Vibration Attenuation of Nonlinear Hysteretic Structures with Fully Unknown Parameters

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Abstract

Natural hazards such as earthquakes have threatened the life of human beings during the history. As a consequence, the vibration mitigation of structures has caught great importance. Active control of structures is one of the rapidly emerging areas in the concept of structural control. This paper presents a control method to deal with this subject when the dynamics of the structure is hysteretic and the parameters of the structure contain uncertainties. The hysteresis behavior of the structure is modeled using Bouc-Wen equation and the uncertainty is considered in its parameters. For control purpose, sliding mode method and its adaptive version are used. The salient point of adaptive sliding mode technique is that it does not use the uncertainty bounds in its controller; this is correspondent to the fact that the estimation of the structural parameters may not be exact. The efficiency of the proposed method is shown with a simulation.

Keywords: Active control of nonlinear structures; structures with uncertain parameters; Bouc-Wen model; vibration mitigation of structures.

1- Introduction

One of the perennial challenges in engineering is to find new and fruitful tools for protecting structures against the damaging effects of natural forces. Meanwhile, the earthquake is one of the events that are not possible to predict its exact time and place, though there has been much research about it. It seems that the way to deal with earthquakes is to immunize the structures against earthquakes. One of the methods that has been the subject of many studies in the field, is the idea of “structural control”, in recent decades, which is used to increase structures’ efficiency and safety against natural hazards. The concept of the structural control was first introduced by Yao [1]. Structural control methods can be classified as passive, semi-active and active control systems. A passive control system consists of an appended or embedded device that modifies the stiffness or the damping of the structure in an appropriate manner without requiring an external power to operate and feed energy to the system [2]. Meanwhile, semi-active control systems are often viewed as controllable passive systems [3].

In an active control system, an external source is applied to the structure in a prescribed manner. In such a system, control forces are developed based on feedbacks from sensors. These sensors measure the excitation and/or the response of the structure [2]. Active control systems are ready always to start activities and control vibrations. In comparison with passive control systems, a number of advantages associated with active control systems can be cited; among them are (a) enhanced effectiveness in response control; (b) relative insensitivity to site conditions and ground motion; (c) applicability to multi-hazard mitigation situations (against wind as well as earthquake); and (d) selectivity of control objectives (human comfort or increased structural safety) [4].

In this paper, the active control of structures is studied.

A mathematical model of a structure is necessary for implementing an active control system in structure. The simplest and ideal model of single degree-of-freedom (SDOF) structural system is a linear dynamic equation composed of a mass connected to a spring and a damper which is subjected to seismic excitation. In practice, structures are placed under movement of went back, when stimulated by vibrations from earthquakes and their went-way is not identical with their back route in each went-back cycles.

This phenomenon indicates that the equation of each class of structures also included “the phenomenon of hysteresis”. In other words, in the real case, the equations governing the motion of structures are not linear and should be considered along with equations including the hysteresis model of structures. So, there is a hysteresis term besides the mass-damper-spring term in the mathematical model of a structure [5].

There are several methods to model the hysteretic behavior, but this phenomenon in structural engineering is usually represented by using the so-called Bouc-Wen model proposed by Bouc and Wen [6], [7]. The model of Bouc-Wen, is the most famous mathematical model of hysteresis. This model has been used experimentally mainly in structural systems. The Bouc- Wen model is able to capture, in an analytical form, a rang of shapes of hysteretic cycles which matches the behavior of a wide class of hysteretic systems [8].

The exact extraction of the parameters of the dynamics model of a structure can not be carried out. After constructing a structure, its parameters are estimated by using special techniques for the purpose of analysis and/or increasing its resistance to natural disturbances such as wind or earthquake. So it is natural that the estimated parameters of a structure may not be exact.

On the other hand, to analyze the nonlinear behavior of a specific structural system, in addition to the parameters of the structure, the parameters of the Bouc-Wen model should be estimated by using identification techniques that lead to the knowledge of an interval for each parameter; not the exact value of the parameters [9]. So the parameters of the whole system, involving the parameters of the structure and the Bouc-Wen model are not exact and contain uncertainty. Some researches have discussed the control of hysteretic structures under the uncertainty of its parameters.

Among the studied methods, we can refer to adaptive back stepping approach for single degree-of-freedom (DOF) and two DOF structures [9], [10], optimal sliding mode control (SMC) [5], Back stepping-based Lyapunov redesign control [2], H_∞ disturbance attenuation and α-degree stability [11]–[14], and energy to- peak method [15]. In these references, the uncertainty in the parameters of the Bouc-Wen model has not been discussed, while the existence of this uncertainty is unavoidable from practical point of view.

This paper proposes a controller, based on the well-known SMC technique, which can stabilize the system in the presence of both structure and Bouc-Wen parameters' uncertainties. Furthermore, since the estimation of a specific parameter may give us an interval for it, instead of its exact or estimated value, the adaptive SMC technique is studied to show its applicability in the area of civil structures.

The organization of this paper is as follows. Section 2 presents the dynamics equation of a hysteretic structure. In section 3, a controller is developed which guarantees the stability of the closed loop system. Simulations' results are given in section 4 to show the efficiency of the presented controllers, and finally section 5 draws the conclusion of the paper.

2- Problem Statement

Consider now a Single degree of freedom (SDOF) structure with an active controller, as illustrated in Fig.1. The passive component of a base-isolated is Hysteretic.

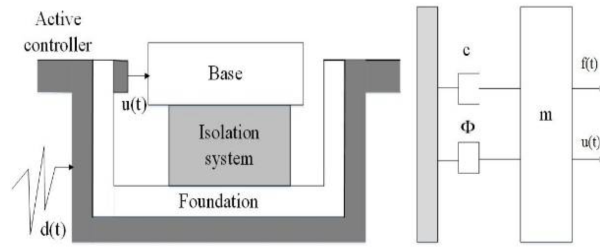

Fig.1. Base isolation system (left) and schematic model (right)

The dynamic equation of a nonlinear SDOF structural system subjected to an earthquake excitation can be formulated as:

$$m\ddot{x}(t)+c\dot{x}(t)+\Phi(x(t))=-ma(t)+u(t) \tag{1}$$

In (1), k , c and m are stiffness, damping and mass coefficient of the structure, respectively. $a(t)$ is the earthquake acceleration, and $u(t)$ is an active control force. $x(t)$, $\dot{x}(t)$ and $\ddot{x}(t)$ are the displacement, velocity and acceleration, respectively. $\Phi(x(t))$ is the nonlinear restoring force that shows nonlinear behavior of the structure, and it is described by Bouc-Wen model as follows:

$$\Phi(x(t))=\alpha kx(t)+(1-\alpha)Dkz(t), \tag{2}$$

$$\begin{aligned}\dot{z}(t)=D^{-1}(A\dot{x}(t)-\beta|\dot{x}(t)||z(t)|^{n-1}\\ z(t)-\lambda\dot{x}(t)|z(t)|^{n}).\end{aligned} \tag{3}$$

This model shows restoring force $\Phi(x(t))$ with an elastic component αkx and a hysteretic component $(1-\alpha)kDz(t)$, in which $D>0$ refers to displacement efficiency constant and $\alpha\in(0,1)$ is pre-efficiency stiffness ratio. $z(t)$ is a non-dimensional variable that is the solution of nonlinear first-order differential equations(3).

In (3), A, β and λ are non-dimensional parameters and control of the size and shape of the hysteresis loop and $n \geq 1$ handles the smoothness and governs the transition from elastic into plastic response. Note that an upper bound exists for earthquake perturbation.

The dynamical modeling of (1)-(3) is valid when all parameters of the structure are exactly known. However, there is always a certain level of inaccuracy on parameter estimation and there may be found some deviations from their exact values in estimating these parameters. So (1)-(3) do not explain the exact behavior of the structure. If uncertainties in structural parameters are taken into account, (1)-(3) can be reformulated as follows:

$$\begin{aligned}&(m+\Delta m)\ddot{x}(t)+(c+\Delta c)\dot{x}(t)\\&\quad+(\alpha+\Delta\alpha)(k+\Delta k)x(t)\\&\quad+(1-(\alpha+\Delta\alpha))(D+\Delta D)(k+\Delta k)z(t)\\&\quad=-(m+\Delta m)a(t)+u(t)\end{aligned} \tag{4}$$

$$\begin{aligned}\dot{z}&=(D+\Delta D)^{-1}((A+\Delta A)\dot{x}(t)\\&\quad-(\beta+\Delta\beta)|\dot{x}(t)||z(t)|^{(n+\Delta n)-1}\times z(t)\\&\quad-(\lambda+\Delta\lambda)\dot{x}(t)|z(t)|^{(n+\Delta n)}),\end{aligned} \tag{5}$$

in which m, c, k, α, D, A, β, n and λ are nominal values of the parameters and their deviation from exact values are estimated as Δm, Δc, Δk, $\Delta\alpha$, ΔD, ΔA, $\Delta\beta$, Δn and Δn. Control purpose here is to design the input force $u(t)$ such a way that structural system in the presence of parameter uncertainties and earthquake perturbation has an acceptable response.

Before proceeding to the controller design, the state-space representation of the system equations is given here. Taking $x_1(t)=x(t)$, $x_2(t)=\dot{x}(t)$ and $z(t)$ as state variables, the state-space equations of the system dynamic (4) and (5) are:

$$\dot{x}_1(t)=x_2(t), \tag{6}$$

$$\begin{aligned}\ddot{x}(t)&=-a+\frac{1}{(m+\Delta m)}u(t)-\frac{(c+\Delta c)}{(m+\Delta m)}x_2(t)\\&\quad-\frac{(k+\Delta k)}{(m+\Delta m)}(\alpha+\Delta\alpha)x_1(t)\\&\quad-\frac{(k+\Delta k)}{(m+\Delta m)}(1-(\alpha+\Delta\alpha))(D+\Delta D)z(t)\end{aligned} \tag{7}$$

$$\begin{aligned}\dot{z}(t)&=(D+\Delta D)^{-1}((A+\Delta A)x_2(t)\\&\quad-(\beta+\Delta\beta)|x_2(t)||z(t)|^{(n+\Delta n)-1}\\&\quad-(\lambda+\Delta\lambda)x_2(t)|z(t)|^{(n+\Delta n)})\end{aligned} \tag{8}$$

3- Control System Design

Let us define a time-varying surface $s(t)$ in the state-space R^n by the scalar equation $s=0$, where

$$s=\left(\frac{d}{dt}+\lambda_0\right)x=\dot{x}+\lambda_0 x \tag{9}$$

With λ_0 as a strictly positive constant which identifies the slope of sliding surface. The dynamic motion on the sliding surface is

$$s=\dot{x}+\lambda_0 x,\quad s=0\Rightarrow\dot{x}=-\lambda_0 x_1. \tag{10}$$

While the motion is on the sliding surface, the system dynamics can be expressed as:

$$\begin{aligned}\dot{s}&=\ddot{x}+\lambda\dot{x}\\&=-a+\frac{1}{m+\Delta m}u(t)-\frac{(c+\Delta c)}{m+\Delta m}x_2\\&\quad-\frac{(k+\Delta k)}{m+\Delta m}(\alpha+\Delta\alpha)x_1-\frac{(k+\Delta k)}{m+\Delta m}\\&\quad\times(D+\Delta D)(1-(\alpha+\Delta\alpha))z(t)+\lambda_0 x_2\end{aligned} \tag{11}$$

The term $\frac{1}{m+\Delta m}$ in (11) can be written as

$$\frac{1}{m+\Delta m}=\frac{1}{m}\left(1-\frac{\Delta m}{m+\Delta m}\right) \qquad (12)$$

Replacing (13) in (11) yields

$$\begin{aligned}\dot{s}=&\lambda_0 x_2-a+\frac{1}{m}\left(1-\frac{\Delta m}{m+\Delta m}\right)u(t)\\&-\frac{1}{m}\left(1-\frac{\Delta m}{m+\Delta m}\right)(c+\Delta c)x_2\\&-\frac{1}{m}\left(1-\frac{\Delta m}{m+\Delta m}\right)(\alpha+\Delta\alpha)(k+\Delta k)x_1\\&-\frac{1}{m}\left(1-\frac{\Delta m}{m+\Delta m}\right)(k+\Delta k)(D+\Delta D)\\&\times(1-(\alpha+\Delta\alpha))z(t)\end{aligned} \qquad (13)$$

By taking $\dot{s}(t)=0$ and solving (13) for the control input when the uncertain terms are neglected, we obtain the equivalent control as (14). $u_{eq}(t)$ can be interpreted as the continuous control law that would maintain if the dynamics were exactly known.

$$u_{eq}=\dot{x}(c-\lambda_0 m)+\alpha kx(t)+(1-\alpha)Dkz(t) \qquad (14)$$

Attaining equivalent control, we seek for a term to be added to (14) for stabilizing the whole system. We take the control force $u(t)$ as:

$$u=u_{eq}+mv=x_2(c-\lambda_0 m)+\alpha kx_1+(1-\alpha)Dkz+mv \qquad (15)$$

in which the signal $v(t)$ will be interpreted later. Substituting (15) into (13) with some mathematical manipulations result in

$$\begin{aligned}\dot{s}=&\frac{1}{m+\Delta m}[-(\Delta c-\lambda_0.\Delta m)x_2\\&-(\alpha.\Delta k+\Delta\alpha\Delta k+k\Delta\alpha)x_1]\\&+\frac{1}{m+\Delta m}[D.\Delta k(1-(\alpha+\Delta\alpha))+\Delta D.k\\&\times(1-(\alpha+\Delta\alpha))+\Delta D\Delta k(1-(\alpha+\Delta\alpha))\\&+\Delta\alpha D.k]z-\frac{\Delta mv}{m+\Delta m}-a+v.\end{aligned} \qquad (16)$$

Now, we are going to implement the SMC technique. By taking into consideration (16), we define δ as

$$\begin{aligned}\delta=&\frac{1}{m+\Delta m}[-(\alpha.\Delta k+\Delta\alpha\Delta k+k\Delta\alpha)x_1\\&-(\Delta c-\lambda_0.\Delta m)x_2+((D.\Delta k+\Delta D.k\\&+\Delta D\Delta k)\times((1-(\alpha+\Delta\alpha)))+\Delta\alpha D.k)\\&\times z-\Delta mv]-a\end{aligned} \qquad (17)$$

Then, an upper bound for $\delta(t)$ as a function of $x_1(t)$, $x_2(t)$ and $v(t)$ can be determined as:

$$\begin{aligned}\|\delta\|=&\left|\frac{1}{m+\Delta m}\right|(|\alpha.\Delta k+\Delta\alpha\Delta k+k\Delta\alpha||x_1|\\&+|\Delta c-\lambda_0.\Delta m||x_2|+(|D.\Delta k+\Delta D.k\\&+\Delta D\Delta k)\times|1-(\alpha+\Delta\alpha)|+|\Delta\alpha D.k||z_0|)\\&-\tilde{a}+\left|\frac{\Delta m}{m+\Delta m}\right|\|v\|,\end{aligned} \qquad (18)$$

Where $|a(t)|\le\tilde{a}$ and $\|.\|$ is a ∞-norm. By taking $\kappa=|\Delta m/m+\Delta m|$, it is obvious that $\kappa<1$. In (18), z_0 is defined as $z_0=\sqrt[n]{A/(\beta+\lambda)}$, It has been proved in [8] that when the parameters of the Bouc-Wen model belongs to the set $\{A>0,|\lambda|<\beta\}$, the Bouc-Wen equation variable $z(t)$ is bounded and by assumption $z(0)=0$, the upper bound of $z(t)$ is z_0. The aforementioned set can tackle a wide range of hysteretic behaviors of practical systems [8].

Taking the Lyapunov function candidate as $V=\frac{1}{2}s^2$ and calculating its time derivative, we have:

$$\dot{V}=s\dot{s}=sv+s\delta\le sv+|s|(\rho(x_1,x_2)+\kappa\|v\|). \qquad (19)$$

We select $v(t)$ as $\frac{-\beta}{1-\kappa}\operatorname{sgn}(s)$ where $\beta = b_1|x_1| + b_2|x_2| + b_3|z_0| + b_4$ with some $b>0$ and b_1, b_2 and b_3 satisfying the following inequalities:

$$b_1 \geq \left|\frac{|\alpha.\Delta k + \Delta\alpha\Delta k + k\Delta\alpha|}{m+\Delta m}\right|,$$
$$b_2 \geq \left|\frac{\Delta c - \lambda_0.\Delta m}{m+\Delta m}\right|,$$
$$b_3 \geq \left|\frac{(D.\Delta k + \Delta D.k + \Delta D.\Delta k)\times(1-(\alpha+\Delta\alpha)) + \Delta\alpha D.k}{m+\Delta m}\right|,$$
$$b_4 \geq |\tilde{a}|.$$

Then, we have $\beta \geq \rho + b$, and thus $\dot{V} \leq -b|s|$.

The obtained sliding mode controller needs the bounds of system uncertainties and uses these bounds in the controller. But, since the parameters of the structure as well as the parameters of the Bouc-Wen model are estimated, one may choose the boundary values of the uncertainties scrupulously which in turn would cause the sliding mode controller to be over-conservative. Hence, an adaptive sliding mode control (ASMC), is developed here. The proposed ASMC does not need the bounds of system uncertainties during the control process and instead, uses some auxiliary variables which adapt themselves during the control procedure. For this purpose, we define variables $\dot{\hat{b}}_1$, $\dot{\hat{b}}_2$, $\dot{\hat{b}}_3$ and $\dot{\hat{b}}_4$ by following differential equation

$$\begin{aligned}\dot{\hat{b}}_1 &= |x_1 s|,\\ \dot{\hat{b}}_2 &= |x_2 s|,\\ \dot{\hat{b}}_3 &= |zs|,\\ \dot{\hat{b}}_4 &= |s|.\end{aligned} \tag{20}$$

Where $\hat{b}_i \geq 0$ for $i = 1,2,3,4$. Now consider the control law (14) with $v = -(b + 2|x_1|\hat{b}_1 + 2|x_2|\hat{b}_2 + 2|z|\hat{b}_3 + 2\hat{b}_4)\operatorname{sgn}(s)$ along with Lyapunov function nominee $V = \frac{1}{2}\Big[s^2 + (b_1-\hat{b}_1)^2 + (b_2-\hat{b}_2)^2 + (b_3-\hat{b}_3)^2$ $+(b_4-\hat{b}_4)^2\Big]$ Time derivative of V with some mathematical manipulations yields:

$$\begin{aligned}\dot{V} = {} & \frac{-\alpha.\Delta k - \Delta\alpha.\Delta k - \Delta\alpha.k}{m+\Delta m} x_1 s - b_1|x_1 s| \\ & - \frac{\Delta c - \lambda_0.\Delta m}{m+\Delta m} x_2 s - b_2|x_2 s| \\ & + \left(\frac{(-D\Delta k + \Delta Dk + \Delta D\Delta k).(1-(\alpha+\Delta\alpha)}{m+\Delta m}\right. \\ & \left. + \frac{\Delta\alpha Dk}{m+\Delta m}\right) zs - b_3|zs| - \tilde{a}s - b_4|s| \\ & + \left(\frac{\Delta m}{m+\Delta m} - 1\right) b|s| + \left(\frac{2\Delta m}{m+\Delta m} - 1\right) \\ & \times(\hat{b}_1|x_1 s| + \hat{b}_2|x_2 s| + \hat{b}_3|zs| + \hat{b}_4|s|).\end{aligned} \tag{21}$$

$$R_1 = \frac{-\alpha.\Delta k - \Delta\alpha.\Delta k - \Delta\alpha.k}{m+\Delta m} x_1 s - b_1|x_1 s|$$
$$R_2 = -\frac{\Delta c - \lambda_0.\Delta m}{m+\Delta m} x_2 s - b_2|x_2 s|$$
$$R_3 = \frac{(D.\Delta k + \Delta D.k + \Delta D.\Delta k)(1-(\alpha+\Delta\alpha)) + kD\Delta\alpha}{m+\Delta m} \times zs - b_3|zs|$$
$$R_4 = -\tilde{a}s - b_4|s|$$

Then, R_1, R_2, R_3 and R_4 are non-positive, because

$$\begin{aligned}R_1 &= \frac{-\alpha.\Delta k - \Delta\alpha.\Delta k - \Delta\alpha.k}{m+\Delta m} x_1 s - b_1|x_1 s| \\ &\leq \left|\frac{-\alpha.\Delta k - \Delta\alpha.\Delta k - \Delta\alpha.k}{m+\Delta m}\right||x_1 s| - b_1|x_1 s| \\ &= \left(\left|\frac{-\alpha.\Delta k - \Delta\alpha.\Delta k - \Delta\alpha.k}{m+\Delta m}\right| - b_1\right)|x_1 s| \\ &\leq 0,\end{aligned} \tag{22}$$

$$
\begin{aligned}
R_2 &= -\frac{\Delta c-\lambda_0.\Delta m}{m+\Delta m}x_2 s - b_2\left|x_2 s\right| \\
&\le \left|\frac{\Delta c-\lambda_0.\Delta m}{m+\Delta m}\right|\left|x_2 s\right| - b_2\left|x_2 s\right| \\
&= \left(\left|\frac{\Delta c-\lambda.\Delta m}{m+\Delta m}\right| - b_2\right)\left|x_2 s\right| \le 0
\end{aligned}
\qquad (23)
$$

$$
\begin{aligned}
R_3 &= \left(\frac{(D.\Delta k+\Delta D.k+\Delta D.\Delta k)(1-(\alpha+\Delta\alpha))}{m+\Delta m} + \frac{\Delta\alpha.D.k}{m+\Delta m}\right)zs - b_3\left|zs\right| \\
&\le \left|\frac{(D.\Delta k+\Delta D.k+\Delta D\Delta k)(1-(\alpha+\Delta\alpha))}{m+\Delta m} + \frac{\Delta\alpha.D.k}{m+\Delta m}\right|\left|zs\right| - b_3\left|zs\right| \\
&= \left(\left|\frac{(D.\Delta k+\Delta D.k+\Delta D\Delta k)(1-(\alpha+\Delta\alpha))}{m+\Delta m} + \frac{\Delta\alpha.D.k}{m+\Delta m}\right| - b_3\right)\left|zs\right| \le 0,
\end{aligned}
\qquad (24)
$$

$$
\begin{aligned}
R_4 &= \tilde{a}s - b_4\left|s\right| \le \left|\tilde{a}\right|\left|s\right| - b_4\left|s\right| \\
&= (\left|\tilde{a}\right| - b_4)\left|s\right| \le 0.
\end{aligned}
\qquad (25)
$$

The terms $\left(\frac{2\Delta m}{m+\Delta m}-1\right)$ in equations are negative when Δm satisfies the inequality $m > \Delta m$. As this condition is likely to be satisfied in practice, hence the stability of the controller is proved.

4- Numerical Simulation and Results

In order to show the efficiency of the proposed method, a numerical simulation is presented in this section. The acceleration of 1940 El-Centro and 1994 Newhall earthquakes are applied to the structure as the ground vibration. The time history of acceleration of these earthquakes are illustrated in figure2. The parameters of the structure are selected as $m = 156\times10^3\,Kg$, $c = 2\times10^4\,Nsm^{-1}, k = 6\times10^6\,Nm^{-1}, \alpha = 0.6,$ $D = 0.6m,$ $A = 1$, $\beta = 0.1$ and $n = 3$. These parameters have been chosen from [9].

We choose $\lambda_0 = 0.5$ for the sliding surface. The upper bound for system uncertainty are taken to be $\Delta p = \pm p$ where $p \in \{m, c, k, \alpha, D, n, A, \beta, \lambda\}$. The chattering phenomenon originating from the sgn(s) function of the controllers has been avoided by using it continuous estimation $s/(s+\varepsilon e^{-\pi t})$ with $\varepsilon = 0.1$.

To evaluate the system performance, the simulation is carried out for a worst case where $\Delta m = 0.2m$, $\Delta c = -0.2c$, $\Delta k = 0.2k$, $\Delta\alpha = 0.2\alpha$, $\Delta D = 0.2D$, $\Delta n = -0.2n$, $\Delta A = 0.2A$, $\Delta\beta = -0.2\beta$, and $\Delta\lambda = 0.2\lambda$. It should be noted that these bounds of the parameter uncertainties do not need to be known for ASMC design. Figures 2-5 show the displacement and velocity of the uncontrolled structure, the structure controlled by SMC, and the structure controlled by ASMC.

Figures 2 and 3 show the structure responses to the El-Centro earthquake and figures 3 and 4 show the structure responses to the Newhall earthquake. From these figures, it can be seen that the proposed SMC and ASMC provide improved results when they are compared with the uncontrolled structures. The comparison of the SMC with the ASMC indicates that both methods have acceptable results. It should be noted that when the response of the uncontrolled structure is compared with the controlled structure during the first seconds of starting the vibrations, the uncontrolled structure outperforms the controlled structure.

This is due to the fact that in the controlled structure, the adaptation of the variables and control law take some time and after a few seconds, the performance of the controlled structures show a very good reduction on both displacement and velocity responses. figures 6 and 7 show the control force time histories of the SMC and ASMC methods when the structure is subjected to El-Centro and Newhall earthquakes, respectively. From these figures, the consumed control force in the ASMC method is less than the consumed control force in the SMC.

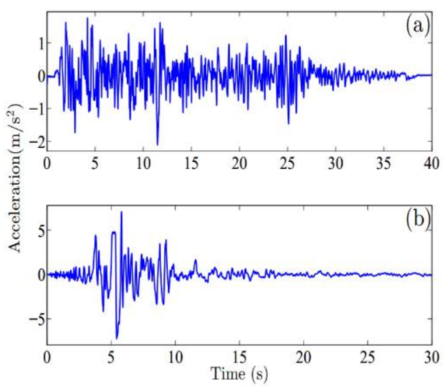

Fig.2. Time history of external excitations; (a) El-Centro, (b) Newhall earthquakes

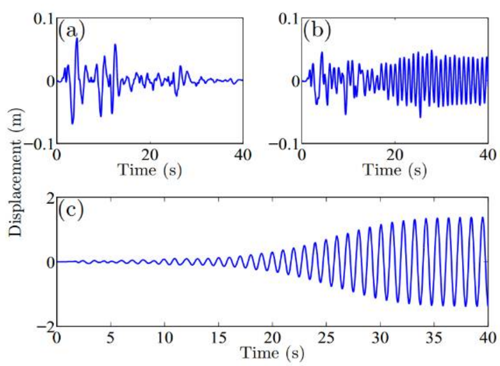

Fig.3. Displacement of the structure subjected to El-Centro; (a) controlled with SMC, (b) controlled with ASMC, (c) uncontrolled.

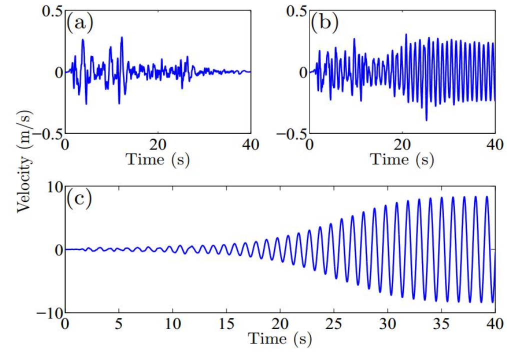

Fig.4. Velocity of the structure subjected to El-Centro; (a) controlled with SMC, (b) controlled with ASMC, (c) uncontrolled.

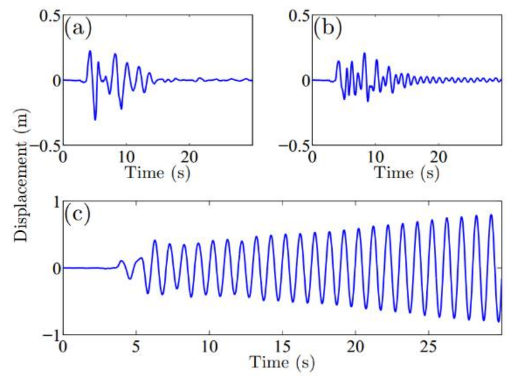

Fig.5. Displacement of the structure subjected to Newhall; (a) controlled with SMC, (b) controlled with ASMC, (c) uncontrolled.

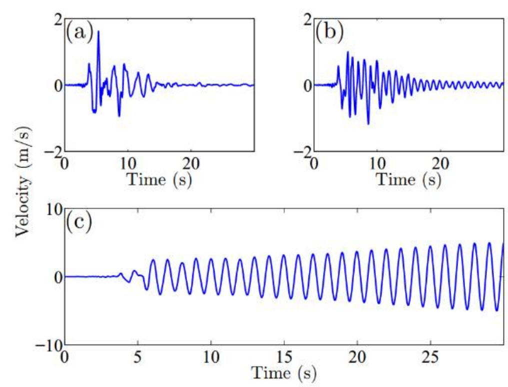

Fig.6. Velocity of the structure subjected to Newhall; (a) controlled with SMC, (b) controlled with ASMC, (c) uncontrolled.

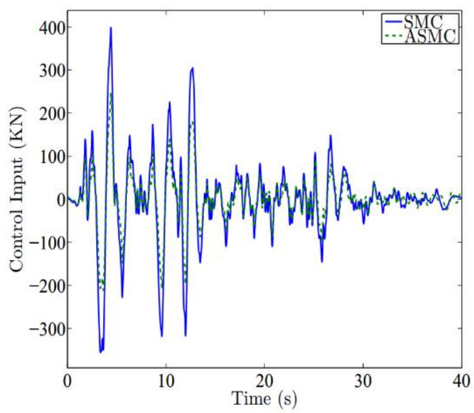

Fig.7. Control Force (KN), when the structure is subjected to El-Centro earthquake.

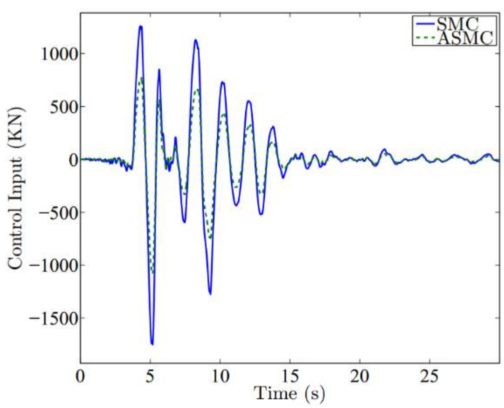

Fig.8. Control Force (KN), when the structure is subjected to Newhall earthquake.

5- Conclusion

An active controller was presented in this paper to mitigate the effects of earthquake on structures. The nonlinear dynamics of a SDOF structure were studied and its nonlinearity was modeled using Bouc-Wen equation. In practice, the parameters of the structure and Bouc-Wen model are estimated by identification methods and as a result, some uncertainties got involved into the parameters. The structure studied here was the one which its parameters as well as the coefficients of Bouc-Wen equation contain uncertainty. The proposed sliding mode controller proved the stability of the closed loop system. Also an adaptive sliding mode controller was proposed to cope with the structural control when the upper bounds of the parameter uncertainties are not identified. A simulation is carried out using records of well-known El-Centro and Newhall earthquakes. The results showed that both presented methods can efficiently reduce the displacement and velocity of the nonlinear structure.

Appendix A

The author(s) can insert an appendix with a meaningful title here. In equation $v = \frac{-\beta}{1-\kappa}\text{sgn}(s)$, the sign function in the sliding mode method causes to chattering in the control diagram. For solving this problem in simulation, we use the following procedure to eliminate this defect:

Sliding mode control of uncertain parameters and external disturbances is known to be resistant. A switching function is used in the control law which causes to control signals chattering. To reduce chattering it can be presented a boundary layer around the sliding surface. Inside boundary layer, there is a discontinuous switching function with a continuous function to avoid discontinuity of control signals. However, different choices of the boundary layer width lead to contradictory effects including small / large boundary layer that causes to more or less effectively reduction of chattering phenomenon, but the results are more or less precise controlled.

Proposing mode dependent boundary layer can effectively reduce chattering while ensuring the control precision simultaneously [16]. We use the following relationships for designing sliding variable:

$$\dot{\upsilon} = x_1 \quad or \quad \upsilon = \int_0^t x_1 d\tau \tag{26}$$

$$s = Cx + c_0\upsilon, \quad C = [c_1, c_2, ..., 1] \tag{27}$$

$$= x_n + c_{n-1}x_{n-1} + ... + c_1x_1 + c_0\int_0^t x_1 d\tau$$

$$= x_1^{(n-1)} + c_{n-1}x_1^{(n-2)} + ... + c_1x_1 + c_0\int_0^t x_1 d\tau \tag{28}$$

Where the coefficients $c_i s$ are chosen so that the differential equation (28) is stable. The purpose of adding the integral in (28) is for the particular case when the system is $n=1$. Differential equations (27) and (28) can be rewritten in state space as follows:

$$\dot{z} = Fz + Gs, \; where \; z = \begin{bmatrix} \int_0^t x_1 d\tau \\ x_1 \\ . \\ . \\ x_{n-1} \end{bmatrix} \in R^n \tag{29}$$

Matrix F and G in the canonical controller form are:

$$F = \begin{bmatrix} 0 & 1 & . & . \\ . & 0 & 1 & . \\ . & . & . & . \\ -c_0 & . & . & -c_{n-1} \end{bmatrix} \in R^{n\times n}, \quad G = \begin{bmatrix} 0 \\ . \\ 0 \\ 1 \end{bmatrix} \in R^n \tag{30}$$

Where the differential equation (28) is table choosing of constant coefficients $c_i s$, the matrix F in (29) is stable.

There are several results about (29) that will be used repeatedly in later sections.

Firstly, due to the stable matrix F in (30), there are a positive constant m and α such that there

$$\left\| e^{F(t-\tau)} \right\| \le me^{-\alpha(t-\tau)} \qquad \forall t \ge \tau \tag{31}$$

α is determined as a control parameter with value by selection of $c_i s$ in (28).

Secondly, with respect to any positive constant $\sigma \ge -\mathrm{Re}[\lambda_i(\mathrm{F})] > 0$ for all i that $\mathrm{Re}[\lambda_i(\mathrm{F})]$ represents the real part of the eigen values F, there are positive definite matrix $P \in R^{n\times n}$ that implies the Liapanove inequality:

$$(-F-\sigma I)^T P + P(-F-\sigma I) \le 0, \quad \sigma \ge -\mathrm{Re}[\lambda_i(F)] > 0 \qquad \forall i \tag{32}$$

Finally, this case is applied in (29)

$$z(t) = e^{Ft}z(0) + \int_0^t e^{F(t-\tau)}Gs(\tau)d\tau \tag{33}$$

In the design of the control input, the steady-state equation (29) shows that if the sliding variable s can be led to zero by some control designs the state is reduced to zero; therefore, the choice of control law in sliding mode is with “switching / sign” function for reaching to zero that switching function equals to:

$$f_0(s) = \mathrm{sgn(s)} = \begin{cases} 1, & s>0 \\ -1, & s<0. \end{cases} \tag{34}$$

In the practical implementation of control and switching, incomplete switching of discontinuous function $f_0 = \mathrm{sgn}(s)$ is the cause of chattering in the control signals. This chattering may cause damage to the actuator or high-frequency non-modeled dynamic stimulation. To reduce this problem, a boundary layer is proposed around the sliding surface $s = 0$ for smoothing (large boundary layer width) of control signal.

Instead of using a discontinuous function $f_0 = \mathrm{sgn}(s)$ is replaced by u in a continuous function:

$$f_1(s) = \frac{s}{|s| + \varepsilon_0 e^{-\pi t}}, \quad \sigma > \pi \geq 0, \varepsilon_0 > 0 \qquad (35)$$

Where $\varepsilon_0 e^{-\pi t}$ is large boundary layer width that is exponential drop to zero when $\pi \neq 0$ and $\pi = 0$ is held constant. The significant point is that the use of larger boundary width can be effective in reduction of chattering phenomenon in the control force, in sliding mode method. By applying a continuous function (35) in the control law, chattering is removed in sliding mode control design in control diagram.

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