Journal of Artificial Intelligence in Electrical Engineering, Vol. 4, No. 15, December 2015

Robust Control of a Quadrotor

Kamal Torabi [1], Alireza Ghafari Kashani [2]
[1]Department of Electrical Engineering. Ahar Branch, Islamic Azad University, Ahar, Iran
eng.torabi_k@yahoo.com
[2]Department of Electrical Engineering. Ahar Branch, Islamic Azad University, Ahar, Iran
ghafari_azad@yahoo.com

Abstract

In this paper, a robust tracking control method for automatic take-off and trajectory tracking of a quadrotor helicopter is presented. The designed controller includes two parts: a position controller and an attitude controller. The attitude controller is designed by using the sliding mode control (SMC) method to track the desired pitch and roll angles, which are the output of position controller. The position controller is also a design using SMC and the attitude controller is faster than position controller.

Keywords: Quadrotor helicopter, Robust control, SMC, Trajectory tracking.

1- Introduction

The development of Unmanned Aerial Vehicles (uav) has generated great interest in the automatic control area in the last few decades. These kinds of vehicles have been used in tasks such as search and rescue, building exploration, security, and inspection. The uav are most useful, mainly, when these desired tasks are executed in dangerous and inaccessible environments.

In the last few years the UVA in the quadrotor configuration has been highlighted in a lot of papers. This vehicle is based on vtol (Vertical Take-Off and Landing) concepts and it is usually used to develop control laws. This kind of helicopter tries to reach a stable hovering and flight, using the equilibrium forces produced by four rotors[1]. One of the advantages of the quadrotor configuration is its load capacity. Moreover, this helicopter is highly maneuverable, which allows take-off and landing as well as flight in tough environments. One drawback, is that this type of uav presents a weight and energy consumption augmentation due to the extra motors. Nevertheless, these kinds of uav's have a high nonlinear and time-varying behavior and they are constantly affected by aerodynamic disturbances. In addition, uavs are usually models subject to unmodelled dynamics and parametric uncertainties. Therefore, an advanced control strategy is required to achieve good performance in autonomous flight or at least to help the piloting of the vehicle, with high maneuverability and robustness with respect to external disturbances. Concerning this matter, it must be taken into account that these vehicles are underactuated mechanical systems, which complicate the control design stage even more. Techniques developed for fully actuated robots cannot be directly applied to this class of systems [2]. Therefore, nonlinear modelling techniques and modern nonlinear control theory are usually employed to achieve autonomous flight with high performance [1]. Many efforts have been made to control quadrotor-based helicopters and some strategies have been developed to solve the path following problems for this type of system. In [3], the aerodynamic forces and moments acting on this model were considered. The path following problem was solved using exact linearization techniques and noninteracting control via dynamic feedback. In [4], the rotor dynamics were considered in the model.

The unmanned helicopter is one of the most complex systems and its dynamics involves nonlinearity, uncertainties, and coupling (see, e.g., [5–6]). Many works have been done on the controller design for the quadrotor helicopter [14]. PD^2 feedback control method was employed to stabilize the attitude of a quadrotor aircraft in [7]. By combining a model predictive control strategy and nonlinear $H\infty$ control approach, Raffo et al. [8] discussed the path following problem. In [9], a station-keeping and tracking controller was designed to deal with underactuation and strong coupling in pitch-yaw-roll. In [10], the singular perturbation theory was used to design a hierarchical controller for a miniature helicopter. Precision hovering and trajectory tracking were achieved for a multi-vehicle quadrotor helicopter as shown in [11]. Previous experimental results mainly focused on the hovering control or trajectory tracking control problems for quadrotor helicopters (see, e.g., [7, 10, 11]). Research on the automatic takeoff and landing problems for this kind of helicopter remains challenging because of the ground effect that the helicopter is subject to. The lift thrusts produced by the rotor blades will change when the helicopter flies near the ground. Nonlinear feedback controllers were applied for a scale model helicopter and a conventional helicopter to achieve the take-off flight and the automatic landing in [12] and [13], respectively. But, the effects of uncertainty existing in the take-off and landing control, were not fully discussed in previous papers. One of the solutions to improve controller performances in take-off, hovering and trajectory tracking missions is to design a controller which is robust against these disturbances. In this paper, a robust controller is designed by the time-scale separation approach with two parts: a position controller and an attitude controller. In order to achieve the motion control of the quadrotor, the position controller is applied to generate the desired pitch and roll angles regarding the information of the position tracking errors and tracking the desired altitude reference.

2- Quadrotor Model

This section shows the model of quadrotor based on description of the dynamics. As shown in Fig.1, the four-rotor helicopter has 6-DOF: three translational components and three rotational components, and has four control inputs: four thrusts produced by the four rotors.

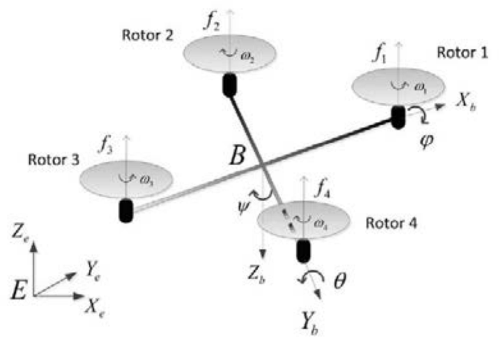

Fig.1. Schematic view of quadrotor aircraft

As shown in Fig. 1, let E = {X_e, Y_e, Z_e} denote the inertial frame, and B={X_b, Y_b, Z_b} denote the frame attached to the vehicle. The Euler angles φ, θ, and ψ, representing the roll, the pitch and the yaw, respectively, determine the orientation of matrix R : E → B

$$R = \begin{bmatrix} c\theta c\psi & s\varphi s\theta c\psi + c\varphi s\psi & c\varphi s\theta c\psi + s\varphi s\psi \\ c\theta s\psi & s\varphi s\theta s\psi + c\varphi c\psi & c\varphi s\theta s\psi + s\varphi c\psi \\ -s\theta & s\varphi c\theta & c\varphi c\theta \end{bmatrix} \quad (1)$$

Where $C_\theta = \cos\theta$ and $S_\theta = \sin\theta$. Let $P = [x\ y\ z]^T$ denote the center of gravity of the vehicle in frame E while the vector $\Omega = [p\ q\ r\ ]^T$ represents the angular velocity in frame B.
Let the vector $[x,y,z]^T$ denote the position of the center of the gravity of the quadrotor and the vector $[u,v,w]^T$ denote its linear velocity in the earth-frame.

The vector [p,q,r]' represents the quadrotor's angular velocity in the body-frame. m denotes the total mass. g represents the acceleration of gravity. l denotes the distance from the center of each rotor to the center of gravity. The orientation of the quadrotor is given by the rotation matrix R:E-B, where R depends on the three Euler angles $[\varphi,\theta,\psi]^T$, which represent the roll, the pitch and the yaw, respectively. And $(-\frac{\pi}{2} \le \varphi \le \frac{\pi}{2})$, $(-\frac{\pi}{2} \le \theta \le \frac{\pi}{2})$, $(-\frac{\pi}{2} \le \psi \le \frac{\pi}{2})$. The transformation matrix from $[\Phi,\theta,\psi]^T$ to $[p,q,r]^T$ is given by

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -s\theta \\ 0 & c\varphi & s\varphi c\theta \\ 0 & -s\varphi & c\varphi c\theta \end{bmatrix} \begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \quad (2)$$

The dynamical model of the quadrotor can be described by the following equations: The quadrotor is controlled by four motors. The two pairs of propellers (1, 3) and (2, 4) as described in Fig. 1 rotate in opposite directions. Increasing or decreasing the four propeller's speed together generates the vertical motion. Changing the propellers (1,3) speed conversely produces the pitch rotation. Meanwhile, the roll rotation can be realized by giving propellers (2, 4) converse control signals. Yaw rotation results from the difference in the counter-torque between each pair of propellers. And the translational motion of the aircraft is generated by an offset in roll and pitch. According to Newton's second law, the dynamics equations are given by:

$$F = m\ddot{P} \quad (3)$$
$$M = \frac{dH}{dt} \quad (4)$$

Where, F is the external force acting on the quadrotor, M denotes the external torque of the quadrotor, H is the angular momentum of the quadrotor relative to the inertial frame E, and m is the mass of the quadrotor. Let l denote the distance between the center of the rotor and the geometric center of quadrotor (arm length), J_r denote the inertial of rotor, fi (i = 1, 2, 3, 4) denote the thrust of rotor i, b and d denote coefficients of the thrust and the drag, and g is the acceleration of gravity. Then $f_i = b\omega^2_i$. The inertial matrix can be expressed as a diagonal matrix: $I = diag\ (I_x\ I_y\ I_z)$ Similarly, the rotational resistance moment coefficient matrix is: $K_{dm} = diag\ (K_{dmx}\ K_{dmy}\ K_{dmz})$. Therefore, equation (3) can be rewritten in details as:

$$\begin{aligned} m\ddot{x} &= (s\psi s\varphi + c\psi s\theta c\varphi)\sum_{i=1}^{4} T_i - \frac{1}{2} c_x A_c \rho \dot{x}|\dot{x}| - \sum_{i=1}^{4} H_{xi} + d_x \\ m\ddot{y} &= (-c\psi s\varphi + s\psi s\theta c\varphi)\sum_{i=1}^{4} T_i - \frac{1}{2} c_{xy} A_c \rho \dot{y}|\dot{y}| - \sum_{i=1}^{4} H_{yi} + d_y \\ m\ddot{z} &= -mg + c\psi c\varphi \sum_{i=1}^{4} T_i + d_z \end{aligned} \tag{5}$$

The rotational motion equation can be derived from (4) the following equation:

$$\begin{aligned} \dot{p} &= I_{XX}\ddot{\varphi} = \dot{\theta}\dot{\psi}(I_{yy} - I_{zz}) + J_r\dot{\theta}\Omega_r + l(-T_2 + T_4) \\ &\quad -h\left(\textstyle\sum_{i=1}^{4} H_{yi}\right) + (-1)^{i+1}\textstyle\sum_{i=1}^{4} R_{mxi} + d_\varphi \\ \dot{q} &= I_{yy}\ddot{\theta} = \dot{\varphi}\dot{\psi}(I_{zz} - I_{xx}) + J_r\dot{\varphi}\Omega_r + l(T_1 - T_3) \\ &\quad +h\left(\textstyle\sum_{i=1}^{4} H_{xi}\right) + (-1)^{i+1}\textstyle\sum_{i=1}^{4} R_{myi} + d_\theta \\ \dot{r} &= I_{zz}\ddot{\psi} = \dot{\theta}\dot{\varphi}(I_{xx} - I_{yy}) + J_r\dot{\Omega}_r + l(H_{x2} - H_{x4}) \\ &\quad +l(H_{y1} - H_{y3}) + d_\psi \end{aligned} \tag{6}$$

$$\begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & s\varphi t\theta & c\varphi t\theta \\ 0 & c\varphi & -s\varphi \\ 0 & s\varphi sec\theta & c\varphi sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{7}$$

Define:

$$\begin{cases} U_1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ U_2 = b(-\Omega_2^2 + \Omega_4^2) \\ U_3 = b(\Omega_1^2 - \Omega_3^2) \\ U_4 = b(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \\ \Omega_r = -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4 \end{cases} \tag{8}$$

Furthermore, as the yaw angle is controlled independently in the practical system, we can suppose $\psi = 0$ here. In the following section, U_i (i = 1, 2, 3, 4), as defined in (8), are used as control inputs of the quadrotor. We obtain the following simplified model:

$$\begin{aligned} \ddot{x} &= (s\psi s\varphi + c\psi s\theta c\varphi)\frac{1}{m} U_1 \\ \ddot{y} &= (-c\psi s\varphi + s\psi s\theta c\varphi)\frac{1}{m} U_1 \\ \ddot{z} &= -g + c\psi c\varphi \frac{1}{m} U_1 \\ \ddot{\varphi} &= \dot{\theta}\dot{\psi}\left(\frac{I_{yy} - I_{zz}}{I_{xx}}\right) + \dot{\theta}\Omega_r\left(\frac{J_r}{I_{xx}}\right) + \frac{l}{I_{xx}} U_2 \\ \ddot{\theta} &= \dot{\varphi}\dot{\psi}\left(\frac{I_{zz} - I_{xx}}{I_{yy}}\right) - \dot{\varphi}\Omega_r\left(\frac{J_r}{I_{yy}}\right) + \frac{l}{I_{yy}} U_3 \\ \ddot{\psi} &= \dot{\theta}\dot{\varphi}\left(\frac{I_{xx} - I_{yy}}{I_{zz}}\right) + \frac{l}{I_{zz}} U_4 \end{aligned} \tag{9}$$

3-Controller Design

Because of the quadrotor dynamics, a nested loops control strategy is appropriate (9) From Eqs. (7) and (9), it can be seen that the rotational motion is independent of the translational motion, while the opposite is not true. Thus, an inner control loop can be designed to ensure asymptotic tracking of desired attitude, altitude and heading. Meanwhile, an outer control loop can be designed for quadrotor navigation, as shown in Fig. 2. In this section, the quadrotor's state space model for controller design is presented.

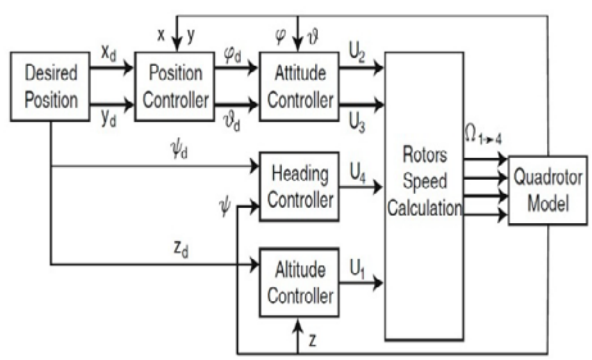

Fig. 2. Control architecture

3-1- State Space Model

The state vector is defined as (9):

$$X = [x, \dot{x}, y, \dot{y}, z, \dot{z}, \varphi, \dot{\varphi}, \theta, \dot{\theta}, \psi, \dot{\psi}]^T \quad (10)$$

The control input vector is defined as:

$$U = [U_1, U_2, U_3, U_4]^T \quad (11)$$

The state space model is given by:

$$\dot{X} = f(X, U) \quad (12)$$

Where,

$$f(X,U) = \begin{pmatrix} \dot{x} \\ u_x \frac{1}{m} U_1 \\ \dot{y} \\ u_y \frac{1}{m} U_1 \\ \dot{z} \\ g - (\cos\varphi \cos\theta) \frac{1}{m} U_1 \\ \dot{\varphi} \\ \dot{\theta}\dot{\psi} a_1 + \dot{\theta} a_2 \Omega_r + b_1 U_2 \\ \dot{\theta} \\ \dot{\varphi}\dot{\psi} a_3 - \dot{\varphi} a_4 \Omega_r + b_2 U_3 \\ \dot{\psi} \\ \dot{\theta}\dot{\varphi} a_5 + b_3 U_4 \end{pmatrix} \quad (13)$$

$$\begin{pmatrix} a_1 = (I_{yy} - I_{zz})/I_{xx} \\ a_2 = J_r/I_{xx} \\ a_3 = (I_{zz} - I_{xx})/I_{yy} \\ a_4 = J_r/I_{yy} \\ a_5 = (I_{xx} - I_{yy})/I_{zz} \\ b_1 = l/I_{xx} \\ b_2 = l/I_{yy} \\ b_3 = l/I_{zz} \end{pmatrix} \quad (14)$$

$$\begin{cases} u_x = (\cos\varphi \sin\theta \cos\psi + \sin\varphi \sin\psi) \\ u_y = (\cos\varphi \sin\theta \sin\psi - \sin\varphi \cos\psi) \end{cases} \quad (15)$$

The following equations are used for the translational motion of quadrotor from Eq. (8):

$$\begin{aligned} \ddot{x} &= u_x \frac{1}{m} U_1 + d_{10} \\ \ddot{y} &= u_y \frac{1}{m} U_1 + d_{12} \\ \ddot{z} &= g - (\cos\varphi \cos\theta) \frac{1}{m} U_1 + d_8 \end{aligned} \quad (16)$$

Where, d_i , i = 8,10,12, represent the effect of the wind on the quadrotor translational accelerations, aerodynamics and drag effect and other external disturbances. The virtual control input, which can be defined for translational system, is a function of the thrust and Euler angles.

3-2- Control using Sliding-Mode Technique

The aim of developing a sliding mode controller is to achieve robustness against uncertainties, unmodeled dynamics and bounded disturbances. The sliding mode control can be used upon regulatory variables to bring the new variables to the equilibrium state. The mapping (13) is used to design the sliding-mode controller for the rotations subsystem of the OS4 helicopter. By considering the state (10) and the system in (13), one can synthesize the control law, forcing the system to follow the desired trajectory.

3-2-1. Attitude Control

As the first step, we consider the tracking error:

$$e_1 = x_{1d} - x_1 \quad (17)$$

Then we use the Lyapunov theorem by considering the Lyapunov function e_1 positively definite and its time derivative negatively semi-definite:

$$V(e_1) = \frac{1}{2}e_1^2$$
$$\dot{V}(e_1) = e_1(\dot{x}_{1d} - x_2) \quad (18)$$

The stabilization of e_1 is obtained by introducing a virtual control input x_2:

$$x_2 = \dot{x}_{1d} + k_1 e_1 \quad (19)$$

The equation (18) is then:

$$\dot{V}(e_1) = e_1(\dot{x}_{1d} - (\dot{x}_{1d} + k_1 e_1)) = -k_1 e_1^2 \quad (20)$$

Let us proceed to a variable change by making:

$$s_2 = x_2 - \dot{x}_{1d} - k_1 e_1 \quad (21)$$

As the second step, we consider the augmented Lyapunov function:

$$V(e_1, s_2) = \frac{1}{2}(e_1^2 + s_2^2) \quad (22)$$

The chosen law for the attractive surface is the time derivative of (21) satisfying ($s\dot{s} < 0$):

$$\dot{s}_2 = -k_1 sign(s_2) - k_2 s_2$$
$$\dot{s}_2 = \dot{x}_2 - \ddot{x}_{1d} - k_1\dot{e}_1 \quad (23)$$
$$\dot{s}_2 = a_1 x_4 x_6 + a_2 x_4 \Omega_r + b_1 U_2 - \ddot{x}_{1d} - k_1(-k_1 e_1 - s_2)$$

With: k1, k2 > 0. The symbol sign stands for signum function. The control input U_2 is then extracted ($\ddot{x}_{1,2,3d} = 0$), satisfying $\dot{V}(z_1, z_2)<0$:

$$U_2 = \left(\frac{1}{b_1}\right)[-a_1 x_4 x_6 - a_2 x_4 \Omega_r - k_1^2 e_1$$
$$-k_1 sign(s_2) - k_2 s_2] \quad (24)$$

The term $k_2 z_2$ with $k_2 > 0$ is added to stabilize e_1. The same steps are followed to extract U_3 and U_4:

$$U_3 = (\frac{1}{b_2})(a_3 x_2 x_6 - a_4 x_2 \Omega_r - k_3^2 e_3 - k_3 sign(s_3) - k_4 s_3)$$
$$U_4 = (\frac{1}{b_3})(-a_5 x_2 x_4 - k_5^2 e_5 - k_5 sign(s_4) - k_6 s_4) \quad (25)$$

With:

$$e_3 = x_{3d} - x_3$$
$$s_3 = x_4 - \dot{x}_{3d} - k_3 e_3$$
$$e_5 = x_{5d} - x_5 \quad (26)$$
$$s_4 = x_6 - \dot{x}_{5d} - k_5 e_5$$

3-2-2-Altitude Control

The autonomous take-off and landing algorithm adapts the altitude reference z_d to follow the dynamics of the quadrotor for taking-off or landing. The altitude controller keeps the distance of the quadrotor to the ground at a desired value.

The same steps calculation u2 are followed to extract U_1:

$$U_1 = \left(\frac{m}{\cos x_1 \cos x_3}\right)[g + k_8 s_7 + k_7^2 e_7 + k_8 sign(s_7)] \quad (27)$$

3-2-3- Position Control

Position control keeps the helicopter over the desired point. Here it means the (x, y) horizontal position with regard to a starting point. Horizontal motion is achieved by orienting the thrust vector towards the desired direction of motion. This is done by rotating the vehicle itself in the case of a quadrotor.

In practice, one performs position control by rolling or pitching the helicopter in response to a deviation from the y_d or x_d references, respectively. Thus, the position controller outputs the attitude references phi d and teta d, which are tracked by the attitude controller (see Fig.2). The same steps of calculation of u2 are followed to extract u_x and u_y:

$$\begin{cases} u_x = \left(\frac{m}{U_1}\right)[-k_{10}s_5 + k_9^2 e_9 - k_9 sign(s_5)] \\ u_y = \left(\frac{m}{U_1}\right)[-k_{12}s_6 + k_{11}^2 e_{11} - k_{11} sign(s_6)] \end{cases} \quad (28)$$

4-Simulation results

In this section, the performance of the proposed approach is evaluated. The corresponding algorithm is implemented in Matlab/Simulink simulation environment. The model parameter values of the quadrotor system are adopted from [4] and listed in Table 1. To explore the effectiveness of the developed controller, two simulation experiments have been performed on the quadrotor. For these simulations, we considered only the angular rotations subsystem in order to be able to verify the development on the real system. The controller above contains 20 parameters listed in Table 1, Table 2. Then, simulation results for the inner and outer loop controllers are presented.

Table.1.Simulation parameters of Sliding-mode controller

Parameter	Value	unit
Ω_r	0	rad/s
m	0.65	kg
g	9.8	m/s^2
l	0.23	m
I_{xx}	0.0075	$kg.m^2$
I_{yy}	0.007	$kg.m^2$
I_{zz}	0.013	$kg.m^2$
J_r	0.000006	$kg.m^2$

4-1-Inner Loop Results

Figure 3 shows the closed loop response of the roll, pitch, yaw and altitude to arbitrary values. The steady state is reached within approximately 0.5 s, using bounded control inputs. As it can be seen in Figs. 4, the disturbances' effect, model uncertainty and measurement noise have been compensated during the scenario with the proposed controller. The wind effect has been applied and it has been rejected by the robust action of the controller. Noise and uncertainties on model have also had negative effect on the system, but their bad effects have been reduced through the proposed controller

Table.2.Simulation Coefficients of Sliding-mode controller

Parameter	Value
k1	10
k2	20
k3	10
k4	20
k5	10
k6	20
k7	1
k8	2
k9	1
k10	1.2
k11	1
k12	1.2

4-2- Outer Loop Results

Figure 5 shows the closed loop response of the x and y to arbitrary values. The steady state is reached within approximately 3 s using bounded control inputs. As it can be seen in Fig. 6, the disturbances effect, model uncertainty and measurement noise have been compensated during the scenario with the proposed controller. The wind effect has been applied and it has been rejected by the robust action of the controller. Noise and uncertainties on model have also had negative effects on the system, but their bad effects have been reduced through the proposed controller.

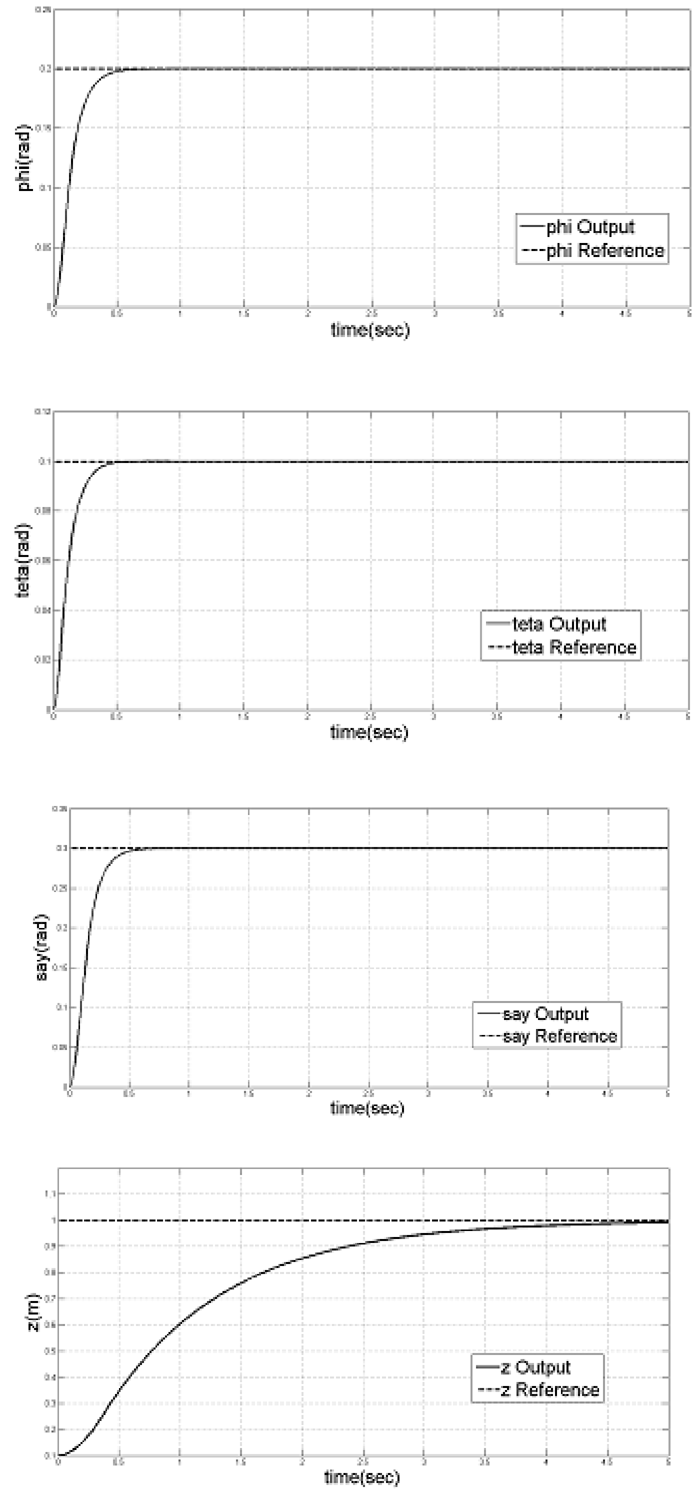

Fig. 3. The roll, pitch and yaw angles and Altitude to Arbitrary values.

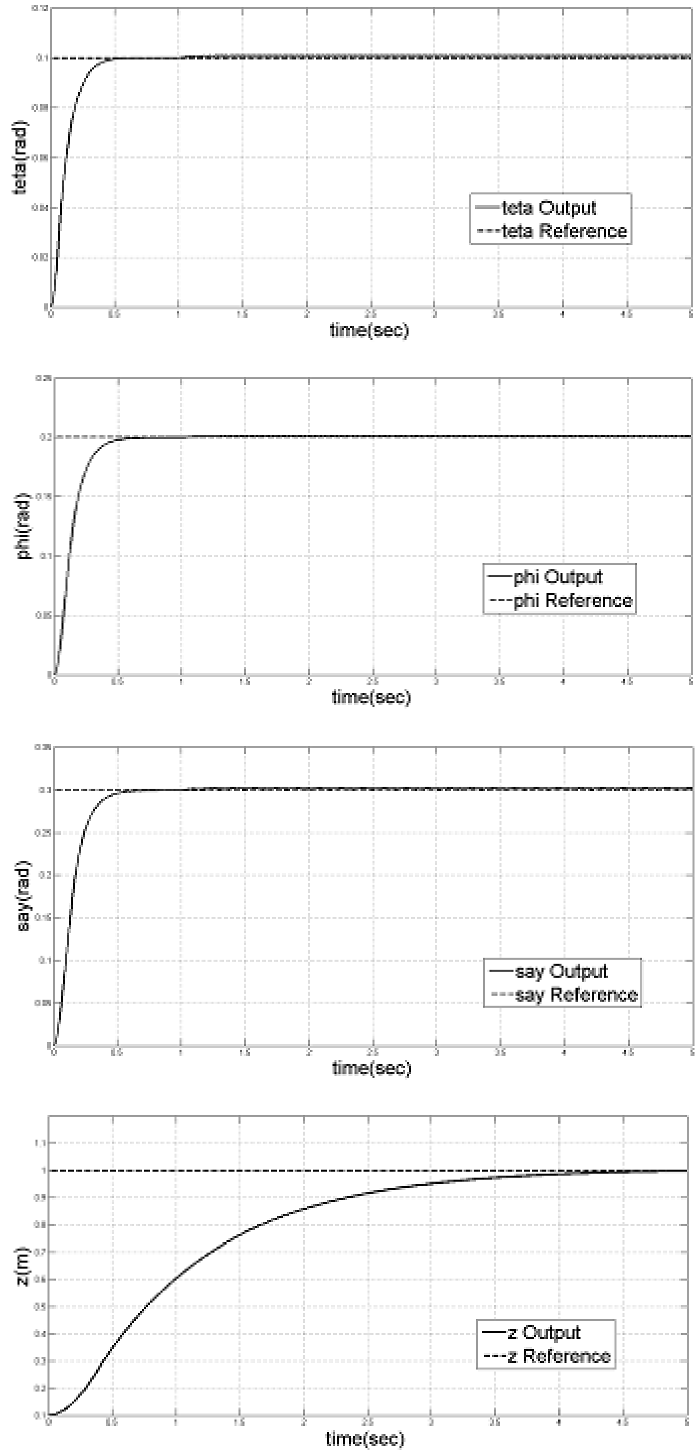

Fig. 4. The roll, pitch and yaw angles and Altitude to Arbitrary values.

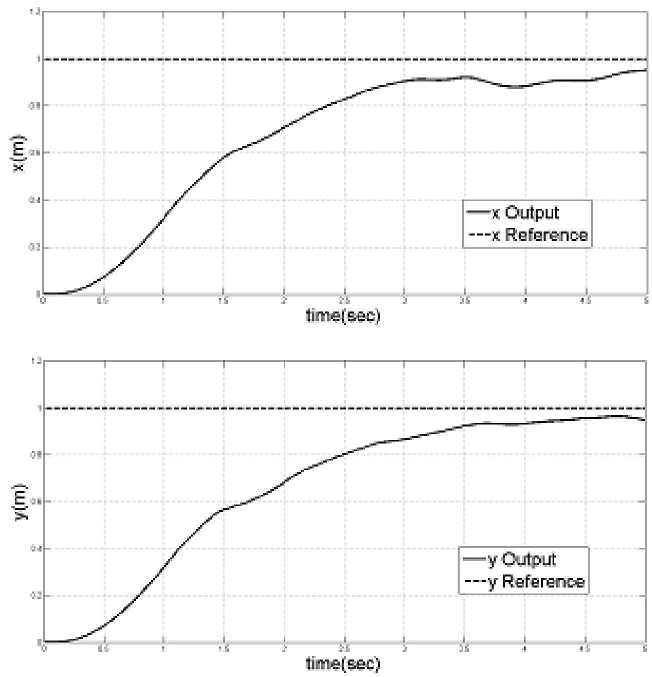

Fig. 5. The x and y to Arbitrary values.

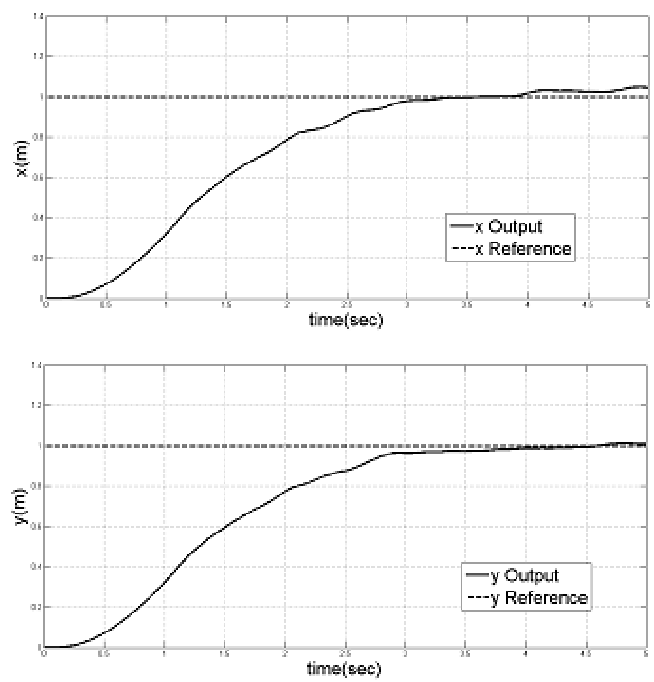

Fig . 6. The disturbances effect, model uncertainty and measurement noise and maintain the x and y to arbitrary values.

5-Conclusions

A robust controller was proposed, based on the time-scale separation approach to achieve the automatic take-off, hovering and trajectory tracking missions for a quadrotor helicopter. The designed controller consists of a position controller and an attitude controller. The position controller generates the desired pitch and roll angles based on the tracking errors of the longitudinal and lateral positions and is applied to follow the height reference for the vertical position. Based on the robust compensation technique, the attitude controller is designed to achieve the desired tracking of the attitude angles. It was proven that attitude tracking errors are proven to ultimately converge to the given neighborhoods of the origin. The proposed controller demonstrates robustness of the system's output against harsh wind disturbances and uncertainties.

References

[1] [P. Castillo, R. Lozano, and A. E. Dzul, (2005). Modelling and control of mini-flying machines. London, UK: Springer-Verlag.

[2] [I. Fantoni, and R. Lozano, (2002). Non-linear control for underactuated mechanical systems. London: Springer-Verlag.

[3] V. Mistler, A. Benallegue, and N.K. M'Sirdi, (2001). "Exact linearization and noninteracting control of a 4 rotors helicopter via dynamic feedback," In Proc. IEEE int. workshop on robot and human inter. Communic.

[4] S. Bouabdallah, P. Murrieri, and R. Siegwart, (2004). "Design and control of an indoor micro quadrotor," In Proc. IEEE Int. conf. on rob. and automat., Vol. 5, New Orleans, USA, pp. 4393_4398.

[5] P. Castillo, A. Dzul, R. Lozano, (2004). "Real-time stabilization and tracking of a four-rotor mini rotorcraft," IEEE Trans. Control Syst. Tech. 12(4), 510–516.

[6] Z. Zuo, (2010). "Trajectory tracking control design with command-filtered compensation for a quadrotor," IET Control Theory Applic. 4(11), 2343–2355.

[7] A. Tayebi, S. McGilvray, (2006). "Attitude stabilization of a VTOL quadrotor aircraft," IEEE Trans. Control Syst. Tech. 14(3), 562–571.

[8] G.V. Raffo, M.G. Ortega, and F.R. Rubio, (2010). "An integral predictive/nonlinear H∞ control structure for a quadrotor helicopter," Automatica 46(1), 29–39.

[9] A. Das, K. Subbarao, and F. Lewis, (2009). "Dynamic inversion with zero-dynamics stabilization for quadrotor control," IET Control Theory Applic. 3(3), 303–314.

[10] S. Bertrand, N. Guenard, T. Hamel, H. Piet-Lahanier, and L. Eck, (2011). "A hierarchical controller for miniature VTOL UAVs: design and stability analysis using singular perturbation theory," Control Eng. Pract. 19(10), 1099–1108.

[11] G.M. Hoffmann, H. Huang, S.L. Waslander, and C.J. Tomlin, (2011). "Precision flight control for a multi-vehicle quadrotor helicopter testbed," Control Eng. Pract. 19(9), 1023–1036.

[12] J.C.A. Vilchis, B. Brogliato, A. Dzul, R. Lozano, (2003). "Nonlinear modelling and control of helicopters," Automatica 45(10), 1583–1596.

[13] K. Peng, G. Cai, B.M. Chen, M. Dong, K.Y. Lum, and T.H. Lee, (2009). "Design and implementation of an autonomous flight control law for a UAV helicopter," Automatica 39(9), 2333–2338.

[14] Moradi, Mohammad Mehdi. "The reduction coefficient of PID controller by using PSO algorithm method for Flexible single-arm robot system." Journal of Artificial Intelligence in Electrical Engineering 3.12 (2015): 44-54.