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Models of Dynamic Resource Allocation

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Abstract. In DEA to obtain an efficient representation in CCR and BCC models for each DMU only one linear plan would be solved; however, it is possible in centralized resource allocation (CRA) to solve all the DMUs by one linear planning. In this paper, models of centralized resource allocation in input-oriented and in dynamic case will be offered and by using dynamic SBM, the representation of DMUs in each time period and in general and based on CRA in input oriented case will be obtained. After that, the theorems of the offered models will be shown and at least, an applicable example will be represented.

Keywords: DEA, Dynamic DEA, SBM, CRA

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1. Introduction

Data Envelopment Analysis (DEA) is a non-parametric procedure for analyzing the operation of Decision Making Units (DMU) in the last two decades. This technique bases on the input and the output data obtained in companies and/or decision making units. To measure the efficiency having the production function is very significant. Since many years ago to1957 when Farel offered non-parametric procedure, a parametric procedure for estimating the production function was used. The model which presented by Farel was only used for computing by several inputs and only one output. After that in 1978, Charles, Cooper and Rhodes, by using Farel's model, presented a new model with the possibility of several inputs and outputs named CCR. This model was one of the main and the most public DEA models. Following that, many scientists made a lot of efforts in presenting different models. A model named BCC was offered by three scientists, Bahker, Cooper and Charles in 1984.This model was applicable for efficiency measurement and score efficiency. Tone (2001) presented the SBM model and he analyzed the relation of this model with the CCR model. Then consulting Tsutsui (2010), he could analyze his model in different periods and times and he offered a dynamic SBM model. Golany (1933) carried out a research in the DEA based on Resource Allocation which was input-oriented. This model based on the DEA standard model, but some problems were detected. Lately, Golany and Tamir (1995) offered a model considering the limitations of the upper borderline of all inputs. Beasely (2003) presented his non-linear resource allocation model which was able to simultaneously compute the inputs and outputs for each DMU and in the next period it was used to maximize the average of efficiency. Lozano and Villa (2004) suggested a model titled Centralized Resource Allocation (CRA) by which firstly instead of solving an independent LP for each DMU, one could simultaneously solve a single LP for all the DMUs, secondly all the DMUs could be shown on a large page and lastly, instead of decreasing the inputs of each DMU individually, all the inputs of the DMUs were decreased. In this paper, the dynamic SBM model by using input oriented CRA is suggested and the efficient representation of the DMUs in each period and in general is computed. In the second section the DEA and the CRA are presented and in the third section the input oriented SBM and CCR models in CRA are offered. In the fourth section the dynamic CRAs are analyzed, in the fifth section the numerical examples are indicated, and the last section belongs to the results and conclusions.

2. Preliminaries

In this section the CCR and the SBM envelopment models in traditional DEA would be firstly offered. Then, these models will be shown based on the CRA model.

Consider n DMUs with m inputs and S outputs. The input and output vectors of DMU_j $(j = 1, ..., n)$ are $X_j = (x_{1j,...,x_{mj}})^t, Y_j = (y_{1j,...,y_{sj}})^t$ where $X_j > 0, \quad Y_j > 0$.

By using the constant returns to scale, convexity, and possibility postulates, the non-empty production possibility set (PPS) is defined as follows:

$$T_c = \left\{(X,Y): X \geq \sum_{j=1}^{n} \lambda_j X_j, Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \lambda_j \geq 0, j = 1\ldots, n\right\}$$

Charles, Cooper and Rhodes analyzed input oriented DMU0 by using input decreasing strategy and considering the above set of minimizing possibility.

$$\begin{aligned}
Min \quad & \theta \\
S.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta\, x_{io}, \qquad i = 1,\ldots,m \\
& \sum_{j=1}^{n} \lambda_j y_{kj} \geq y_{ko}, \qquad k = 1,\ldots,p \qquad (1) \\
& \lambda_j \geq 0, \qquad j = 1,\ldots,n.
\end{aligned}$$

Definition 1. DMU_p is CCR-efficient if and only if $\theta^* = 1$.
Tone (2001) indicated the SBM model by using the definition of slack-variable.

$$\gamma^* = Min \quad \frac{1 - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{io}}}{1 + \frac{1}{s}\sum_{k=1}^{p}\frac{s_k^+}{y_{ko}}}$$

$$\begin{aligned} S.t \quad & \sum_{j=1}^{n}\lambda_j x_{ij} + s_i^- = x_{io}, \qquad i = 1,\dots,m \\ & \sum_{j=1}^{n}\lambda_j y_{kj} - s_k^+ = y_{ko}, \qquad k = 1,\dots,p \qquad (2) \\ & s_i^- \geq 0, s_k^+ \geq 0, \qquad i = 1,\dots,m, \qquad k = 1,\dots,p, \\ & \lambda_j \geq 0, \qquad j = 1,\dots,n. \end{aligned}$$

Definition 2. DMU_p is SBM-efficient if and only if $\gamma^* = 1$.
Let j, r=1,...,n, be indexes for DMUs; i=1,...,m, be index for inputs; k=1,...,p be index for outputs; x_{ij}, amount of input i consumed by DMU j; y_{kj}, quantity of output k produced by DMU_j; θ, radial contraction of total input vector; s_i, slack along the input dimension i; tk, additional increase along the output dimension k;$^{(\lambda_1 r, \lambda_2 r, \dots, \lambda_n)}$ vector for projecting DMU r.
The CCR model based on CRA-I in phase I which was shown by Lozano and Villa (2004) for the first time:
Model Phase 1/$Radial/Input - Oriented$

$$\begin{aligned} & Min \quad \theta \\ & s.t \quad \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}x_{ij} \leq \theta\sum_{j=1}^{n}x_{ij} \qquad i = 1,...,m, \\ & \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}\, y_{kj} \geq \sum_{r=1}^{n}y_{kr} \qquad k = 1,..., p, \qquad (3) \\ & \sum_{j=1}^{n}\lambda_{jr} = 1 \qquad r = 1,...,n, \\ & \lambda_{jr} \geq 0 \qquad j = 1,...,n, r = 1,...,n, \theta : free. \end{aligned}$$

Let θ^* be the optimum of the previous model,then the phase II model can be formulated as:
Model Phase $II/Radial/Input-Oriented$

$$
\begin{aligned}
&Max \quad \sum_{i=1}^{m} s_i + \sum_{k=1}^{p} t_k \\
&s.t \quad \sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr} x_{ij} = \theta^* \sum_{j=1}^{n} x_{ij} - s_i \qquad i=1,\dots,m, \qquad (4)\\
&\sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr} y_{kj} = \sum_{r=1}^{n} y_{kr} + t_k \qquad k=1,\dots,p,\\
&\sum_{j=1}^{n} \lambda_{jr} = 1 \qquad r=1,\dots,n,\\
&\lambda_{jr}, s_i, t_k \geq 0 \qquad i=1,\dots,m, r=1,\dots,n, j=1,\dots,n, k=1,\dots,p.
\end{aligned}
$$

Presentation in CRA model:
The representation of each DMU in the CRA model is presented as follows:
The corresponding vector $(\lambda^*_{1r}, \lambda^*_{2r}, \dots, \lambda^*_{nr})$ defines for each DMUr and the operating point at which it should aim.The inputs and outputs of each such point can be computed as:

$$
\begin{aligned}
&\hat{x}_{ir} = \sum_{j=1}^{n} \lambda^*_j x_{ij} \qquad i=1,\dots,m,\\
&\hat{y}_{kr} = \sum_{j=1}^{n} \lambda^*_j y_{kj} \qquad k=1,\dots,p.
\end{aligned}
$$

3. The SBM-I model based on CRA in phase I:

In this section the SBM-I model based on CRAand considering 2n of decision making unit, m input and p output by assuming the variable score output is suggested as follows:

$$\rho = Min \quad \frac{1-\frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{\sum_{j=1}^{n}x_{ij}}}{1+\frac{1}{k}\sum_{k=1}^{p}\frac{s_k^+}{\sum_{r=1}^{n}y_{kr}}}$$

$$s.t \quad \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}x_{ij}+s_i^- = \sum_{j=1}^{n}x_{ij} \qquad i=1,\ldots,m, \qquad (5)$$

$$\sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}y_{kj}-s_k^+ = \sum_{r=1}^{n}y_{kr} \qquad k=1,\ldots,p,$$

$$\sum_{j=1}^{n}\lambda_{jr}=1 \qquad r=1,\ldots,n.$$

Theorem 1: Assume that is the optimal value in model (3) and p is the optimal value in model (5) then: $P^* \leqslant \theta^*$
demonstration:

$$\sum_{j=1}^{n}x_{ij} = \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^{*}x_{ij}+s_i^{*}+(1-\theta^{*})\sum_{j=1}^{n}x_{ij}$$

$$\sum_{r=1}^{n}y_{kr} = \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^{*}y_{kj}-s_k^{*}$$

is defined: $\overline{\lambda}=\lambda^*, \overline{s}_i = s_i^* + (1-\theta^*)\sum_{j=1}^{n}x_{ij}, \overline{s}_k = s_k^*$

The above optimal solution is applied on the SBM objective function:

$$\rho^* \le \rho \to \rho = \frac{1-\frac{1}{m}\sum_{i=1}^{m}\frac{s_i^- + (1-\theta^*)\sum_{j=1}^{n}x_{ij}}{\sum_{j=1}^{n}x_{ij}}}{1+\frac{1}{k}\sum_{k=1}^{p}\frac{s_k^+}{\sum_{r=1}^{n}y_{kr}}} = \frac{1-\frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{\sum_{j=1}^{n}x_{ij}}-(1-\theta^*)}{1+\frac{1}{k}\sum_{k=1}^{p}\frac{s_k^+}{\sum_{r=1}^{n}y_{kr}}} = \rho^* - (1-\theta^*) \ge 0 \to \rho^* - 1 + \theta^* \ge 0$$

$$\to \frac{\theta^* - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{\sum_{j=1}^{n}x_{ij}}}{\theta^* + \frac{1}{k}\sum_{k=1}^{p}\frac{s_k^+}{\sum_{r=1}^{n}y_{kr}}} \le \theta^* \xrightarrow{0\le\rho^*\le 1} \rho^* \le \theta^*$$

Theorem 2: For each DMU_r,' $(\hat{x}_{1r}, \hat{x}_{2r}, ..., \hat{x}_{mr}, \hat{y}_{1r}, \hat{y}_{2r}, ..., \hat{y}_{pr})$ means the efficient pareto:

$$\hat{x}_{ir} = \sum_{j=1}^{n} \lambda_j^* x_{ij} \qquad i = 1, ..., m$$

$$\hat{y}_{kr} = \sum_{j=1}^{n} \lambda_j^* y_{kj} \qquad k = 1, ..., p$$

Demonstration: For each DMUr by the use of model (4), it is indicated that the representative points $(\hat{x}_{1r}, \hat{x}_{2r}, ..., \hat{x}_{mr}, \hat{y}_{1r}, \hat{y}_{2r}, ..., \hat{y}_{pr})$ are the efficient paretos:

We assume that the assumption of the theorem is wrong, so, it may lead to a contradiction.

If $(\hat{x}_{1r}, \hat{x}_{2r}, ..., \hat{x}_{mr}, \hat{y}_{1r}, \hat{y}_{2r}, ..., \hat{y}_{pr})$ may not be a technical efficiency then BCC-I should be used. There is the vector $(\lambda_{1r}, \lambda_{2r}, ..., \lambda_{nr})$ in the casethat: $\sum_{j=1}^{n} \lambda_{jr} = 1$

It is defined:

$$\hat{\hat{x}}_{ir} = \sum_{j=1}^{n} \lambda_j^* x_{ij} \leq \hat{x}_{ir} \qquad i = 1, ..., m$$

$$\hat{\hat{y}}_{kr} = \sum_{j=1}^{n} \lambda_j^* y_{kj} \geq \hat{y}_{kr} \qquad k = 1, ..., p$$

in the case that the inequality be strict at least for one$^{i'}$ or one$^{k'}$ output. Assume that the input be for each

$\hat{\hat{x}}_{i'r} = \sum_{j=1}^{n} \lambda_j^* x_{i'j} \leq \hat{x}_{i'r}$, $^{i'}$ then in the DMUr vector relationship: $(\lambda_{1r}, \lambda_{2r}, ..., \lambda_{nr})$.

The assumed optimality $(\lambda_{1r}^*, \lambda_{2r}^*, ..., \lambda_{nr}^*)$ leads to the solution of the following objective function measurements:

$$\sum_{i=1}^{m} s_i^* + \sum_{k=1}^{p} t_k^* + \sum_{i=1}^{m} (\hat{x}_{ir} - \hat{\hat{x}}_{ir}) + \sum_{k=1}^{p} (\hat{\hat{y}}_{kr} - \hat{y}_{kr}) \rangle \sum_{i=1}^{m} s_i^* + \sum_{k=1}^{p} t_k^*$$

which is more than the premier optimality and this is the contradiction.

In the same case,it is possible for the k'output and for each $\hat{\hat{y}}_{kr}=\sum_{j=1}^{n}\lambda_j^* y_{kj}\geq\hat{y}_{kr}$ one to obtain the results that is similar to the above ones.

4. Models of dynamic CRA:

The envelopment model of dynamic input oriented BCC based on CRA in time period is suggested as follows:

$$\begin{aligned}
&Min \quad \theta^t \\
&s.t \quad \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^t x_{ij}^t \leq \theta^t \sum_{j=1}^{n} x_{ij}^t \qquad i=1,\dots,m, t=1,\dots,T, \\
&\sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^t y_{kj}^t \geq \sum_{r=1}^{n} y_{kr}^t \qquad k=1,\dots,p, t=1,\dots,T, \qquad (6)\\
&\sum_{j=1}^{n}\lambda_{jr}^t = 1 \qquad r=1,\dots,n, t=1,\dots,T, \\
&\lambda_{jr}^t \geq 0 \qquad j=1,\dots,n, r=1,\dots,n, t=1,\dots,T, \theta: free.
\end{aligned}$$

Considering the SBM model which was shown for the assessment of independent efficiency in time period by Tone and Tsutsui (2011), the input oriented SBM based on dynamic CRA is offered as follows:

$$\begin{aligned}
&Min \quad \frac{1-\frac{1}{m}\sum_{t=1}^{T}\sum_{i=1}^{m}\frac{s_i^t}{\sum_{j=1}^{n} x_{ij}^t}}{1+\frac{1}{k}\sum_{t=1}^{T}\sum_{k=1}^{p}\frac{s_k^t}{\sum_{r=1}^{n} y_{kr}^t}} \\
&s.t \quad \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^t x_{ij}^t + s_i^t = \sum_{j=1}^{n} x_{ij}^t \qquad i=1,\dots,m, t=1,\dots,T, \qquad (7)\\
&\sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^t y_{kj}^t - s_k^t = \sum_{r=1}^{n} y_{kr}^t \qquad k=1,\dots,p, t=1,\dots,T, \\
&\sum_{j=1}^{n}\lambda_{jr}^t = 1 \qquad r=1,\dots,n, t=1,\dots,T.
\end{aligned}$$

Considering SBM and CRA models in T1,,Tn time periods and by the use of Tone and Tsutsui (2011) the SBM model based on dynamic CRA is presented as follows:

$$Min \quad 1-\frac{1}{T}\sum_{t=1}^{T} w^t \left[\frac{1-\frac{1}{m}\sum_{t=1}^{T}\sum_{i=1}^{m}\frac{s_i^t}{\sum_{j=1}^{n} x_{ij}^t}}{1+\frac{1}{k}\sum_{t=1}^{T}\sum_{k=1}^{K}\frac{s_k^t}{\sum_{r=1}^{n} y_{kr}^t}}\right]$$

$$s.t \quad \sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^t x_{ij}^t + s_i^t = \sum_{j=1}^{n} x_{ij}^t \qquad i=1,\dots,m, t=1,\dots,T, \qquad (8)$$

$$\sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}^t y_{kj}^t - s_k^t = \sum_{r=1}^{n} y_{kr}^t \qquad k=1,\dots,p, t=1,\dots,T,$$

$$\sum_{j=1}^{n}\lambda_{jr}^t = 1 \qquad r=1,\dots,n, t=1,\dots,T.$$

Definition 3: Good link: This link concerns the constancy of the profit and transferring the obtained surplus to the next time. According to the Tone model, the good links act like the output behavior, and the link value may not be less than the observed value. The relative deficiency of the links in this class is taken into account as deficiency.

Bad link: This link refers to the fixed liability and storage. According to the Tone model, a bad link acts as an input and the value of it is not more than the observed value. The relative surplus of the links in this class is taken into account as deficiency. Fix link: This link refers to the non-controllability of DMU and its value in the fixed observed value. Similar to free link, this link in the case of time units association influences the score efficiency indirectly.

Free link: This link refers to the issue that the DMU can be used freely. The value of it may decrease or increase according to the observed value. According to four definitions of good, bad, fix, and free links, the model in T1, ,Tn time periods, using Tone and Tsutsui's (2011) idea is presented as follows

$$\rho^* = Min \quad \frac{1}{T}\sum_{t=1}^{T} w^t [1 - \frac{1}{m+nfree}(\sum_{i=1}^{m} \frac{w_i^- s_{it}^-}{\sum_{j=1}^{n} x_{ijt}} + \sum_{i=1}^{nfree} \frac{s_{it}^{free}}{\sum_{j=1}^{n} z_{ijt}^{free}})]$$

$$s.t \quad \sum_{j=1}^{n} z_{ijt}^{free} \lambda_j^t = \sum_{j=1}^{n} z_{ijt}^{free} \lambda_j^{t+1} \qquad i=1,\dots,m, t=1,\dots,T, \qquad (9)$$

$$\sum_{j=1}^{n} z_{ijt}^{free} = \sum_{j=1}^{n} z_{ijt}^{free} \lambda_j^t + \sum_{j=1}^{n} s_{ijt}^{free} \qquad i=1,\dots,m, t=1,\dots,T,$$

$$\sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_j^t x_{ij}^t + s_i^t = \sum_{j=1}^{n} x_{ij}^t \qquad i=1,\dots,m, t=1,\dots,T,$$

$$\sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr}^t y_{kj}^t - s_k^t = \sum_{j=1}^{n} y_{kr}^t \qquad k=1,\dots,p, t=1,\dots,T,$$

$$\lambda_j^t \ge 0, s_i^t \ge 0, s_k^t \ge 0, s_{ijt}^{free} : free \quad i=1,\dots,m, j=1,\dots,n, k=1,\dots,p, t=1,\dots,T.$$

Considering the above models the following theorems are existed for T1,,Tn time periods.

Theorem 3: Assume that θ^* is the optimal value of model (6) and p^{*^t} is the optimal value of model (7) then $p^{*^t} \leqslant \theta^{*^t}$

Theorem 4: The improved following dynamic activity means efficient pareto:

$$\hat{x}_{ir}^t = \sum_{j=1}^{n} \lambda_j^{*t} x_{ij}^t \qquad i=1,\dots,m, t=1,\dots,T$$

$$\hat{y}_{kr}^t = \sum_{j=1}^{n} \lambda_j^{*t} y_{kj}^t \qquad k=1,\dots,p, t=1,\dots,T$$

5. Numerical Examples

In this section, considering 5 decision making units with two inputs and one output, the representation based on model (5) in CRA-I case and in phase I by itself is computed. (The input & the output data in table one represent Tone's(2011) research data.)

Table 1: the inputs and outputs data

DMU	Input 1	Input2	Output 1	Output 2
A	4	3	2	3
B	6	3	2	3
C	8	1	6	2
D	8	1	6	1
E	2	4	1	4

To obtain the inputs and the outputs representations on the above table based on CRA-I in phase (I), model (5) is used. The obtained results are shown in table 2.

Table 2: the results of the model

DMU	SBM			
	Input1	Input2	Output1	Output2
	$\hat{x}_1$	$\hat{x}_2$	$\hat{y}_1$	$\hat{y}_2$
A	7.5385	1.2308	5.6154	2.1538
B	4	3	2	3
C	4	3	2	3
D	4	3	2	3
E	7.3846	1.3077	5.3846	2.1538

Considering unit A: according to model (5) the efficient representation is as follows: Input 1increased 3.54 units and input 2decreased 1.76 units. Output 1increased 3.62 units and output 2decreased 0.846 units. The efficiency representation of DMUb in input 1 increased 2 units, it is fixed in input 2 and also the outputs.

In DMUc the 1st input decreased, the 2nd input increased, the 1st output decreased and the 2nd output increased. The efficiency representation of DMUd is like that of DMUc. In unit E the first input encounters 5.38 units increasing and the 2nd input faces 2.69 units decreasing. The

1st output encounters 4.40 units increasing and the 2nd output faces 1.846 units decreasing.

Case study

According to Tone & Tsutsui's (2010) data in analyzing SBM dynamic model, the dynamic SBM based on CRA-I in phase I is used. In the following table there are 8decision making units with one input, one output and one link in the four time periods. (T1, T2, T3, T4).

Table 3: the inputs and outputs data during four different time periods

DMU	Input				Output				Link			
	T1	T2	T3	T4	T1	T2	T3	T4	T1	T2	T3	T4
A	10	11	12	13	50	50	50	50	10	10	10	10
B	30	33	36	39	150	150	150	150	20	25	30	35
C	20	22	24	26	50	100	150	180	30	30	30	30
D	30	33	36	39	100	120	150	180	15	20	25	30
E	30	33	36	39	150	135	120	105	20	25	30	35
F	10	11	12	13	100	90	40	35	10	10	10	10
G	30	33	36	39	100	180	95	200	20	30	40	50
H	20	22	24	26	100	40	150	100	10	10	10	10

Table 4: the representation in four time periods

DMU	SBM-CRA											
	Input				Output				Link			
	$\hat{X}$				$\hat{Y}$				$\hat{Z}$			
	T1	T2	T3	T4	T1	T2	T3	T4	T1	T2	T3	T4
A	10	11	24	26	1.00	90	1.50	1.00	10	10	10	10
B	10	11	12	26	1.00	90	55	1.00	1	10	10	10
C	10	11	24	26	1.00	90	1.50	1.00	10	10	10	10
D	10	11	24	26	1.00	90	1.50	1.80	10	10	10	30
E	10	11	12	26	1.00	90	50	1.80	10	10	10	30
F	10	24.5	24	26	1.00	1.45	1.50	1.00	10	22.3	10	10
G	10	33	12	26	1.00	1.80	50	1.00	10	30	10	10
H	10	11	24	26	1.00	90	1.50	1.40	10	10	10	20

According to model (9) the representation of DMUa in four time periods is as follows:
The inputs increased in the first two time periods and in the next one increased.
The outputs orderly decreased and increased and the links were fixed. In BMUb all the inputs, outputs and links decreased. In unit C the first two units decreased and the second two units were fixed. The outputs and the links decreased in all the periods. In units D & E, like unit Cthe outputs and the links decreased. The first input in unit f was fixed and three other inputs increased, the outputs decreased and the links are fixed in the first period and decreased in the 2nd period. They are increased in the next two periods. In units G and H the inputs, outputs and links had fluctuation.

6. Conclusion

Since finding a suitable target in Data Envelopment Analysis is very significant, in the basic DEA models, which used the technology of efficiency with different scores, it is possible for this issue to solve n different problems of linear planning. Moreover, finding a suitable dynamic target is vital; so, whenever the DMUs and the periods increase finding the suitable target becomes harder. In this paper, by using the CRA and the SBM models in the case of dynamic CRA, a suitable target is offered for all the decision making units by solving only one linear planning problem in all periods.Future researches: Using uncertain and missing data to find out the representations of the units in the dynamic case, and also pointing out the type of score efficiency by using CRA dynamic models are suggested.

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