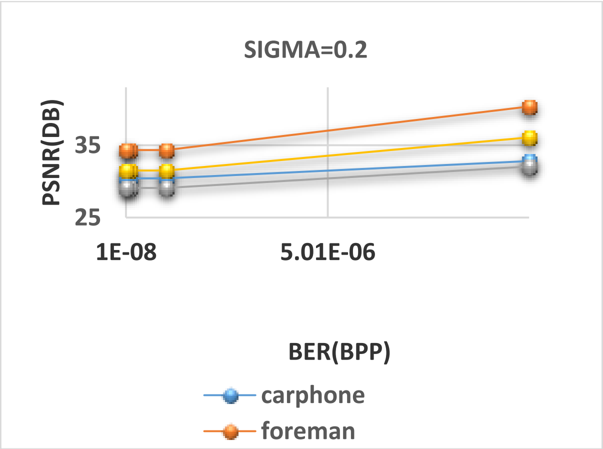

Fig. 3. PSNR values of the central decoder for videos tested with sigma=0.2 and error bit rate

Table 1
The average PSNR (dB) of the central and the side decoders for the selected video sequences.

Video/Sigma		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Miss-America	Side1	31.50	31.86	32.93	33.86	34.56	35.43	37.04	37.50	38.40	38.45
	Side2	31.50	31.90	32.93	33.96	34.43	35.43	37.04	37.50	38.40	38.45
	Central	31.50	31.86	32.93	33.86	34.56	35.43	37.04	37.50	38.40	38.45
Foreman	Side1	37.61	37.67	38.15	39.45	38.81	39.80	40.77	42.51	42.51	43.88
	Side2	37.61	37.52	38.15	39.53	39.81	40.24	41.77	42.51	43.88	43.88
	Central	37.61	37.67	38.15	39.35	39.73	40.24	41.77	42.51	42.80	43.88
Akiyo	Side1	34.04	34.54	35.20	36.21	38.02	38.82	36.49	35.75	38.82	41.56
	Side2	33.96	34.54	35.13	36.35	38.38	38.82	36.49	35.75	38.82	41.56
	Central	34.00	34.48	35.20	36.21	38.02	38.82	36.49	35.75	38.82	41.56
Carphone	Side1	31.50	31.73	32.44	33.42	34.71	37.60	35.93	35.93	39.77	39.77
	Side2	31.50	31.72	32.44	33.42	34.71	38.41	35.93	39.77	39.77	38.40
	Central	31.50	31.72	32.44	33.42	34.71	37.60	35.93	38.67	39.77	38.40

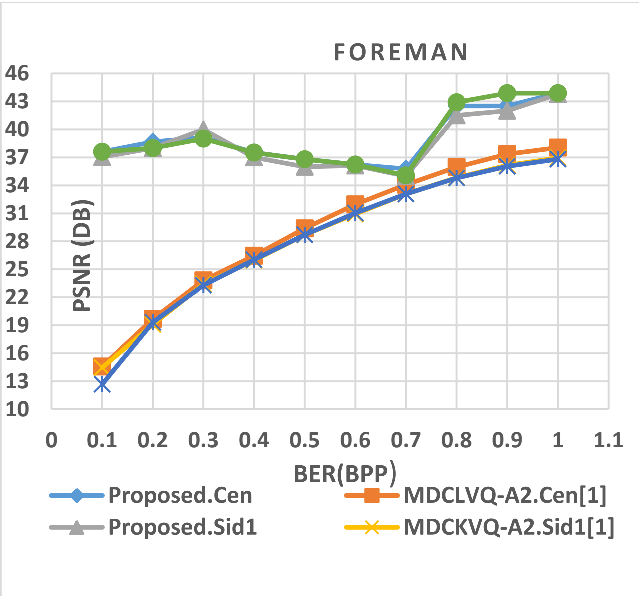

Fig. 4. PSNR values for the Foreman video by the proposed MDCLVQ-A_2 and referenced algorithms.

5. Conclusion

The multiple descriptions lattice vector quantization (MDLVQ) has become a popular choice for robust of data transmission over unreliable network channels. The main advantage of using MD video coding is that it increases the robustness of video transmission over error-prone communication channels while conserving the bandwidth. However, was not considered discrete network conditions. MDCLVQ-A_2 scheme implemented in the real network environment. So that, packets sent independently and packets lost independently. To assess the performance of the MDCLVQ-A_2 implementation scheme, this was applied to several test videos. The experimental results shown that the PSNR values are increase to Compared scheme. As the value of PSNR is increased, the quality of the reconstructed videos increase. Unlike the previous schemes, some of the packets videos were received at the receiver always. As a result, it improves the robustness of video transmission and increases error resilience over error-prone communication channels.

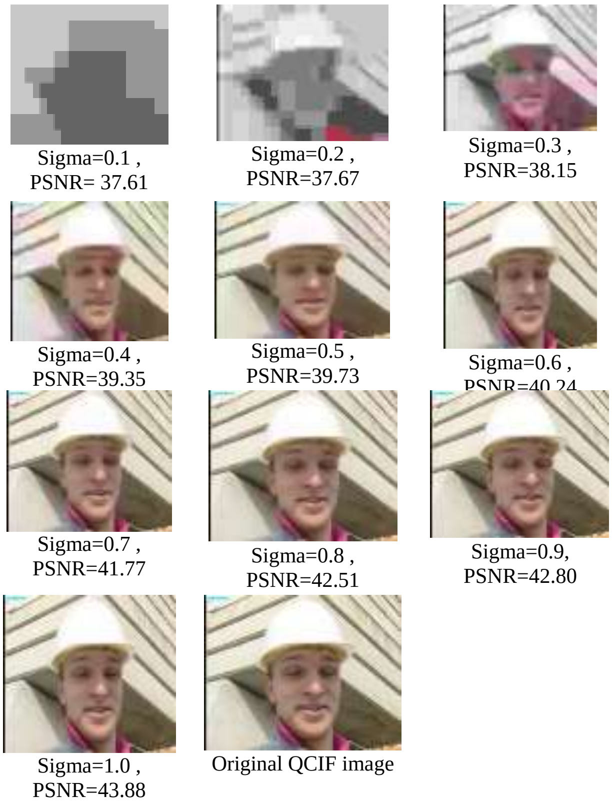

Fig. 5. The 1st frames of the QCIF Foreman sequence reconstructed by the central decoder corresponding to sigma = 0.1 to 1.

References

[1] Akhtarkavan, E. and M.F.M. Salleh, Multiple descriptions coinciding lattice vector quantizer for wavelet image coding. IEEE Trans. on Image Process, 2012. 21(2).

[2] Kusetogullari, H., Dynamic multiple description wavelet based image coding using enhanced particle swarm optimization. IEEE Journal of Selected Topics in Signal Processing, 2015. 9(1): p. 125-138.

[3] Sheshjavani, A.G. and B. Akbari. A study on repeat-request chunks in pull-based peer-to-peer video-on-demand streaming. in Electrical Engineering (ICEE), 2014 22nd Iranian Conference on. 2014. IEEE.

[4] Immich, R., et al. Adaptive motion-aware fec-based mechanism to ensure video transmission. in Computers and Communication (ISCC), 2014 IEEE Symposium on. 2014. IEEE.

[5] Sgardoni, V. and A.R. Nix, Raptor code-aware link adaptation for spectrally efficient unicast video streaming over mobile broadband networks. IEEE Transactions on Mobile Computing, 2015. 14(2): p. 401-415.

[6] Chuang, K.P., W.T. Lee, and H.W. Wei. Dynamically Adjust Packet Size and Block Size to Achieve Maximum Effectiveness of Forward Error Correction. in Intelligent Information Hiding and Multimedia Signal Processing (IIH-MSP), 2014 Tenth International Conference on. 2014. IEEE.

[7] Kazemi, M., S. Shirmohammadi, and K.H. Sadeghi, A review of multiple description coding techniques for error-resilient video delivery. Multimedia Systems, 2014. 20(3): p. 283-309.

[8] Ali, F.A., et al., Bandwidth efficient adaptive forward error correction mechanism with feedback channel. Journal of communications and networks, 2014. 16(3): p. 322-334.

[9] Wu, T.-Y., et al., An enhanced structure of layered forward error correction and interleaving for scalable video coding in wireless video delivery. IEEE Wireless Communications, 2013. 20(4): p. 146-152.

[10] Ghahremani, S. and M. Ghanbari, Error resilient video transmission in ad hoc networks using layered and multiple description coding. Multimedia Tools and Applications, 2016: p. 1-17.

[11] Akhtarkavan, E. and M.F.M. Salleh, Multiple descriptions coding for H.264/AVC using coinciding A2 lattice vector quantizer, in IEEE 2011 International Conference on Signal and Image Processing Applications (ICSIPA 2011). 2011: Kuala Lumpur, Malaysia.

[12] Sun, Y.-C. and W.-J. Tsai, Rate-distortion optimized mode selection method for multiple description video coding. Multimedia tools and applications, 2014. 72(2): p. 1411-1439.

[13] Akhtarkavan, E., M. Salleh, and O. Sidek, Multiple Descriptions Video Coding Using Coinciding Lattice Vector Quantizer for H. 264/AVC and Motion JPEG2000. World Applied Sciences Journal, 2013. 21(2): p. 157-169.

[14] Goyal, V.K. and J. Kovacevic, Generalized multiple description coding with correlating transforms. IEEE Transactions on Information Theory, 2001. 47(6): p. 2199-2224.

[15] Goyal, V.K., Multiple description coding: Compression meets the network. IEEE Signal Processing Magazine, 2001. 18(5): p. 74-93.

[16] Servetto, S.D., V.A. Vaishampayan, and N. Sloane. Multiple description lattice vector quantization. in Data Compression Conference, 1999. Proceedings. DCC'99. 1999. IEEE.

[17] Conway, J., NJA sloane," sphere Packings, Lattices and Groups. Grundlehren der mathematischen Wissenschaften, 1998. 290.

[18] Vaishampayan, V.A., N. Sloane, and S. Servetto, Multiple description vector quantization with lattice codebooks: Design and analysis. IEEE Trans. on Information Theory, 2001. 47: p. 1718-1734.

[19] Conway, J.H., E.M. Rains, and N.J.A. Sloane, On the existence of similar sublattices. Canad. J. Math, 1999. 51: p. 1300-1306.