Design of fuzzy sliding mode controller with moving sliding surface for robot manipulator

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Abstract*:*

In sliding mode control, the sliding movement can be divided into two phases: reaching phase and sliding phase. In each phase, we face a series of problems. In the sliding phase, switching leads to the occurrence of undesirable chattering phenomenon, so that such high frequency oscillations stimulate the unmodeled dynamics of the system and may cause damage to the controlled device. In this paper, a fuzzy-sliding model controller (FSMC) is presented to solve this problem. On the other hand, during the reaching phase, SMC is sensitive to parameter uncertainty and external disturbance. In the continuation of the paper, a sliding mode fuzzy controller (SMFC) with a moving sliding surface to minimize or even eliminate the reaching phase is introduced.

Keywords*:* *Sliding-mode control; Fuzzy control; chattering phenomenon; moving sliding surface.*

1. Introduction

The dynamic equations of the robot manipulator are non-linear and interdependent. In addition, these equations include structural and non-structural uncertainties. Sliding mode control (SMC) was investigated as an efficient and robust method in controlling system uncertainties and external disturbances by Utkin in 1978 [1]. Sliding mode control is one type of variable-structure control scheme. Normally, two steps, namely the reaching and sliding phases, are necessary in the design of sliding mode controller. Therefore, the sliding mode controller usually consists of an equivalent law and a switching law. The equivalent law is given so that the states can stay on sliding surface. The switching law is used to drive the state trajectory to the sliding manifold [2]. Despite its good robust properties, this controller has some problems that we use intelligent control techniques to solve in this paper.

In the past three decades, fuzzy systems have replaced conventional technologies in many applications, especially in control systems. One major feature of fuzzy logic is its ability to express the amount of ambiguity in human thinking. Thus, when the mathematical model of one process does not exist, or exists but with uncertainties, fuzzy logic is an alternative way to deal with the unknown process [3]. But, the huge amounts of fuzzy rules for a high-order system makes the analysis complex.

One of the problems of sliding mode control is the occurrence of chattering phenomenon. Various methods have been proposed to eliminate this problem, including defining a boundary layer around the sliding surface. It can be reduced and even eliminated in some cases, but this is at the cost of increasing the permanent error. The use of a fuzzy sliding mode controller (FSMC) can minimize the mentioned problem in such a way that near the sliding surface of a fuzzy controller comes into action [4].

Another disadvantage of sliding mode control is that the system is sensitive to uncertainties in the reaching phase. One of the methods to minimize or completely eliminate the reaching phase is to use a moving sliding surface (MSS) [5-7]. The design of this moving surface is done using fuzzy knowledge. In the second part of this paper, the dynamic equations of a robot manipulator are examined and in the third part, a fuzzy sliding mode controller is designed. In the fourth section, a moving sliding surface has been designed using the fuzzy technique. Finally, the simulation results on the system are given in the fifth section.

2. Preliminaries and Problem Formulation

We consider a robot manipulator, described by the following equation [8,9].

$$M(q)\,\ddot{q} + C(q,\dot{q}) + G(q) + F(\dot{q},\tau) = \tau \qquad (1)$$

where $M(q) \in \mathbf{R}^{\mathbf{n}\times\mathbf{n}}$ is the inertial matrix which is a symmetric positive definite matrix, $C(q,\dot{q})) \in \mathbf{R}^{\mathbf{n}\times\mathbf{n}}$ is the Coriolis matrix and $G(q) \in \mathbf{R}^{\mathbf{n}}$ is the gravity vector. $q,\dot{q},\ddot{q} \in \mathbf{R}^{\mathbf{n}}$ are the position, velocity and angular acceleration vectors of the robot joints. $\tau \in \mathbf{R}^{\mathbf{n}}$ is the torque vector applied to the joints. Also, $F(\dot{q},\tau)$ represents the friction vector whose i-th component is in the following form:

$$f_i(\dot{q},\tau_i) = \; b_i\dot{q}_i + f_{ci}sgn(\dot{q}_i) + [1 - sgn(\dot{q}\_i\,)]sat(\tau\_i; f\_si\,) \qquad (2)$$

b_{i}, $f_{\mathrm{ci}}$and f_{si} are adhesion, coulomb and static friction models, respectively. The function $sat(.;.)$ is defined as follows:

$$sat(\tau; f_{si}) = \begin{cases} f_{si} & \tau_i > f_{si} \\ \tau_i & -f_{si} \le \tau_i \le f_{si} \\ -f_{si} & \tau_i < -f_{si} \end{cases}$$

The robot manipulator has the following properties:

Property 1: The matrix $\dot{M}(q) - 2C(q,\dot{q})$ is an antisymmetric matrix, that is, it satisfies the following condition for every non-zero vector x:

$$q^{\mathrm{T}}\left(\dot{M}(q) - 2C(q,\dot{q})\right)q = 0 \qquad (3)$$

Property 2: $\dot{q}^T F(\dot{q},\tau) > 0 \;\; \forall\tau \in R^n$

Property 3: The components of the gravity torque vector G(q) have an upper limit so that:

$$sup\{|g_i(q)|\} \le \bar{g}_i \qquad \bar{g}_i \ge 0$$

where $g_i$is the i-th component of vector G.

The maximum torque that the joint actuator can provide is τ^{max}, as a result:

$$|\tau_i| \le \tau^{max}\,, i = 1,2,\dots,n \qquad (4)$$

$$\tau^{max} > \bar{g}_i + f_{s_i}$$

In most cases, the matrices M and G can be easily determined, but it is difficult to accurately determine C. Therefore, the matrix C is considered as follows:

$$C = \hat{C} + \Delta C \qquad (5)$$

In this paper, our goal is to design a fuzzy sliding model control that tracks the position vector q of the desired state vector $q_d$in the presence of uncertainty and disturbance.

3. fuzzy Sliding-Mode Control

In order to design a sliding mode controller, two essential steps should be carefully investigated, namely, the selection of sliding mode surface and the design of control law.

The selection of sliding mode surface is based on desired motion of the system. considering the simplicity of design, we define a sliding surface as:

$$s=e+\lambda e \qquad (6)$$

where $e = -\tilde{q} = q - q_d$_d and λ is a positive definite matrix. By defining the reference velocity vector as $\dot{q}_r = \dot{q}_d - \lambda e$, the sliding surface can be defined as follows:

$$s = \dot{q} - \dot{q}_r \qquad (7)$$

In order for the states of the system to reach the sliding surface and remain on it, the following condition, which is known as the sliding condition, must be satisfied [1,10]:

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{dt}}[s^T M s] < -\eta(s^T s)^{1/2} \tag{8}$$

where η is a positive definite matrix.

By stating the following lemma, we design a sliding mode controller for a robot manipulator.

Lemma 1: By defining the sliding surface (6) and the following control law for the system (1), the sliding condition (8) is satisfied.

$$\tau = \hat{\tau} - K sgn(s) \tag{9}$$

Where

$$\hat{\tau} = M\ddot{q}_r + \hat{C}\dot{q}_r + G \tag{10}$$

$$K_i \geq \|\Delta C\dot{q}_r\| + \Gamma_i \tag{11}$$

$\Gamma \in \boldsymbol{R}^n$ is a design parameter and should be designed in such a way that:

Γ _i≥F_up+ η _i (12)

Proof 1: Consider the following Lyapunov function:

$$V = \frac{1}{2}s^T M s \tag{13}$$

Considering that the matrix M is a positive definite matrix, therefore, if $s \neq 0, V > 0$, and deriving from V, we will have:

$$\dot{V} = s^T M\dot{s} + \frac{1}{2}s^T \dot{M} s \tag{14 }$$

Using relation (7), we have:

$$\dot{V} = s^T(M\ddot{q} - M\ddot{q}_r) + \frac{1}{2}s^T\dot{M}s \tag{15}$$

By putting relation (1) in (15) and property 1, the following result is obtained:

$$\dot{V} = s^T(\tau - C\dot{q}_r - G - F - M\ddot{q}_r) \tag{16}$$

By putting relations (9) and (10) in the above relation, it is obtained:

$$\dot{V} = -s^T(\Delta C\dot{q}_r + F) - \sum_{i=1}^{n} K_i|s_i| \tag{17}$$

The above relation shows that the derivative of the Lyapunov function satisfies the sliding condition (8).

In order to reduce the phenomenon of chattering, we define a boundary layer with thickness φ around the sliding surface. For this purpose, we replace the saturation function sat, which is defined as follows, with sgn in equation (9).

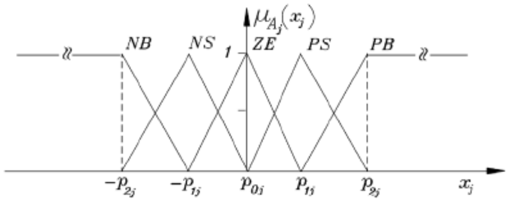
$$sat\left(\frac{s}{\varphi}\right) = \begin{cases} sgn\left(\frac{s}{\varphi}\right) & |s| \geq |\varphi| \\ \frac{s}{\varphi} & |s| < |\varphi| \end{cases} \tag{18}$$

In the following, we will design a simple Sugeno type fuzzy controller. The fuzzy rule base includes the rules if then two inputs one output as follows:

IF x_1 is $A_1^{l_1}$ and x_2 is $A_2^{l_2}$THEN y is $B^{l_1 l_2}$ (19)

For each input fuzzy set $A_j^{l_j}$ and output fuzzy set $B^{l_1 l_2}$ exist an input membership function $\mu_{A_j^{l_j}}(x_j)$ and output membership function $\mu_{B^{l_1 l_2}}(x_j)$, respectively. $l_j = -(N_1 - 1)/2, \ldots, -(N_j - 1)/2$ and N_j is the individual number of membership functions corresponding to input j. The output variable of the fuzzy controller can also have an individual number, N, of membership functions $\mu_{B^l}(y)$ with $l = -(N-1)/2, \ldots, -(N-1)/2$.

In the following, we consider an SFC with a single fuzzifier, the number $N_j$of triangular membership functions for each input with j=1,2 (Figure 1), the number N of single membership functions for the output (Figure 2), the fuzzy rule base defined by (19) (Table 1), inferring the product of the average centers and de-fuzzifier.



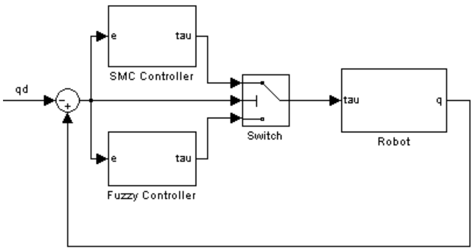


Fig.1. Membership functions for inputs $\tilde{q}$ and $\dot{\tilde{q}}$

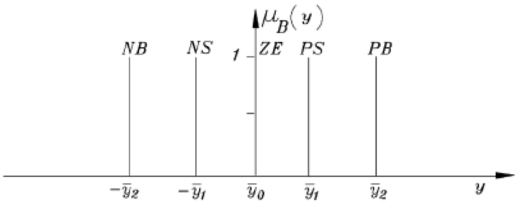


Fig.2. Individual membership functions for output $\boldsymbol{\Phi}(\tilde{q},\dot{\tilde{q}})$

Table1. Rule base

x_2 \ x_1	NB	NS	ZE	PS	PB
NB	NB	NB	NS	ZE	ZE
NS	NB	NB	NS	ZE	ZE
ZE	NS	NS	ZE	PS	PS
PS	ZE	ZE	PS	PB	PB
PB	ZE	ZE	PS	PB	PB

In order to use the advantages of sliding mode and fuzzy controllers simultaneously and to minimize the disadvantages of each of them, we propose the following collaborative controller:

$$\tau = \begin{cases} \hat{\tau} - Ksgn(s) & |\tilde{q}_i| \geq \alpha \\ \boldsymbol{\Phi}(\tilde{q},\dot{\tilde{q}}) + G(q) & |\tilde{q}_i| < \alpha \end{cases} \qquad (20)$$

where α is a positive parameter. If the error is greater than α, the sliding mode controller will operate, and if the error is less than this value, the fuzzy controller will operate. In the vicinity of the sliding surface, the phenomenon of chattering occurs, which stimulates both unmodeled high-frequency dynamics and increases the input torque, so by using a fuzzy controller in relation (20) Overcame these problems. In addition, the overall structure of fuzzy sliding mode control technique in the robot manipulator can be shown in Figure 3.

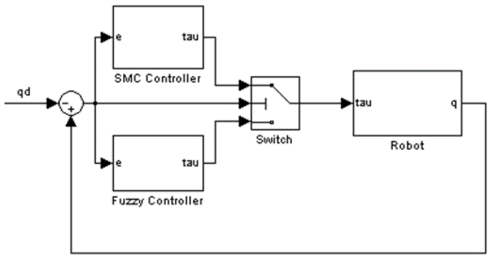


Fig. 3. Overall fuzzy sliding mode control scheme

4. Moving Sliding Surface Design

One of the problems of the classic sliding mode controller is that it is robust only in the sliding phase of the uncertainty and the disturbance, and it is not robust in the reaching phase. One of the proposed solutions is to minimize the reaching phase by rotating or shifting the sliding surface, which is called the moving sliding surface (MSS) [11,12].

For the robot with dynamic equation (1), the moving sliding surface is considered as follows:

$$s(e,\dot{e},t) = \dot{e} + \lambda e - \gamma \qquad (21)$$

The rotation of the surface is done by changing λ, which is the slope of the surface, and the displacement is done by changing the value of γ. In second-order systems, if the initial condition is in quadrant one or three, we will shift the sliding surface, and if it is in quadrant two or four, we will rotate it.

Based on the above statements, the control law of the sliding mode with a boundary layer and a moving sliding surface is as follows:

$$\tau = \hat{\tau} - K\ sat(\frac{\dot{e} + \lambda e - \gamma}{\varphi}) \tag{22}$$

We will use fuzzy logic to adjust the values of λ and γ and adjust them based on the error and error changes. With two inputs and two outputs, the fuzzy rules in the simple Sugeno method are as follows:

$\boldsymbol{IF}\ \tilde{q}\ is\ A_i\ and\ \dot{\tilde{q}}\ is\ B_i$ THEN

$$\tau = \hat{\tau} - K\ sat(\frac{\dot{e} + \lambda_i e - \gamma_i}{\varphi}) \tag{23}$$

First, for each of the inputs $\tilde{q}$ and $\dot{\tilde{q}}$, we define six membership functions $\{NL, NS, NZ, PZ, PS, PL\}$ according to Figure 4, then the Sugeno fuzzy rule base to obtain λ_i and $\gamma_i$as Tables 2 and 3 is considered.

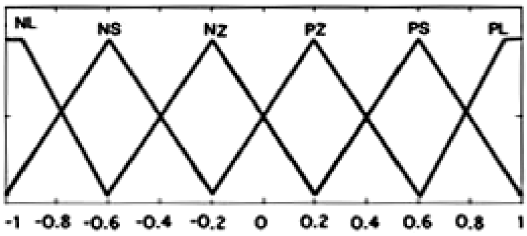


Fig. 4. Membership functions for inputs $\tilde{q}$ and $\dot{\tilde{q}}$

Table 2: Rule base forλ_i

$\dot{e}$	e					
	PL	PS	PZ	NZ	NS	NL
PL	0.6	0.6	0.6	0.6	0.6	0.6
PS	0.6	0.6	0.6	5	5	5
PZ	0.6	0.6	5	8	8	8
NZ	8	8	8	5	0.6	0.6
NS	5	5	5	0.6	0.6	0.6
NL	0.6	0.6	0.6	0.6	0.6	0.6

Table 3: Rule base for γ_i

$\dot{e}$	e					
	PL	PS	PZ	NZ	NS	NL
PL	-10	-8	-4	0	0	0
PS	-8	-4	-2	0	0	0
PZ	-3	-2	0	0	0	0
NZ	0	0	0	0	2	4
NS	0	0	0	0	4	8
NL	0	0	0	4	6	10

5. Simulation Results

The proposed methods in this article are implemented on a robot with the following parameters:

$m_1 = 10\ \ \hat{m}_2 = 5\ \ l_1 = 1\ \ l_2 = 0.5\ \ l_{c_1} = 0.5\ \ \hat{l}_{c_2} = 0.25$

$I_1 = {}^{10}/_{12}\ \ \hat{I}_2 = {}^{5}/_{12}$

$0 \le \Delta m_2 \le 2\ \ 0 \le \Delta l_{c_2} \le 0.25\ \ 0 \le \Delta I_2 \le 0.5$

The desired state vector is considered as $q_d = [\pi\ \ -\pi]^T$ and the design parameters of the sliding mode controller considered as $\lambda = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$, $K = \begin{bmatrix} 75 & 0 \\ 0 & 110 \end{bmatrix}$.

Due to the fact that if the input torques exceed a certain limit, the problem of saturation of the actuator of the joint will arise, so we are facing a limitation in the application of input torques. For the robot model that we have simulated, the maximum torque applied to the first joint is 150 and for the second joint is 15.

Figures 5, 6, and 7 show the results of classic sliding model control simulations, and figures 8, 9, and 10 show the results of fuzzy-sliding model cooperative controller simulations. Finally, figures 11, 12, and 13 show the controller simulation results with a moving sliding surface. As can be seen from the figures, the problem of chattering around the sliding surface has been solved in the cooperative controller, and the sliding surface is much smoother than the sliding surface of the classical sliding mode controller. Figures 11 and 12 clearly show that the sliding phase in the controller with the moving sliding surface has reached its minimum possible.

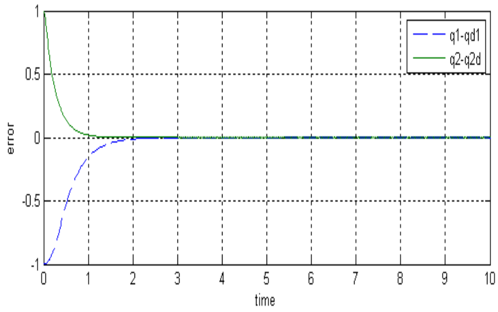


Fig.5. Classic sliding mode control tracking error

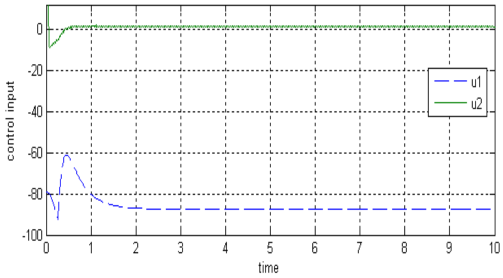


Fig. 6. Control inputs of classical sliding mode control

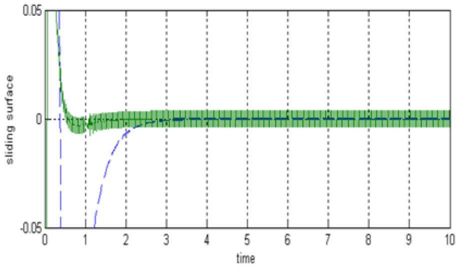


Fig.7. Close-up view of classic sliding mode control sliding surfaces

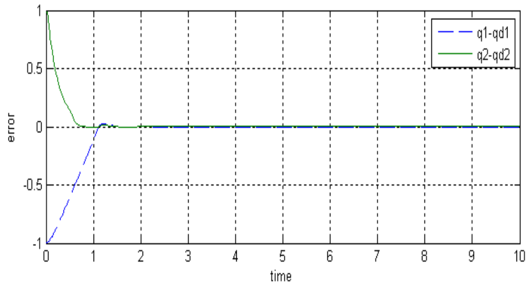


Fig.8. Tracking error of fuzzy-sliding mode cooperative control

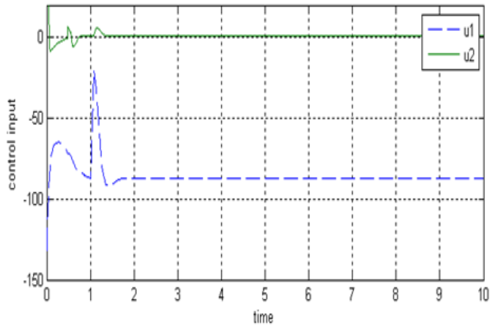


Fig. 9. Control inputs of fuzzy-sliding mode cooperative control

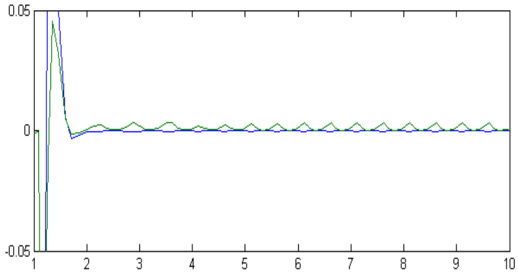


Fig. 10. Close-up view of fuzzy-sliding cooperative control sliding surface

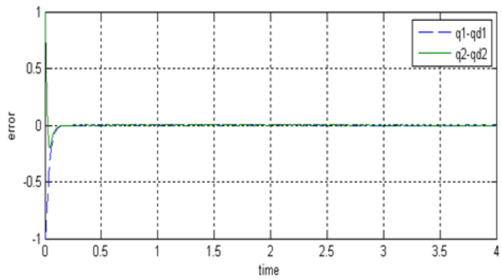


Fig. 11. Tracking error of sliding mode control with moving surface

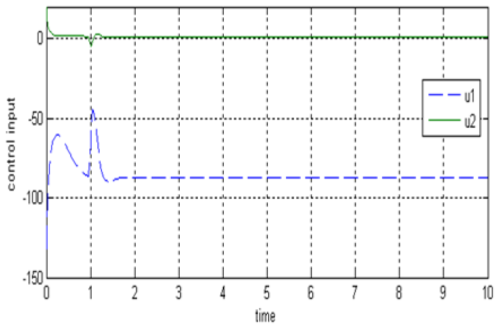


Fig.12. Control inputs of sliding mode control with moving surface

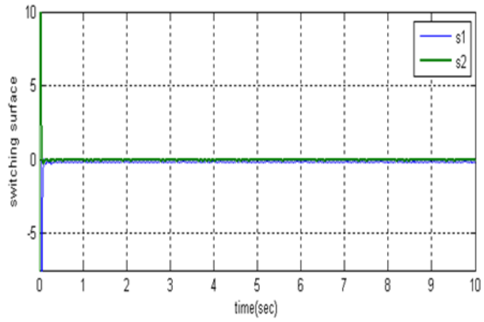


Fig. 13. Close-up view of the moving sliding surface

Conclusions

In this paper, by using fuzzy knowledge, we solved two major problems of classical sliding model control. The first problem was the chattering around the sliding surface, which we minimized by using a sliding mode-fuzzy cooperative controller. In this method, the sliding mode control works first, and the proximity of the sliding surface of the fuzzy control is implemented. The second problem was that the sliding model control was not robust in the reaching phase, and by defining a moving sliding surface, we were able to minimize the sliding phase.

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