Control of Flexible Link Robot using a Closed Loop Input-Shaping Approach

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Abstract

This paper is has addressed the Single Flexible Link Robot. The dynamical model is derived using Euler-Lagrange equation and then a proper controller is designed to suppress a vibration based-on Input-Shaping (IS) method. But, IS control method is an open loop strategy. Due to the weakness of open loop control systems, a closed loop IS control system is proposed. The achieved closed loop control system becomes an input delay system. To control this delay system, a robust linear state feedback with proper gain matrix is designed based-on an LMI method. Finally, the simulation results are illustrated to verify closed loop control system behavior.

Keywords: Flexible link robot, Input-Shaping control method, Linear Matrix Inequality (LMI), Delay system

1- Introduction

Most current mechanical link robots in the industry are designed and manufactured in a way that the mechanical strength of the link is maximum. In this way, the links are rigid and inflexible and therefore the bend and mechanical vibrations of the links and the final administrators (tools) are minimized and an appropriate precision is created in locating and directing the tools. To supply for this amount of mechanical rigidity, a very heavy mass of materials (metals in most cases) should be used to manufacture links and bonds. Therefore, using this approach will lead to high robot manufacturing costs and additionally, the robots will become heavy and giant. Robot heaviness not only causes high construction costs, but also it will lead to inconsistency regarding performance speed and energy consumption amounts [1-4]. The limitations of heavy and traditional robots have provoked scholars to consider the difficulties in rigidity of the links in smaller robots but this will lead the links to be flexible. The flexibility in robot links causes mechanical bend and vibrations and thus the precision will be reduced in locating and positioning [4-6]. Flexible link robots are faster than the traditional heavy robots and can do much more maneuvers. Also the cost of construction and power consumption is less in such robots due to the small size of these robots and using smaller stimulators. But, along these advantages, flexible robots have a big disadvantage. The great disadvantage of big flexible robots is their mechanical vibrations and this is due to the presence of flexibility and smallness of the rigidity of the links [6-8].

The arm link is thin and light in a flexible robot and the load on the tip of the link should be directed though it. Due to the use of a thin and light link in such a robot, the arm link is flexible. Therefore, the creation of torques by the motor and arm rotation will cause the arm to bend and the robot will encounter link vibrations while the load is being moved and the performance goals of the robot will be challenged. In order to utilize this robot, we should remove vibration problem. Thus, in designing control systems for such robots we should notice the link flexibility and extract a precise dynamic model and use it through the process of control system design. Below we will deal with the extraction of flexible robots' model.

To utilize flexible robots efficiently we should consider the mechanical vibrations in flexible arm robot links in an appropriate control form. Therefore, it is very important to notice active and passive control of the vibrations in such robots. The passive control method does not require certain equipments or software in comparison with active control methods. Meanwhile, practically when we use an active control method a control loop with several equipments should be utilized [9]. To control a robot of one degree of freedom with a single link, several control methods were proposed based on passive and active methods. In reference [10], an overall review of the previous works has been represented regarding flexible arm robots' control methods. To control the flexible robot, four control targets have been introduced [10]: the final administrator position regulation issue, final administrator location at an optimal time issue, tracing with least vibrations, and final administrator route tracing issue. To do so, several control methods were proposed as: proportionate derivative (PD) control method [11, 12], Lead-Lag-PID control method [13, 14], linearization feedback control method [15, 17], adaptive control [18, 19], neural network based control methods [20, 21], single chaotic control method [16, 22, 23], sliding mode control method [5, 24, 25], consistent reverse dynamic control method within the realm of time and frequency, optimal control methods [7, 26, 28], resistant control methods [29, 30], input signal shaping method [4, 31, 32].

Input-shaping method is one of order production (order formation) methods that can have many functions effectively and appropriately. Input-shaping method can be operationalized in immediate functions to achieve a proper consistency level. Input-shaping is designed to reduce or omit vibrations resulted from effects of inputs. On the whole, if the input of a flexible system is stimulated by a reference order, the result in the output would be small vibrations. If the natural frequencies and the vibration system fatality coefficient is determined, we can change any reference order in a way that the residual vibrations in the system response would be less or it would disappear completely. The preliminary research resulting in the development of input-shaping control method was first proposed by Smith in 1957 [33]. Additionally, John Calvert posed a vibration filter based on time delay [34]. After that a zero vibration and derivative shaper (ZVD shaper) was proposed for the first time [35]. In other projects input-shaping design for multiple vibration mode systems was considered [36-39]. Furthermore, several studies were carried out to design input-shaping for nonlinear systems [40-42]. In other research projects, the function of input-shaping to control systems with variable parameters has been recognized [43]. Due to the simplicity of the implementation of input-shaping method structure, this method has been successfully used in many functions [44-51].

Regarding the simplicity and the efficient effects of delay control methods such as input-shaping control method, we have used input-signal-shaping control method in this research. Input-shaping control method has several weak points and many of such weak points are due to open loop strategy of the control strategy. Thus, several research projects have been proposed to use feedback and to create control loops to reduce problems with this method [31-52]. Here we try to propose an appropriate closed loop control input-shaping control method using status feedback in order to control the robot with a single-link degree of freedom one. To achieve a consistent control loop we will use a linear matrix inequality (LMI) method.

2- A flexible single link robot modeling

In this section, an analytic model of a single-link flexible arm robot will be presented in the form of a status area through a finite set of natural modes. This model will be used in forthcoming sections to do simulation, frequency analysis, and also to design the control systems. On the whole, two models could be used in analysis and design of flexible link robots' control: permanent link model based on the descriptions of partial derivatives deferential equations (PDE) [8, 53] and compressed constituent model based on ordinary deferential equations (ODE) with limited degrees of freedom [30, 54, 55]. Here the integrals and permanent functions are extended before using Hamilton rule or Euler-Lagrange function extension method. Therefore, during this trend we will extract ODE equations instead of PDE equations. To model the flexible arm robot, we suppose that the arm link could be rotated by an electric motor in the connection point and the link has a fixed position in the second connection part. Due to the presence of flexibility, it can be supposed that the link can be bent, but its rigidity is so much that the link can not have twirling or changes in length and also it is considered that the link end is free and there exists a package with a weight of m in the end of the link. Here we presuppose that the arm link and the package have inertia Torque of I_b and I_p, respectively, and the inertia Torque of the router set, shaft, and the roller is equal to I_r. Also the link length is equal to L, the length mass capacity is equal to ρ and the package weight at the end of the link is equal to m_p and it is presupposed that all these parameters are fixed.

To describe the movement dynamic behaviors of the flexible arm robot it would be necessary to define proper coordinate frameworks. In figure (1), the descriptive frameworks of the flexible robot have been represented. Also the required variables for describing movement behaviors can be observed. The framework {0}, is the reference and fixed framework. The framework {1} refers to the arm body framework and it is fixed in arm muscles. But they rotate regarding the framework {0} and axis Z of Z0 by the variable of degree of freedom θ. The degree of freedom of θ is stimulated by a control torque τ using an electric motor. Due to the flexibility feature of the arm, the arm link can be bent or have vibrated movements within its own body framework or within the framework {1}. To describe the link's vibration movements, the latitudinal vibrations variable has been defined as w (x, t) in a way that the dependent latitudinal vibrations are defined through position x and time t.

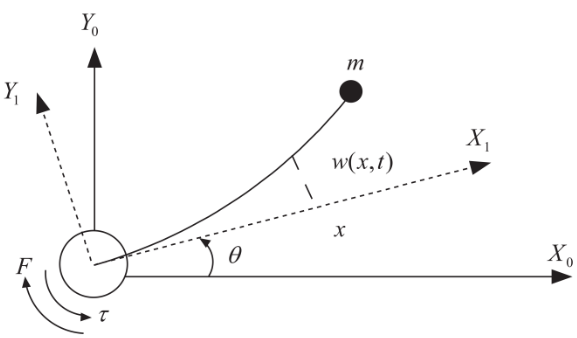

Fig.1. The degree of freedom and the definition frameworks for a flexible single-link arm robot

All movements in this system entail link rotary movements and latitudinal vibration movements and are represented as θ(t) and w (x, t), respectively. The position of x on the link is as follows regarding the framework {0}:

$$\vec{p}(x,t) = \begin{bmatrix} x\cos\theta(t) - w(x,t)\sin\theta(t) \\ x\sin\theta(t) + w(x,t)\cos\theta(t) \end{bmatrix} \qquad (1)$$

To simplify the sinus functions we used $S_\theta = \sin\theta$, and $C_\theta = \cos\theta$. The velocity of point x can be calculated as follows:

$$\dot{\vec{p}}(x,t) = \begin{bmatrix} -(x\dot{\theta} + w_t)S_\theta - w\dot{\theta}C_\theta \\ (x\dot{\theta} + w_t)C_\theta - w\dot{\theta}S_\theta \end{bmatrix} \qquad (2)$$

Using the velocity vector, the kinetic energy of the system can be calculated as follows:

$$E_k = \frac{1}{2}I_r\dot{\theta}^2 + \frac{1}{2}\int_0^\ell \rho\big(x^2\dot{\theta}^2 + w_t^2 + 2w_t x\dot{\theta} + w^2\dot{\theta}^2\big)dx \qquad (3)$$

Also the potential energy could be calculated using the equation below:

$$E_p = \frac{1}{2}EI\int_0^\ell w_{xx}^2(x,t)dx \qquad (4)$$

This energy is elastic due to the bend energy. In this equation, there is not any ground gravity potential energy because the movement of the arm is within the plate and the arm rotation axis is along with the axis z and ground gravity vector. We can use a Lagrange function and utilize Euler-Lagrange formula to calculate system dynamic. Lagrange function and Euler-Lagrange formula are as follows:

$$\frac{d}{dt}\left[\frac{\partial\mathcal{L}}{\partial\dot{q}}\right] - \frac{\partial\mathcal{L}}{\partial q} = W\tau \qquad (5)$$

Where, $\mathcal{L} = E_k - E_p$ is the system Lagrange function. To use the formula above we define the generalized degrees of freedom vector to express q. First, the latitudinal vibrations movement is analyzed as follows:

$$w(x,t) = \sum_{i=1}^{n}\phi_i(x)\eta_i(t) = \phi^T\eta \qquad (6)$$

Where, the vector is related to the generalized degrees of freedom vector to express:

$$\eta = [\eta_1 \quad \eta_2 \quad \cdots \quad \eta_n]^T \in R^n \\ \phi = [\phi_1 \quad \phi_2 \quad \cdots \quad \phi_n]^T \in R^n \tag{7}$$

Therefore, the vector for the whole system freedom degrees can be defined as follows:

$$q = [\theta \quad \eta^T]^T \in R^{n+1} \\ = [\theta \quad \eta_1 \quad \eta_2 \quad \cdots \quad \eta_n]^T \tag{8}$$

Using the expansion based on equation (6), to calculate the potential energy functions and the kinetic energy and through the use of Euler-Lagrange formula, we can calculate the dynamic of this system in the form of second order ODE equations and in the form of matrixes as follows:

$$M(q)\ddot{q} + C(q,\dot{q}) + K(\dot{q})q = W\tau \tag{9}$$

Where, the matrixes M and K represent inertia matrix and system rigidity, respectively and are shown as below. The dynamic model above is a nonlinear model because the matrixes for the model mentioned depend on q and q':

$$M(q) = \begin{bmatrix} m_{11}(\eta) & m_{12} \\ m_{12}^T & m_{22} \end{bmatrix} \\ \in R^{(n+1)\times(n+1)} K(q) \\ = \begin{bmatrix} 0 & 0_{1\times n} \\ 0_{n\times 1} & k_{22}(\dot{\theta}) \end{bmatrix} \\ \in R^{(n+1)\times(n+1)} C(q,\dot{q}) \\ = \begin{bmatrix} c_1(\eta,\dot{\eta},\dot{\theta}) & 0_{1\times n} \end{bmatrix}^T \\ \in R^{(n+1)\times 1} W = [1 \ \ 0_{1\times n}]^T \\ \in R^{(n+1)\times 1} \tag{10}$$

Where, the sub-matrixes are as follows due to the fact that is the natural frequency related to the mode:

$$m_{11} = I_r + I_b + \rho\ell\eta^T\eta m_{22} \\ = diag[\rho\ell \quad \rho\ell \quad \cdots \quad \rho\ell] \in R^{n\times n} m_{12} \\ = [\mu_1 \quad \mu_2 \quad \cdots \quad \mu_n] \in R^{1\times n} \ k_{22} \\ = diag\left[\rho\ell\left(\omega_i^2 - \dot{\theta}^2\right)\right]_{i=1}^n \in R^{n\times n} \\ \mu_i = \rho\int_0^\ell x\phi_i(x)dx \, c_1 = 2\rho\ell\dot{\theta}\eta^T\dot{\eta} \tag{11}$$

Model (9) is a nonlinear model and to simplify the model we presuppose that the velocity of changes in freedom degree of θ is small and it is considered that $\dot{\theta} \approx 0$.

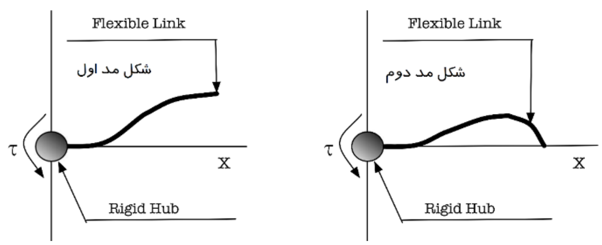

Fig.2. First and second mode shape functions

Through defining the vector for status variables in the form of $= [q \ \ \dot{q}]^T \in R^{2(n+1)}$, model (10) can be rewritten as follows:

$$\dot{X} = AX + Buy = CX \tag{12}$$

Where, the matrixes for status area form are as follows:

$$A = \begin{bmatrix} 0 & I \\ 0 & -M^{-1}K \end{bmatrix}; B = \begin{bmatrix} 0 \\ W \end{bmatrix} C \\ = [1_{1\times(n+1)} \ \ 0_{1\times(n+1)}] \tag{13}$$

To calculate the items in matrixes above we should define the basic mode shape functions of $\phi_i(x)$. Ofcourse, these functions should have certain characteristics. Here we used mode shape functions below:

$$\phi(x) = [\phi_1(x) \quad \phi_2(x)]^T \in R^2 \phi_1(x) \\ = 3\left(\frac{x}{\ell}\right)^2 - 2\left(\frac{x}{\ell}\right)^3 \phi_2(x) \\ = \left(\frac{x}{\ell}\right)^2 - \left(\frac{x}{\ell}\right)^3 \tag{14}$$

The graphs of shape functions above are represented in figure (2).

3- Designing flexible arm robot control systems

Controlling flexible systems is specifically important because one of the major challenges is to control vibrations in flexible systems. To limit the amount of vibrations in flexible mechanical systems we can consider three common methods: (1) creating slow movements, (2) using smart movement orders, and (3) using feedback control with a certain controlling structure in a way that we are sure that the vibration dynamics are not stimulated in a flexible system. In many functions, we consider fast movements. Thus, the first approach is not appropriate. Using the second method we try to delete vibration dynamic stimulators and thus we can utilize certain filters to do so. Based on the third approach, several control methods with closed loop structures have been proposed. The major goal of this research is to use and integrate the second and third approaches. The second approach has several weak points and many of these weaknesses are due to the open loop feature of the approach. Thus, we can use feedback and create control loops to try to reduce the weaknesses above. Here an input-shaping control of a closed loop control seems appropriate to control the robot with a single-link degree of freedom.

3-1 ZVD Input shaping control method

In input shaping technique, the linear integration of total delay signals of the main entry is calculated to apply with the vibration system. Suppose that r(t) is the major signal and $r_{sh}(t)$ is the shaped signal. On the whole, based on input-shaping method, through integration of the delay signals of the main entry, the signal is shaped and is produced as follows:

$$r_{sh}(t) = \sum_{k=0}^{N} A_k r(t - t_{d_k}) \qquad (15)$$

To normalize and calculate a unique resolution, the following conditions on A_k are taken into consideration:

$$\sum_{k=0}^{N} A_k = 1,\ A_k \geq 0, k = 0,1,2,3,\ldots \qquad (16)$$

To determine the appropriate amount for the parameter of the size A_k and the delay parameter t_{dk} we should have a precise knowledge of the natural frequencies of vibration dynamics. We can calculate input shaper actuator using the transformation function of $G_{sh}(S)$ as follows.

$$R_{sh}(S) = G_{sh}(S)R(S) = \sum_{k=0}^{N} A_k e^{t_{d_k} S} R(s) \qquad (17)$$

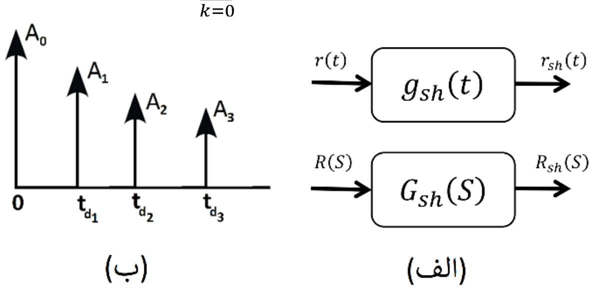

Fig.3. a- input shaping actuator in the range of time and Lapillus; b- hit sequence responding input shaping actuator hit

Therefore, the transformation function and input shaping actuator hit will be as follows:

$$G_{sh}(S)=\sum_{k=0}^{N}A_k e^{t_{d_k}S}\,g_{sh}(t)=\sum_{k=0}^{N}A_k\delta(t-t_{d_k}) \quad (18)$$

In figure (3), the input shaping actuator and its hit response have been represented. As it can be observed, the response function of input shaping actuator hit includes a series of hit sequences. If r(t) is considered as an optimal order input, the calculations related to input shaping and its revision within the realm of time can be carried out through Canolosion integral of $g_{sh}(t)*r(t)$ as follows:

$$r_{sh}(t)=\int_0^{+\infty}g_{sh}(\tau)r(t-\tau)d\tau \quad (19)$$

ZVD is one of the input shaping methods. As it was observed above, the parameters of input shaping control are A_k and t_{dk}. ZVD input shaping control actuator is as follows:

$$g_{ZV}(t)=A_0\delta(t)+A_1\delta(t-t_{d_1})+A_2\delta(t-t_{d_2}) \quad (20)$$

Suppose that this vibration system has a vibration dynamic with a natural frequency of ω_n and collapse coefficient of ξ. Thus, the natural frequency could be alive and the liveliness of the vibration dynamic will be equal to $\omega_d=\omega_n\sqrt{1-\xi^2}$ and $\sigma=\xi\omega_n$, respectively. Through enforcing the sequence of shaping actuator hits, the residual vibrations responding the vibration system could be calculated as follows:

$$V(\xi,\omega_n)=e^{-\sigma t_N}\sqrt{C^2(\xi,\omega_n)+S^2(\xi,\omega_n)} \quad (21)$$

Where,

$$C(\xi,\omega_n)=\sum_{k=0}^{N}A_k e^{\sigma t_{d_k}}\cos(\omega_d t_{d_k})\quad S(\xi,\omega_n)=\sum_{k=0}^{N}A_k e^{\sigma t_{d_k}}\sin(\omega_d t_{d_k}) \quad (22)$$

In ZVD shaping method, the following performance conditions are taken into consideration [35]:

$$V(\xi,\omega_n)=0\quad \frac{d}{d\omega}V(\xi,\omega)\Big|_{\omega=\omega_n}=0 \quad (23)$$

Through resolving the equations above and also considering condition (14), the matrix for ZVD shaping parameters defined in the form of M_{ZVD} will be represented as follows:

$$M_{ZVD}=\begin{bmatrix}0 & t_{d_1} & t_{d_2}\\ A_0 & A_1 & A_2\end{bmatrix}=\begin{bmatrix}0 & T/2 & T\\ \frac{\gamma^2}{\gamma^2+2\gamma+1} & \frac{2\gamma}{\gamma^2+2\gamma+1} & \frac{1}{\gamma^2+2\gamma+1}\end{bmatrix} \quad (24)$$

In equations above, we have $T=2\pi/\omega_d$ and $\gamma=e^{\xi\pi/\sqrt{1-\xi^2}}$. Where, ξ and ω_d represent survival coefficient and vibrating frequency.

3-2 Closed loop input shaping control method

Feedback controlling is one of most successful strategies because using system behavior feedbacks and the creation of control loops have been able to be used in different applications and it has always been noticed by the researchers. Here a control system based on figure (4) is utilized in a way that input shaping actuator is located within this control loop. According to this control structure, the consistency of the performance of the input shaping actuator will improve and additionally, the effects of the noises can be reduced.

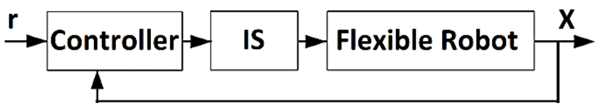

Fig.4. Closed loop control system including an input shaping actuator

Consider status (12) of location model. If we add ZVD input shaping actuator, the status location model will be as below:

(25) $$\dot{X} = AX + B_0u(t) + \sum_{k=1}^{2} B_k u\left(t - t_{d_k}\right)\; y = Cx$$

Where, $B_k = BA_k$ and the matrixes of A_k and delay times of t_{dk} are calculated based on equation 24. A linear status feedback control for the delay system could be proposed as follows:

(26) $u(t) = KX(t)$

Therefore, the dynamic for the closed loop system would be:

$$\dot{X} = \tilde{A}X + \sum_{k=1}^{2} \tilde{B}_k X\left(t - t_{d_k}\right) \tag{27}$$

Where, $\tilde{A} = A + B_0K$ and $\tilde{B}_k = B_kK$ are equal. Consider the new variable of $z(t) = E_1x(t) + E_2u(t)$. If the optimal amount for the status feedback is determined in a way that the following cost function is the minimum, the closed loop control system would be consistent.

$$J(x,K) = \int_0^\infty z^T(t)z(t)dt \tag{28}$$

First the matrixes R and R_1 are defined as $R_1 = E_1^T E_1$ and $R = \tilde{E}^T\tilde{E}$, where we have $\tilde{E} = E_1 + E_2K$.

Premise: if there are a positive finite P matrix and the positive amounts of ϵ, α_k, for k = 1,2, we will have:

$$\tilde{A}^TP + P\tilde{A} + \epsilon P + \sum \alpha_k^2 K^T R_2 K + \sum \alpha^{-2} P B_k R_2^{-1} B_k^T P + R < 0 \tag{29}$$

Where, the following function will be a candidate Lyapanov function:

$$V(X) = X^TPX + \sum \alpha_k^2 \int_{t-t_{dk}}^{t} X^T(\tau)K^TR_2KX(\tau)d\tau \tag{30}$$

Therefore, the closed loop control system will have a concurrent consistency with status feedback controller (26).

Consider the definitions for the following matrixes:

(31) $\hat{R}_1 = \sum \alpha_k^2 R \quad ; \quad \hat{R}_2 = \sum \alpha_k^{-2} B_k R_2^{-1} B_k^T$

Thus, we can rewrite the inequality (29) as below:

$$\tilde{A}^TP + P\tilde{A} + \epsilon P + K^T\hat{R}_1K + P\hat{R}_2P + \tilde{E}^T\tilde{E} < 0 \tag{32}$$

As we multiply the matrix P^{-1} in both sides of the inequality and utilize the definition matrixes below:

$$L = P^{-1} \quad ; \quad Y = KL\hat{A} = AL + BY \quad ; \quad \hat{E} = E_1L + E_2Y \tag{33}$$

And use Schur integrated matrix inequality, we can rewrite the inequality (32) as follows:

$$\begin{bmatrix} \hat{A}+\hat{A}^T+\epsilon L+\hat{R}_1 & Y^T & \hat{E}^T \\ \times & -\hat{R}_2^{-1} & 0 \\ \times & \times & -I \end{bmatrix} < 0 \qquad (34)$$

If we have the positive definite matrixes of L and Y for the positive amounts of ϵ, α_k, where there exists a matrix inequality (34), the matrix $P = L^{-1}$ and status feedback control (26) with the matrix $K = YL^{-1}$ could guarantee the consistency of the closed loop control system.

4- Results of simulations and conclusion

Here we have represented the results of simulations to assess the flexible robot model and to assess the proposed controller. To carry out the simulation, the amounts of the parameters were considered to be $EI = 2.4$, link mass longitudinal capacity was equal to $\rho = 0.265\frac{kg}{m}$, and the mass of link tip was equal to $m = 0.3\ kg$, also the inertia torque of shaft-rotor related to the motor was considered to be equal to $0.8\ kg.m^2$. The reference amount for the arm's rotary angle has been decided to be equal to $0.5\ rad$. To present the results of simulations, the outputs, arm joint rotation angle, arm tip movement rotation angle, and also the produced torques by the controllers were measured. As it was observed, the joint angle movement and also the link tip angular movement within about 5 seconds could reach the amount of the reference with a smooth movement. Additionally, the signal of the produced torque has also been smooth and electric motors could produce it.

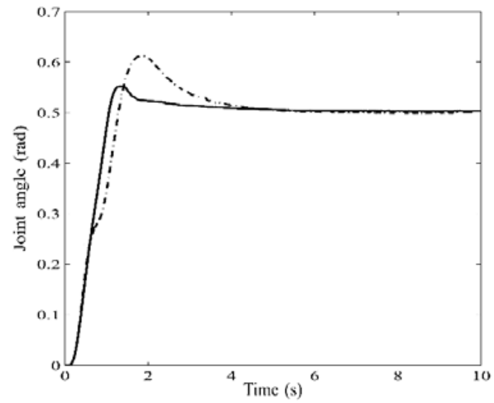

Fig.5. Joint angle movement after the application of the controller and comparing the performance of the controller regarding m = 0 kg and m = 0.3 kg

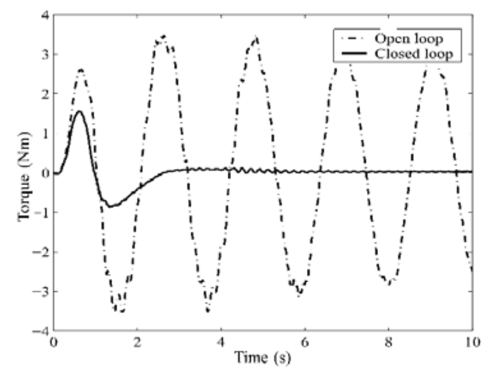

Fig.5. Torque in open loop status and control loop

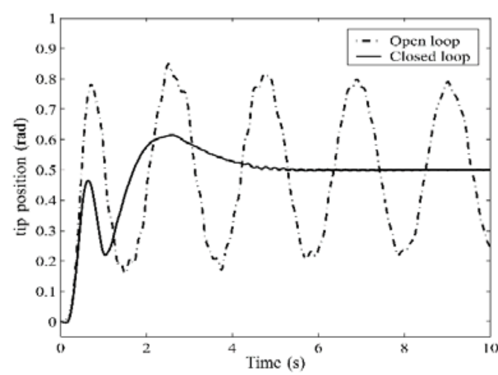

Fig.7. arm tip movement rotation angle in open loop status with a control loop including input shaping

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