CS-MRI: The Application of Compressed Sensing for Magnetic Resonance Imaging

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Receive Date: 03 September 2023 Revise Date: 12 June 2024 Accept Date: 22 August 2024

Abstract

Magnetic resonance imaging (MRI) is a non-aggressive medical imaging modality that use magnetic resonance property of hydrojen atom core in body and can show a wide range of tissues with high resolution. The main disadvantage of MRI which limits its application, is the slow data acquisition speed. The use of compressed sensing (CS) in MRI, known as CS-MRI, reduces data acquisition time in MRI by reducing the required samples. One important challenge in CS-MRI, is to develop a sparsity inducing model which can reflect the image priors appropriately and hence yields high quality recovery results. In this study, principles of MRI, application of CS in MRI, and CS-MRI reconstruction techniques are discussed in brief.

Keywords Compressed sensing, Image recovery, Magnetic resonance imaging, Reconstruction algorithm

1. Introduction

Magnetic resonance imaging (MRI) is a non-aggressive medical imaging modality that use magnetic resonance property of hydrojen atom core in body and can show a wide range of tissues with high resolution. The main disadvantage of MRI which limits its application, is the slow data acquisition speed for the long sampling period required by the traditional Nyquist sampling Theorem [1]. The slow data acquisition speed may result in prolongation of imaging time and so increase patients discomfort and motion artifacts [2]. Therefore, improving the speed of MRI is of particular importance. One of the methods of increasing the data acquisition speed is to upgrade the hardware used for data collection. However, the physical (gradient amplitude and slew-rate) and physiological (nerve stimulation) constraints limit the speed at which data can be collected [3]. Thus, the only effective method to decrease the imaging time is to reduce the amount of acquired data. However, decreasing the sampling rate violates the Nyquist sampling theorem and leads to aliasing artifacts in reconstructed MR images. Therefore, many researchers are looking for methods to reduce amount of acquired data without degrading the image quality [3]. Among these methods, there is parallel imaging (PMRI) which is hardware-based and uses redundancy in k-space. SENSE [4] and GRAPPA [5] are two common parallel imaging methods. A known limitation of these methods is that with the increase of the acceleration factor, the amplitudes of reconstruction artifacts grow rapidly [6]. By introducing compressed sensing (CS) theory in 2006 by Candès et al. [7, 8], and Donoho [9], as a surrogate of traditional Nyquist sampling theorem, it is provided to reconstruct the MR images without artifacts from far fewer data. The result of applying CS in MRI, which is known as CS-MRI, is to reduce the time required for imaging, reduce costs and patient comfort. A successful application of CS has three requirements [3, 10]:

- The image should have a sparse representation in a certain transform domain.
- The artifacts in reconstruction caused by k-space undersampling should be incoherent in the sparsifying transform domain.
- The image should be reconstructed by a nonlinear method that enforces both sparsity of the image and consistency of the reconstruction with the acquired samples.

The rest of the paper is organized as follows: In section 2, principles of magnetic resonance imaging are briefly presented. In section 3, application of CS in MRI are explained. In section 4, some of the CS-MRI reconstruction methods are reviewed. Finally, the conclusions are provided in section 5.

2. Magnetic Resonance Imaging

The operation of the MRI system is based on the nuclear magnetic resonance (NMR) property. The MR signal is generated by protons in the body, mostly those in water molecules. A strong static magnetic field B_0 polarizes the protons and leads to the creation of a net magnetic moment in the direction of the static magnetic field. Applying a radio frequency (RF) excitation field B_1 to the polarization vector rotates it and produces a magnetization component m transverse to the static field. This magnetization process takes place at a frequency proportional to the static field strength. The transverse component of the magnetization emits a RF signal that can be received by a coil [11]. The transverse component at position r is represented by the following complex quantity:

$$m(r) = |m(r)|\, e^{-j\emptyset(r)} \quad (1)$$

The $m(r)$ component represents many characteristics of the tissue. In fact, the desired MR image to be reconstructed is $m(r)$, which is an image of the spatial distribution of the transverse component of magnetization [11].

It can be shown that the received signal equation in MRI is obtained as follows [12]:

$$S(\boldsymbol{k}(t)) = \iiint m(\boldsymbol{r}) e^{-j2\pi \boldsymbol{k}(t).\boldsymbol{r}} d\boldsymbol{r} \quad (2)$$

In other words, the received signal $S(\boldsymbol{k}(t))$ is the Fourier transform of the image $m(\boldsymbol{r})$ sampled at the spatial frequency $\boldsymbol{k}(t)$ [11]. In two-dimensional imaging, the received signal equation in MRI is simplified as follows:

$$S\big(k_x(t), k_y(t)\big) = \iint m(x,y)\, e^{-j2\pi k_x(t)x} e^{-j2\pi k_y(t)y} dxdy \quad (3)$$

In general, the frequency domain (k_x, k_y) is referred to as k-space. The k-space trajectory is controlled by gradients. The gradients can be chosen in such a way that k-space is adequately sampled. In this case, the complete MR image can be reconstructed as the inverse Fourier transform of the acquired data [12].

Traditionally, the k-space sampling pattern is designed in such a way that the Nyquist conditions are met, which depends on resolution and field of view (FOV). Image resolution is determined by the sampling area of k-space. A larger sampling area results in higher resolution. The field of view is determined by the sampling density in the desired area. Figure 1 shows the relationship between image domain and k-space. Violation of the Nyquist criterion causes image artifacts in linear reconstructions. The appearance of such artifacts depends on the details of the sampling pattern [11].

In recent years, a variety of techniques were proposed to accelerate data acquisition in MRI. Due to the physical and physiological constraints, the speed of k-space traversal and thus the data acquisition speed is limited. Therefore, many researchers are looking for methods to reduce the amount of acquired data without degrading the quality of the image. The basic idea behind many of these methods is the use of spatial and/or temporal redundancy of the MR images [10]. One of the effective methods to achieve this goal is the compressed sensing, which by taking advantage of the sparsity of MR images, provides the possibility of complete reconstruction of images from a small subset of k-space.

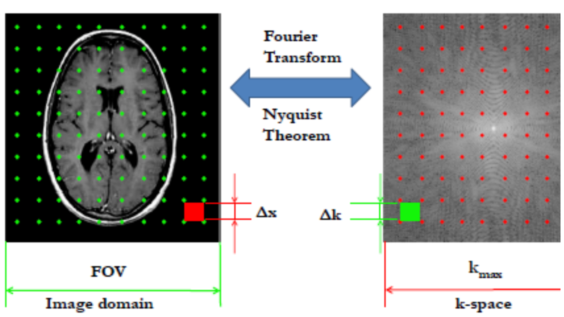

Fig.1.The relationship between image domain and k-space [13]

3. Compressed Sensing MRI

Equation (3) shows the mathematical model related to MR imaging. The vector form of this equation can be written as follows:

$$\boldsymbol{S} = \mathbf{F}\boldsymbol{m}, \quad (4)$$

where $\boldsymbol{m} \in \mathbb{C}^N$ is the MR image in vectorized form, $\mathbf{F} \in \mathbb{C}^{N\times N}$ is the Fourier transform matrix, and $\boldsymbol{S} \in \mathbb{C}^N$ is the k-space data vector. In order to speed up imaging, undersampling is done. But when the k-space is undersampled, the Nyquist conditions are violated, causing image artifacts in the reconstruction. Using compressed sensing in MRI, only a small subset of k-space data will be required. In this case, the equation (4) is written as follows:

$$\mathbf{y} = \mathbf{WFu} \quad \Rightarrow \quad \mathbf{y} = \mathbf{F}_u\mathbf{u}, \quad (5)$$

where $\mathbf{W} \in \mathbb{R}^{M\times N}$, with $M < N$, is a binary mask consisting of zeros and ones that selects M rows of the matrix $\mathbf{F}$; $\mathbf{y} \in \mathbb{C}^M$ is undersampled k-space data (measurement vector); $\mathbf{F}_u \in \mathbb{C}^{M\times N}$ is the undersampling Fourier operator; and $\mathbf{u} \in \mathbb{C}^N$ is the desired image. Sampling in MRI is a special case of compressed sensing where the sampled linear combinations are Fourier coefficients. In this case, the compressed sensing method is claimed to be able to accurately reconstruct the original image from a small subset of k-space. Figure (2) shows the requirements of this method. As can be seen from the Figure, the CS approach has three requirements: (a) the desired image have a sparse representation in a known transform domain, (b) the aliasing artifacts due to k-space undersampling be incoherent in that transform domain, (c) a nonlinear reconstruction algorithm be used to enforce both sparsity of the image and consistency with the acquired data [3]. If these requirements are met, it is actually possible to recover the desired image. The recovery problem of the image $\mathbf{u}$ from $\mathbf{y}$ is formulated as the following unconstrained optimization problem:

$$\min_{\mathbf{u}} \ \frac{1}{2}\|\mathbf{y} - \mathbf{F}_u\mathbf{u}\|_2^2 + \lambda\Re(\mathbf{u}) \quad (6)$$

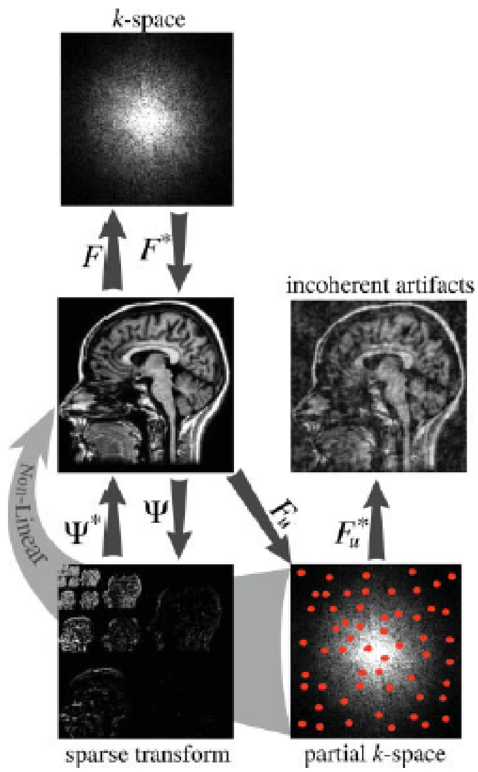

Fig.2.CS-MRI circuit diagram and requirements [3].

where $\Re(.)$ is called sparsity inducing model, $\|\mathbf{y}-\mathbf{F}_u\mathbf{u}\|_2$ is known as data fidelity model, and λ is a regularization parameter. The sparsity inducing model plays a key role in achieving high quality images. Early reconstruction methods utilize the sparsity of the image in some predefined transform domains. In this case, the image $\mathbf{u}$ can be represented as $\mathbf{u}=\boldsymbol{\Psi}\mathbf{x}$ where $\boldsymbol{\Psi}$ is a transform domain or sparsifying basis and $\mathbf{x}\in\mathbb{R}^N$ is the coefficient vector that most entries of which are zero or close to zero. This means that $\|\mathbf{x}\|_0 \ll N$, where $\|\mathbf{x}\|_0$ is the l_0 norm of the vector and represents the number of its non-zero components. In other words, the sparsity model can be as the l_0 norm of transform coefficients $\|\boldsymbol{\Psi}^T\mathbf{u}\|_0$ and or $\|\boldsymbol{\Psi}^T\mathbf{u}\|_1$ as a relaxation of the l_0 norm. Examples of the transform domain can be discrete cosine transform (DCT) [3], discrete wavelet transform (DWT) [14], EWT [15], contourlet transform [16], and finite difference domain [17]. Some methods combine sparse transforms to exploit the advantage of each other. In general, these models are built based on the assumption that images are locally smooth except at the edges and demonstrate high effectiveness in reconstructing smooth areas. However, they cannot deal well with image details and fine structures, and tend to over-smooth images [18]. To improve the sparsity, patch-based models were proposed. The main idea is to decompose the image into overlapped patches and represent each patch by a few elements from a basis set called dictionary, which is learned from images. The learned dictionaries enjoy the advantage of being better adapted to image local structures, thereby enhancing the sparsity and lead to reduce artifacts in CS-MRI. However, in the process of dictionary learning and sparse coding, each patch is considered independently, which ignores the relationships between similar patches [19].

Recently, it has been shown that nonlocal self-similarity based models, called nonlocal sparsity models, are effective in preserving details and demonstrate great advantages in image reconstruction [20]. The nonlocal self-similarity depicts the repetitiveness of higher level patterns (e.g., textures and structures) globally positioned in images [18]. To exploit the nonlocal self-similarity prior, the image is divided into overlapped patches. Then, for each patch within the search window, a set of similar patches are searched to form a data matrix, called a group. The patches in the group are correlated; the strong correlations allow one to develop a much more accurate sparsity inducing model by exploiting nonlocal redundancies [21].

4. CS-MRI Reconstruction Approaches

In this section, some techniques proposed for CS-MRI reconstruction are reviewed. In [22], fast composite splitting algorithm (FCSA) for MR image reconstruction was presented, which tries to solve the following CS-MRI reconstruction problem:

$$\min_{\mathbf{u}} \ \frac{1}{2}\|\mathbf{F}_u\mathbf{u}-\mathbf{y}\|_2^2+\alpha\left\|\mathbf{\Psi}^T\mathbf{u}\right\|_1+\beta\|\mathbf{u}\|_{TV}, \quad (7)$$

where $\mathbf{\Psi}$ is the wavelet transform matrix, and $\|.\|_{TV}$ is denoted total variation (TV) norm which is used to induce the sparsity of the image in the finite difference domain. In this method, the problem (7) is divided into two sub-problems of regularizing the l_1 norm and regularizing the TV norm. The final reconstructed image is obtained from the weighted average of the answers of both sub-problems. Despite using the hybrid sparse model in the reconstruction problem in order to better reflect the characteristics of the image, the reconstructed images have low PSNR. The FCSA method is based on the fast iterative shrinkage-thresholding algorithm (FISTA) and therefore has a good convergence speed.

In order to remove the blocking effects caused by TV norm regularization and preserve the details of MR images, a reconstruction method based on TV and NLTV was presented in [17]. In this method, the MR image is first reconstructed by solving the problem (7). Then, the final image is obtained by solving problem (8) in one iteration.

$$\min_{\mathbf{u}} \ \frac{1}{2}\|\mathbf{F}_u\mathbf{u}-\mathbf{y}\|_2^2+\alpha\left\|\mathbf{\Psi}^T\mathbf{u}\right\|_1+\beta\|\mathbf{u}\|_{NLTV} \quad (8)$$

In [23], the use of contourlet as a sparsifying transform along with the FISTA algorithm was proposed to solve the CS-MRI reconstruction problem (9). This method is known as fast iterative contourlet thresholding algorithm (FICOTA). Contourlet transform performs better than wavelet transform in displaying edges and curved lines of images, but increases the amount of computation.

$$\min_{\mathbf{u}} \ \frac{1}{2}\|\mathbf{F}_u\mathbf{u}-\mathbf{y}\|_2^2+\alpha\left\|\mathbf{\Psi}^T\mathbf{u}\right\|_1 \quad (9)$$

In [24], instead of contourlet, discrete nonseparable shearlet transform (DNST) was used as a sparsifying transform. The use of DNST improves the reconstruction quality of the FICOTA method and at the same time increases its execution time. In [25], an algorithm named projected iterative soft thresholding algorithm (pISTA) and its accelerated version pFISTA were presented to solve the CS-MRI reconstruction problem (9). These algorithms use the following repetition pattern:

$$\mathbf{u}^{k+1}=\mathbf{\Psi}^{*}T_{\mu\lambda}(\mathbf{\Psi}(\mathbf{u}^k+\mu{\mathbf{F}_u}^T(\mathbf{y}-\mathbf{F}_u\mathbf{u}^k))), \quad (10)$$

where $\mathbf{\Psi}$ is a base or a tight frame (a frame that has redundancy). In such frames, the dimensions of an image are much smaller than its coefficients under the frame. Since these algorithms are implemented without saving frame coefficients, they greatly reduce memory consumption. In [26], an iterative algorithm based on p-thresholding was proposed to solve the reconstruction problem (9). This algorithm uses the following repetition pattern:

$$\mathbf{x}^{k+1}=G_{(\lambda,p)_k}(\mathbf{x}^k+\mathbf{\Psi}{\mathbf{F}_u}^T\mathbf{y}-\mathbf{\Psi}{\mathbf{F}_u}^T\mathbf{F}_u\mathbf{\Psi}\mathbf{x}^k), \quad (11)$$

where G denotes the thresholding function and is defined as $G_{(\lambda,p)_k}(\mathbf{x}^k)=sgn(\mathbf{x}^k)\max\{0,|\mathbf{x}^k|-\lambda_k|\mathbf{x}^k|^{p-1}\}$. Another method was proposed in [27] to reconstruct

MR images in the tight frame domain. This method uses the following model:

$$\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{F}_u vec(\mathbf{\Psi x}) - \mathbf{y}\|_2^2 + \lambda \sum_i w_i \sigma_i^2(\mathbf{x}), \qquad (12)$$

where $\mathbf{x}$ denotes the image coefficients under frame $\mathbf{\Psi}$, $vec(.)$ is the vectorization operator of a matrix, $\sigma_i(\mathbf{x})$ is the i-th singular value of $\mathbf{x}$, and w_i is the weight assigned to $\sigma_i(\mathbf{x})$.

In [15], the EWT-ISTA method was presented to reconstruct MR images, which uses the exponential wavelet transform or EWT as a sparsifying transform with the aim of increasing the sparsity of the images in the wavelet domain. MR images are usually sparse in the wavelet domain. The rationale of EWT to improve sparsity is that if significant coefficients are enhanced and non-significant coefficients are attenuated, through a nonlinear transformation, then sparsity is improved. The used nonlinear transformation is an exponentiation that has a parameter n. The larger n, the higher sparsity degree of coefficients will be. Meanwhile, as n increases, the non-zero coefficients are strongly weakened, which leads to a decrease in the reconstruction quality. Also, this method has relatively slow convergence due to the use of ISTA algorithm. The improved version of the method [15] was presented in [28], which is a combination of EWT, FISTA and SISTA and is known as EWISTA. In fact, this method uses the sparsity of EWT, the convergence speed of FISTA and the parameters adjustment in SISTA to increase the reconstruction quality and reduce the calculation time.

Wavelet transform provides a sparse representation for smooth images. But if the image does not meet this feature, the resulting representation may not be optimal. To solve this limitation, a graph-based additive wavelet transform (GBRWT) was proposed in [29] to sparsify MR images. In this method, the reference image is first divided into several blocks and the corresponding graph is formed in such a way that the blocks of the image are considered as vertices and their difference as edges of the graph. The shortest path on the graph will produce a smooth image. If the image pixels are arranged according to this short path, a sparser representation is obtained by applying the wavelet transform. Finally, the problem of CS-MRI reconstruction (9) is solved by using GBRWT and ADMM algorithm.

In [30], instead of predefined sparsifying transforms, block-based adaptive dictionary was used to achieve better reconstruction performance in CS-MRI. These dictionaries are more adaptable to the local structures of image, improve the sparsity and lead to the reduction of artifacts in CS-MRI. In the method [30], which is known as dictionary learning MRI (DLMRI), dictionary learning and image reconstruction from undersampled k-space data are used simultaneously in a single model. This model is formulated as follows:

$$\min_{\mathbf{u},\mathbf{D},\mathbf{x}} \frac{1}{2}\|\mathbf{F}_u\mathbf{u} - \mathbf{y}\|_2^2 + \eta \sum_i \|\mathbf{R}_i\mathbf{u} - \boldsymbol{D}\mathbf{x}_i\|_2^2 \quad s.t. \quad \|\mathbf{x}_i\|_0 \le T_0, \qquad (13)$$

where $\mathbf{R}_i$ is the matrix that extracts block $\boldsymbol{u}_i$ from image $\mathbf{u}$. The problem (13) is solved by alternating minimization method at two steps. To learn the dictionary $\boldsymbol{D}$, K-singular value decomposition (K-SVD) algorithm [31] is used, which has a high computational complexity.

In [32], a dictionary updating (DU) based MR image reconstruction method called DUMRI was presented. In dictionary learning based MRI, several iterations between sparse coding and dictionary updating in the dictionary learning process lead to wasted imaging time. To reduce the imaging time without degrading the reconstruction quality, the authors proposed a dictionary updating method that avoids the iterations of the dictionary learning process. Also, in order to improve the reconstruction quality, the block matching and three-dimensional collaborative filtering (BM3D) denoising method was used. One of the disadvantages of this method is the low speed of running and the complexity of calculations, which cannot be suitable for the practical application of MRI.

In order to provide a sparser representation of MR images and thus improve the reconstruction results, a patch-based non-local operator called PANO proposed in [33], which uses the similarity of image patches. This operator is defined as $\boldsymbol{A}_j = \boldsymbol{\Psi}_{3D}\boldsymbol{R}_{v_j}\mathbf{R}_i$, where $\mathbf{R}_i$ is the matrix that extracts the patch $\boldsymbol{u}_i$ from the image u (so that $\boldsymbol{u}_i = \mathbf{R}_i\mathbf{u}$), $\boldsymbol{R}_{v_j}$ selects the c number of blocks with the most similarity to form the v_jth group where $v_j = \{i_1, \dots, i_c\}$, and $\boldsymbol{\Psi}_{3D}$ is a three-dimensional transform. The MR image is obtained by solving the problem (14), taking into account the sparsity of the coefficients obtained by applying the PANO operator to it:

$$\min_{\mathbf{u}} \ \frac{1}{2}\|\mathbf{F}_u\mathbf{u} - \mathbf{y}\|_2^2 + \alpha\sum_j\|\boldsymbol{A}_j\mathbf{u}\|_1 \qquad (14)$$

But using the similarity of patches requires having the original image. Since no ground truth image is available for similarity learning, the authors proposed to learn the similarity from a guide image estimated from the measurements and showed that learning the similarity is not sensitive to the initial guide image.

In [34], a CS-MRI reconstruction method based on group sparse representation and statistical estimation, called as group-based eigenvalue decomposition and estimation (GEDE), was presented. This method emphasizes on sparser representation of the image and more accurate estimation of sparse coefficients. In this method, a sparse representation is developed based on the local and non-local features of the image; By performing SVD on the group as $[\mathbf{D}_i, \boldsymbol{\gamma}_i, \boldsymbol{\vartheta}_i] = svd(\mathbf{U}_{G_i})$, where $\mathbf{D}_i$ denotes the dictionary corresponding to local sparsity, $\boldsymbol{\vartheta}_i$ is the dictionary corresponding to nonlocal sparsity, and $\boldsymbol{\gamma}_i$ is the vector of sparse representation coefficients. Also, linear minimization mean square error (LMMSE) has been used to accurately estimate sparse coefficients. The reconstruction problem is formulated as follows:

$$\min_{\mathbf{u},\boldsymbol{\gamma}} \frac{1}{2}\|\mathbf{F}_u\mathbf{u} - \mathbf{y}\|_2^2 + \eta\sum_i\{\|\widetilde{\mathbf{R}}_{G_i}\mathbf{u} - \mathbf{D}_i\boldsymbol{\gamma}_i\boldsymbol{\vartheta}_i^T\|_2^2 + \lambda\mathbb{L}(\boldsymbol{\gamma})\}, \qquad (15)$$

where the $\mathbb{L}(\boldsymbol{\gamma})$ represents the LMMSE, and $\mathbf{D}_i$ and $\boldsymbol{\vartheta}_i^T$ are the left and right dictionaries of the group, respectively.

In [1], a MR image reconstruction method based on low-rank structure using non-local sparsity of MR images was presented. In this method, low-rank regularization becomes a nuclear norm minimization problem and is solved by SVT and ADMM methods.

In [35], robust CS-MRI based on the combined nonconvex regularization model was proposed to enhance the details recovery and reduce artifacts. In this framework, the bias-based sharpness enhancement prior was integrated into smoothed l_0 (SL0) model. The nonconvex optimization problem was solved by a two-cycle iterative algorithm.

5. Conclusions

MRI is a non-aggressive medical imaging modality that can show a wide range of tissues with high resolution. A complete MR image is reconstructed using the acquired data. In MRI, the data acquisition is done at a relatively slow speed. This result in prolongation of imaging time and so increase patients discomfort and motion artifacts. Therefore, improving the speed of MRI is of particular importance. Considering the physical and physiological constraints, the only effective method to decrease the imaging time is to reduce the amount of acquired data. However, decreasing the sampling rate violates the Nyquist sampling theorem and leads to aliasing artifacts in reconstructed MR images. In order to reduce amount of acquired data without degrading the image quality, the compressed sensing theorem has been proposed. The use of compressed sensing in MRI reduces the number of samples and the time required for imaging. As a result, it leads to a reduction in costs and patient comfort.

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