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# Nonlinear $H_\infty$ Control for Uncertain Flexible Joint Robots with Unscented Kalman Filter

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## ABSTRACT

*Todays, use of combination of two or more methods was considered to control of systems. In this paper is presented how to design of a nonlinear $H_\infty$ (NL-$H_\infty$) controller for flexible joint robot (FJR) based on bounded UKF state estimator. The UKF has more advantages to standard EKF such as low bios and no need to derivations. In this research, based on spong primary model for FJRs, same as rigid robots links position are selected as differential equations variables. Then this model was reformed to $\boldsymbol{NL-H_\infty}$ differential equations. The results of simulations demonstrate that mixed of $\boldsymbol{NL-H_\infty}$ controller and UKF estimator lead to conventional properties such as stability and good tracking. Also, Simulation results show the efficiency and superiority of the proposed method in compare with EKF.*

## 1. INTRODUCTION

The control of link position for flexible manipulators is considered by control researchers [2]. This problem for rigid manipulators was solved in several approaches. But, for flexible robots which have uncertainties and disturbed input signal, there is no any suitable solution. Flexibility in this type of robots is emerged by mechanical elements. In most applications of flexible manipulators, for reach to simple controller, supposed which robot is rigid [2]. While this assumption is a preventive to fine control of link position, and also can generate unwanted behavior such as vibration. Nowadays, for controlling of new applications such as space manipulators, mechanical hands and human coworker robots which must be controlled high precision, rigid assumption can not become useful for flexibility description [3].

For many last years, using of harmonic drivers for backlash reduction was a usual method. This gearbox has the high stiffness characteristic, so that generates flexibility in robots manipulator's joint. This type of robots called Flexible Joint Robot (FJR) . The differential equations of FJR more complex than rigid's equations. Also, feedback linearization is not useful in this case (FJRs). In recent years, several researchers have proposed several approaches to control of FJR robots. These methods can be split to, two major groups, linear and nonlinear controllers. The other problem is uncertainties in FJR's describer parameters, such as unbalancing and asymmetrically , mass and length tolerances , nonlinearity of harmonic drive stiffness and other parameter changing in mechanical parts, that might generated with lapse [1,4].

To counter this problem, researchers were proposed a type of controllers, which called robust controller. These controllers are resistant against uncertainties and disturbance.

Nonlinear $H_\infty$ (NL-$H_\infty$) controller is one of the best robust controllers, because that has nonlinear structure [6]. NL-$H_\infty$ needs for full state of controlled system, therefore, for using of that must be prepared all internal states [5]. In FJRs, states are related to links and actuators position and velocities. More instrument equipments are need for measuring of full states. On the other hand, equipment addition can created complexity in mechanical structure . In practical applications can reduced number of sensors by using of state estimators.

Kalman Filter (KF) for estimating of states in linear systems is usual. But that is not convenient to FJRs, because FJRs have nonlinear behaviors and nonlinear equations. Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) are proposed for nonlinear systems such as FJR robots [12]. EKF cannot use for high precision control, because that has the extra bios, when as UKF with less bios and don't use of local linearization has more performance to EKF [8].

This paper has the four sections. In section 2, we have a discussion about FJRs equations. Then in section 3 uncertain nonlinear systems and preliminary of NL-$H_\infty$ controller are studied. The UKF with five sigma points is perused in next section. At the final section, NL-$H_\infty$ controller with UKF state s estimator are simulated on the single link FJR. For achieving to good tracking and least error of tracking with least overshooting at step response by NL-$H_\infty$ , penalty coefficients for link position and tracking error states are selected nonzero and also link position as measured output entered to UKF block.

## 2. UNCERTAIN NONLINEAR SYSTEMS

Based on spong primary model for FJRs, same as rigid robots links position are selected as differential equations variables. Moreover, actuators position as new variables are added to exiting variables. Hence, FJRs equations for n-links manipulator have 2n (n number of links) variables. Let $\boldsymbol{\theta_i} : \boldsymbol{i} = 1,2,...,\boldsymbol{n}$ show the i'th link position and $\boldsymbol{\theta_{i+n}} : \boldsymbol{i} = 1,2,...,\boldsymbol{n}$ show the i'th actuator position. $\boldsymbol{Q}$ is the vector of links position and actuators position as below:

$$Q = \left[\theta_1, \theta_2, ..., \theta_n, \theta_{n+1} | \theta_{n+2}, \theta_{n+3}, ..., \theta_{2n-1}, \theta_{2n}\right]^T = \left[q_1^T, q_2^T\right]^T \quad (1)$$

where $\bar{\boldsymbol{q}}_1$ is the vector of links position and $\bar{\boldsymbol{q}}_2$ is the vector of actuators position. Based on above assumptions, spong was proposed FJRs model as follows [9]:

$$\begin{aligned} &\boldsymbol{I}(\bar{\boldsymbol{q}}_1)\ddot{\bar{\boldsymbol{q}}}_1 + \bar{\boldsymbol{C}}(\bar{\boldsymbol{q}}_1, \dot{\bar{\boldsymbol{q}}}_1)\dot{\bar{\boldsymbol{q}}}_1 + \boldsymbol{G}(\bar{\boldsymbol{q}}_1) + \boldsymbol{K}(\bar{\boldsymbol{q}}_1 - \bar{\boldsymbol{q}}_2) = 0 \quad (2)\\ &\boldsymbol{J}\ddot{\bar{\boldsymbol{q}}}_2 - \boldsymbol{K}(\bar{\boldsymbol{q}}_1 - \bar{\boldsymbol{q}}_2) - \bar{\boldsymbol{u}} + \boldsymbol{B}\dot{\bar{\boldsymbol{q}}}_2 = 0 \end{aligned}$$

where $\boldsymbol{I}$ is the links inertia matrix, $\boldsymbol{J}$ is motors inertia matrix, $\bar{\boldsymbol{C}}$ is the vector of all gravitational, centrifugal and Coriolis torques and $\bar{\boldsymbol{u}}$ is the input vector torques. In this model we suppose stiffness of the i'th joints ($\boldsymbol{k_i} : \boldsymbol{i} = 1,2,...,\boldsymbol{n}$) has linear behavior such as linear spring. Thus, they can be created the diagonal positive define matrix $\boldsymbol{K}$.

## 3. $NL - H_\infty$ CONTROLLER

Suppose states of nonlinear plant are ready and those describe by following equations:

$$\begin{aligned} &\dot{\boldsymbol{x}}(\boldsymbol{t}) = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}_1(\boldsymbol{x})\boldsymbol{\omega} + \boldsymbol{g}_2(\boldsymbol{x})\boldsymbol{u}\\ &\boldsymbol{z}(\boldsymbol{t}) = \boldsymbol{h}_1(\boldsymbol{x}) + \boldsymbol{l}(\boldsymbol{x})\boldsymbol{u} \quad (3)\\ &\boldsymbol{y}(\boldsymbol{t}) = \boldsymbol{h}_2(\boldsymbol{x}) \end{aligned}$$

Where $x(t) \in R^n$ is the state with $x(0) = 0$, $z(t) \in R^s$ is the penalty output to be regulated. $y(t) \in R^p$ is the measured output. $u \in R^m$ is the control input, and $\omega$ denotes the plant disturbance *f (x)*, *h*1*(x)*, *h*2*(x)*, *l1(x)*,*g*1*(x)* and *g*2*(x)* are all known smooth mappings defined in $\boldsymbol{X}$ with *f (*0*)* = 0.
to simplify the analysis and to provide a reasonable expression of the controller, we make the following assumption about the nonlinear plan :

$$\begin{aligned} &\boldsymbol{h}_1^{\boldsymbol{T}}(\boldsymbol{x})\boldsymbol{l}(\boldsymbol{x}) = 0 \\ &\boldsymbol{l}^{\boldsymbol{T}}(\boldsymbol{x})\boldsymbol{l}(\boldsymbol{x}) = \boldsymbol{R}_2 \end{aligned} \tag{4}$$

Where $\boldsymbol{R}_2$ is a constant and nonsingular. Then control law can be written as follow:
$\boldsymbol{u} = \boldsymbol{k}(\boldsymbol{x})$
IN THIS EQUATION $\boldsymbol{k}(\boldsymbol{x})$ is a smooth function which satisfying $\boldsymbol{k}(0) = 0$.
This controller has two aimds, closed loop stability and to attenuate effect of external disturbances to penalty outputs ($\boldsymbol{z}(\boldsymbol{t})$). Closed loop stability means that the plant at $\boldsymbol{u} = 0$ with $\boldsymbol{w} = 0$ is asymptotically stable, And the disturbance attenuation means that the external disturbances are locally attenuate by a real and positive number such as $\gamma$ if there exists a neighborhood $\boldsymbol{X}$ of the point $x(0) = 0$ that satisfies blow condition:

$$\int_0^T z^T(s)z(s)ds \le \gamma^2 \int_0^T d^T(s)d(s)ds \tag{5}$$

to solve the $\boldsymbol{NL-H_\infty}$ problem**,** the Hamiltonian function is formed as below:

$$\begin{aligned} H_a(x,p,d,u) = p^T (f(x) \\ + g_1(x).d + g_2(x).u) \\ + \frac{1}{2}(\|h(x) + l(x).u\|^2 - \gamma^2\|d\|^2) \end{aligned} \tag{6}$$

Under assumption (6), the above Hamiltonian function can be written as follows:

$$\begin{aligned} H_a(x,p,d,u) = p^T f(x) + \frac{1}{2}h^T(x)h(x) + \\ [p^T g_1(x) \quad p^T g_2(x)]\begin{bmatrix} d \\ u \end{bmatrix} + \frac{1}{2}[d \quad u]R\begin{bmatrix} d \\ u \end{bmatrix} \end{aligned} \tag{7}$$

where $\boldsymbol{R} = \boldsymbol{diag}(-\gamma^2\boldsymbol{I}, \boldsymbol{R}_2)$ and $\boldsymbol{d}^* = \frac{1}{\gamma^2}\boldsymbol{g}_1^{\boldsymbol{T}}\boldsymbol{p}$ and $\boldsymbol{u}^* = -\boldsymbol{R}_2^{-1}\boldsymbol{g}_2^{\boldsymbol{T}}\boldsymbol{p}$ are selected as optimal inputs. Then with $\boldsymbol{p}^{\boldsymbol{T}} = \boldsymbol{V}_{\boldsymbol{x}}$ equation (7) become as following:

$$\mathbf{V_x f(x)} + \frac{1}{2}\mathbf{h}_1^{\mathbf{T}}\mathbf{h}_1 + \frac{1}{2}\mathbf{V_x}\left(\frac{\mathbf{g}_1\mathbf{g}_1^{\mathbf{T}}}{\gamma^2} - \mathbf{g}_2\mathbf{R}_2^{-1}\mathbf{g}_2\right)\mathbf{V_x^T} = 0 \tag{8}$$

if there exist a positive define function such as $\boldsymbol{V}(\boldsymbol{x})$ which satisfies above differential equation (8), then the control signal law can be written as following:
$\mathbf{u} = -\mathbf{R}_2^{-1}\mathbf{g}_2^{\mathbf{T}}(\mathbf{x})\mathbf{V_x^T}$

## 4. UKF STATE ESTIMATOR

In the nonlinear systems often aren't available some states, or measuring of those are difficult. Usual at this type of systems, state estimators can be useful. Kalman filter (KF) as a state estimator is used at linear systems. But KF hasn't good performance at systems with high nonlinearity. Instead, Extended Kalman Filter (EKF) can be useful for state estimating at such systems. But, EKF often has a high level bias for nonlinear systems. For solving this problem, a new approach for state estimating was proposed. Unscented Kalman Filter (UKF) is same new approach. UKF has several performance to EKF such as, low bios, don't need to derivations. Also this type estimator can be useful for work points which are farther from $x = 0$. Suppose discrete time equation of the nonlinear plant is as below:

$$\begin{aligned} X_{k+1} &= F(X_k, U_k) + v_k \\ \mathbf{Y_k} &= H(\mathbf{X_k}) + \mathbf{n_k} \end{aligned} \tag{9}$$

Where $X_k$ is state vector, $U_k$ is input vector and $\mathbf{Y_k}$ is measured output vector. $v_k$ and $\mathbf{n_k}$ are process and measuring noise, which assume are Gaussian noise. Standard UKF has the algorithm as follows:

Initial with:

$$\hat{\mathbf{X}}_0 = \mathbf{E}[\mathbf{X}_0] \quad ; \quad \mathbf{P}_0 = \mathbf{E}\left[\left(\mathbf{X}_0 - \hat{\mathbf{X}}_0\right)\left(\mathbf{X}_0 - \hat{\mathbf{X}}_0\right)^{\mathbf{T}}\right]$$

For $\mathbf{k} \in \{1, 2, ...., \infty\}$

Calculate Sigma points:

1: $\mathbf{X_{k-1}} = \left[\hat{\mathbf{X}}_{\mathbf{k}-1} \quad \hat{\mathbf{X}}_{\mathbf{k}-1} + \left(\gamma\sqrt{\mathbf{P_{k-1}}}\right) \quad \hat{\mathbf{X}}_{\mathbf{k}-1} - \left(\gamma\sqrt{\mathbf{P_{k-1}}}\right)\right]$

Time update:

2: $\mathbf{X}^{*}_{\mathbf{k|k}-1} = \mathbf{F}[\mathbf{X}_{\mathbf{k}-1}, \mathbf{U}_{\mathbf{k}-1}]$

3: $\hat{\mathbf{X}}^{-}_{\mathbf{k}} = \sum_{\mathbf{i}=0}^{2\mathbf{n}} \mathbf{W}^{\mathbf{m}}_{\mathbf{i}} \mathbf{X}^{*}_{\mathbf{i,k|k}-1}$

4: $\mathbf{P}^{-}_{\mathbf{k}} = \sum_{\mathbf{i}=0}^{2\mathbf{n}} \mathbf{W}^{\mathbf{c}}_{\mathbf{i}} \left[\mathbf{X}^{*}_{\mathbf{i,k|k}-1} - \hat{\mathbf{X}}^{-}_{\mathbf{k}}\right]\left[\mathbf{X}^{*}_{\mathbf{i,k|k}-1} - \hat{\mathbf{X}}^{-}_{\mathbf{k}}\right]^{\mathbf{T}} + \mathbf{R}^{\mathbf{v}}$

5: $\mathbf{X_{k|k-1}} = \left[\hat{\mathbf{X}}^{-}_{\mathbf{k}} \quad \hat{\mathbf{X}}^{-}_{\mathbf{k}} + \left(\gamma\sqrt{\mathbf{P}^{-}_{\mathbf{k}}}\right) \quad \hat{\mathbf{X}}^{-}_{\mathbf{k}} + \left(\gamma\sqrt{\mathbf{P}^{-}_{\mathbf{k}}}\right)\right]$

6: $\mathbf{Y_{k|k-1}} = \mathbf{H}[\mathbf{X_{k|k-1}}]$

7: $\hat{\mathbf{Y}}^{-}_{\mathbf{k}} = \sum_{\mathbf{i}=0}^{2\mathbf{n}} \mathbf{W}^{\mathbf{m}}_{\mathbf{i}} \mathbf{Y}_{\mathbf{i,k|k}-1}$

$\mathbf{R^v}$ = process noise covariance

Measurement updates equations:

8: $\mathbf{P}_{\hat{\mathbf{y}}_\mathbf{k}\hat{\mathbf{y}}_\mathbf{k}} = \sum_{\mathbf{i}=0}^{2\mathbf{n}} \mathbf{W}^{\mathbf{c}}_{\mathbf{i}} \left[\mathbf{Y}_{\mathbf{i,k|k}-1} - \hat{\mathbf{y}}^{-}_{\mathbf{k}}\right]\left[\mathbf{Y}_{\mathbf{i,k|k}-1} - \hat{\mathbf{y}}^{-}_{\mathbf{k}}\right]^{\mathbf{T}} + \mathbf{R}^{\mathbf{n}}$

9: $\mathbf{P}_{\mathbf{x_k y_k}} = \sum_{\mathbf{i}=0}^{2\mathbf{n}} \mathbf{W}^{\mathbf{c}}_{\mathbf{i}} \left[\mathbf{X}_{\mathbf{i,k|k}-1} - \hat{\mathbf{x}}^{-}_{\mathbf{k}}\right]\left[\mathbf{Y}_{\mathbf{i,k|k}-1} - \hat{\mathbf{y}}^{-}_{\mathbf{k}}\right]^{\mathbf{T}}$

10: $\mathbf{K_k} = \mathbf{P}_{\mathbf{x_k y_k}} \mathbf{P}^{-1}_{\hat{\mathbf{y}}_\mathbf{k}\hat{\mathbf{y}}_\mathbf{k}}$

11: $\hat{\mathbf{X}}_{\mathbf{k}} = \hat{\mathbf{X}}^{-}_{\mathbf{k}} + \mathbf{K_k}\left(\mathbf{y_k} - \hat{\mathbf{y}}^{-}_{\mathbf{k}}\right)$

12: $\mathbf{P_k} = \mathbf{P}^{-}_{\mathbf{k}} - \mathbf{K_k} \mathbf{P}_{\hat{\mathbf{y}}_\mathbf{k}\hat{\mathbf{y}}_\mathbf{k}} \mathbf{K}^{\mathbf{T}}_{\mathbf{k}}$

$\mathbf{R^n}$ = measurement noise covariance

**Algorithm 1:** Standard UKF algorithm

Where UKF parameters define as below:

$$\begin{aligned} &\mathbf{W}^{\mathbf{m}}_0 = \lambda/(\mathbf{n}+\lambda) \\ &\mathbf{W}^{\mathbf{c}}_0 = \lambda/(\mathbf{n}+\lambda) + (1-\alpha^2+\beta) \\ &\mathbf{W}^{\mathbf{c}}_{\mathbf{i}} = 1/\{2(\mathbf{n}+\lambda)\} \\ &\mathbf{i} = 1,2,...,2\mathbf{n} \\ &\mathbf{W}^{\mathbf{m}}_{\mathbf{i}} = 1/\{2(\mathbf{n}+\lambda)\} \\ &\mathbf{i} = 1,2,...,2\mathbf{n} \end{aligned} \tag{10}$$

and scaling parameters are $\lambda = \alpha^2(\mathbf{n}+\kappa) - \mathbf{n}$ and $\gamma = \sqrt{(\mathbf{n}+\lambda)}$.

In the above parameters $\alpha$ is a positive constant, which define sigma points distance from operating point ($1\mathbf{e}^{-4} < \alpha < 1$), and $\kappa$ is the second scaling constant. $\beta$ is disturbance constant ($\beta = 2$ is optimal value for Gaussian noise).

## 5. STATE SPACE/ FJR / MODELING

In this research a $NL-H_\infty$ controller is designed to attenuate disturbance and uncertainties effects and also having good tracking. Signal error is selected as $e = \int_0^t (x_1 - x_d)d\tau$, and also state vector as $\bar{x} = [\bar{q}_1 \;\; \dot{\bar{q}}_1 \;\; \bar{q}_2 \;\; \dot{\bar{q}}_2 \;\; \bar{e}]$. So FJR space state equations can be written as follows:

$$\begin{aligned}
\dot{\bar{x}}_1 &= \bar{x}_2\\
\dot{\bar{x}}_2 &= -I(\bar{x}_1)^{-1}\left(C(\bar{x}_1,\bar{x}_2)\bar{x}_2 + g(\bar{x}_1) + K(\bar{x}_1 - \bar{x}_3)\right)\\
\dot{\bar{x}}_3 &= \bar{x}_4\\
\dot{\bar{x}}_4 &= J^{-1}\left(B\bar{x}_4 - K(\bar{x}_1 - \bar{x}_3) - \bar{x}_4 k_m k_b R^{-1} + \bar{V}_m k_m R^{-1}\right)\\
\dot{\bar{x}}_5 &= \bar{x}_1 - \bar{x}_d
\end{aligned}$$

also, penalty function is preferred as $h(x) = [0 \;\; 0 \;\; 0 \;\; 0 \;\; Q\bar{x}_5]^T$. Usually in practical manipulators driver (DC motor) dynamic has a high frequency pole, which its effect can be canceled and only $k_m$, $k_b$, $R$ are spotted at final equations. Therefore, final equations are written based on below figure:

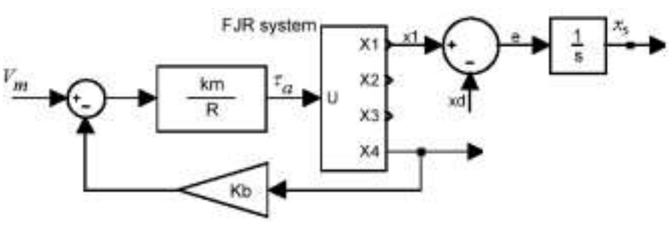


Fig.1. n-link FJR with n DC motors

To control FJR manipulator with $u = k(x)$ law is deeds for whole states of plant. In this research, it is supposed that measurable output is only link position vector ($\bar{x}_1$), that means $\bar{y} = \bar{x}_1$. So the other states must be obtained using of the UKF state estimator as figure (2).

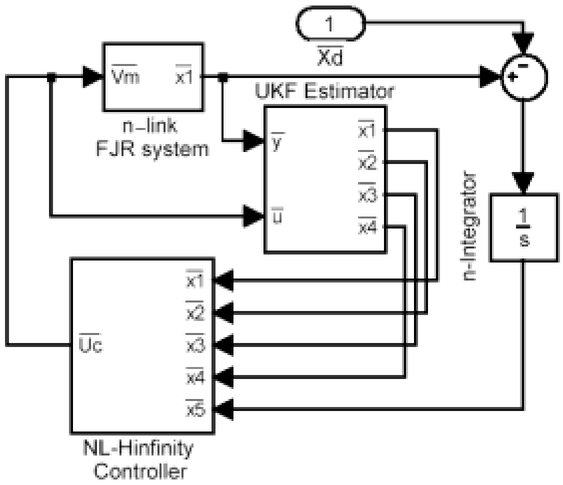


**Fig.2.** n-link FJR $NL-H_\infty$ **controller** with UKF estimator

$\bar{x}_5$ isn't an internal state of FJR. So UKF only creates $\bar{x}_1, \bar{x}_3, \bar{x}_3, \bar{x}_4$ vectors, based on FJR equations without $\dot{x}_5$ .

## 6. ILLUSTRATION EXAMPLE

to verify the efficiency of the discussed method , that simulated on the single-link FJR.

Space state equation of single-link FJR as follows:

$$\begin{aligned}
\dot{x}_1 &= x_2\\
\dot{x}_2 &= \frac{-MgL}{I}\sin(x_1) - \frac{k}{I}(x_1 - x_3)\\
\dot{x}_3 &= x_4\\
\dot{x}_4 &= \frac{k}{J}(x_1 - x_3) + \frac{1}{J}(R^{-1}k_m V_m - R^{-1}k_m k_b x_4 - bx_4)\\
\dot{x}_5 &= x_1 - x_d
\end{aligned}$$

link, DC motor and UKF estimator parameters are shown in Tables 1,2,3.

**Table1.** Arm parameters and tolerances

| Parameters | Nominal values | Tolerance |
|---|---|---|
| Mass | M=1 | 5% |
| Joint stiffness | K=100 | 12% |
| Length | L=1 | 3% |
| Gravity coefficient | g=9.8 | 1% |
| Inertia | I=1 | 4% |
| Motor inertia | J=1 | 3% |

**Table2.** DC Motor parameters and tolerances

| Parameters | Nominal values | Tolerance |
|---|---|---|
| Coil resistance | r=10 ohm | 4% |
| Coil inductance | l=0.1 H | 5% |
| Torque constant | km=9 | 3% |
| Back emf constant | kb=1 | 3% |
| Friction constant | B=.1 | 15% |
| | | |

**Table3.** UKF estimator parameters

| Parameters | values |
|---|---|
| $\beta$ | 2 |
| $\alpha$ | 1 |
| $\kappa$ | 0 |
| **n** | 4 |

So, we have:

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} \mathbf{x}_2 \\ -9.8\sin(\mathbf{x}_1) - 100(\mathbf{x}_1 - \mathbf{x}_3) \\ \mathbf{x}_4 \\ 100(\mathbf{x}_1 - \mathbf{x}_3) - \mathbf{x}_4 \\ \mathbf{x}_1 \end{bmatrix}$$

$$\mathbf{g}_1(\mathbf{x}) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} \quad \mathbf{g}_2(\mathbf{x}) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.9 \\ 0 \end{bmatrix}$$

and also penalty vector is selected as following:

$$\mathbf{z} = \begin{bmatrix} 60\mathbf{x}_1 \\ 0 \\ 0 \\ 0 \\ 300\mathbf{x}_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ (10)^{1/2} \\ 0 \end{bmatrix} .\mathbf{u} \qquad (11)$$

in (11), $60\mathbf{x}_1$ reduces step response overshoot and $300\mathbf{x}_5$ guarantees fast response to step command. It is well known, there is no analytically solution for control signal law in general for $NL-H_\infty$ , so that is calculated by numerical approach [7] order to 3 based on $u = Q_1.\mathbf{x} + \mathbf{Q}_3.\mathbf{x}^{[3]}$.

Results of simulation are shown at figures (3-7). At the all simulations, results of state measuring are compared with state estimating by UKF estimator.

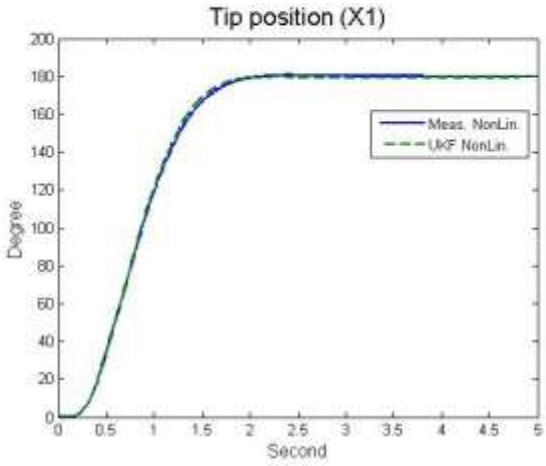


**Fig .3.**step response $\mathbf{x_d}=180'$ by $NL-H_\infty$

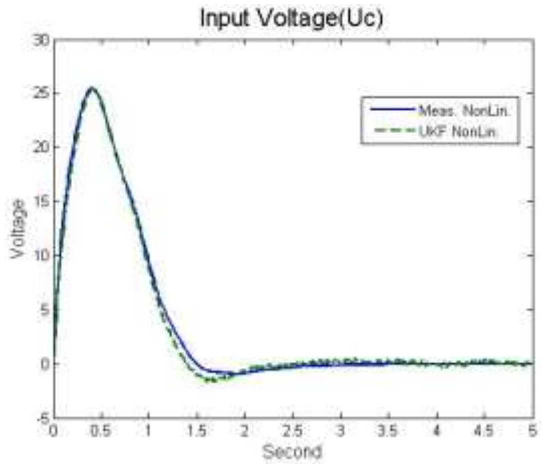


**Fig .4.**signal control $\mathbf{x_d}=180'$ by $NL-H_\infty$

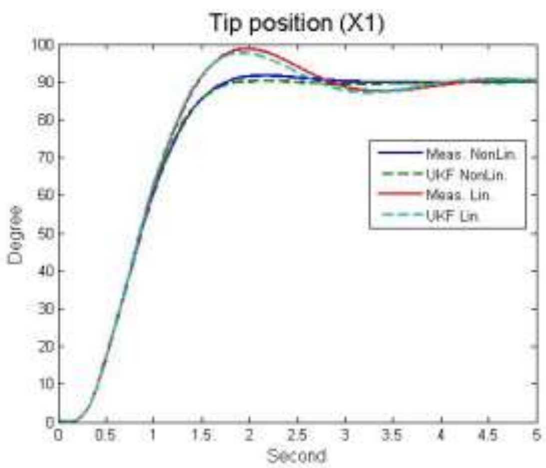


**Fig .5.**step response $\mathbf{x_d}=90'$ by $NL-H_\infty$ **and** $\mathbf{L-H_\infty}$

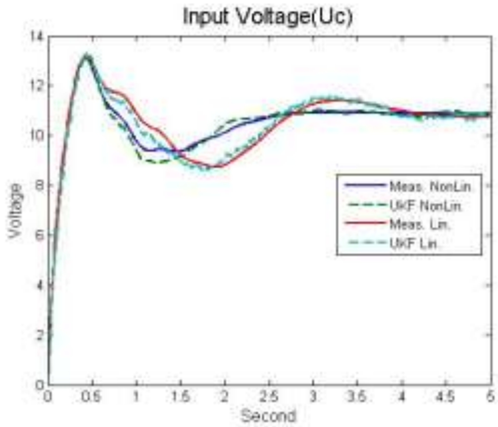


**Fig .6.**signal control $\mathbf{x_d} = 90'$ by $\boldsymbol{NL-H_\infty}$ and $\mathbf{L-H_\infty}$

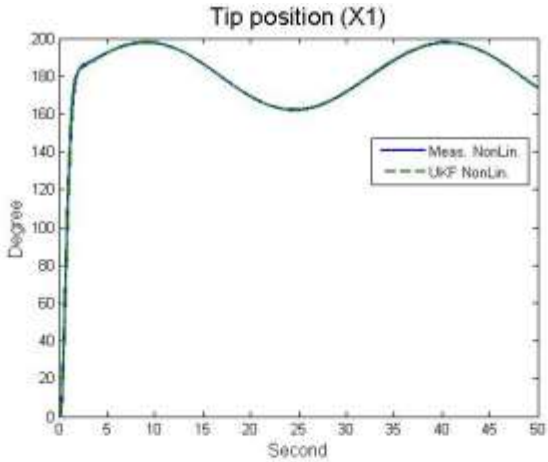


**Fig .7.**response to $\mathbf{x_d} = \pi + \sin(\frac{\pi}{10}\mathbf{t})$ by $\boldsymbol{NL-H_\infty}$

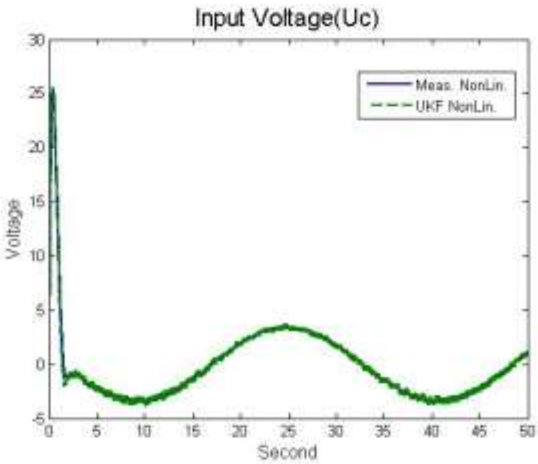


**Fig .8.**signal control to $\mathbf{x_d} = \pi + \sin(\frac{\pi}{10}\mathbf{t})$ by $\boldsymbol{NL-H_\infty}$

Effects of modeling uncertainty and external disturbances were processed. The simulations results are shown figs (9-12). Figure 9, shows tip position of link by measuring and in figure 10 by using UKF estimator. Disturbance effect was simulated by using step and sin form disturbances so that inputs command was 90' with 2 second delay.

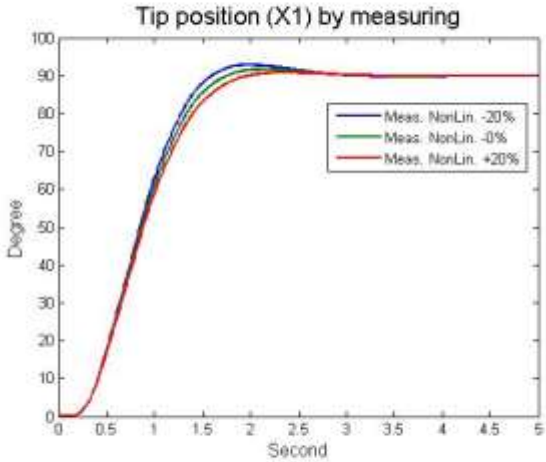


**Fig .9.**Response to $\mathbf{x_d} = 90'$ by $\boldsymbol{NL-H_\infty}$ **with** $\mathbf{M} = \mathbf{M} \pm 20\%$

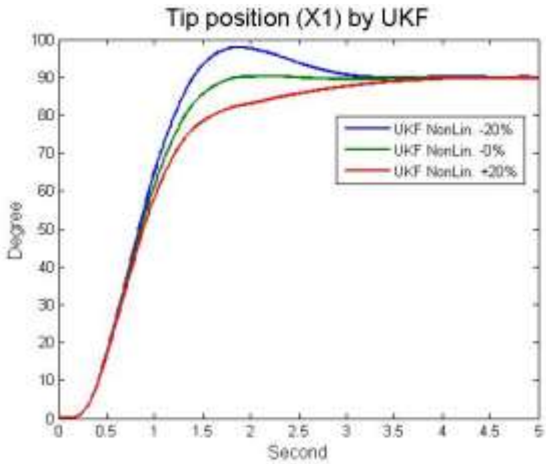


**Fig .10.**Response to $\mathbf{x_d} = 90'$ by $\boldsymbol{NL-H_\infty}$ **with** $\mathbf{M} = \mathbf{M} \pm 20\%$

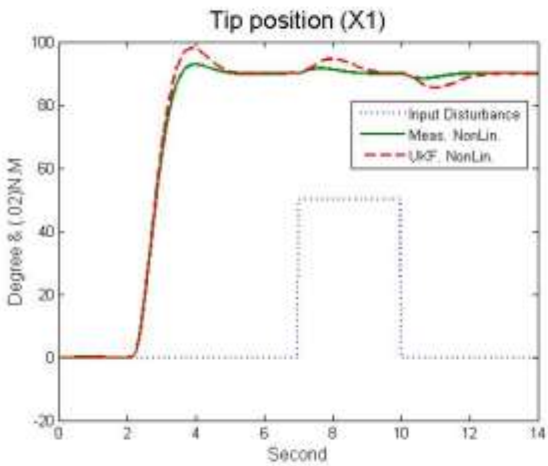

**Fig.11.** Response to $x_d = 90'$ by $NL - H_\infty$ with step form disturbance, $f_d = 1^{N.m}$

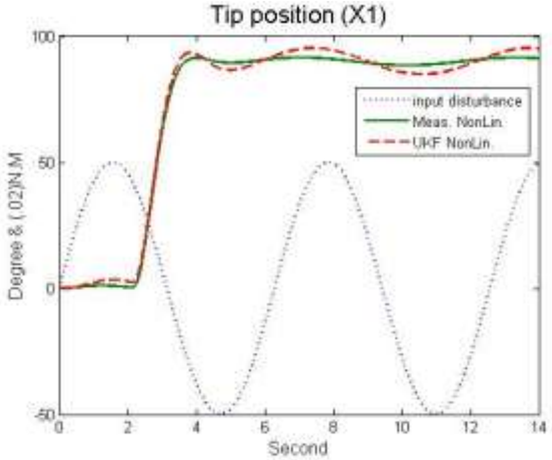


**Fig.12.** Response to $x_d = 90'$ by $NL - H_\infty$ with sin form disturbance, $f_d = 1^{N.m}\sin(t)$

Figures (3 -12) show that combination of $NL - H_\infty$ and UKF estimator have ability to control of uncertain and distributed nonlinear system.

## 6. CONCLUSIONS

In this paper a UKF estimator was proposed for robust control of n-link FJRs by $NL - H_\infty$. The UKF has several performance to EKF such as, low bios, don't need to derivations. The results of simulations show that composite of $NL - H_\infty$ controller and UKF estimator have ability to stable and control of such systems which have disturbance and uncertainties effects and also having good tracking.

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