

# Adaptive Inverse Control of Flexible Link Robot Using ANFIS Based on Type-2 Fuzzy

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## Abstract

*This paper presents a novel adaptive neuro-fuzzy inference system based on interval Gaussian type-2 fuzzy sets in the antecedent part and Gaussian type-1 fuzzy sets as coefficients of linear combination of input variables in the consequent part. The capability of the proposed ANFIS2 for function approximation and dynamical system identification is remarkable. The structure of ANFIS2 is very similar to ANFIS but in ANFIS2, a layer is added aiming at type reduction. An adaptive learning rate based back propagation with convergence guaranteed, is used for parameter learning. Finally, the proposed ANFIS2 is used to control a flexible link robot arm. Simulation results show the proposed ANFIS2 with Gaussian type-1 fuzzy set as a coefficient of linear combination of input variables in the consequent part with good performance and high accuracy but more training time.*

**Keywords:**ANFIS, Interval Type-2 Fuzzy Sets, Inverse Control, Flexible Link Robot.

## 1- Introduction

High computation ability, adaptability and parallel processing are the important advantages of neural networks [1]. Using the knowledge of expert man, as if-then rules and having real concept of parameters are the advantages of fuzzy systems. Among hybrid fuzzy neural networks, ANFIS is very popular and widespread. ANFIS is very simple and intelligible so it has affected many areas such as geography, medical Sciences, meteorological science, chemical and petroleum engineering, etc. [2, 3]. A flexible link arm is a distributed parameter system of

infinite order, but must be approximated by a lower-order model and be controlled by a finite-order controller due to onboard computer limitations, sensor inaccuracy, and system noise. The so-called “control spillover” and “observation spillover” effects occur then, which under certain conditions can lead to instability [4].

In recent ten years, type-2 fuzzy logic with more capabilities and more flexibility than type-1 fuzzy logic has been investigated. Castillo et al. investigated type-2 fuzzy logic in more details [5]. Huang and Chen [6] used the combination of quantum inspired bacterial foraging algorithm (QBFA) and

recursive least squares (RLS) to tune a type-2 fuzzy system. Tavoosi et al. proposed a different architecture of interval type-2 Takagi-sugeno-kang fuzzy neural network [7]. They proposed an ANFIS based on type-2 fuzzy sets. Shahnazi [8] used type-2 fuzzy systems to approximate the unknown nonlinearities in MIMO systems' control problem. He derived all the adaptive laws via Lyapunov's synthesis approach.

Not many studies have been done on fuzzy systems with type-1 (or type-2) fuzzy sets in the consequent part. In most of papers, the consequent part is singleton [9] or interval type-1 fuzzy sets [10-12], up to now. Next, some of the works in this area will be reviewed. In [13] interval type-2 fuzzy integrators in ensembles of ANFIS models for the time series prediction is used. Genetic algorithm is used to optimize the proposed model.

The equations of Type-2 ANFIS and its optimizations are not presented. In [14], interval type-2 adaptive network-based fuzzy inference system with type-2 non-singleton fuzzification have been introduced. Interval type-1 fuzzy sets have been used as consequent parameters. Mendez and Hernandez [15] presented a type-2 fuzzy ANFIS through which interval type-1 non-singleton fuzzy numbers are the inputs and type-2 TSK FLS is the output and the consequent parameters are estimated by the recursive least-squares (RLS) method. They didn't provide further details of learning equations. Bhattacharyya et al. [16] proposed a type-2 fuzzy ANFIS that an interval type-2 fuzzy logic is used to combine the different

outputs of the ANFIS classifiers to produce a final optimal result.

Tavoosi and Badamchizadeh [17] proposed a type-2 Takagi-Sugeno-Kang fuzzy neural network with linear consequent part to system identification and modeling. Rule pruning was the novelty of that paper. Higher learning speed was the goal by reducing the parameter in both antecedent and consequent parts. Tavoosi et al. [18] presented a new method to stability analysis of a class of type-2 fuzzy system. Jahangiri et al. [19] proposed a method for stability analysis of a class of neural networks. In [20] a new method to MIMO type-2 fuzzy stability analysis has been presented. Robot manipulators have become increasingly important in the field of flexible automation. So, modeling and control of robots in automation will be very important. Some literatures used fuzzy logic to robot control [21-24].

This paper presents a novel ANFIS based on type-2 fuzzy named ANFIS2. The proposed method uses Interval Gaussian type-2 fuzzy sets in the antecedent part and Gaussian type-1 fuzzy sets as coefficients of linear combination of input variables in the consequent part. The paper is organized as follows. In section 2, type-2 fuzzy systems is viewed. In section 3, the structure of ANFIS2 is investigated. Parameter identification is given at the end of this section. In section 4, learning convergence of ANFIS2 based on Lyapunov theory is derived. In section 5, the simulation studies are presented for identification of three nonlinear systems. Finally, Section 6 gives the conclusions of the advocated design methodology.

## 2- A Review on Type-2 Fuzzy Systems

In dealing with a lot of uncertainties, the performance and efficiency of type-1 fuzzy systems is not suitable. The membership degree of type-1 fuzzy sets is a crisp number while the membership degree of type-2 fuzzy sets is a type-1 fuzzy number.

Some difficulties of type-1 fuzzy logic can be solved by using type-2 fuzzy logic. In some systems such as time-series prediction, the exact membership degree is determined in a very difficult manner due to their complexity and their noisy information [25]. So using type-2 fuzzy systems for describing behavior of these systems can be useful. In [26], some disadvantages of type-1 fuzzy sets are mentioned.

Fig. 1 shows the Gaussian primary membership function and Gaussian secondary membership function. For example, if  $m = 0$ ,  $\sigma = 1$  and  $x = 1$  then degree of membership is 0.6, if this membership degree is too fuzzy or 0.6 then primary membership is Gaussian type-1 fuzzy set with  $m = 0$ ,  $\sigma = 1$  and secondary membership is Gaussian type-1 fuzzy set with  $m = 0.6$ ,  $\sigma = 0.1$ .

Note that, when secondary membership is not Gaussian type-1 fuzzy set and it is equal to one and in other words secondary membership function is interval set with one magnitude, then fuzzy set is called interval type-2 fuzzy set.

Two cases of interval type-2 fuzzy sets are shown in Fig. 2. In Fig. 2-a, a case of a fuzzy set characterized by a Gaussian membership function with mean  $m$  and a standard deviation that can take values in  $[\sigma_1, \sigma_2]$  and

in Fig. 2-b, a case of a fuzzy set with a Gaussian membership function with a fixed standard deviation  $\sigma$ , but an uncertain mean, taking values in  $[m_1, m_2]$  and are shown.

In this paper Gaussian membership function with fixed standard deviation  $\sigma$  and uncertain mean is used (Fig. 2. b).

## 3- Adaptive Neuro Fuzzy Inference system by Type-2 Fuzzy Sets (ANFIS2)

Similar to type-1 TSK fuzzy systems, the output of type-2 TSK fuzzy systems is a function of their inputs. But, in type-2 fuzzy systems the output and its coefficients are type-1 fuzzy sets. In this paper, the proposed ANFIS2 has seven layers whose structure is shown in Fig 3.

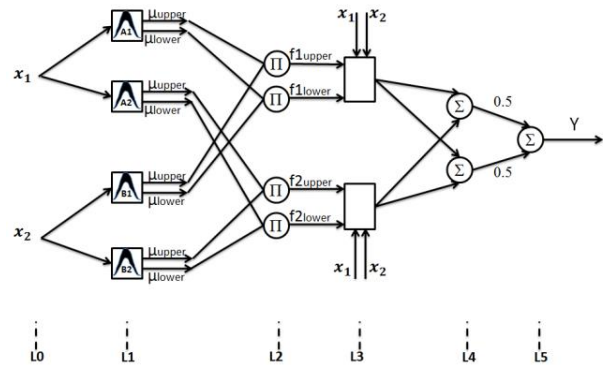


Fig.3.The structure of ANFIS2

The two rules of ANFIS2 can be described as follows:

$$\begin{aligned}
 R^1: & \text{if } x_1 \text{ is } \tilde{A}_1 \text{ and } x_2 \text{ is } \tilde{B}_1 \text{ then} \\
 & \tilde{y}_1 = \tilde{r}_1 + \tilde{p}_1 x_1 + \tilde{q}_1 x_2 \\
 R^2: & \text{if } x_1 \text{ is } \tilde{A}_2 \text{ and } x_2 \text{ is } \tilde{B}_2^k \text{ then} \\
 & \tilde{y}_2 = \tilde{r}_2 + \tilde{p}_2 x_1 + \tilde{q}_2 x_2
 \end{aligned} \tag{1}$$

Where  $x_i (i = 1,2)$  are inputs,  $\tilde{y}_k (k = 1,2)$  is output of the  $k$ th rule which is type-1 fuzzy set (since it is a linear combination of Gaussian type-1 fuzzy sets),

$\tilde{A}_i^k$  are antecedent interval type-2 fuzzy sets,  $\tilde{r}_k, \tilde{p}_k$  and  $\tilde{q}_k$  ( $k = 1, 2$ ) are Gaussian type-1 fuzzy sets. For simplicity in description, we select only two inputs and two rules but the proposed ANFIS2 can be generalized to n-inputs and m-rules ( $n, m \in \mathbb{N}$ ).

The forward-propagation procedure is described as follows:

**Layer 0:** This layer is inputs layer. The number of nodes in this layer is equal to the number of inputs.

$$\bar{\mu}_{k,i}(x_i) = \begin{cases} {}^1\mu_{k,i}(x_i, [\sigma_{k,i}, {}^1m_{k,i}]), & x_i < {}^1m_{k,i} \\ 1, & {}^1m_{k,i} \leq x_i \leq {}^2m_{k,i} \\ {}^2\mu_{k,i}(x_i, [\sigma_{k,i}, {}^2m_{k,i}]), & x_i > {}^2m_{k,i} \end{cases} \quad (4)$$

$$\underline{\mu}_{k,i}(x_i) = \begin{cases} {}^2\mu_{k,i}(x_i, [\sigma_{k,i}, {}^2m_{k,i}]), & x_i \leq \frac{{}^1m_{k,i} + {}^2m_{k,i}}{2} \\ {}^1\mu_{k,i}(x_i, [\sigma_{k,i}, {}^1m_{k,i}]), & x_i > \frac{{}^1m_{k,i} + {}^2m_{k,i}}{2} \end{cases} \quad (5)$$

Where,  $\bar{\mu}_{k,i}$  is upper membership degree and  $\underline{\mu}_{k,i}$  is lower membership degree.

**Layer 2:** This is rule layer. Each output node represents the lower ( $\underline{f}^k$ ) and upper ( $\bar{f}^k$ ) firing strength of a rule:

$$\underline{f}^k = \prod_{i=1}^n \underline{\mu}_{k,i} \quad ; \quad \bar{f}^k = \prod_{i=1}^n \bar{\mu}_{k,i} \quad (6)$$

**Layer 3:** This is consequent layer.

$$\begin{aligned} \tilde{y}_1 &= \tilde{r}_1 + \tilde{p}_1 x_1 + \tilde{q}_1 x_2 \\ \tilde{y}_2 &= \tilde{r}_2 + \tilde{p}_2 x_1 + \tilde{q}_2 x_2 \\ &\vdots \\ \tilde{y}_k &= \tilde{r}_k + \tilde{p}_k x_1 + \tilde{q}_k x_2 \end{aligned} \quad (7)$$

$\tilde{r}_k, \tilde{p}_k$  and  $\tilde{q}_k$  ( $k = 1, 2$ ) are consequent coefficient that they are Gaussian type-1 fuzzy sets. Note that (7) can be extended

**Layer 1:** This layer is fuzzification layer. The output of this layer as follows:

$$\begin{aligned} {}^1\mu_{k,i}(x_i, [\sigma_{k,i}, {}^1m_{k,i}]) \\ = e^{-.5\left(\frac{x_i - {}^1m_{k,i}}{\sigma_{k,i}}\right)^2} \end{aligned} \quad (2)$$

$$\begin{aligned} {}^2\mu_{k,i}(x_i, [\sigma_{k,i}, {}^2m_{k,i}]) \\ = e^{-.5\left(\frac{x_i - {}^2m_{k,i}}{\sigma_{k,i}}\right)^2} \end{aligned} \quad (3)$$

Where  $m_{k,i} \in [{}^1m_{k,i}, {}^2m_{k,i}]$  is uncertain mean for  $k$ th rule and  $i$ th input.

to  $\tilde{y}_k$  ( $k = 1, \dots, n$ ). In this paper for simplicity we have  $k = 2$ .

**Layer 4:** This layer is used for consequent lower-upper firing points [27] (see(8)).  
**Layer 5:** The single node in this layer computes the output.

$$\hat{y} = \frac{\hat{y}_l + \hat{y}_r}{2} \quad (9)$$

Gradient descent with adaptive learning rate back propagation is used for learning phase [7].

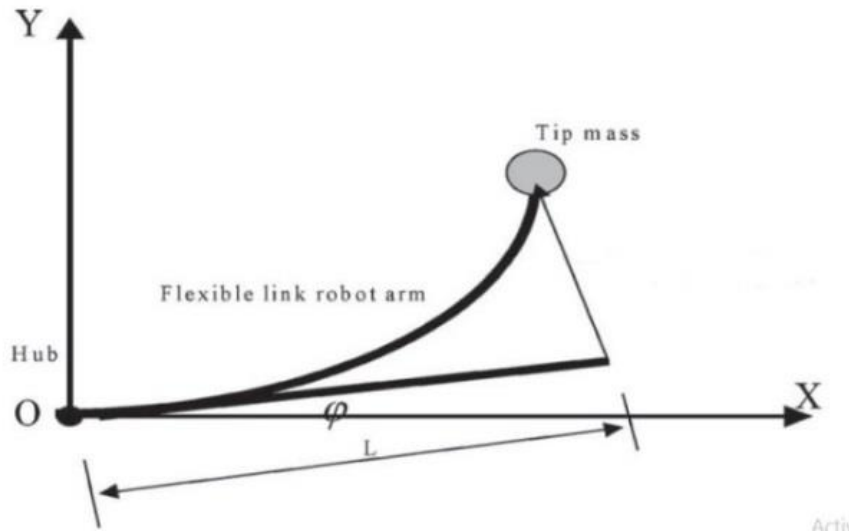
#### 4- Flexible Link Robot Arm

Consider a single-link robotic manipulator coupled to a brushed direct current motor with a no rigid joint. When the joint is modeled as a linear torsional spring, from the

Euler–Lagrange equation, the equations of motion for such an electromechanical system can be derived as:

$$\begin{aligned}
 J_1\ddot{q}_1 + F_1\dot{q}_1 + K\left(q_1 - \frac{q_2}{N}\right) + mgd \cos q_1 &= 0 \\
 J_2\ddot{q}_2 + F_2\dot{q}_2 + \frac{K}{N}\left(q_1 - \frac{q_2}{N}\right) &= K_t i \\
 LDi + Ri + K_b\dot{q}_2 &= u
 \end{aligned}
 \tag{10}$$

Where,  $q_1$  and  $q_2$  are the angular positions of the link and the motor shaft,  $i$  is the armature current, and  $u$  is the armature voltage. The inertias  $J_1$ ,  $J_2$ , the viscous friction constants  $F_1$ ,  $F_2$ , the spring constant  $K$ , the torque constant  $K_t$ , the back-emf constant  $K_b$ , the armature resistance  $R$  and inductance  $L$ , the link mass  $M$ , the position of the link's center of gravity  $d$ , the gear ratio  $N$  and the acceleration of gravity  $g$  can all be unknown. A flexible link robot arm is shown in Fig. 4.



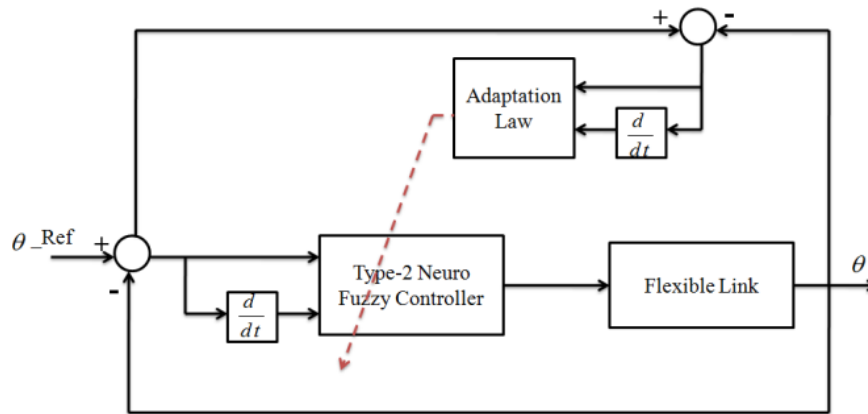
**Fig.4.** Configuration of flexible-link robot arm.

### 5- Simulation Results

In this section, a flexible link robot arm is controlled using ANFIS2. The structure of the robot arm and ANFIS2 based controller is shown in Fig 5. Where the reference signal (desired angle in here) is applied to system then the error between reference signal and the output of robot system (angle of link in here) is calculated. This error must be

minimized, so ANFIS2 is adapted to minimize the error.

In order to illustrate the effectiveness of the proposed results, the simulation will be conducted to control systems, where  $J_1 = 1.625 \text{ kg m}^2$ ,  $J_2 = 1.625 \text{ kgm}^2$ ,  $R = 0.5 \text{ } \Omega$ ,  $K_t = 0.9 \text{ N m/A}$ ,  $K = 0.5868$ ,  $K_b = 0.9 \text{ N m/A}$ ,  $M = 4.34 \text{ kg}$ ,  $L = 25.0 \times 10^{-3} \text{ H}$ ,  $g = 9.8 \text{ N/kg}$ ,  $F_1 = 1.625 \times 10^{-2} \text{ N m s/rad}$ ,  $F_2 = 1.625 \times 10^{-2} \text{ N m s/rad}$ ,  $N = 2$ ,  $d = 0.5 \text{ m}$



**Fig.5.** The structure of the robot arm and ANFIS2 based controller

The performance of ANFIS2 with sine angle reference is shown in fig. 6. The performance of ANFIS2 with step angle reference is shown in fig. 7. The performance of ANFIS2 with ramp angle reference is shown in fig. 8. Figures 5-8 show that adaptive inverse control based on ANFIS2 is suitable and robust strategy to control a flexible link robot arm.

## 6- Conclusion

In this paper, a novel ANFIS2 was proposed for identification of nonlinear dynamical systems. The proposed ANFIS2 is based on interval Gaussian type-2 fuzzy sets in the antecedent part and Gaussian type-1 fuzzy sets as coefficients of linear combination of input variables in the consequent part that it helps to improve modeling of highly nonlinear systems. Adaptive learning rate helps to prevent the ANFIS2 from trapping into a local minima and it helps to fast convergence of training algorithm. The test results show the importance and necessity of ANFIS2 to

modeling the inverse of uncertain systems and control it.

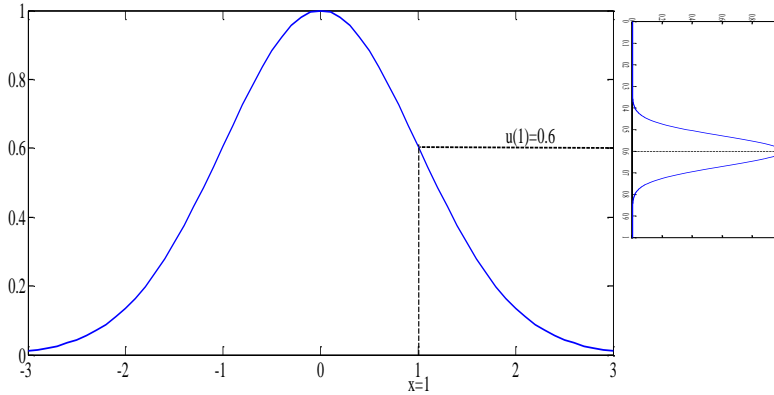
## References

- [1] J. Tavoosi, A. A. Suratgar, M. B. Menhaj, Nonlinear system identification based on a self-organizing type-2 fuzzy RBFN, *Engineering Applications of Artificial Intelligence* 54 (2016), 26-38.
- [2] J. Tavoosi, M. Alaei, B. Jahani, Temperature Control of Water Bath by using Neuro-Fuzzy Controller, 5th Symposium on Advance in Science & Technology, 2011.
- [3] J. Tavoosi, M. Alaei, B. Jahani, M.A. Daneshwar, A novel intelligent control system design for water bath temperature control, *Australian Journal of Basic and Applied Sciences* 5 (12) 1879-1885, 2011.
- [4] G.C. Calafiore, A subsystems characterization of the zero modes for flexible mechanical structures, in: *Proceedings of 36th IEEE Conference On Decision and Control*, San Diego, CA, 1997, pp. 1375-1380.
- [5] O. Castillo, P. Melin, *Type-2 Fuzzy Logic: Theory and Applications*, Springer-Verlag Berlin Heidelberg, 2008.
- [6] S. Huang, M. Chen, Constructing optimized interval type-2 TSK neuro-fuzzy systems with noise reduction property by quantum inspired BFA, *Neurocomputing* 173 (2015) 1839-1850.

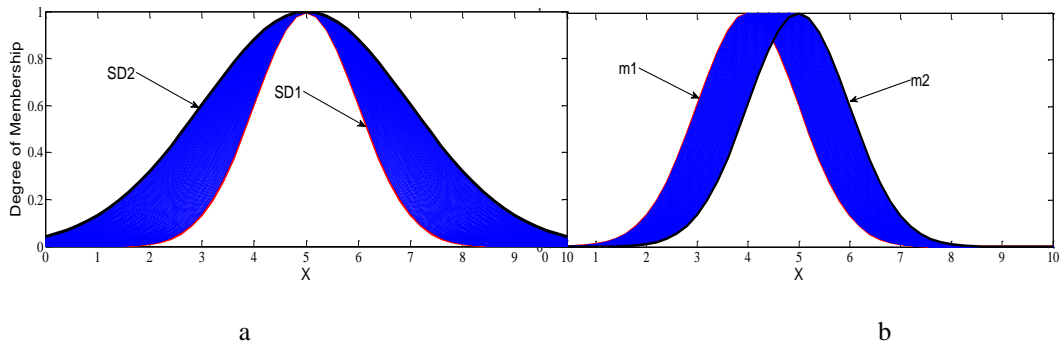
- [7] J. Tavoosi, A. A. Suratgar, M. B. Menhaj, Stable ANFIS2 for Nonlinear System Identification, *Neurocomputing* 182 (2016) 235–246.
- [8] R. Shahnazi, Observer-based adaptive interval type-2 fuzzy control of uncertain MIMO nonlinear systems with unknown asymmetric saturation actuators, *Neurocomputing* 171 (2016) 1053-1065.
- [9] S.I. Han, J.M. Lee, Recurrent fuzzy neural network backstepping control for the prescribed output tracking performance of nonlinear dynamic systems, *ISA Transactions* 53 (2014) 33–43.
- [10] J Tavoosi, MA Badamchizadeh, S Ghaemi, Adaptive Inverse Control of Nonlinear Dynamical System Using Type-2 Fuzzy Neural Networks, *Journal of Control* 5 (2) (2011).
- [11] YP Asad, A Shamsi, H Ivani, J Tavoosi, Adaptive Intelligent Inverse Control of Nonlinear Systems with Regard to Sensor Noise and Parameter Uncertainty (Magnetic Ball Levitation System Case Study), *International Journal on Smart Sensing and Intelligent Systems* 9 (1) 148-169.
- [12] T.C. Lin, C.H. Kuo, V.E. Balas, Real-time fuzzy system identification using uncertainty bounds, *Neurocomputing* 125 (2014) 195–216.
- [13] J. Soto, P. Melin, Genetic Optimization of Type-2 Fuzzy Integrators in Ensembles of ANFIS Models for Time Series Prediction, *Recent Advances on Hybrid Approaches for Designing Intelligent Systems Studies in Computational Intelligence* 547 (2014) 79–97.
- [14] H.M. Vaghefi, M.R. Sandgani, M. A. Shoorehdeli, Interval Type-2 Adaptive Network-based Fuzzy Inference System (ANFIS) with Type-2 non-singleton fuzzification, *13th Iranian Conference on Fuzzy Systems* (2013) 1-6.
- [15] G.M. Mendez, D.L.A Hernandez, Interval Type-2 ANFIS. In: *Innovations in Hybrid Intelligent Systems*, Springer, Heidelberg (2007) 64–71.
- [16] S. Bhattacharyya, D. Basu, A. Konar, D.N. Tibarewala, Interval type-2 fuzzy logic based multiclass ANFIS algorithm for real-time EEG based movement control of a robot arm, *Robotics and Autonomous Systems* 68 (2015) 104–115.
- [17] J. Tavoosi, M.A. Badamchizadeh, A Class of Type-2 Fuzzy Neural Networks for Nonlinear Dynamical System Identification, *Neural Computing & Application* 23 (3) (2013) 707–717.
- [18] J Tavoosi, AA Suratgar, MB Menhaj, Stability analysis of recurrent type-2 TSK fuzzy systems with nonlinear consequent part, *Neural Computing and Applications* (2015) 1-10.
- [19] F. Jahangiri, A. Doustmohammadi, M.B. Menhaj, An adaptive wavelet differential neural networks based identifier and its stability analysis, *Neurocomputing* 77(2012)12–19.
- [20] J Tavoosi, AA Suratgar, MB Menhaj, Stability Analysis of a Class of MIMO Recurrent Type-2 Fuzzy Systems, *International Journal of Fuzzy Systems* (2016) 1-14.
- [21] J. Tavoosi, A. ShamsiJokandan, M. A. Daneshwar, A new method for position control of a 2-DOF robot arm using neuro-fuzzy controller, *Indian Journal of Science and Technology* 5 (3) (2012) 2253-2257.
- [22] M.B.B. Sharifian, A. Mirlo, J. Tavoosi, M. Sabahi, Self-adaptive RBF neural network PID controller in linear elevator, *International Conference on Electrical Machines and Systems (ICEMS)*, 2011.
- [23] JafarTavoosi, Majid Alaei, BehrouzJahani, Neuro – Fuzzy Controller for Position Control of Robot Arm, *5th Symposium on Advances in Science and Technology (5thsastech)*, 2011.
- [24] M.B.B SharifianJafarTavoosi, AhadMirloo, PMSM Position and Speed Estimation by Moving Horizon Estimation (MHE), *ICEE19\_207*, 2011.
- [25] T. Dereli, A. Baykasoglu, K. Altun, A. Durmusoglu, I.B. Turksen, Industrial applications of type-2 fuzzy sets and systems: A concise review, *Computers in Industry* 62 (2011) 125–137
- [26] J.R. Castro, O. Castillo, L.G. Martínez, an Interval Type-2 Fuzzy Logic Toolbox for Control Applications, *IEEE International Conference on Fuzzy Systems*, London (2007) 1–6.
- [27] C.J. Chen, S.M. Yang, Z.C. Wung, System Identification by Neuro-Fuzzy Model with Sugeno and Mamdani Fuzzy Rules, *Journal of Aeronautics, Astronautics and Aviation, Series A41* (4) (2009) 263 – 270.

$$\hat{y}_l = \frac{\sum_{k=1}^N \bar{f}^k(m_{r_k}\sigma_{r_k} + m_{p_k}\sigma_{p_k}x_1 + m_{q_k}\sigma_{q_k}x_2) + \sum_{k=N+1}^M \underline{f}^k(m_{r_k}\sigma_{r_k} + m_{p_k}\sigma_{p_k}x_1 + m_{q_k}\sigma_{q_k}x_2)}{\sum_{k=1}^N \bar{f}^k(\sigma_{r_k} + \sigma_{p_k} + \sigma_{q_k}) + \sum_{k=N+1}^M \underline{f}^k(\sigma_{r_k} + \sigma_{p_k} + \sigma_{q_k})} \quad (8)$$

$$\hat{y}_r = \frac{\sum_{k=1}^L \underline{f}^k(m_{r_k}\sigma_{r_k} + m_{p_k}\sigma_{p_k}x_1 + m_{q_k}\sigma_{q_k}x_2) + \sum_{k=L+1}^P \bar{f}^k(m_{r_k}\sigma_{r_k} + m_{p_k}\sigma_{p_k}x_1 + m_{q_k}\sigma_{q_k}x_2)}{\sum_{k=1}^L \underline{f}^k(\sigma_{r_k} + \sigma_{p_k} + \sigma_{q_k}) + \sum_{k=L+1}^P \bar{f}^k(\sigma_{r_k} + \sigma_{p_k} + \sigma_{q_k})}$$

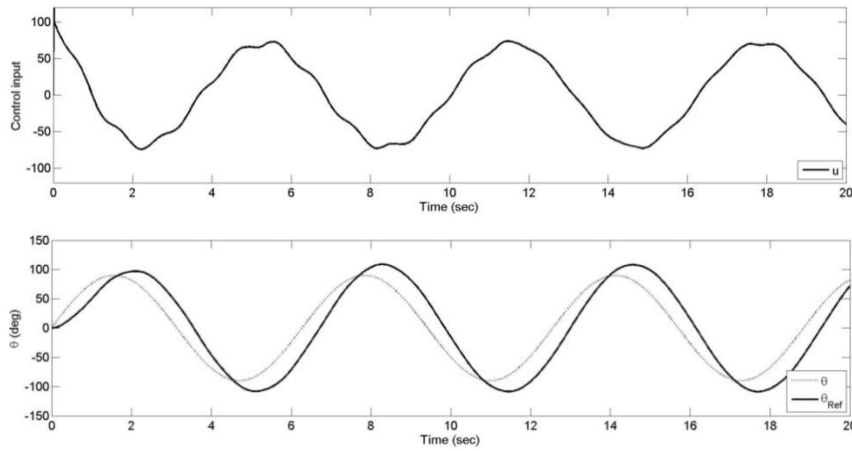


**Fig.1.** Gaussian primary and secondary membership functions

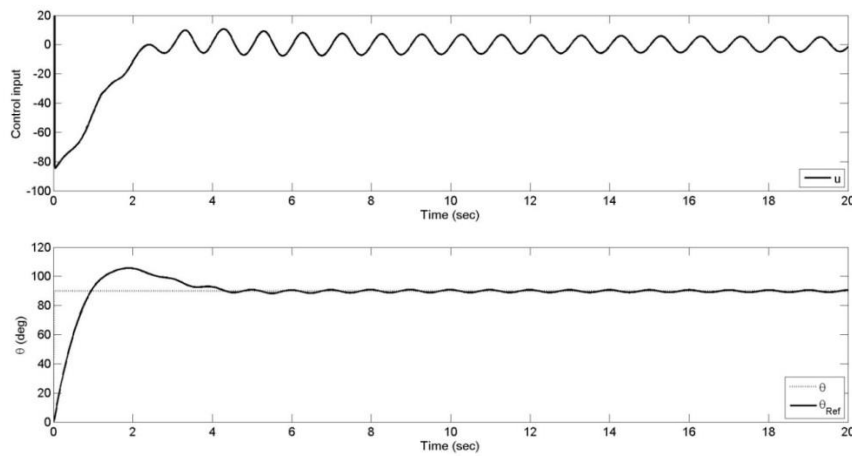


**Fig.2. a)** Uncertainty in standard deviation **b)** uncertainty in mean

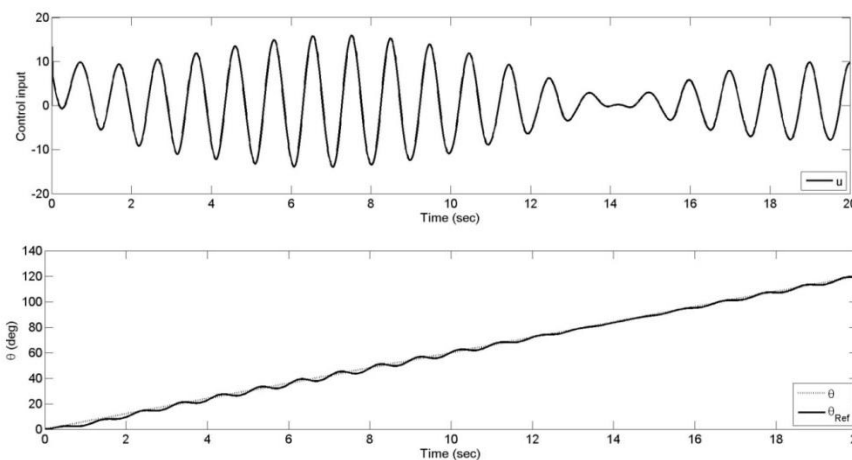




**Fig.6.**ANFIS2 based controller with sine angle reference



**Fig.7.**ANFIS2 based controller with step angle reference



**Fig.8.**ANFIS2 based controller with ramp angle reference